CM1103: Solutions 2 – Sets

1. | | | |
|---|---|---|
| (a) $\{1,3,4,5,7\}$ | (b) $\{3\}$ | (c) $\{1,4,5,7\}$ |
| (d) A or $\{3,4\}$ | (e) $\emptyset$ | (f) $\{3\}$ |
| (g) $\emptyset$ | (h) B or $\{1,3,5,7\}$ | (i) $\emptyset$ |
| (j) $\{1,3,5,7,a,b,c\}$ | (k) $\emptyset$ | |

   (l) Depends on the choice of U e.g. taking U as all lower case letters gives the answer 23

2. (a) Explicitly: {Bread, Apples, Brie}
   Set builder notation: $\{x : x \text{ is a food I like}\}$

   (b) Explicitly: $\{51, 53, 55, 57, 59, 61, 63, 65, 67, 69\}$
   Set builder notation: $\{x : 50 < x < 70 \text{ and } x \text{ is odd}\}$

   In general, set builder notation is much easier to use and frequently involves no work at all to specify a set. Sometimes, it would be impossible to express a set explicitly. For example, how long would it take you to express the set $\mathbb{Z}$ explicitly?!

3. $A \cup B \neq A \cap B$. This is proven by a counterexample (n.b. always chose a simple counterexample). E.g. let $A = \{1\}$ and $B = \{2,3\}$. Then $A \cup B = \{1,2,3\}$ and $A \cap B = \emptyset$. Hence it is not true in general than $A \cup B = A \cap B$.

4. $\{a,b\}$, $\{a,c\}$, $\{b,c\}$

5. $-1, -2, -3, \ldots$.

6. Four: $\{1,2,3,4\}$, $\{2,3,4\}$, $\{1,2,4\}$, $\{2,4\}$, and four: $\{1,2,3\}$, $\{2,3\}$, $\{1,2\}$, $\{2\}$.

7. Which of the following statements are true:

   (a) True – order of elements doesn't matter in sets

   (b) False – each element can only appear once in a set

   (c) True

   (d) False – not true if $A = \emptyset$ (remember the difference between $\subset$ and $\subseteq$)

8. Highlight the set $(A \cup B) - C$ in a Venn diagram. *Solution given in tutorial.*

9. (a) True. Illustrate this by constructing the Venn diagrams of the expressions on either side of the equals sign and observing that they are the same.

   (b) False. Use a counterexample: one possibility is to let $A = B = \{1\}$ and $C = \{2\}$. Note that there are many possible counterexamples. For example, any choice of A, B and C in which $A = B$ and $A \cap C = \emptyset$ will do the job.

   (c) False. Counterexample: Let $A = \{1,2\}$ and $B = \{2,3\}$. Then $A - B = \{1\}$ but $B - A = \{3\}$.

10. Decide whether the following is true:

$$A - (B \cup C) = (A - B) \cup (A - C)$$

by drawing Venn diagrams to illustrate the sets $B \cup C$, $A - B$, $A - C$ and each side of the statement. *Solution given in tutorial.*

11. Given that $A = \{1\}$, $B = \{1,2\}$ and $C = \{\{1\},\{1,2\}\}$.

(a) $\mathcal{P}(A) = \{\emptyset, \{1\}\}$ (b) $\mathcal{P}(B) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$
(c) $\mathcal{P}(A \cup B) = \mathcal{P}(B)$ (d) $\mathcal{P}(A \cap B) = \mathcal{P}(A)$
(e) $\mathcal{P}(C) = \{\emptyset, \{\{1\}\}, \{\{1,2\}\}, \{\{1\},\{1,2\}\}\}$
(f) $\mathcal{P}(A \cup C) = \mathcal{P}\left(\{1,\{1\},\{1,2\}\}\right)$
$= \{\emptyset, \{1,\{1\},\{1,2\}\}, \{1\}, \{\{1\}\}, \{\{1,2\}\}, \{1,\{1\}\}, \{1,\{1,2\}\}, \{\{1\},\{1,2\}\}\}$
(g) $\mathcal{P}(\emptyset) = \{\emptyset\}$ (h) $\mathcal{P}(\mathcal{P}(\emptyset)) = \{\emptyset, \{\emptyset\}\}$

12. All partitions of the set $\{1,2,3\}$ are $\{\{1\},\{2\},\{3\}\}$, $\{\{1\},\{2,3\}\}$, $\{\{1,3\},\{2\}\}$, $\{\{1,2\},\{3\}\}$, $\{\{1,2,3\}\}$

13. Stages of the proof can be constructed as follows:

   i. As $A \subseteq B$, all elements of A are elements of B ($x \in A \Rightarrow x \in B$)

   ii. As $B \subseteq C$, all elements of B are elements of C ($x \in B \Rightarrow x \in C$)

   iii. Hence every element of A is an element of C i.e. $A \subseteq C$.