

CM1103 Week 6: Exercises 1 – Logic

- Construct truth tables for the following propositions and statements [Note: $p \Leftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$]
 - $p \wedge \neg q$
 - $p \vee (q \Rightarrow r)$
 - $p \Rightarrow (\neg q \Rightarrow r)$
 - $p \vee (q \Leftrightarrow r)$
- Let p be 'I am hungry', q be 'my plate is empty' and r be 'the canteen is open'. Write propositional statements using p , q and r for the following
 - I am hungry and my plate is empty
 - If the canteen is open, my plate is not empty
 - I am not hungry if the canteen is open
 - I am hungry if the canteen is not open or if the canteen is open and my plate is empty
- Show that " $p \Rightarrow r$ " is logically equivalent to " $\neg r \Rightarrow \neg p$ ".
- Show that " $\neg(p \wedge q)$ " is logically equivalent to " $\neg p \vee \neg q$ ".
- Find example propositions to demonstrate that " $p \Rightarrow r$ " is *not* logically equivalent to " $r \Rightarrow p$ ".
- Consider the statement "I am in Cardiff only if I am in Wales".
 - Convert this statement into propositional logic using your own symbols.
 - Write down the converse and contrapositive of this statement.
 - From the original statement, can we correctly infer the converse? Can we correctly infer the contrapositive? Explain your answers.
- Let p be the proposition "I bought a lottery ticket". Let q be the proposition "I won the jackpot". Express the following sentences in English
 - $\neg p$
 - $p \vee q$
 - $p \Rightarrow q$
 - $p \wedge q$
 - $p \Leftrightarrow q$
 - $\neg p \Rightarrow \neg q$
 - $\neg p \wedge \neg q$
 - $\neg p \vee (p \wedge q)$
- Let p be the proposition "Today is Friday". Let q be the proposition "Today is a holiday". Let r be the proposition "I have an exam". Express the following propositional statements in mathematical symbolic form
 - Today is not a holiday
 - Today is Friday and a holiday
 - If today is Friday, then I have an exam
 - Today is a holiday and I have an exam
 - Today is not Friday, it is a holiday, and I have no exam
- A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie.

You meet two inhabitants: Homer and Bozo. Homer tells you, 'At least one of the following is true: that I am a knight or that Bozo is a knight.' Bozo claims, 'Homer could say that I am a knave.'

Can you determine who is a knight and who is a knave?

[From <http://philosophy.hku.hk/think/logic/knight.php>]