CM1103 Week 8: Exercises 3 – Counting

1. List *and* count the ways that two winners from the set { **A**ndrew, **B**ob, **C**lare, **D**eb } can be awarded prizes when:

   (a) each person may only win one prize and the two prizes are identical;

   (b) each person may only win one prize and the two prizes are different;

   (c) each person is allowed to win both prizes and the two prizes are identical;

   (d) each person is allowed to win both prizes and the two prizes are different.

2. Evaluate the following

   (a) $P(6,3)$ (b) $P(6,6)$ (c) $P(8,2)$ (d) $P(8,1)$

   (e) $C(6,3)$ (f) $C(6,6)$ (g) $C(8,2)$ (h) $C(8,1)$

3. Explain why the following two values are the same:

   - The number of subsets of $\{1, 2, 3, 4, 5\}$.

   - The number of distinct outcomes that can occur if a coin is flipped 5 times in a row.

4. What does $n!$ mean? Without a calculator find

   (a) $1!$ (b) $2!$ (c) $3!$ (d) $4!$ (e) $5!$

5. What is

   (a) an r-permutation of a set of cardinality n?

   (b) an r-combination of a set of cardinality n?

6. Give the formula for

   (a) the number of r-permutations of a set of cardinality n, denoted $P(n, r)$

   (b) the number of r-combinations of a set of cardinality n, denoted $C(n, r)$

7. List *all* the 2-permutations and *all* the 2-combinations of the set $\{w, x, y, z\}$. Hence, by counting, state the values of $P(4, 2)$ and $C(4, 2)$. Check your answers are correct using the formulae from the previous question.

8. In general, which is largest: $P(n, r)$ or $C(n, r)$? Are $P(n, r)$ and $C(n, r)$ ever equal? Explain your answer.

9. Twelve astronauts have been short-listed to go on an expedition. Only three will be selected.

   (a) How many different teams of three can be chosen?

   (b) Amongst the twelve short-listed are husband and wife Jim and Alma. Jim and Alma want to go either together or not at all. Taking this into account, how many teams of three can be chosen?

10. (a) State the inclusion-exclusion rule for two sets

    (b) Let $A = \{x : x \in \mathbb{Z} \text{ and } 2 \leq x < 5\}$, $|B| = 20$, $A \cap B = \{2, 4\}$. What is $|A \cup B|$?

    (c) State the inclusion-exclusion rule for three sets

11. A particular lottery has the following rules:

- You choose 5 numbers, with no repeats, from 34 numbers.
- You then choose an additional bonus number from 14 numbers. This step does not depend on how you chose the first numbers.
- To win the jackpot, your first five numbers must match the first five numbers drawn from 34, in any order, AND your bonus number must match the one drawn from 14.

If you wanted to be guaranteed to win this lottery jackpot, what is the minimum number of tickets you would need to purchase?

12. (a) Calculate how many strings of length 3 can be made using the letters **a, b, c, d, e**, if:
    i. repetition is not allowed;
    ii. repetition is allowed.

    (b) A shop sells 8 different types of fruit. Assuming their supply is not constrained, and repetition is allowed, how many ways are there of choosing:
    i. 4 pieces;
    ii. 10 pieces.

    (c) How many strings can be made by rearranging the letters of *agreeableness*?