Dependable Hybrid Systems Design: Prelude

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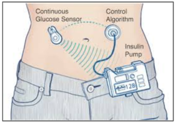
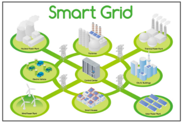
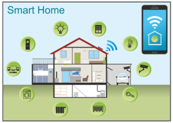
Outlines

Hybrid Systems

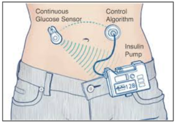


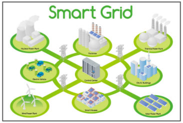


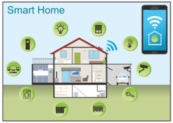


- Hybrid = Continuous + Discrete behaviors

Hybrid Systems

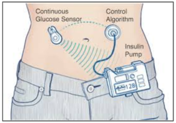


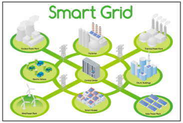


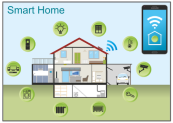


- Hybrid = Continuous + Discrete behaviors
- Open-loop vs. Close-loop

Hybrid Systems





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- Open-loop vs. Close-loop
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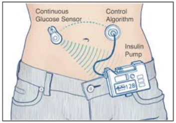
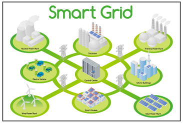
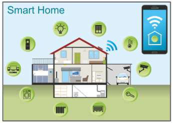
Hybrid Systems





- Hybrid = Continuous + Discrete behaviors
- Open-loop vs. Close-loop
- Ubiquitous
- Safety

Outlines

Derivatives

- The rate of change of function f(x), w.r.t. x

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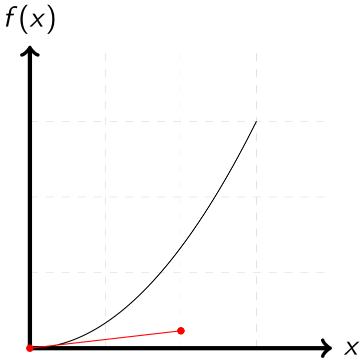
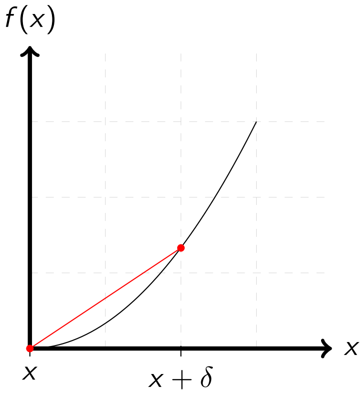
- Slope of the tangent line!



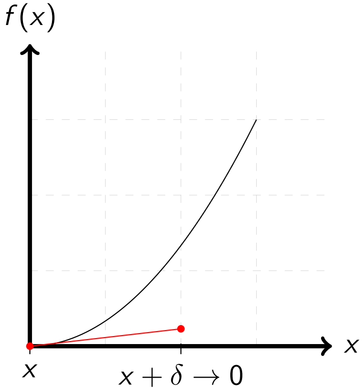
Derivatives

- The rate of change of function f(x), w.r.t. x
- Slope of the tangent line!
- $$\frac{df}{dx} \approx \frac{f(x+\delta) - f(x)}{\delta}$$



Derivatives

- The rate of change of function f(x), w.r.t. x
- Slope of the tangent line!
- $$\frac{df}{dx} = \lim_{\delta \to 0} \frac{f(x+\delta) - f(x)}{\delta}$$



Power Rule

Try $f(x) = x^n$, (Power Rule): $\frac{df}{dx} = nx^{n-1}$

$$\begin{aligned}
\frac{df}{dx} &\approx \frac{f(x+\delta) - f(x)}{\delta} && \textbf{(approximation)} \\
&= \frac{1}{\delta}[(x+\delta)^n - x^n] && \textbf{(rewriting)} \\
&= \frac{1}{\delta}(x^n + nx^{n-1}\delta + \frac{n(n-1)}{2}x^{n-2}\delta^2 ... - x^n) && \textbf{(Pascal-triangle)} \\
&= \frac{1}{\delta}(nx^{n-1}\delta + O(\delta^2)) \\
&= nx^{n-1} + O(\delta)
\end{aligned}$$

Chain Rule

$$\frac{d}{dx}(f(g(x))) = \dot{f}(g(x)) \cdot \dot{g}(x)$$

Ex:

- f(x) = cos(x^3)
- f(x) = 2^x
- f(x) = e^{x^2} sin(x)

Differential Equations

Ex: x is the size of a population of procreate bunnies...

- Population grows at a rate λ proportional to its population size: $\frac{dx}{dt} = \lambda x$
- What is population as a function over time?

$$\begin{aligned} \frac{dx}{dt} &= \lambda x \\ \rightarrow \frac{dx}{x} &= \lambda dt \\ \rightarrow \int \frac{dx}{x} &= \int \lambda dt \\ \rightarrow ln(x) &= \lambda t + C \\ \rightarrow x &= e^{\lambda t + C} \end{aligned}$$

- Determine C by initial conditions, e.g. (x_0 at t_0)

Outlines

Review of Event-B

- Context: static properties of Event-B models
  - Sets: user-defined types
  - Constants: static object in development
  - Axioms: presumed properties about sets and constants
  - Theorems: derived properties about sets and constants

Review of Event-B

- Machine: behavioral properties of Event-B models
  - Variables: states
  - Invariants: properties of variables that always need to hold
  - Theorems: derived properties about variables
  - Events: possible state changes

Review of Event-B

- Proof obligations: must be proved to show that Event-B models fulfill their specified properties.
  - INV: invariant preservation
  - FIS: action feasibility
  - ...

Develop Theories in Event-B

- Theory plugin: more modularize and reusable polymorphic "Context"
- Developed at University of Southampton, still under development
- Installation: `http://rodin-b-sharp.sourceforge.net/updates`
  - Modelling Extensions → Theory Feature
- Let us develop a theory for real numbers
- Fork: `https://github.com/veriatl/LORIA_WEEK1`
  - Open model "theory-axiom-reals"

Exercise One (*)

- Prove: $a + b + c = c + b + a$ on real numbers

? How to write this theorem

? What is the key to prove this theorem

? How to use theory plugin to prove this

Exercise Two (**)

- Develop the power operator a^b

? What are its arguments and results

? What is its semantics

Exercise Three (***)

- Open model "ex-pattern-const-DE"

? What this model does

? What is its invariant

? What operators are needed to express this invariant, and what are their semantics

? How to prove your invariant

Caveats

- Axioms inconsistency $\rightarrow$ Introduce when necessary, Prove when you can
- Big fat theories $\rightarrow$ Modular theories