1 Introduction

1.1 Kaggle French Motor Third-Party Liability (MTPL) Dataset

The *freMTPL2freq* dataset, published on Kaggle as part of the French Motor Claims Datasets,[1] is a widely recognized resource in actuarial science and insurance analytics. This dataset contains information on motor third-party liability (MTPL) insurance policies, specifically focusing on claim frequency modeling.

Dataset Overview

- **Source:** Kaggle – French Motor Claims Datasets (*freMTPL2freq*)
- **Number of policies:** 677,991
- **Observation period:** One-year exposure per policy
- **Main objective:** Predict the frequency of insurance claims per policy

Data Structure and Key Features The dataset contains a variety of explanatory variables that describe the policyholders, the insured vehicles, and the insurance contracts. These features include:

- **Policyholder characteristics:** Age of the driver, gender, and driving experience.
- **Vehicle attributes:** Vehicle age, vehicle brand, and vehicle power.
- **Policy details:** Type of coverage, bonus-malus level (No-Claim Discount), and geographic area of residence.
- **Exposure:** The fraction of the year during which the policy was active.
- **Claim information:** Number of claims filed within the exposure period.

Relevance The *freMTPL2freq* dataset is particularly suitable for analyzing

2 Exposure

General claim frequency modeling has traditionally assumed that the occurrence of claims follows a Poisson distribution, implying that claims are proportional to the time at risk and independent of the observation period. However, there is a noticeable seasonality effect: November shows the highest claim frequency, while January and February show the lowest. Furthermore, the impact of COVID-19 curfews has introduced anomalies that are difficult to explain. Post-COVID, there has been a consistent decline in claim frequency.

To address seasonality and the pre/post-COVID differences, we leverage country-wide claim count summaries available on a quarterly basis.

3 Postcode-Level Territorial Modelling

Since some postcodes have limited exposure due to biased data, we model the expected claim count for each postcode and interpret the deviation as the territorial impact.

Observations indicate a negative exponential relationship between the geographical distribution of claims and the travel distance (measured in hours under optimal traffic conditions). To incorporate this, we employ a distance-weighted Bayesian smoothing approach using the following function:

$$f_p = \frac{c_p + \alpha \sum_i c_i e^{-\beta \cdot d_{i,p}}}{e_p + \alpha \sum_i e_i e^{-\beta \cdot d_{i,p}}}$$

where:

- f_p = smoothed claim frequency for postal code p

[1] `https://www.kaggle.com/datasets/floser/french-motor-claims-datasets-fremtpl2freq`

Table 1: Description of the generated dataset variables

Variable	Meaning	Type
`postcode`	Hungarian postcode	Categorical
`licence_age`	Age of the driving licence (years)	Numeric
`n_drivers`	Number of drivers	Integer
`young_driver`	Age of the youngest driver	Numeric
`old_driver`	Age of the oldest driver	Numeric
`domestic`	Expected kilometres driven in Hungary next year	Numeric
`foreign`	Expected kilometres driven outside Hungary next year	Numeric
`is_financed`	Vehicle financed (1 = yes, 0 = no)	Binary (0/1)
`is_rh`	Right-hand steering wheel (1 = yes, 0 = no)	Binary (0/1)
`fuel`	Fuel type (e.g., petrol, diesel, electric)	Categorical
`odometer`	Current odometer reading (km)	Numeric
`daily_commute`	Daily commute time (minutes)	Numeric
`exam`	Recent technical exam within the last 180 days (1 = yes, 0 = no)	Binary (0/1)
`non_payment`	Termination of previous insurance due to non-payment (1 = yes, 0 = no)	Binary (0/1)
`casco`	Casco (comprehensive insurance coverage) (1 = yes, 0 = no)	Binary (0/1)
`is_retired`	Policyholder is retired (1 = yes, 0 = no)	Binary (0/1)
`is_disabled`	Policyholder is disabled (1 = yes, 0 = no)	Binary (0/1)
`seasonal_tyre`	Seasonal tyre usage (1 = yes, 0 = no)	Binary (0/1)

- c_p = observed number of claims in postal code p
- e_p = exposure (weight) for postal code p
- $d_{i,p}$ = distance between postal codes i and p (in hours, optimal traffic conditions)
- α = smoothing parameter

This approach allows for a more stable estimation of claim frequencies for postcodes with limited data by borrowing strength from neighboring areas.

4 Custom Loss Function

To optimize profit, we introduce a custom loss function that adjusts the objective function based on predicted price (P), observed claim (A), and market price (M). This function prioritizes cases that have a more significant financial impact using the following formula:

$$L(P) = \max(M, A) - P + (M - A - (\max(M, A) - P)) \cdot \sigma(P, M - \epsilon, k)$$

where:

- k = sharpness parameter controlling the transition speed
- ϵ = small positive value to define the transition point
- $\sigma(P, M - \epsilon, k)$ = smooth step function defined as:

$$\sigma(P, M - \epsilon, k) = \frac{1}{1 + e^{-k(P-(M-\epsilon))}}$$

The step function σ mimics a soft thresholding behavior, enabling a smoother gradient than a hard cutoff and thus improving convergence during optimization and provides a convex objective function.

This loss function design aims to balance underpricing and overpricing risks, focusing the model's attention on cases with a higher impact on profitability. The smooth step function σ enables a gradual transition, making the optimization process more stable. We've tested a Blended Loss Function a Maximum-Based Relative Loss and a Smooth Margin-Penalized Loss. The goal here is to keep it relatively simple, a convex function is aimed to help the optimisation run smoothly.

In our setup, the optimal pricing is intentionally set slightly below the market price to encourage competitive offers while still ensuring profitability. To reflect the asymmetric financial risk, we design the σ function so that underpricing is heavily penalized: as the predicted price P drops below the market price M, the loss grows rapidly, tending toward infinity as P approaches zero. This ensures the model strongly avoids severe underpricing. On the other hand, if the predicted price exceeds the market price, the penalty grows much more slowly; although overpricing is undesirable, it carries a comparatively smaller financial risk. Importantly, the σ function remains smooth and continuous, which facilitates stable optimization and helps avoid numerical instability during training. The sharpness parameter k in the Smooth Margin-Penalized Loss controls how steeply the penalty escalates for underpricing errors.

Recent research by Burka, Kovács, and Szepesváry (2021) demonstrates the potential of machine learning methods — such as random forests and neural networks — to outperform traditional GLMs in MTPL claim classification tasks. Their findings underscore the importance of incorporating complex interactions and nonlinearities in tariff models. Motivated by their methodology and evaluation framework, our approach also emphasizes model interpretability and economic relevance, especially under asymmetric financial risks.

Building on these insights, we constructed a baseline GLM model using manually engineered feature transformations. This included binning for categorical conversion and, where appropriate, optional polynomial terms — for instance, to capture nonlinear effects in variables such as the age of the driver's license.

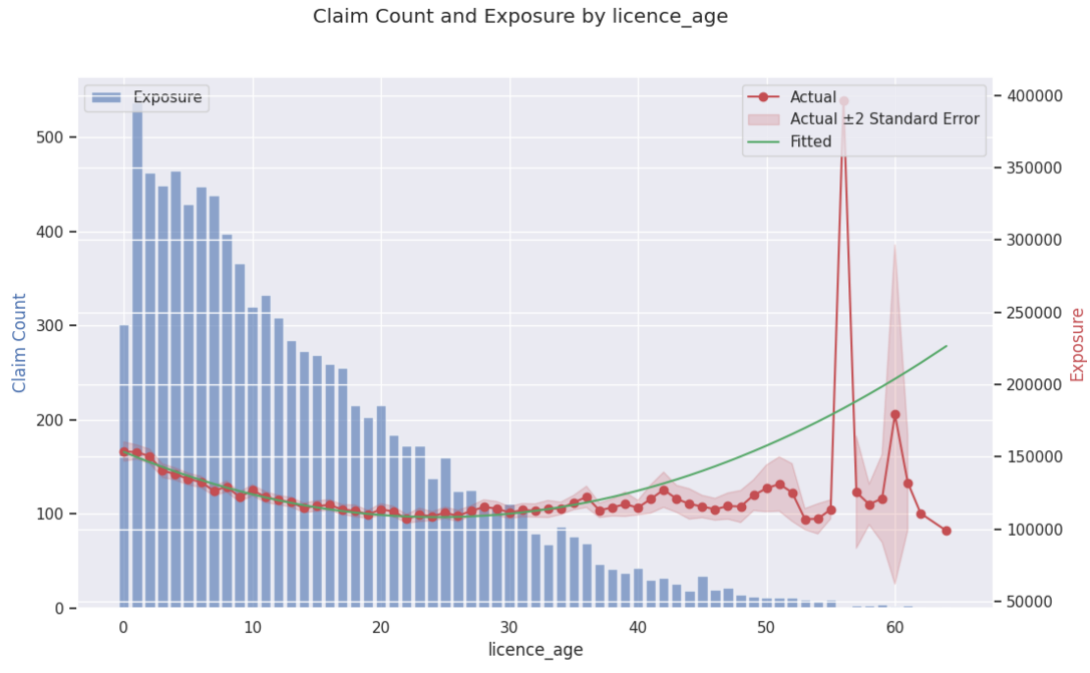


Figure 1: The quadratic polynomial fits well, but licenses older than 35 years must be binned to X = 36

Where necessary, outlying high or low values were also manually grouped to ensure robustness and reduce sensitivity to extreme cases.

Tested Loss Functions

We experimented with the following loss functions involving the predicted price P, the observed claim amount A, and the market benchmark price M:

- **Blended Loss Function:**

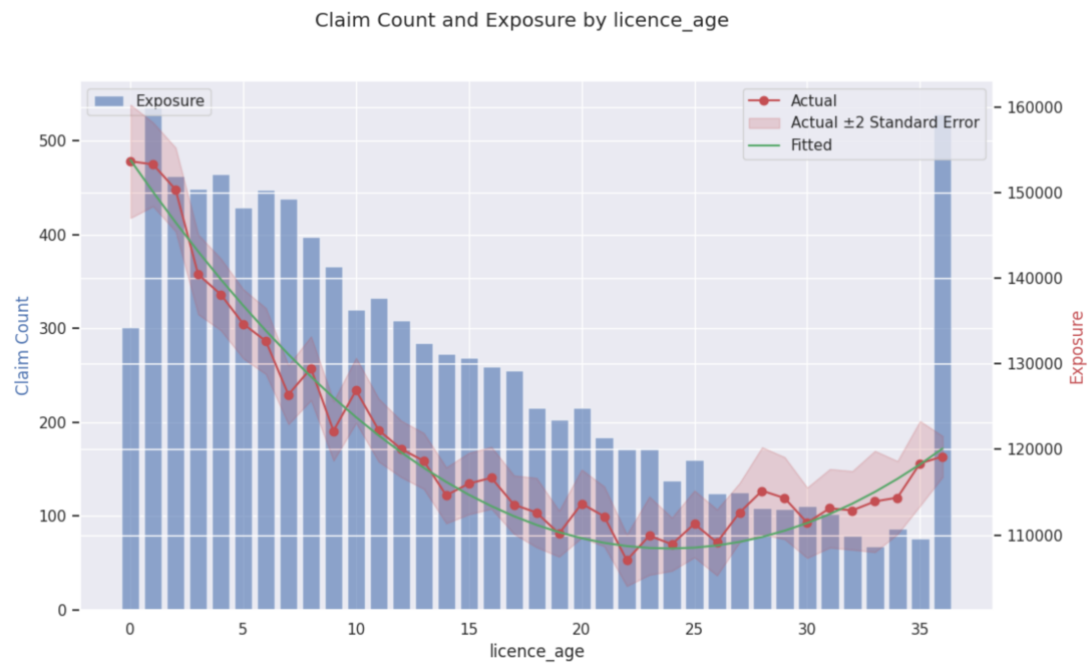


Figure 2: Older licenses align well with the quadratic fit

This loss interpolates between squared error to the true claim A and to the market price M, using a mixing parameter $\alpha \in [0, 1]$:

$$L_{\text{blend}}(P; A, M, \alpha) = (1-\alpha)(P-A)^2 + \alpha(P-M)^2$$

- **Maximum-Based Relative Loss:**

  This function penalizes the squared deviation from the higher of the true and market values:

$$L_{\max}(P; A, M) = (P - \max(A, M))^2$$

- **Smooth Margin-Penalized Loss:**

  This function uses a sigmoid weighting centered at the market price M, smoothly interpolating between penalizing underpricing and overpricing:

$$L_{\text{smooth}}(P; A, M, k) = \frac{1}{1+e^{-k(M-P)}} \cdot (M-P)^2 + \left(1 - \frac{1}{1+e^{-k(M-P)}}\right) \cdot (P-A)^2$$

  where $k > 0$ controls the sharpness of the transition between the two penalty regimes.

These loss functions exhibit distinct behaviors under different pricing scenarios. The figures below illustrate their shapes when applied to (i) a profit-making policy, (ii) a loss-making policy, and (iii) a properly priced policy. Each plot compares the three losses as a function of the predicted price P, holding A and M fixed.

We have developed an XGBoost model, for the simulated dataset for illustration purpose: here the model prefers the Age:

Note that the first decision tree select only from this variable.

Blattberg and George's paper *Estimation under Profit-Driven Loss Functions* does not focus on insurance contexts; rather, it assumes that costs are given and known in advance. Its main contribution lies in its discussion of price sensitivity. The authors also find that estimation based on the mean squared error (MSE) systematically underestimates customers' true price sensitivity. When estimating the price sensitivity parameter ($\hat{\beta}$), they show that their profit-driven loss function is concave in the true price sensitivity (β), which highlights how standard approaches can bias pricing decisions away from profit-maximizing strategies.

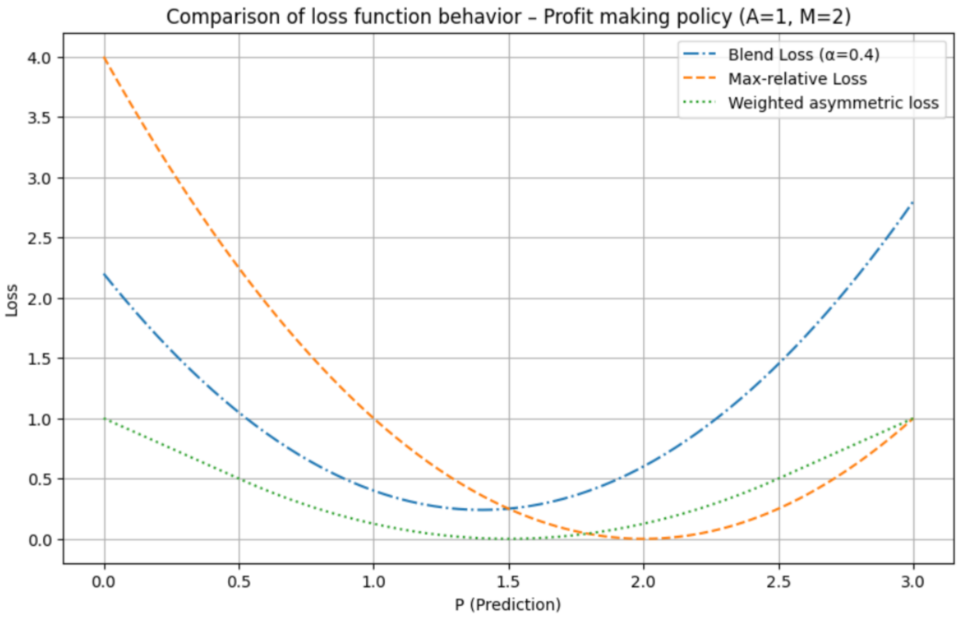


Figure 3: Loss function behavior – profit-making policy $(A = 1, M = 2)$

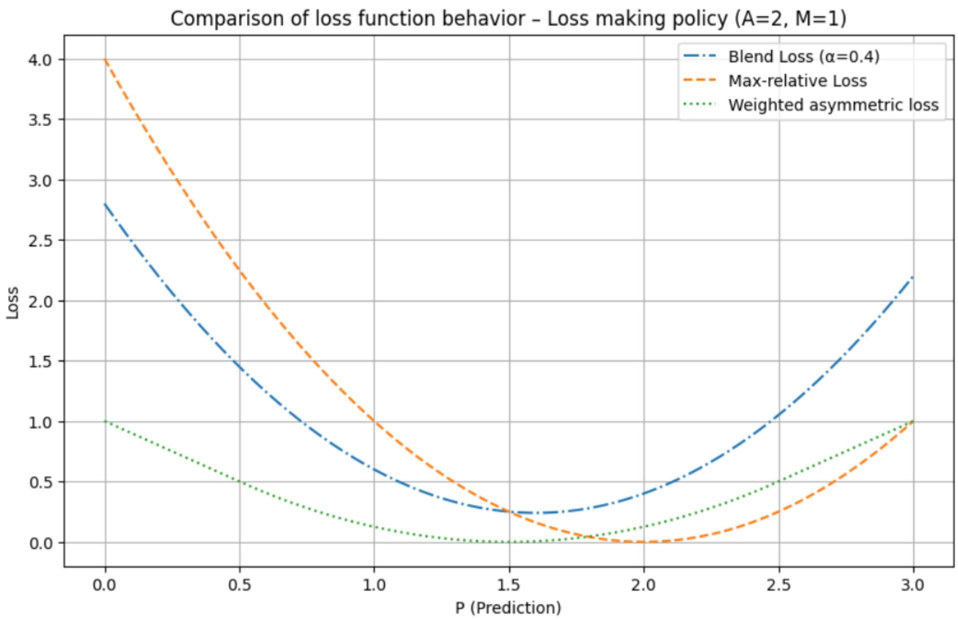


Figure 4: Loss function behavior – loss-making policy $(A = 2, M = 1)$

The paper by Betancourt, Hortaçsu, Öry, and Williams titled *Dynamic Price Competition: Theory and Evidence from Airline Markets* also highlights the importance of cost estimation in modeling pricing behavior. Although the context differs from insurance or simple static pricing models, the insights are relevant for profit-optimized pricing strategies. Their dynamic framework incorporates cost considerations directly into firms' pricing decisions over time, emphasizing that accurately understanding and estimating costs can substantially improve the predictive performance of pricing models.

In their 2023 working paper, Seetharaman, Shi, and Sundaramoorthi propose a *decision-theoretic loss function* tailored for pricing models, emphasizing profit maximization over traditional prediction accuracy. Their approach integrates the firm's profit objectives directly into the model training process, ensuring that the resulting pricing decisions are aligned with maximizing expected profits.

The core idea is to define a loss function that penalizes pricing decisions based on their impact on profit, rather than merely the discrepancy between predicted and actual demand. Mathematically, for a given feature vector x, the model predicts a demand function $\hat{d}(p, x)$. The optimal price p^* is determined

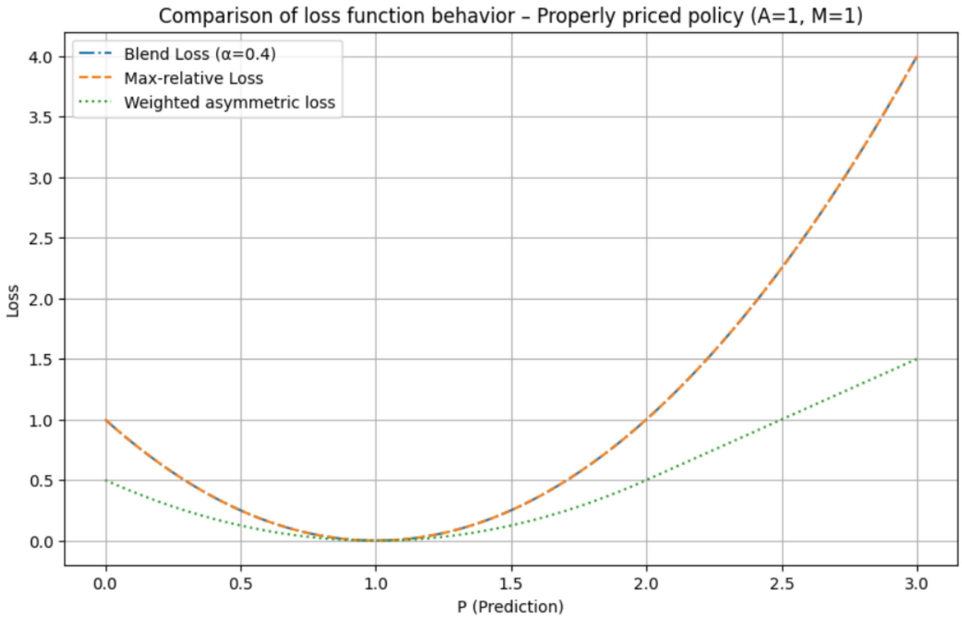


Figure 5: Loss function behavior – properly priced policy ($A = 1, M = 1$)

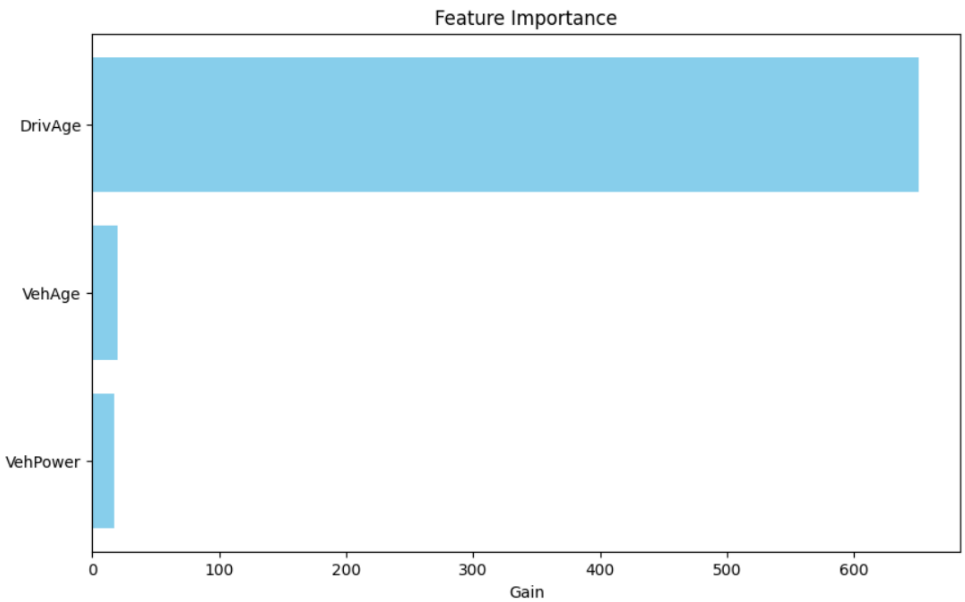


Figure 6: Feature Importance

by maximizing the expected profit:

$$p^* = \arg\max_p (p - c) \cdot \hat{d}(p, x)$$

where c represents the unit cost. The loss function is then defined as the negative of the actual profit achieved at this optimal price:

$$\mathcal{L}(x) = -(p^* - c) \cdot d_{\text{true}}(p^*, x)$$

Here, $d_{\text{true}}(p^*, x)$ denotes the true demand at price p^* for features x. By minimizing this loss across the dataset, the model is trained to make pricing decisions that directly contribute to profit maximization.

This decision-theoretic framework aligns with the work of Muus, van der Scheer, and Wansbeek (2002), who advocate for integrating estimation uncertainty into profit-based decision-making. In their study on direct marketing, they derive an optimal Bayesian decision rule that accounts for the firm's

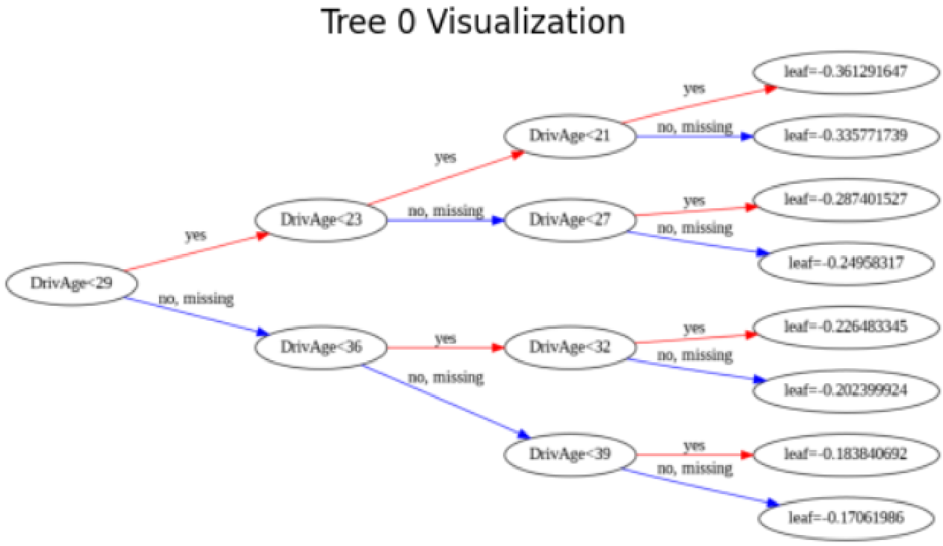
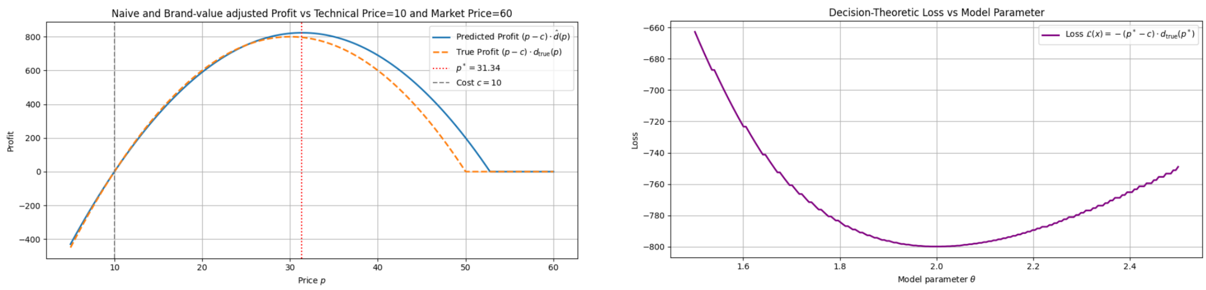


Figure 7: Tree 0 Visualization

profit function, emphasizing that neglecting estimation uncertainty can lead to suboptimal profits. Their approach involves approximating the integral resulting from the Bayesian decision rule using methods like Laplace approximation or Markov chain Monte Carlo integration, demonstrating that such integration leads to higher profits in empirical examples.

Both studies underscore the importance of incorporating profit considerations directly into the modeling process, moving beyond traditional loss functions that focus solely on prediction accuracy.



(a) Expected and true profit as a function of price p. The predicted profit is based on a model $\hat{d}(p, x)$, and the true profit is based on $d_{\text{true}}(p, x)$. The optimal price p^* and unit cost c are marked.

(b) Decision-theoretic loss $\mathcal{L}(x) = -(p^* - c) \cdot d_{\text{true}}(p^*, x)$ as a function of the model parameter θ. More negative values indicate better (more profitable) pricing decisions.

Figure 8: Illustrations of profit-based pricing optimization and decision-theoretic loss.

Note that in Figure 5b, profit maximization is represented as the minimization of negative profit. The parameter θ is used illustratively to represent price elasticity, although in practice, customer price sensitivity is modeled using multiple variables. Figure 5a illustrates a case where the brand value is below the market average.