$$
\begin{aligned}
&\oplus \underset{g_1g_2g_3}{\triangle^3} f\big|_{g=0}\cdot g_1\cdot g_3)\cdot \underset{x_i}{\triangle} g_2\\
&\oplus (\underset{g_3}{\triangle} f\big|_{g=0} \oplus \underset{g_1g_3}{\triangle^2} f\big|_{g=0}\cdot g_1 \oplus \underset{g_2g_3}{\triangle^2} f\big|_{g=0}\cdot g_2\\
&\oplus \underset{g_1g_2g_3}{\triangle^3} f\big|_{g=0}\cdot g_1\cdot g_2)\cdot \underset{x_i}{\triangle} g_3\\
&\oplus (\underset{g_1g_2}{\triangle^2} f\big|_{g=0} \oplus \underset{g_1g_2g_3}{\triangle^3} f\big|_{g=0}\cdot g_3)\cdot \underset{x_i}{\triangle} g_1\cdot \underset{x_i}{\triangle} g_2\\
&\oplus (\underset{g_1g_3}{\triangle^2} f\big|_{g=0} \oplus \underset{g_1g_2g_3}{\triangle^3} f\big|_{g=0}\cdot g_2)\cdot \underset{x_i}{\triangle} g_1\cdot \underset{x_i}{\triangle} g_3\\
&\oplus (\underset{g_2g_3}{\triangle^2} f\big|_{g=0} \oplus \underset{g_1g_2g_3}{\triangle^3} f\big|_{g=0}\cdot g_1)\cdot \underset{x_i}{\triangle} g_2\cdot \underset{x_i}{\triangle} g_3\\
&\oplus \underset{g_1g_2g_3}{\triangle^3} f\big|_{g=0}\cdot \underset{x_i}{\triangle} g_1\cdot \underset{x_i}{\triangle} g_2\cdot \underset{x_i}{\triangle} g_3.
\end{aligned}
$$

By Lemma 3 with x_i replaced by g_{i_1}, g_{i_2}, g_{i_3} the coefficient of $\underset{x_i}{\triangle} g_1$ in the last expression is exactly the expansion of $\underset{g_i}{\triangle} f$. Similarly the coefficient of $\underset{x_i}{\triangle} g_2$ is the expansion for $\underset{g_i}{\triangle} f$, and so forth, down to the last term, the coefficient of $\underset{x_i}{\triangle} g_1\cdot \underset{x_i}{\triangle} g_2\cdot \underset{x_i}{\triangle} g_3$, which identically equals $\underset{g_1g_2g_3}{\triangle^3} f$. Thus

$$
\begin{aligned}
\underset{x_i}{\triangle} f = {}& \underset{g_1}{\triangle} f\cdot \underset{x_i}{\triangle} g_1 \oplus \underset{g_2}{\triangle} f\cdot \underset{x_i}{\triangle} g_2 \oplus \underset{g_3}{\triangle} f\cdot \underset{x_i}{\triangle} g_3\\
&\oplus \underset{g_1g_2}{\triangle^2} f\cdot \underset{x_i}{\triangle} g_1\cdot \underset{x_i}{\triangle} g_2 \oplus \underset{g_1g_3}{\triangle^2} f\cdot \underset{x_i}{\triangle} g_1\cdot \underset{x_i}{\triangle} g_3\\
&\oplus \underset{g_2g_3}{\triangle^2} f\cdot \underset{x_i}{\triangle} g_2\cdot \underset{x_i}{\triangle} g_3\\
&\oplus \underset{g_1g_2g_3}{\triangle^3} f\cdot \underset{x_i}{\triangle} g_1\cdot \underset{x_i}{\triangle} g_2\cdot \underset{x_i}{\triangle} g_3.
\end{aligned}
$$

This is the desired result for $m=3$. It is evident that exactly the same procedure will show in general case for all m. Hence the theorem is proved.

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Fourier Preprocessing for Hand Print Character Recognition

G. H. GRANLUND, MEMBER, IEEE

Abstract—A pattern-recognition method, making use of Fourier transformations to extract features which are significant for a pattern, is described. The ordinary Fourier coefficients are difficult to use as input to categorizers because they contain factors dependent upon size and rotation as well as an arbitrary phase angle. From these Fourier coefficients, however, other more useful features can easily be derived.

By using these derived property constants, a distinction can be made between genuine shape constants and constants representing size, location, and orientation.

The usefulness of the method has been tested with a computer program that was used to classify 175 samples of handprinted letters, e.g., 7 sets of the 25 letters *A* to *Z*. In this test, 98 percent were correctly recognized when a simple nonoptimized decision method was used.

The last section contains some considerations of the technical realizability of a fast preprocessing system for reading printed text.

Index Terms—Character, contour, feature, Fourier, invariant, pattern, recognition, transform.

I. Introduction

Pattern recognition is a very broad field that includes various subjects such as character reading, medical diagnosis, weather prediction, and sound recognition. A general pattern-recognition system which can be taught to perform any arbitrary function is, so far, only a theoretical possibility. Present-day recognition systems, such as those for character recognition, can only deal with limited and well-defined classes of patterns.

A pattern-recognition system is usually considered to have two main parts. First, the pattern is treated by a feature extraction device which has to make measurements of parameters giving maximal information about the pattern. Secondly, this information is fed into a categorizer which makes a classification of the unknown samples. This latter problem is the better understood of the two and has been investigated in some detail. For example, considerable work has been carried out on the optimum decision problem [1], [2]. Feature extraction, however, has proved too difficult to allow a general approach. Nevertheless, the extraction of really significant features is considered more important for the final result than the optimality of the secondary decision process [3].

An important problem in pattern recognition is to find features that describe the processed patterns in an optimal way. Specifically when it concerns character recognition devices, it is important to find features that are invariant for different types of style, size, width of shape, slant of character, etc. These features are still expected to give enough information for an unambiguous classification.

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The author is with the Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Mass. 02139.

A great deal of work has been spent to find features which are invariant in different respects [4]–[6]. Different types of Fourier descriptors have been used because this is a representation that admits a separation of factors dependent upon size, angular orientation, and position [7], [8].

Some difficulty is involved if the Fourier coefficients as such are to be processed in a classifier. The Fourier coefficients contain information about size and orientation, and a phase factor dependent upon starting conditions. The phase factor is generally of no interest while information about size and orientation sometimes is useful.

In this note, features are described that are essentially shape parameters independent of size, angular orientation, position, and starting conditions. These features can be derived from the ordinary Fourier coefficients.

II. Mathematical Considerations

The pattern is, in this part, assumed to be a contour. We assume that the contour function, a closed-curve C, is included in a complex space according to Fig. 1. A point moving around the contour generates a complex-valued function u. Assume that the point is moving at a constant speed v along C. At every time t, there is a complex number u defined which is given by

$$u = u(t).$$

Now the parameter t does not necessarily mean time, but rather a parameter of length along the contour. The function u is periodic, which implies that there exists a value T such that

$$u(t + nT) = u(t).$$

It is now possible to express u as a complex Fourier series. The Fourier coefficients become

$$a_n = \frac{1}{T}\int_0^T u(t) \exp\,[-jn2\pi t/T]dt.$$

It can be shown that the Fourier series is convergent. This gives

$$u(t) = \sum_{-\infty}^{\infty} a_n \exp\,[jn2\pi t/T].$$

For simplicity, we now assume that the velocity v is chosen so that $T=2\pi$. The formulas are then simplified to

$$a_n = \frac{1}{2\pi}\int_0^{2\pi} u(t) \exp\,[-jnt]dt$$

and

$$u(t) = \sum_{-\infty}^{\infty} a_n \exp\,[jnt].$$

The Fourier coefficients of a specific contour are not unique. As one can easily realize, they are dependent upon the starting point of the contour tracking. The Fourier coefficients then differ from one another with respect to a

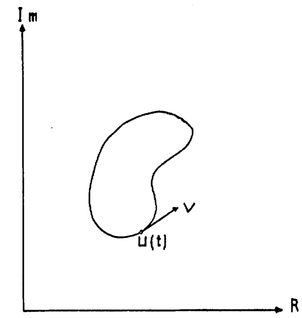

Fig. 1. Example of a contour function.

parameter τ. This means that there is a set of Fourier coefficients for each τ of the function

$$u = u(t + \tau).$$

Now we assume that there exists a certain function

$$u(t) = u^{(o)}(t)$$

and, subsequently, the other functions are given by

$$u(t) = u^{(o)}(t + \tau).$$

The index (o), both now and later, refers to a specific contour function. The resulting Fourier coefficients become

$$\begin{aligned} a_n &= \frac{1}{2\pi}\int_0^{2\pi} u^{(o)}(t+\tau) \exp\,[-jnt]dt \\ &= \exp\,[jn\tau]\cdot\frac{1}{2\pi}\int_0^{2\pi} u^{(o)}(t) \exp\,[-jnt]dt \\ &= \exp\,[jn\tau]a_n^{(o)}. \end{aligned}$$

It is now of great interest to consider the results of certain operations upon the contour such as translation, rotation, and dilation.

A. Translation

We assume $a_n^{(o)}$ to be a specific set of Fourier coefficients from a certain contour. Let us consider a translation of the contour with the complex vector Z. The resulting contour function, expressed in an inverse Fourier series, becomes

$$u(t) = u^{(o)}(t) + Z = \sum_{-\infty}^{\infty} a_n^{(o)} \exp\,[jnt] + Z.$$

We can now identify the Fourier coefficients of the translated contour as

$$a_n = \begin{cases} a_n^{(o)}, & \text{for } n \neq 0 \\ a_o^{(o)} + Z, & \text{for } n = 0. \end{cases}$$

All coefficients except a_o are invariant of translation. a_o is simply the complex vector indicating the position of the center of gravity of the contour.

B. Rotation

We assume here that the center of gravity is positioned at the origin. Rotation of the contour function $u^{(o)}(t)$ around the origin, with an angle ϕ in positive direction, gives another function $u(t)$ where

$$u(t) = \exp[j\phi]u^{(o)}(t).$$

The influence upon the Fourier coefficients is apparent if the function $u(t)$ is again expressed in an inverse Fourier series. The rotated contour then has the Fourier coefficients

$$a_n = \exp[j\phi]a_n^{(o)}.$$

C. Dilation

In a similar way, it can be shown that dilation of the contour with a factor R implies that the Fourier coefficients are multiplied with R.

$$a_n = Ra_n^{(o)}.$$

The general form for the Fourier coefficients of a contour generated by translation, rotation, and dilation of a specific contour is given by

$$a_n = \exp[jn\tau]R\exp[j\phi]a_n^{(o)}$$

where $a_n^{(o)}$ are the Fourier coefficients of the original contour.

These Fourier coefficients in their common form are not particularly useful because they contain information that, in some cases, may be insignificant. What is desired here is some operation that makes a distinction between shape properties and properties dependent upon translation, rotation, and dilation. However, other types of coefficients can be derived which include features more suitable for our purpose.

Let us consider the expression

$$\begin{aligned} b_n &= \frac{a_{1+n}a_{1-n}}{a_1^2} \\ &= \frac{\overset{(o)}{a}_{1+n}\exp[j(1+n)\tau]R\exp[j\phi]\overset{(o)}{a}_{1-n}\exp[j(1-n)\tau]R\exp[j\phi]}{[a_1^{(o)}\exp(j\tau)R\exp(j\phi)]^2} \\ &= \frac{\overset{(o)}{a}_{1+n}\overset{(o)}{a}_{1-n}}{[a_1^{(o)}]^2} = b_n^{(o)}. \end{aligned}$$

The last part of the expression does not contain either τ, R, or ϕ. As these coefficients do not contain a_o, they are also independent of translation. These b_n coefficients might thus be representative for the form of the contour.

In some cases, information of the orientation is necessary and the series of b_n coefficients can be completed by defining a coefficient

$$b_1 = \frac{a_2 \mid a_1 \mid}{a_1^2}.$$

The development of this expression becomes

$$b_1 = \frac{a_2^{(o)}\exp[j2\tau]R\exp[j\phi]R \mid a_1^{(o)} \mid}{[a_1^{(o)}\exp[j\tau]R\exp[j\phi]]^2}$$

$$\begin{aligned} b_1 &= \frac{a_2^{(o)} \mid a_1^{(o)} \mid}{[a_1^{(o)}]^2}\exp[-j\phi] \\ &= b_1^{(o)}\exp[-j\phi] \end{aligned}$$

where b_1 is a parameter depedent upon rotation, but not on translation and dilation.

Another useful set of constants can also be defined. Consider the expression

$$d_{mn}' = \frac{\overset{n}{a}_{1+m}\overset{m}{a}_{1-n}}{a_1^{m+n}}, \qquad \begin{aligned} m &= 1, 2, 3, \cdots \\ n &= 2, 3, 4, \cdots. \end{aligned}$$

As in the case of the b_n coefficients, we assume that the pattern considered is derived by translating, rotating, and dilating a certain pattern in a normalized form. Then the coefficients can be written as

$$\begin{aligned} d_{mn}' &= \frac{[\overset{(o)}{a}_{1+m}\exp[j\tau(1+m)]R\exp[j\phi]]^n}{[a_1^{(o)}\exp[j\tau]R\exp[j\phi]]^n} \\ &\cdot \frac{[\overset{(o)}{a}_{1-n}\exp[j\tau(1-n)]R\exp[j\phi]]^m}{[a_1^{(o)}\exp[j\tau]R\exp[j\phi]]^m} \\ &= \frac{[\overset{(o)}{a}_{1+m}]^n[\overset{(o)}{a}_{1-n}]^m}{[a_1^{(o)}]^{m+n}} = d_{mn}'^{(o)}. \end{aligned}$$

We see that these d_{mn} coefficients are invariant in the same way as the earlier described b_n coefficients. Here, too, a coefficient containing angular information can be defined.

$$d_{m1}' = \frac{a_{1+m} \mid a_1 \mid^m}{a_1^{m+1}}.$$

A development of the expression according to former examples gives

$$\begin{aligned} d_{m1}' &= \frac{\overset{(o)}{a}_{1+m} \mid \overset{(o)}{a_1} \mid^m}{[a_1^{(o)}]^{m+1}}\exp[-jm\phi] \\ &= d_{m1}'^{(o)}\exp[-jm\phi]. \end{aligned}$$

The coefficients d_{m1}' are then invariant of translation and dilation, but rotation with an angle ϕ causes the normalized term $d_{m1}^{(o)}$ to be multiplied with a factor $\exp[-jm\phi]$.

If d_{mn}' applies to numbers m and n that have a factor k in common, the expression d_{mn}' is a power with exponent k. We can thus define

$$d_{mn} = \frac{a_{1+m}^{n/k} a_{1-n}^{m/k}}{a_1^{(m+n)/k}}$$

which gives the relation

$$d_{mn}' = d_{mn}^k$$

to the earlier defined d_{mn}'. If m and n have no factor in common, $(k=1)$, then $d_{mn}=d_{mn}'$, but in other cases, raising to a power causes an unnecessary loss of information which can be avoided by using d_{mn}. The properties of transformation of the d_{mn} constants are similar to those of the d_{mn}' constants.

An important notion is that $d_{nn}=b_n$ which means that the b_n coefficients are special cases of the d_{mn} coefficients. Later on, when b_n and d_{mn} coefficients are concerned, they are generally referred to as d_{mn} coefficients.

III. Qualitative Interpretation of Feature Constants

Let us make a qualitative interpretation of the various constants previously described and evaluate their significance for describing shape features.

The Fourier constants, and other expressions derived from them, indicate the degree of symmetry of the contour. A rotational symmetry of degree m means that a rotation with an angle $2\pi/m$ brings back the same pattern. For such patterns described by $u(t)$, the following expressions are valid.

$$\exp[j2\pi/m]u(t) = u(t + 2\pi/m)$$
$$\exp[j2\pi/m]a_n = \exp[j2\pi n/m]a_n$$
$$a_n[1 - \exp[j2\pi(n-1)/m]] = 0.$$

If $a_n \neq 0$, then

$$1 - \exp[j2\pi(n-1)/m] = 0$$

which means that $n-1$ has to be a multiple of m. For patterns with a rotational symmetry of degree m, significant values of the Fourier coefficients can be expected for indices of type $1 \pm pm$ where p is a positive integer.

If we now consider the previous expression for b_n,

$$b_n = \frac{a_{1+n}a_{1-n}}{a_1^2}.$$

It is apparent that the expression contains Fourier coefficients that should be significant in case of rotational symmetry of degree n. The coefficient for normalization in the denominator is chosen to be a_1 because it is normally the biggest and works well in the invariance transformation.

An arbitrary contour can, to a certain extent, be divided into rotational symmetries of different degrees. However, some different contours can have the same set of coefficients b_n. Let us consider Fig. 2. It shows some coefficients of two triangular curves. They have rotational symmetries of degree two, as well as of degree three, and the computed values of b_2, respectively, b_3 (b_n with $n=2$ and 3) show very little difference, as can be expected. It is difficult to make a distinction between these two shapes based upon the constants b_2 and b_3 alone.

		
b_2	$2.22 \cdot 10^{-2}$	$2.45 \cdot 10^{-2}$
b_3	$7.19 \cdot 10^{-3}$	$7.17 \cdot 10^{-3}$
d_{23}	$1.61 \cdot 10^{-5}$	$-0.77 \cdot 10^{-5}$
d_{32}	$3.52 \cdot 10^{-5}$	$-9.68 \cdot 10^{-5}$

Fig. 2. Triangles with rotational symmetries of different orientation

However, the d_{mn} coefficients can be used to give some helpful information. The d_{mn} coefficients indicated the relations between rotational symmetries of different degrees. This relation is expressed as a magnitude as well as an angle; the latter indicates primarily the angle between axes of different rotational symmetries. In Fig. 2 the coefficients d_{23} and d_{32} are given and a distinction based upon these coefficients is quite possible.

It should be noted that the d_{mn} and b_n coefficients are not complete descriptors of the shape and, therefore, the contour can not be reconstructed from the coefficients.

IV. Character-Recognition Experiments

In order to get an idea of the significance of the features previously described, an experimental test on handwritten letters was carried out.

The function of a character-recognition system using Fourier methods has been simulated with an IBM 360/65 computer. The test was performed upon a set of 175 letters. As the letters are fed to the computer via punched cards describing the pattern, it was rather time-consuming to produce an appropriate set. The same set of letters was therefore used as a learning set as well as a test set.

This fact certainly has an influence upon the final result and brings forth too optimistic results. It is difficult to estimate the discrepancy, and this performed test should merely be taken as a qualitative indication of the usefulness of the features. However, the aim has been to make a learning test set that is representative of what could be "normal" variations of carefully handprinted letters. No changes were made in the letter set after the start of the experiment. The different shapes appear in Fig. 3.

The processing starts with a character-acquisition phase, where each character, expressed as a 35×35 binary matrix, is fed into the computer. Each row of the matrix is represented by one card giving numbers of sequentially white and black elements. The computer then scans the matrix until it reaches the first black element. After that it tracks the contour, and, during the tracking, it evaluates the Fourier coefficients as defined previously. The d_{mn} coefficients are then computed.

In this test, the coefficients used were d_{11}, d_{22}, d_{21}, d_{12}, d_{31}, d_{13}, and d_{44}. It is not certain that these coefficients were the best choice. They were chosen after merely qualitative considerations. Five of the coefficients used contain information of the angular position of the letter, which may give too much dependence upon this orientational parameter. The character-classification scheme used is based on simple probability and distance measurements.

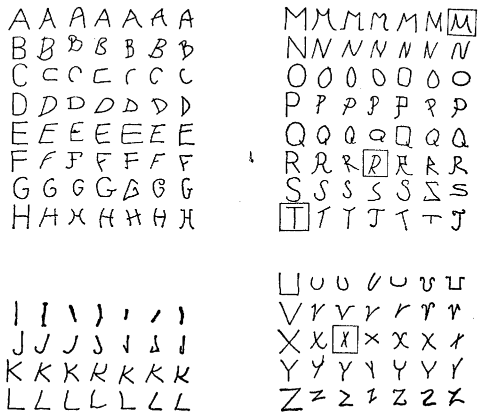

Fig. 3. Shapes of the processed letters.

First a training set was run. This set consisted of 7 handprinted samples of the 25 letters A to Z. W was unfortunately omitted because it is not included in the Swedish alphabet. In order to simplify the probability-function computations, it is assumed that the distribution of a certain d_{mn} constant of a letter class constitutes a two-dimensional Gaussian distribution. As the number of points defining it is as low as seven, the mathematical requirements are obviously not met.

The two-dimensional Gaussian distribution is defined by

$$f_{d_{mn}}(x, y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\left[-\frac{1}{2} Q(x, y)\right]$$

where

$$Q(x, y) = \frac{1}{1-\rho^2}\left[\frac{(x-x_o)^2}{\sigma_1^2} - \frac{2\rho(x-x_o)(y-y_o)}{\sigma_1\sigma_2} + \frac{(y-y_o)^2}{\sigma_2^2}\right].$$

In that expression, x and y mean real and imaginary components of a constant d_{mn}; x_o and y_o are means derived from the coefficients of the learning set; σ_1 and σ_2 are the variances of the variables in the learning set, and ρ is the correlation coefficient.

In the computer program, the correlation ellipse was computed. In Fig. 4, mean values and variance ellipses of coefficient d_{22} are plotted for the first few letters of the alphabet.

When the first set was analyzed, the d_{mn} coefficients of the unknown letters were computed. Any d_{mn} coefficient, together with the related variance ellipse, defines a one-dimensional distribution along the line joining the actual d_{mn} point and the mean. The new distribution function is of the form

$$f_{d_{mn}}(z) = \frac{1}{\sigma_z\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma_z^2} z^2\right]$$

where σ_z is the resulting variance derived from the operation earlier described and z is the distance between actual d_{mn} and the mean of d_{mn}. Assuming—with more or less validity—that the various d_{mn} coefficients are independent makes it possible to define a logarithmic relative probability of a letter belonging to a certain class c of letters.

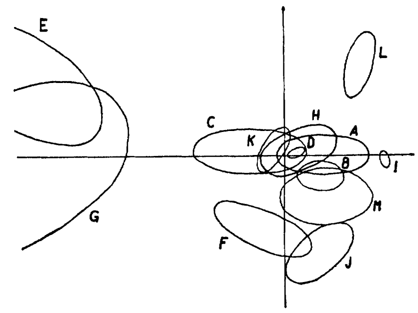

Fig. 4. Example of the distribution of coefficients d_{22} for the first 13 letters of the alphabet.

```
PROBABILITY
  OF
  A                         -84865
  B                         -172974
  C                         1000
  D                         -.2010347'+08
  E                         -1501
  F                         -3035
  G                         991
  H                         -944495
  I                         -.1202279'+10
  J                         -811
  K                         -3977302
  L                         -654
  M                         930
  N                         -552583
  O                         -.1094149'+12
  P                         -2069462
  Q                         -.6672197'+08
  R                         -4635
  S                         -1245
  T                         -271396
  U                         961
  V                         919
  X                         -161162
  Y                         -1346
  Z                         -55214
CONCLUSION:
THE PATTERN REPRESENTS C
```

Fig. 5. Example of a printout from a recognition test.

$$P(c) = \sum_{m,n}\left[\left[-\frac{1}{2\sigma_z^2} z^2\right] - \ln \sigma_{z_{mn}}\right].$$

The index mn refers to a certain d_{mn} coefficient. The factor $\sqrt{\pi 2}$, common to all terms, is omitted in the computations. The decision is then based upon a comparison of $P(c)$ for different classes, and the class category c, assigned to the highest value of $P(c)$, is printed out. A typical printout is shown in Fig. 5. The numbers are normalized such that the highest value of $P(c)$ is set at 1000. The actual letter is C in series no. 2. It is apparent that we have a number of confusion pairs, such as C-G, C-M, C-U, and C-V, which have nearly the same distance measure. That seems reasonable because these letters have essentially a similar shape.

As is indicated in the printout, there is no "reject class" in the classification because an estimation of such a class would require a more careful analysis of the relationships and absolute values of the probabilities.

Of 175 test samples, all except four were recognized correctly. The incorrectly recognized characters were as follows. M in column seven was interpreted as an R; R in column four was interpreted as a P; T in column one was interpreted as an I; and X in column three was interpreted as an R. These letters are indicated with boxes in Fig, 3. The differences between the probabilities of the confusion pairs are, in most cases, small and could be used as rejection criteria.

The probability figures for some of the confusion pairs are given below.

$M7$	$R4$	$T1$	$X3$
H 998	M 989	B 987	C 986
$\underline{M}$ 999	P 1000	I 1000	M 983
$\bar{R}$ 1000	$\underline{R}$ 998	P 982	R 1000
X 1000	T 988	$\underline{T}$ 993	$\underline{X}$ 998

The reason for misrecognition has been that these letters have generated some d_{mn} coefficients that are far from the mean for that letter set. It is apparent from Fig. 3 that this does not necessarily mean that it is difficult to recognize the letter visually. It seems reasonable, however, that a more elaborate decision procedure using a weighting of the coefficients would give a better result.

V. Implementation Considerations

The classification experiments were carried out on a digital computer. However, the computation of Fourier coefficients on a general-purpose digital computer takes a relatively long time due to the great number of operations to be performed. There are also special-purpose computers available that make fast Fourier coefficient computations. They are, however, rather expensive.

In this section an analog system for the preprocessing is suggested that supposedly would be faster and less expensive than a digital system. A complete text-reading system consists of several parts. First, there has to be a scanning system and some system for administration of the material. These systems will not be considered here. It is assumed that the scanner can shift from scanning of the frame to following of a contour when the tracking point reaches the contour. Contour-tracking systems have been developed, for instance by IBM in their 1288 page reader [9]. We assume that the output from the scanner consists of two voltages x and y indicating the position of the tracking point. Referring again to Fig. 1, we have

$$\begin{cases} x = \mathrm{Re}\, u(t) \\ y = \mathrm{Im}\, u(t). \end{cases}$$

What is wanted is a system that makes a computation of the coefficients a_n where

$$\begin{aligned} a_n &= \frac{1}{T}\int_0^T u(t) \exp\,[-jn2\pi t/T]dt \\ &= \mathrm{Re}\, a_n + j\cdot \mathrm{Im}\, a_n \end{aligned}$$

and

$$\mathrm{Re}\, a_n = \frac{1}{T}\int_0^T [\mathrm{Re}\, u(t) \cos n2\pi t/T + \mathrm{Im}\, u(t) \sin n2\pi t/T]\, dt$$

$$\mathrm{Im}\, a_n = \frac{1}{T}\int_0^T [\mathrm{Im}\, u(t) \cos n2\pi t/T - \mathrm{Re}\, u(t) \sin n2\pi t/T]\, dt.$$

The first values to be determined are T, cos nt, and sin nt. This can be done by using a system according to Fig. 6. The control logic causes the spot of the scanner to travel twice around the contour of the character. The first trip is to scale the system with respect to the processed letter. When the spot starts its first trip around the contour, the switch $s1$ is closed, which makes the following integrator start its action. At the time when the spot is back at the starting position, the switch $s1$ is opened and the integration stops. The voltage of the integrator is proportional to the time T to go around the contour. An analog-division unit D transfers the received value to $1/T$. This value can now be fed to the multipliers M controlling the feedback of an oscillator built around two integrators. The period time is given by

$$e\left(s^2 + \frac{n^2}{T^2}\right) = 0$$

which gives a period time T/n for the oscillator.

The initial value of the oscillators is set before the spot starts its second trip around the character. By closing the switches $s2$ and $s3$ the initial values are set to $1/T$ for the cosine integrator and zero for the sine integrator. During the time when the spot is making its second trip around the character, the oscillator is producing the signals $1/T$ cos nt and $1/T$ sin nt.

The remaining part of the integration can be performed by a system according to Fig. 7. As the system consists only of multipliers, adders, and integrators, the function should need no further explanation. The possibility of getting the elements of a_n, as well as of a_{-n}, is indicated.

One oscillator and one integrating system for every desired value n of a_n has to be included. The d_{mn} coefficients to be chosen might be the ones earlier treated, namely d_{11}, d_{22}, d_{21}, d_{12}, d_{31}, d_{13}, and d_{44}.

In order to make a computation of these d_{mn} coefficients, the following a_n coefficients are used: a_{-3}, a_{-2}, a_{-1}, a_1, a_2, a_3, a_4, and a_5. This means 5 oscillators; 2 "double" integrating systems (giving a_{-n} and a_n) and 3 single ones.

The computation of the d_{mn} coefficients may be best done by a digital computer connected on-line to the analog computer previously described. The digital computer might be

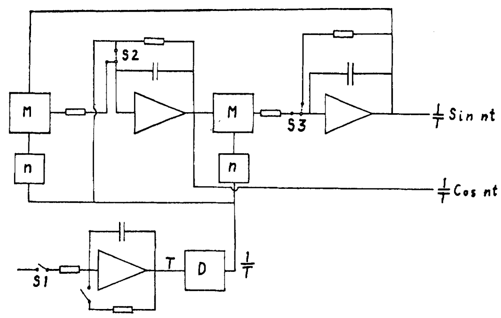

Fig. 6. System for generation of functions of weight.

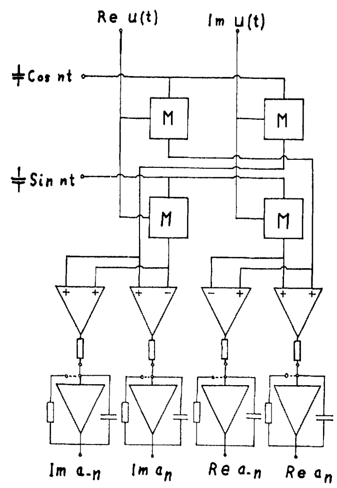

Fig. 7. System for computation of Fourier coefficients.

of a general-purpose type although a special-purpose computer of the same size would do faster work. As the tasks to be performed are quite simple and the amount of data per sample character is rather small, a computer, organized with parallel arithmetic units for each processing step and a small buffer memory between each unit, would be suitable.

The organization, in detail, is dependent upon the decision method used, but this part of the problem will be omitted here.

VI. Conclusions and Extensions

The outlines for a preprocessing system for character recognition has been described. A computer simulation of a recognition system working upon a specified set of unknown letters has given a recognition rate of 98 percent. The result has shown that a letter-recognition system based upon modified Fourier transform would be quite feasible. The system is inherently making a separation of shape properties and properties that in some cases may be insignificant, such as orientation and size. As is indicated in the test results, the features derived seem to be significant for shape properties. A better classification method would probably give better results. In the last part, the realization of a system for fast preprocessing of letters is described. Use of modern electronic components should allow a processing speed in the order of 5000 letters/s.

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Results Obtained Using a Simple Character Recognition Procedure on Munson's Handprinted Data

A. B. SHAHIDUL HUSSAIN, GODFRIED T. TOUSSAINT, STUDENT MEMBER, IEEE, AND ROBERT W. DONALDSON

Abstract—The number of black points in each of the 25 nonoverlapping square regions of a size-normalized character matrix were used to recognize the 3822 uppercase handprinted alphabetic characters from Munson's multiauthor data set. The recognition accuracy obtained using a Bayes' classifier, which assumes statistically independent features, compares favorably with earlier results obtained using recognition systems having complexity comparable to ours. Included are results and a recommendation regarding system evaluation procedures.

Index Terms—Character recognition, character size normalization, classification, experimental methodology, feature extraction, handprinted characters, limited data experiments, Munson's data, pattern recognition, preprocessing.

I. Introduction

When an unknown pattern appears at a recognition system's input, extracted pattern features are used in conjunction with stored data to classify the pattern. In general,

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The authors are with the Department of Electrical Engineering, University of British Columbia, Vancouver 8, B. C., Canada.