CM1103 Week 6: Exercises 1 – Logic

1. Construct truth tables for the following propositions and statements [Note: $p \Leftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$]

   (a) $p \wedge \neg q$ (b) $p \vee (q \Rightarrow r)$ (c) $p \Rightarrow (\neg q \Rightarrow r)$
   (d) $p \vee (q \Leftrightarrow r)$

2. Let p be 'I am hungry', q be 'my plate is empty' and r be 'the canteen is open'. Write propositional statements using p, q and r for the following

   (a) I am hungry and my plate is empty
   (b) If the canteen is open, my plate is not empty
   (c) I am not hungry if the canteen is open
   (d) I am hungry if the canteen is not open or if the canteen is open and my plate is empty

3. Show that " $p \Rightarrow r$ " is logically equivalent to " $\neg r \Rightarrow \neg p$ ".

4. Show that " $\neg(p \wedge q)$ " is logically equivalent to " $\neg p \vee \neg q$ ".

5. Find example propositions to demonstrate that " $p \Rightarrow r$ " is *not* logically equivalent to " $r \Rightarrow p$ ".

6. Consider the statement "I am in Cardiff only if I am in Wales".

   (a) Convert this statement into propositional logic using your own symbols.
   (b) Write down the converse and contrapositive of this statement.
   (c) From the original statement, can we correctly infer the converse? Can we correctly infer the contrapositive? Explain your answers.

7. Let p be the proposition "I bought a lottery ticket". Let q be the proposition "I won the jackpot". Express the following sentences in English

   (a) $\neg p$ (b) $p \vee q$ (c) $p \Rightarrow q$ (d) $p \wedge q$
   (e) $p \Leftrightarrow q$ (f) $\neg p \Rightarrow \neg q$ (g) $\neg p \wedge \neg q$ (h) $\neg p \vee (p \wedge q)$

8. Let p be the proposition "Today is Friday". Let q be the proposition "Today is a holiday". Let r be the proposition "I have an exam". Express the following propositional statements in mathematical symbolic form

   (a) Today is not a holiday
   (b) Today is Friday and a holiday
   (c) If today is Friday, then I have an exam
   (d) Today is a holiday and I have an exam
   (e) Today is not Friday, it is a holiday, and I have no exam

9. A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie.

   You meet two inhabitants: Homer and Bozo. Homer tells you, 'At least one of the following is true: that I am a knight or that Bozo is a knight.' Bozo claims, 'Homer could say that I am a knave.'

   Can you determine who is a knight and who is a knave?

   [From `http://philosophy.hku.hk/think/logic/knight.php`]

CM1103 Week 6: Solutions 1 – Logic

1.

(a)

p	q	$(p \wedge \neg q)$
T	T	F
T	F	T
F	T	F
F	F	F

(b)

p	q	r	$q \Rightarrow r$	$p \vee (q \Rightarrow r)$
T	T	T	T	T
T	T	F	F	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	F
F	F	T	T	T
F	F	F	T	T

(c)

p	q	r	$\neg q$	$\neg q \Rightarrow r$	$p \Rightarrow (\neg q \Rightarrow r)$
T	T	T	F	T	T
T	T	F	F	T	T
T	F	T	T	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	T	F	F	T	T
F	F	T	T	T	T
F	F	F	T	F	T

(d)

p	q	r	$q \Rightarrow r$	$q \Leftarrow r$	$q \Leftrightarrow r$	$p \vee (q \Leftrightarrow r)$
T	T	T	T	T	T	T
T	T	F	F	T	F	T
T	F	T	T	F	F	T
T	F	F	T	T	T	T
F	T	T	T	T	T	T
F	T	F	F	T	F	F
F	F	T	T	F	F	F
F	F	F	T	T	T	T

2. (a) $p \wedge q$

   (b) $r \Rightarrow \neg q$

   (c) $r \Rightarrow \neg p$ (notice how the condition for the if turns up half way through the English sentence)

   (d) $(\neg r \vee (r \wedge q)) \Rightarrow p$

3. Construct a truth table to show logical equivalence:

p	r	$\neg r$	$\neg p$	$p \Rightarrow r$	$\neg r \Rightarrow \neg p$	$(p \Rightarrow r) \Leftrightarrow (\neg r \Rightarrow \neg p)$
T	T	F	F	T	T	T
T	F	T	F	F	F	T
F	T	F	T	T	T	T
F	F	T	T	T	T	T
				$\uparrow$ the same	$\uparrow$	each implies the other

Therefore $p \Rightarrow r$ and $\neg r \Rightarrow \neg p$ are logically equivalent.

4. Use truth tables and compare the columns:

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$(\neg(p \wedge q)) \Leftrightarrow (\neg p \vee \neg q)$
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	T

Therefore $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are logically equivalent.

5. For example, take p to be 'I am in Cardiff' and r to be 'I am in Wales'. In this case $p \Rightarrow r$ is true but $r \Rightarrow p$ is false, so the two are not equivalent. (For logical equivalence, the two things must *always* have the same truth value as one another, regardless of what values are substituted for the statement variables. Here we have found an example where the truth values differ. The one counterexample disproves the *always*, so the two things cannot be logically equivalent.)

   Note that if we were not asked to find an example, but just to demonstrate that $p \Rightarrow r$ and $r \Rightarrow p$ are not logically equivalent, we could choose either to use a counterexample to logical equivalence as above, or to construct the truth tables and show that the $p \Rightarrow r$ and $r \Rightarrow p$ columns are not 'the same'.

6. (a) Let p be "I am in Cardiff" and r be "I am in Wales". Then the statement "I am in Cardiff only if I am in Wales" becomes $p \Rightarrow r$.
   (b) The converse is $r \Rightarrow p$ or 'If I am in Wales then I am in Cardiff'. The contrapositive is $\neg r \Rightarrow \neg p$ or 'If I'm not in Wales then I'm not in Cardiff'.
   (c) The converse cannot be inferred from the original implication statement; the contrapositive can (you can't be in Cardiff and *not* be in Wales).

7. (a) I didn't buy a lottery ticket
   (b) I bought a lottery ticket or I won the jackpot
   (c) I won the jackpot if I bought a lottery ticket
   (d) I bought a lottery ticket and I won the jackpot
   (e) I bought a lottery ticket if and only if I won the jackpot
   (f) If I didn't buy a lottery ticket then I didn't win the jackpot
   (g) I neither bought a lottery ticket nor won the jackpot
   (h) I didn't buy a lottery ticket or I bought a lottery ticket and won the jackpot

   Note that there can be more than one way to phrase each of these statements in English, but make sure that your phrase really is equivalent to the symbolic statement given in each case.

8. (a) $\neg q$ (b) $p \wedge q$ (c) $p \Rightarrow r$ (d) $q \wedge r$ (e) $\neg p \wedge q \wedge \neg r$

9. Let p be the proposition *Homer is a knight*, q be the proposition *Bozo is a knight*, and r and s be the statements they make respectively.

   Then $r \equiv p \vee q$ and $s \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$, giving the truth table:

p	q	r	s
T	T	T	F
T	F	T	T
F	T	T	T
F	F	F	F

   Hence the only consistent answer is both are knaves.