



**Informatikos fakultetas**

# **MATEMATIKA 2**

Namų darbas Nr.1

Variantas Nr. 18

**Atliko:**

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# Darbo užduotis:

## 18 variantas

### Užduotis:

1. Raskite kompleksinio skaičiaus realiąją ir menamąją dalis.
2. Parašykite kompleksinį skaičių  $z = x + iy$  trigonometrine ir rodikline forma; apskaičiuokite jo  $n$ -ąjį laipsnį.
3. Raskite kompleksinio skaičiaus šaknies visas reikšmes ir pavaizduokite jas geometriškai.
4. Išspręskite lygtį.
5. – 16. Apskaičiuokite neapibrėžtinius integralus.

Visų šių uždavinių patikrinimą atlikite su Octave / Matlab (ar Mathcad) programine įranga.

Jei naudosite kitą programinę įrangą, nurodykite – kokią (pateikite internetinę nuorodą ir pavadinimą).

Prie kiekvieno uždavinio pateikite (priklijuokite) atspausdintą sprendimą.

1.	$z = \bar{z}_1 \cdot z_2 + \frac{29z_1}{z_2} - i^{37}, z_1 = 5 - 2i, z_2 = -2 + 5i$	9.	$\int \frac{\cos(1 - 2 \ln 3x)}{x} dx$
2.	$z = \sqrt{2} - \sqrt{2}i, n = 5$	10.	$\int \frac{x dx}{\sqrt{9 - 16x^4}}$
3.	$\sqrt[4]{81}i$	11.	$\int \frac{\cos(1 - 3x)}{4 + \sin^2(1 - 3x)} dx$
4.	$4z^2 - 32z + 65 = 0$	12.	$\int \frac{3x^2 - 5}{x - 4} dx$
5.	$\int \frac{(1 + \sqrt[3]{x})^2}{\sqrt{x}} dx$	13.	$\int \frac{8x + 5}{\sqrt{4x^2 + 20x + 29}} dx$
6.	$\int \frac{dx}{e^{1-9x}}$	14.	$\int \frac{x - 7}{x^2 + 12x + 37} dx$
7.	$\int \frac{x dx}{\cos^2(3 - 8x^2)}$	15.	$\int x \cdot \arcsin x^2 dx$
8.	$\int x^5 \cdot \sqrt[7]{(2 - 3x^6)^3} dx$	16.	$\int (3x^2 - x + 2) \cdot e^{3x} dx$

1.

Matematika 2 Nd.1 Vas 18 Martynas Kuliešius IFF-1/9

$$z = \underbrace{\bar{z}_1 \cdot z_2}_{(1)} + \underbrace{\frac{29 z_1}{z_2}}_{(2)} - \underbrace{i^{37}}_{(3)}$$

$$z_1 = 5 - 2i \quad \bar{z}_1 = 5 + 2i$$

$$z_2 = -2 + 5i \quad \bar{z}_2 = -2 - 5i$$

$$(1) \quad \bar{z}_1 \cdot z_2 = (5 + 2i) \cdot (-2 + 5i) = 5 \cdot (-2) + 5 \cdot 5i + 2i \cdot (-2) + 2i \cdot 5i = -10 + 25i - 4i + 10i^2 = -20 + 21i$$

$$i^2 = -1$$

$$(2) \quad 29 z_1 = 29(5 - 2i) = 145 - 58i$$

$$(3) \quad i^{37} = i^{36+1} = i^{36} \cdot i = (i^4)^9 \cdot i = 1^9 \cdot i = i$$

$$\frac{29 z_1}{z_2} = \frac{145 - 58i}{-2 + 5i} \cdot \frac{-2 - 5i}{-2 - 5i} = \frac{-290 - 425i + 116i + 290i^2}{(-2)^2 + (5)^2} = \frac{-290 - 609i - 290}{4 + 25} = \frac{-580 - 609i}{29} = -20 - 21i$$

$$(-20 + 21i) + (-20 - 21i) - i = -40 - i$$

```

octave:6> z1=5-2i
z1 = 5 - 2i
octave:7> z1n=5+2i
z1n = 5 + 2i
octave:8> z2=-2+5i
z2 = -2 + 5i
octave:9> z=z1n*z2+((29*z1)/z2)-i^37
z = -40 - 1i
  
```

2.

(2)

$$z = \frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}}i$$

$$n = 5$$

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(\sqrt{2})^2 + (-\sqrt{2})^2} = \sqrt{2+2} = 2$$

$$\varphi = \arctg\left(-\frac{\sqrt{2}}{\sqrt{2}}\right) \quad \varphi = -45^\circ$$

trigonometriji forma:

$$z = 2 \cdot \left( \cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right) \right)$$

rodikliji forma:

$$z = 2 \cdot e^{-i\frac{\pi}{4}} \quad z = r \cdot e^{i\varphi}$$

$$z^4 = z^5 = 2^5 \left( \cos\left(5 \cdot \left(-\frac{\pi}{4}\right)\right) + i \sin\left(5 \cdot \left(-\frac{\pi}{4}\right)\right) \right) = 32 \left( \cos\left(-\frac{5\pi}{4}\right) + i \sin\left(-\frac{5\pi}{4}\right) \right)$$

```
octave:13> z=sqrt(2)-sqrt(2)*i
z = 1.4142 - 1.4142i
octave:14> z^5
ans = -22.627 + 22.627i
```



3.

(3)

$$z = \sqrt[4]{81i}$$

$$\omega = \sqrt[n]{z}, \omega_k = \sqrt[n]{r} \left( \cos\left(\frac{\varphi + 2\pi k}{n}\right) + i \sin\left(\frac{\varphi + 2\pi k}{n}\right) \right)$$

$$81 \left( \cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right) \right)$$

$$k=0: \sqrt[4]{81} \left( \cos\left(\frac{\frac{\pi}{2} + 2\pi \cdot 0}{4}\right) + i \sin\left(\frac{\frac{\pi}{2} + 2\pi \cdot 0}{4}\right) \right) = 3 \left( \cos\left(\frac{\pi}{8}\right) + i \sin\left(\frac{\pi}{8}\right) \right) =$$

$$3 \sqrt{\frac{\sqrt{2}}{4} + \frac{1}{2}} + 3i \sqrt{\frac{1}{2} - \frac{\sqrt{2}}{4}} \quad (\approx 2,77 + 1,15i)$$

$$k=1: \sqrt[4]{81} \left( \cos\left(\frac{\frac{\pi}{2} + 2\pi \cdot 1}{4}\right) + i \sin\left(\frac{\frac{\pi}{2} + 2\pi \cdot 1}{4}\right) \right) = 3 \left( \cos\left(\frac{5\pi}{8}\right) + i \sin\left(\frac{5\pi}{8}\right) \right) =$$

$$-3 \sqrt{\frac{1}{2} - \frac{\sqrt{2}}{4}} + 3i \sqrt{\frac{\sqrt{2}}{4} + \frac{1}{2}} \quad (\approx -1,15 + 2,77i)$$

$$k=2: \sqrt[4]{81} \left( \cos\left(\frac{\frac{\pi}{2} + 2\pi \cdot 2}{4}\right) + i \sin\left(\frac{\frac{\pi}{2} + 2\pi \cdot 2}{4}\right) \right) = 3 \left( \cos\left(\frac{9\pi}{8}\right) + i \sin\left(\frac{9\pi}{8}\right) \right) =$$

$$-3 \sqrt{\frac{\sqrt{2}}{4} + \frac{1}{2}} - 3i \sqrt{\frac{1}{2} - \frac{\sqrt{2}}{4}} \quad (\approx -2,77 - 1,15i)$$

$$k=3: \sqrt[4]{81} \left( \cos\left(\frac{\frac{\pi}{2} + 2\pi \cdot 3}{4}\right) + i \sin\left(\frac{\frac{\pi}{2} + 2\pi \cdot 3}{4}\right) \right) = 3 \left( \cos\left(\frac{13\pi}{8}\right) + i \sin\left(\frac{13\pi}{8}\right) \right) =$$

$$3 \sqrt{\frac{1}{2} - \frac{\sqrt{2}}{4}} - 3i \sqrt{\frac{\sqrt{2}}{4} + \frac{1}{2}} \quad (\approx 1,15 - 2,77i)$$

Nežinau kaip pavaizduoti reikia..

4.

(4)

$$4z^2 - 32z + 64 = 0$$
$$D = (-32)^2 - 4 \cdot 4 \cdot 64 = 1024 - 1024 = -16 \quad -16 < 0$$
$$x_1 = \frac{-(-32) \pm \sqrt{-16}}{2 \cdot 4} = \frac{32 \pm 4i}{8} = 4 \pm \frac{1}{2}i = 4 \pm 0,5i$$
$$x_2 = \frac{-(-32) - \sqrt{-16}}{2 \cdot 4} = \frac{32 - 4i}{8} = 4 - \frac{1}{2}i = 4 - 0,5i$$

**Octave nenori skaičiuoti šitos užduoties**

5.

⑤



$$\int \frac{(1 + \sqrt[3]{x})^2}{\sqrt{x}} dx = \int \left( \sqrt{x} + \frac{2}{\sqrt[3]{x}} + \frac{1}{\sqrt{x}} \right) dx = \int \sqrt{x} dx + 2 \int \frac{1}{\sqrt[3]{x}} dx + \int \frac{1}{\sqrt{x}} dx =$$

$$= \frac{6}{7} x^{\frac{7}{6}} + \frac{12}{5} x^{\frac{5}{6}} + 2\sqrt{x} + C$$

①

 **WolframAlpha**® computational intelligence.

$$\int \frac{(1 + \sqrt[3]{x})^2}{\sqrt{x}} dx$$

 NATURAL LANGUAGE
 MATH INPUT

★
√ ∂f (::) √ ∂ω ...

POPULAR

$\frac{\square}{\square}$ 
 $\square^\square$ 
 $\sqrt{\square}$ 
 $\sqrt[3]{\square}$ 
 $\sqrt[n]{\square}$ 
 $\frac{d}{d\square}$ 
 $\frac{d^2}{d^2\square}$ 
 $\int \square$ 
 $\int \square$ 
 $\sum \square$ 
 $\lim \square \rightarrow \square$ 
 $[\square, \square, \square]$ 
 $\left( \begin{smallmatrix} \square & \square \\ \square & \square \end{smallmatrix} \right)$

Indefinite integral ☒ Step-by-step solution

$$\int \frac{(1 + \sqrt[3]{x})^2}{\sqrt{x}} dx = \frac{6x^{7/6}}{7} + \frac{12x^{5/6}}{5} + 2\sqrt{x} + \text{constant}$$

6.

(6)

$$\int \frac{dx}{e^{1-9x}} = \frac{1}{e^1} \cdot \int \frac{1}{e^{-9x}} dx = \frac{1}{e} \cdot \int e^{9x} dx = \frac{1}{e} \cdot \int e^u \frac{1}{9} du = \frac{1}{e} \cdot \frac{1}{9} e^u =$$

$$= \frac{1}{e} \cdot \frac{1}{9} e^{9x} = \frac{1}{9} e^{9x-1} + C$$



$\int \frac{1}{e^{1-9x}} dx$ 

× =

☀ NATURAL LANGUAGE
∫<sub>a</sub><sup>b</sup> MATH INPUT

★ √ ∂f (::) √<sub>v</sub> αω ...

Indefinite integral

Approximate form
☑ Step-by-step solution

$\int \frac{1}{\exp(1-9x)} dx = \frac{1}{9} e^{9x-1} + \text{constant}$



7.

④

$$\int \frac{x dx}{\cos^2(3-8x^2)} = \frac{1}{16} \operatorname{tg}(3-8x^2) + C$$

8 formuli

Elementariųjų funkcijų integralų lentelė

1. $\int du = u + C$	$u - x$
2. $\int u^\alpha du = \frac{u^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	2a. $\int \frac{du}{\sqrt{u}} = 2\sqrt{u} + C$
3. $\int \frac{du}{u} = \ln  u  + C$	2b. $\int \frac{du}{u^2} = -\frac{1}{u} + C$
4. $\int e^u du = e^u + C$	5. $\int a^u du = \frac{a^u}{\ln a} + C$
6. $\int \cos u du = \sin u + C$	7. $\int \sin u du = -\cos u + C$
8. $\int \frac{du}{\cos^2 u} = \operatorname{tg} u + C$	9. $\int \frac{du}{\sin^2 u} = -\operatorname{ctg} u + C$
10. $\int \frac{du}{\sqrt{1-u^2}} = \arcsin u + C$	11. $\int \frac{du}{1+u^2} = \operatorname{arctg} u + C$

 **WolframAlpha** computational intelligence.

$$\int \frac{x}{(\cos(3-8x^2))^2} dx$$

NATURAL LANGUAGE  $\int_2^x$  MATH INPUT

Indefinite integral Step-by-step solution

$$\int \frac{x}{\cos^2(3-8x^2)} dx = -\frac{1}{16} \tan(3-8x^2) + \text{constant}$$

8.

8

$$\int x^5 \cdot \sqrt[4]{(2-3x^6)^3} dx = \int x^5 \cdot (2-3x^6)^{\frac{3}{4}} dx =$$

~~$\int x^5 \cdot \sqrt[4]{(2-3x^6)^3} dx = \int x^5 \cdot \sqrt[4]{(2-3x^6)^3} dx =$~~

~~$\int x^5 \cdot \sqrt[4]{(2-3x^6)^3} dx = \int x^5 \cdot \sqrt[4]{(2-3x^6)^3} dx =$~~

$u = (2-3x^6)$

$$= -\frac{1}{18} \int u^{\frac{3}{4}} du = -\frac{1}{18} \cdot \frac{4}{10} u^{\frac{10}{4}} = -\frac{1}{18} \cdot \frac{4}{10} \cdot (2-3x^6)^{\frac{10}{4}} =$$

$$-\frac{4}{180} (2-3x^6)^{\frac{10}{4}} + C$$

 **WolframAlpha** computational intelligence.

$\int x^5 \sqrt[7]{(2-3x^6)^3} dx$

NATURAL LANGUAGE ☒ MATH INPUT

★ √ ∂f (::) √ ∂ω ...

Indefinite integral ☒ Step-by-step solution

$$\int x^5 \sqrt[7]{(2-3x^6)^3} dx = -\frac{7}{180} (2-3x^6) \sqrt[7]{(2-3x^6)^3} + \text{constant}$$

9.

⑨

$$\int \frac{\cos(1-2\ln(3x))}{x} dx = \int \cos(u) \cdot \frac{1}{x} dx =$$

$$= \int \cos(u) du = \sin(u) = \sin(1-2\ln(3x)) + C$$



$$\int \frac{\cos(1-2\ln(3x))}{x} dx$$

NATURAL LANGUAGE  $\int_{\frac{x}{2}}$  MATH INPUT
★ √ ∂f (::) √ ∞ ∞ | ...

Indefinite integral Step-by-step solution

$$\int \frac{\cos(1-2\log(3x))}{x} dx = -\frac{1}{2} \sin(1-2\log(3x)) + \text{constant}$$

(assuming a complex-valued logarithm)

log(x) is the natural logarithm

10.

(10)

$$\int \frac{x dx}{\sqrt{9-16x^4}} = \boxed{u=x^2} = \frac{1}{2} \int \frac{1}{\sqrt{9-16t^2}} dt = \frac{1}{2} \cdot \int \frac{1}{\sqrt{16(\frac{9}{16}-t^2)}} dt =$$

$$\frac{1}{2} \cdot \frac{1}{4} \cdot \int \frac{1}{\sqrt{\frac{9}{16}-t^2}} dt = \frac{1}{8} \cdot \arcsin\left(\frac{t}{\frac{3}{4}}\right) = \frac{1}{8} \cdot \arcsin\left(\frac{x^2}{\frac{3}{4}}\right) = \frac{\arcsin\left(\frac{4}{3}x^2\right)}{8}$$

(12 formula)


Kai kurie dažnai pasitaikantys integralai

$$12. \int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right) + C \quad 13. \int \frac{du}{a^2+u^2} = \frac{1}{a} \operatorname{arctg}\left(\frac{u}{a}\right) + C$$

 **WolframAlpha** computational intelligence.

$$\int \frac{x}{\sqrt{9-16x^4}} dx$$

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 NATURAL LANGUAGE
∫<sub>10</sub><sup>π</sup> MATH INPUT

★ √ ∂f (::) ∇ αω ...

Indefinite integral
Step-by-step solution

$$\int \frac{x}{\sqrt{9-16x^4}} dx = \frac{1}{8} \sin^{-1}\left(\frac{4x^2}{3}\right) + \text{constant}$$

sin<sup>-1</sup>(x) is the inverse sine function

11.

(11)

$$\int \frac{\cos(1-3x)}{4 + \sin^2(1-3x)} dx = \int \frac{1}{4 + u^2} du$$

$u = \sin(1-3x)$



$$= -\frac{1}{3} \int \frac{1}{4 + u^2} du = -\frac{1}{3} \cdot \frac{1}{2} \arctan\left(\frac{u}{2}\right) = -\frac{1}{6} \arctan\left(\frac{\sin(1-3x)}{2}\right) + C$$



13 formulė

12.  $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\left(\frac{u}{a}\right) + C$     13.  $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \operatorname{arctg}\left(\frac{u}{a}\right) + C$

 **WolframAlpha**® computational intelligence.

$$\int \frac{\cos(1-3x)}{4 + (\sin(1-3x))^2} dx$$

 NATURAL LANGUAGE
  MATH INPUT
 

★ √ ∂f (::) √ √ aω ...

Indefinite integral Step-by-step solution

$$\int \frac{\cos(1-3x)}{4 + \sin^2(1-3x)} dx = -\frac{1}{6} \tan^{-1}\left(\frac{1}{2} \sin(1-3x)\right) + \text{constant}$$

$\tan^{-1}(x)$  is the inverse tangent function



12.

(12)

$$\int \frac{3x^2 - 5}{x - 4} dx =$$
$$\begin{array}{r} 3x^2 + 0x - 5 \\ -(3x^2 - 12x) \\ \hline 12x - 5 \\ -(12x - 48) \\ \hline 43 \end{array} \quad \begin{array}{r} x - 4 \\ 3x + 12 \end{array}$$
$$= \int 3x + 12 + \frac{43}{x - 4} = \int 3x dx + \int 12 dx + \int \frac{43}{x - 4} dx =$$
$$= \frac{3x^2}{2} + 12x + 43 \ln|x - 4| + C$$

 **WolframAlpha** computational intelligence.

$$\int \frac{3x^2 - 5}{x - 4} dx$$

NATURAL LANGUAGE MATH INPUT

★ √ ∂f (::) ∇ α<sub>ω</sub> ...

Indefinite integral

☒ Step-by-step solution

$$\int \frac{3x^2 - 5}{x - 4} dx = \frac{3x^2}{2} + 12x + 43 \log(x - 4) - 72 + \text{constant}$$

(assuming a complex-valued logarithm)

**13.**







14.

$$\begin{aligned}
 & \int \frac{x-7}{x^2+12x+37} dx = \int \frac{\frac{1}{2} \cdot (2x+12) - 13}{x^2+12x+37} dx = \\
 & = \int \frac{\frac{1}{2} \cdot (2x+12)}{x^2+12x+37} dx - \int \frac{13}{x^2+12x+37} dx = \\
 & = \frac{1}{2} \ln|x^2+12x+37| - 13 \arctan(x+6) + C
 \end{aligned}$$

 **WolframAlpha** computational intelligence.

$$\int \frac{x-7}{x^2+12x+37} dx$$

 NATURAL LANGUAGE
 MATH INPUT

★ √ ∂f (::) √ ∂ω | ...

Indefinite integral
☒ Step-by-step solution

$$\int \frac{x-7}{x^2+12x+37} dx = \frac{1}{2} \log(x^2+12x+37) - 13 \tan^{-1}(x+6) + \text{constant}$$

$\tan^{-1}(x)$  is the inverse tangent function

$\log(x)$  is the natural logarithm

15.

16

$$\int \frac{x}{\sqrt{1-x^2}} \cdot \arcsin x^2 \, dx =$$

$$\left\{ \begin{aligned} u &= \arcsin x^2 & du &= \frac{dx}{\sqrt{1-x^2}} \cdot 2x = \frac{2x}{\sqrt{1-x^2}} dx \\ dv &= x dx & v &= \frac{x^2}{2} \end{aligned} \right.$$

$$= \frac{x^2}{2} \arcsin x^2 - \int \frac{x^2}{2} \cdot \frac{2x}{\sqrt{1-x^2}} dx = \frac{x^2 \arcsin x^2}{2} - \int \frac{x^3}{\sqrt{1-x^4}} dx =$$

$$\left\{ \begin{aligned} u &= 1-x^4 & du &= -4x^3 dx \\ dx &= \frac{du}{-4x^3} \end{aligned} \right.$$

$$= \frac{x^2 \arcsin x^2}{2} + \frac{1}{2} \sqrt{1-x^4} + C$$

 **WolframAlpha** computational intelligence.

$\int x \arcsin(x^2) dx$

NATURAL LANGUAGE  $\int_{\text{E}}$  MATH INPUT

★ √ ∂ ∫ ∴ √ ∂ ∞ ...

Indefinite integral Step-by-step solution

$$\int x \sin^{-1}(x^2) dx = \frac{1}{2} \left( \sqrt{1-x^4} + x^2 \sin^{-1}(x^2) \right) + \text{constant}$$

$\sin^{-1}(x)$  is the inverse sine function



16.

(16)

$$\int (3x^2 - x + 2) e^{3x} dx =$$

$$u = 3x^2 - x + 2 \quad du = (6x - 1) dx$$

$$dv = e^{3x} dx \quad v = \frac{e^{3x}}{3}$$

$$(3x^2 - x + 2) \cdot \frac{e^{3x}}{3} - \int \frac{e^{3x}}{3} \cdot (6x - 1) dx = (3x^2 - x + 2) \cdot \frac{e^{3x}}{3} - \int \frac{6xe^{3x} - e^{3x}}{3} dx =$$

$$(3x^2 - x + 2) \cdot \frac{e^{3x}}{3} - \frac{1}{3} \int 6xe^{3x} - e^{3x} dx =$$

$$= (3x^2 - x + 2) \cdot \frac{e^{3x}}{3} - \frac{1}{3} \left( 2xe^{3x} - \frac{2e^{3x}}{3} - \frac{e^{3x}}{3} \right) = x^2 e^{3x} - x e^{3x} + e^{3x} + C$$



$\int (3x^2 - x + 2) e^{3x} dx$

NATURAL LANGUAGE MATH INPUT

Indefinite integral

Approximate form ☒ Step-by-step solution

$$\int (3x^2 - x + 2) e^{3x} dx = e^{3x} (x^2 - x + 1) + \text{constant}$$

# **Informacijos šaltiniai ir pagalbinės programos:**

**Octave internetinė skaičiuoklė**  
**<https://octave-online.net/>**

**WolframAlpha internetinė skaičiuoklė**  
**<https://www.wolframalpha.com/>**

**Symbolab internetinė skaičiuoklė**  
**<https://www.symbolab.com/>**

**Matematikos formulynas, suteiktas dėstytojos**

**Indų matematiniai video įrašai Youtube**