CM1103 Week 7: Exercises 2 – Sets

1. Let $A = \{3, 4\}$, $B = \{1, 3, 5, 7\}$, $C = \{\}$ and $D = \{a, b, c\}$. Specify the sets given below.

   (a) $A \cup B$ (b) $A \cap B$ (c) $(A - B) \cup (B - A)$
   (d) $A \cap A$ (e) $A \cap \overline{A}$ (f) $A - \overline{B}$
   (g) $A \cap C$ (h) $B \cup C$ (i) $A \cap D$
   (j) $B \cup D$ (k) $(A \cup B \cup D) \cap C$ (l) $|\overline{D}|$

2. Express each of the following sets by enumerating the elements and also by using set builder notation. Which do you think is the most useful method of expression?

   (a) Your favourite foods
   (b) All odd numbers between 50 and 70

3. Let A and B be any sets. Does $A \cup B = A \cap B$? Prove your assertion.

4. List all of the subsets of the set $\{a, b, c\}$ of cardinality 2.

5. Give a member of the set $\mathbb{Z} - \mathbb{N}$.

6. How many subsets of $\{1, 2, 3, 4\}$ are there which contain both the element 2 and the element 4? How many subsets of $\{1, 2, 3, 4\}$ contain the element 2 but *not* the element 4?

7. Which of the following statements are true:

   (a) $\{2, 1, 3\} = \{3, 2, 1\}$ (b) $\{1, 2, 3, 1, 2, 3\}$ is a legal set
   (c) $\{6, 7, 8\} \subseteq \{1, 2, 3, 4, 5, 6, 7, 8\}$ (d) $\emptyset \subset A$ for all sets A

8. Highlight the set $(A \cup B) - C$ in a Venn diagram.

9. Let A, B and C be any sets. For each of the following, decide if the statement is true or false. If false, give a counterexample.

   (a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   (b) If $A \subseteq (B \cup C)$ then $A \subseteq B$ and $A \subseteq C$
   (c) Set difference is commutative, i.e. $A - B = B - A$

10. Decide whether the following is true:

$$A - (B \cup C) = (A - B) \cup (A - C)$$

    by drawing Venn diagrams to illustrate the sets $B \cup C$, $A - B$, $A - C$ and each side of the statement.

11. Let $A = \{1\}$, $B = \{1, 2\}$ and $C = \{\{1\}, \{1, 2\}\}$. Find the following powersets

    (a) $\mathcal{P}(A)$ (b) $\mathcal{P}(B)$ (c) $\mathcal{P}(A \cup B)$ (d) $\mathcal{P}(A \cap B)$
    (e) $\mathcal{P}(C)$ (f) $\mathcal{P}(A \cup C)$ (g) $\mathcal{P}(\emptyset)$ (h) $\mathcal{P}(\mathcal{P}(\emptyset))$

12. Find all partitions of the set $\{1, 2, 3\}$

13. Show that 'is a subset of' is transitive i.e. that, for any sets A, B and C, if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.