

# Dependable Hybrid Systems Design: Prelude

Zheng Cheng   Dominique Méry

Nov, 2020

# Outlines

## Introduction to Hybrid Systems

### Review of Calculus

Derivatives

Power Rule

Chain Rule

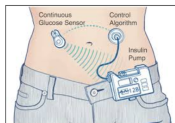
Differential Equations

### Design Hybrid Systems in Event-B

Review of Event-B

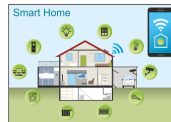
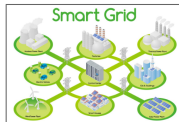
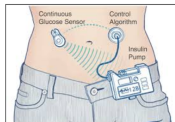
Develop Theories in Event-B

# Hybrid Systems



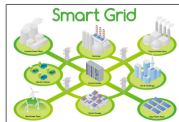
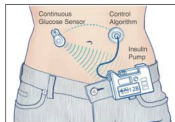
► **Hybrid** = Continuous + Discrete behaviors

# Hybrid Systems



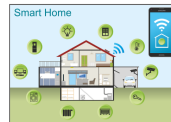
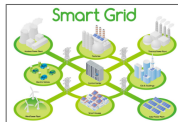
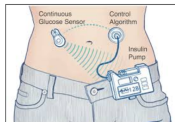
- ▶ **Hybrid** = Continuous + Discrete behaviors
- ▶ Open-loop vs. **Close-loop**

# Hybrid Systems



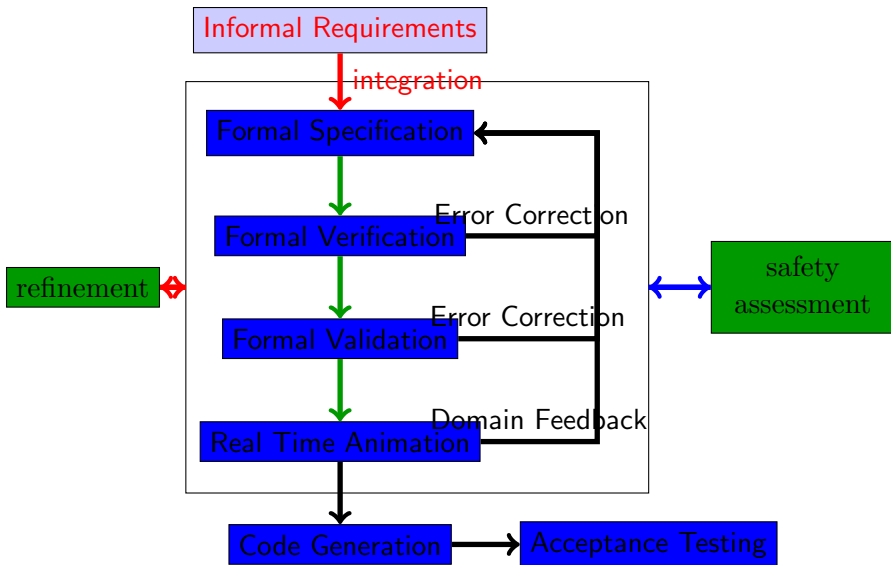
- ▶ **Hybrid** = Continuous + Discrete behaviors
- ▶ Open-loop vs. **Close-loop**
- ▶ Ubiquitous

# Hybrid Systems



- ▶ **Hybrid** = Continuous + Discrete behaviors
- ▶ Open-loop vs. **Close-loop**
- ▶ Ubiquitous
- ▶ **Safety**

# Tool Support for Design Safe Hybrid Systems



# Tool Support for Design Safe Hybrid Systems

- ▶ Requirement engineering(formal specification)
- ▶ Theorem Proving(formal verification)
- ▶ Model Checking(formal validation)
- ▶ Simulation(real-time animation)



# Requirement engineering

- ▶ Workflow
  - ▶ Rewriting informal requirement into formal specification(preconditions and postconditions)
- ▶ e.g. Event-B, Z, TLA+

# Theroem proving

- ▶ Workflow
  - ▶ Hybrid system safety as a theorem:  
 $\{\text{Precondition}\} \text{ System } \{\text{Postcondition}\}$
  - ▶ Interacting with a mechanized theorem prover to generate proofs
- ▶ e.g. KeYmaera, HHL, Z3
- ▶ Expensive, time consuming

# Model Checking

- ▶ Workflow
  - ▶ Building model, specify its initial states of interest, and safety property
  - ▶ Checking every execution of model starting in an initial state always stays within the set of safe states
- ▶ e.g. HyTech, SpaceEx, Flow\*
- ▶ Undecidable in general

# Simulation

- ▶ Workflow
  - ▶ Building model(behaviors, environment, interactions)
  - ▶ Computational evaluation of a model instance over time
  - ▶ Checking evaluation result
- ▶ e.g. Ptolemy, Matlab(Simulink,Stateflow), SAMSON
- ▶ Inconclusive result (valid only on the chosen input)

# Outlines

## Introduction to Hybrid Systems

## Review of Calculus

- Derivatives

- Power Rule

- Chain Rule

- Differential Equations

## Design Hybrid Systems in Event-B

- Review of Event-B

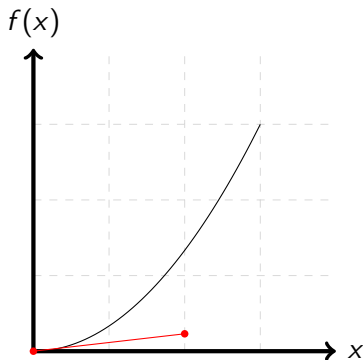
- Develop Theories in Event-B

# Derivatives

- ▶ The rate of change of function  $f(x)$ , w.r.t.  $x$

# Derivatives

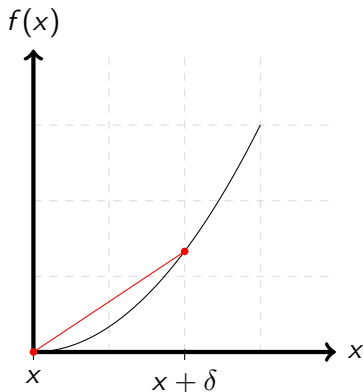
- ▶ The rate of change of function  $f(x)$ , w.r.t.  $x$
- ▶ Slope of the tangent line!



# Derivatives

- ▶ The rate of change of function  $f(x)$ , w.r.t.  $x$
- ▶ Slope of the tangent line!
- ▶

$$\frac{df}{dx} \approx \frac{f(x + \delta) - f(x)}{\delta}$$

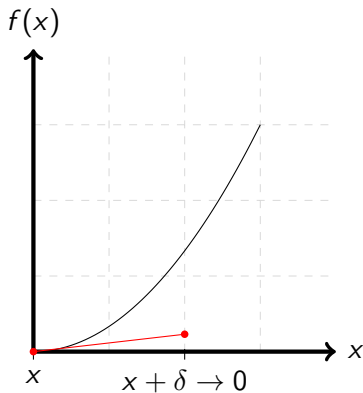




# Derivatives

- ▶ The rate of change of function  $f(x)$ , w.r.t.  $x$
- ▶ Slope of the tangent line!
- ▶

$$\frac{df}{dx} = \lim_{\delta \rightarrow 0} \frac{f(x + \delta) - f(x)}{\delta}$$



# Power Rule

Try  $f(x) = x^n$ , (Power Rule):  $\frac{df}{dx} = nx^{n-1}$

$$\begin{aligned}\frac{df}{dx} &\approx \frac{f(x + \delta) - f(x)}{\delta} && \text{(approximation)} \\&= \frac{1}{\delta} [(x + \delta)^n - x^n] && \text{(rewriting)} \\&= \frac{1}{\delta} (x^n + nx^{n-1}\delta + \frac{n(n-1)}{2}x^{n-2}\delta^2 \dots - x^n) && \text{(Pascal-triangle)} \\&= \frac{1}{\delta} (nx^{n-1}\delta + O(\delta^2)) \\&= nx^{n-1} + O(\delta)\end{aligned}$$

# Chain Rule

$$\frac{d}{dx}(f(g(x))) = \dot{f}(g(x)) \cdot \dot{g}(x)$$

Ex:

- ▶  $f(x) = \cos(x^3)$
- ▶  $f(x) = 2^x$
- ▶  $f(x) = e^{x^2} \sin(x)$

# Differential Equations

Ex:  $x$  is the size of a population of procreate bunnies...

- ▶ Population grows at a rate  $\lambda$  proportional to its population size:  $\frac{dx}{dt} = \lambda x$
- ▶ What is population as a function over time?

$$\frac{dx}{dt} = \lambda x$$

$$\rightarrow \frac{dx}{x} = \lambda dt$$

$$\rightarrow \int \frac{dx}{x} = \int \lambda dt$$

$$\rightarrow \ln(x) = \lambda t + C$$

$$\rightarrow x = e^{\lambda t + C}$$

- ▶ Determine  $C$  by initial conditions, e.g. ( $x_0$  at  $t_0$ )

# Outlines

## Introduction to Hybrid Systems

## Review of Calculus

Derivatives

Power Rule

Chain Rule

Differential Equations

## Design Hybrid Systems in Event-B

Review of Event-B

Develop Theories in Event-B

# Review of Event-B

- ▶ Context: static properties of Event-B models
  - ▶ Sets: user-defined types
  - ▶ Constants: static object in development
  - ▶ Axioms: presumed properties about sets and constants
  - ▶ Theorems: derived properties about sets and constants

# Review of Event-B

- ▶ Machine: behavioral properties of Event-B models
  - ▶ Variables: states
  - ▶ Invariants: properties of variables that always need to hold
  - ▶ Theorems: derived properties about variables
  - ▶ Events: possible state changes

# Review of Event-B

- ▶ Proof obligations: must be proved to show that Event-B models fulfill their specified properties.
  - ▶ INV: invariant preservation
  - ▶ FIS: action feasibility
  - ▶ ...



# Develop Theories in Event-B

- ▶ **Theory plugin**: more modularize and reusable polymorphic “Context”
- ▶ Developed at University of Southampton, still under development
- ▶ Installation:  
`http://rodin-b-sharp.sourceforge.net/updates`
  - ▶ Modelling Extensions → Theory Feature
- ▶ Let us develop a theory for real numbers
- ▶ Fork: `https://github.com/veriat1/LORIA\_WEEK1`
  - ▶ Open model “theory-axiom-reals”

# Exercise One (\*)

► Prove:  $a + b + c = c + b + a$  on real numbers

? How to write this theorem

? What is the key to prove this theorem

? How to use theory plugin to prove this

## Exercise Two (\*\*)

- ▶ Develop the power operator  $a^b$
- ? What are its arguments and results
- ? What is its semantics

## Exercise Three (\*\*\*)

- ▶ Open model “ex-pattern-const-DE”
- ? What this model does
- ? What is its invariant
- ? What operators are needed to express this invariant, and what are their semantics
- ? How to prove your invariant

# Caveats

- ▶ Axioms inconsistency  $\rightarrow$  Introduce when necessary, Prove when you can
- ▶ Big fat theories  $\rightarrow$  Modular theories