Review: Logic

- **Negation** of *p*:
  'not'

p	$\neg p$
T	F
F	T

- **Conjunction** of *p* and *q*:
  'and'

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

- **Disjunction** of *p* and *q*:
  'or'

P	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

- **Exclusive or** (XOR):

  $p \oplus q$

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

- **Implication**: $p \Rightarrow q$

  "if p then q", "q if p"

  Contrapositive of $p \Rightarrow q$: $\neg q \Rightarrow \neg p$

  logically equivalent

  Converse of $p \Rightarrow q$: $q \Rightarrow p$

  not equivalent

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

- **Biconditional**: $p \Leftrightarrow q$

  "p if, and only if q"

  $p \Leftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

- **Tautology**: a compound proposition that is always true.

  e.g. $p \vee \neg p$ $\quad$ $False \Rightarrow p$

- **Contradiction**: a compound proposition that is always false.

  e.g. $p \wedge \neg p$

- Logical equivalence

  $p \equiv q$ when p and q have the same truth table

- De Morgan's Laws

  $\neg(p \vee q) \equiv \neg p \wedge \neg q$

  $\neg(p \wedge q) \equiv \neg p \vee \neg q$

- Other logical equivalences

- Show/prove logical equivalence
  - Compare truth tables
  - Use known logical equivalences

Commutative	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Associative	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Identity	$p \wedge T \equiv p$	$p \vee F \equiv p$
Negation	$p \vee \neg p \equiv T$	$p \wedge \neg p \equiv F$
Idempotent	$p \wedge p \equiv p$	$p \vee p \equiv p$
De Morgan	$\neg(p \vee q) \equiv \neg p \wedge \neg q$	$\neg(p \wedge q) \equiv \neg p \vee \neg q$
Double negation	$\neg(\neg p) \equiv p$	

Review: Sets

- Definitions

  $x \in S$, $x \notin S$,

  $A \subseteq B$, $A \subset B$, $A = B$

  $|S|$, $\{\}$ or $\emptyset$, $\mathbb{U}$

- How to write sets
  - Enumeration
  - Set builder notation

- Special sets

  $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, and their relationships

- Operations on sets

  Union of A and B $\quad A \cup B$

  elements are either in A or in B or in both

  Intersection of A and B $\quad A \cap B$

  elements are in both A and B

  Difference of A minus B $\quad A - B$

  elements in A but not in B

  Complement of A $\quad \overline{A} = \mathbb{U} - A$

  Power set of S: $\mathcal{P}(S)$

  elements are subsets of S

  $\{\, subset\ with\ size\ 0,\ subsets\ with\ size\ 1,$
  $\quad subsets\ with\ size\ 2, \ldots, subset\ with\ size\ |S|\, \}$

  Cartesian Product of A and B $\quad A \times B$

  $\{\, (a, b) : a \in A\ and\ b \in B \}$

- A partition of a set A

  $$P = \{A_1, A_2, \cdots, A_n\}$$

  satisfying three conditions

- Inclusion-exclusion principle

  For two finite sets:

  $$|A \cup B| = |A| + |B| - |A \cap B|$$

  For three finite sets:

  $$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| \\ -|B \cap C| + |A \cap B \cap C|$$

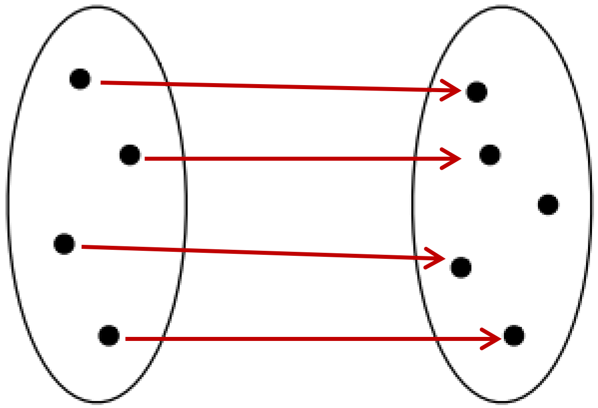
- Venn diagram

- Set properties

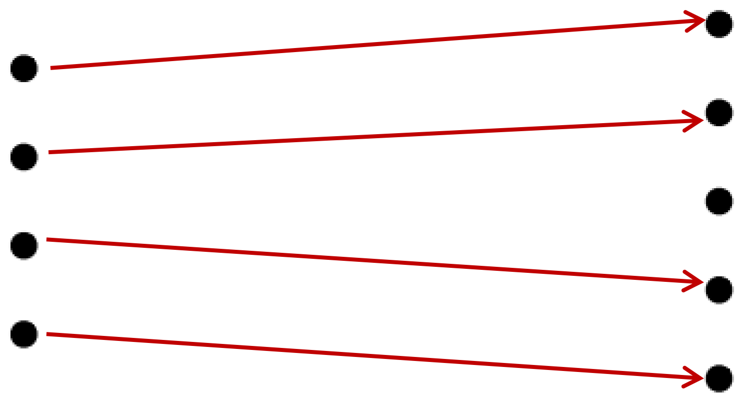
Commutative	$A \cap B = B \cap A$	$A \cup B = B \cup A$
Associative	$(A \cap B) \cap C = A \cap (B \cap C)$	$(A \cup B) \cup C = A \cup (B \cup C)$
Distributive	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
Identity	$A \cap \mathbb{U} = A$	$A \cup \emptyset = A$
Negation	$A \cup \overline{A} = \mathbb{U}$	$A \cap \overline{A} = \emptyset$
Idempotent	$A \cap A = A$	$A \cup A = A$
De Morgan	$\overline{(A \cap B)} = \overline{A} \cup \overline{B}$	$\overline{(A \cup B)} = \overline{A} \cap \overline{B}$
Complement	$\overline{\mathbb{U}} = \emptyset$	$A - B = A \cap \overline{B}$

- Functions:

A mapping from A to B, satisfying that each element in A maps to one element in B

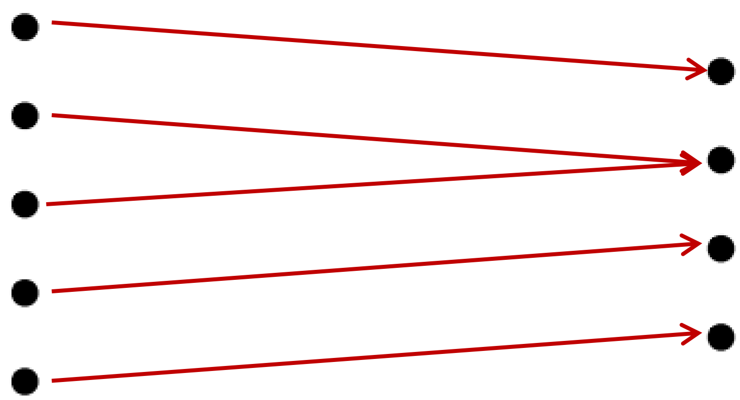


- Injective function/injection



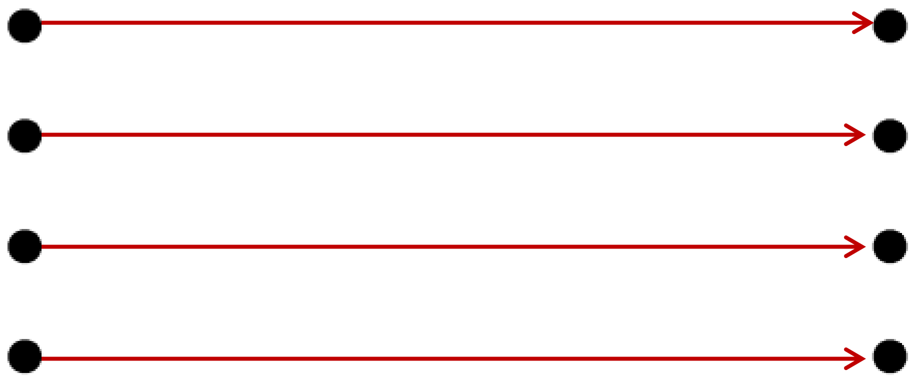
$$|A| \leq |B|$$

- Surjective function/surjection



$$|A| \geq |B|$$

- Bijective function: both injective and surjective



$|A| = |B|$

- |A|=|B| if and only if there is a bijection *f*: A->B
- $|\mathbb{Q}|=|\mathbb{Z}|=|\mathbb{N}|=\aleph_0$ ⇒ The smallest cardinality that an infinite set can have
- $|\mathbb{R}|>|\mathbb{N}|$
- A set is countable when $|S| \leq |\mathbb{N}|$