1 Mathematical Formalism

- $\mathbf{v}_j$: representation vector of modellable asset j. This will stay the same for all periods.
- $\mathbf{w}_{t-1}$: modellable weight vector at the beginning of period t.
- $\mathbf{b}_{t-1}$: representation vector of benchmark asset at the beginning of period $t-1$.

$$\mathbf{b}_{t-1} = \sum_j w_{j,t-1}\mathbf{v}_j$$

- $\mathbf{u}_{i,t-1}$: representation vector of non-modellable asset i at the beginning of period t.

$$\mathbf{u}_{i,t} = \mathbf{u}_{i,t-1} + \beta_{i,t}\mathbf{b}_{t-1}$$

- p_{t-1}: negative ES (NES) of non-modellable portfolio at the beginning of period t.

$$\begin{aligned} p_t &= p_{t-1} + r_t \\ &= p_{t-1} + \sum_i \left[\mathrm{NES}\left(\mathbf{u}_{i,t}\right) - \mathrm{NES}\left(\mathbf{u}_{i,t-1}\right)\right] \end{aligned}$$

The final NES is

$$p_f = p_0 \sum_t^{t_f+1} r_t$$

We want to maximize p_f for a given time frame, which is equivalent to maximizing

$$R = \frac{1}{t_f}\sum_{t=1}^{t_f+1} r_t$$