IFF-8/8 Rokas Šliuskis

Individualios užduotys

Variantas 6

# 1.

a) 11. $f(n) = n^9 - 2n^3 - 4$

$g(n) = n^8 - 9n^3 - 10n$

$$\lim_{n \to \infty} \frac{n^9 - 2n^3 - 4}{n^8 - 9n^3 - 10n} = \frac{n - \frac{2}{n^5} - \frac{4}{n^8}}{1 - \frac{9}{n^5} - \frac{10}{n^7}} = \left[\frac{\infty}{1}\right] = \infty$$

Ats.: $f(n) = \Omega(g(n))$ arba $g(n) = O(f(n))$

b) 37. $f(n) = (\log_2(n^2))^2$

$g(n) = \sqrt{n^3} \log_3 n$

$$\lim_{n \to \infty} \frac{(\log_2(n^2))^2}{\sqrt{n^3} \log_3 n} = 4 \lim_{n \to \infty} \frac{\frac{1}{(\log_n 2)^2}}{\frac{\sqrt{n^3}}{\log_n 3}} =$$

$$= \cancel{4 \lim_{n \to \infty}} \; 4 \frac{\ln(3)}{\ln(2)} \lim_{n \to \infty} \frac{\ln(n)}{\sqrt{n^3}} = \left[\frac{\infty}{\infty}\right] =$$

$$= 4 \frac{\ln(3)}{\ln(2)} \lim_{n \to \infty} \frac{2 \cancel{\sqrt{x}} \, x^{\frac{1}{2}}}{3 \, x^{\frac{5}{2}}} = 4 \frac{2\ln(3)}{3\ln(2)} \lim_{n \to \infty} \frac{1}{x^{\frac{3}{2}}} =$$

$$= \left[\frac{1}{\infty}\right] = 0$$

Ats.: $f(n) = O(g(n))$ arba $g(n) = \Omega(f(n))$

# 2.

18. $O(n^3 + (\log_3(n^2))^2)$

$f(n) = n^3$

$g(n) = (\log_3(n^2))^2$

$$\lim_{n\to\infty} \frac{n^3}{(\log_3(n^2))^2} = \lim_{n\to\infty} \frac{n^3}{\frac{4(\ln(x))^2}{(\ln(3))^2}} =$$

$$= \frac{(\ln(3))^2}{4} \lim_{n\to\infty} \frac{n^3}{(\ln(x))^2} = \left[\frac{\infty}{\infty}\right] = \frac{(\ln(3))^2}{4} \lim_{n\to\infty} \frac{3x^2}{\frac{2\ln(x)}{x}} =$$

$$= \frac{3(\ln(3))^2}{8} \lim_{n\to} \frac{x^3}{\ln(x)} = \left[\frac{\infty}{\infty}\right] \frac{3(\ln(3))^2}{8} \lim_{n\to\infty} 3x^3 = \infty$$

$f(n) = \Omega(f(n))$ arba $g(n) = O(f(n))$

Ats.: $O(n^3 + (\log_3(n^2))^2) = O(n^3)$

# 3.

6. $\sum_{i=1}^{n} (i^5 + 9i^4 + 10)$

$$\int_0^n (x^5 + 9x^4 + 10)dx \le \sum_{i=1}^{n} (i^5 + 9i^4 + 10) \le \int_1^{n+1} (x^5 + 9x^4 + 10)dx$$

$$\left(\frac{x^6}{6} + \frac{9x^5}{5} + 10x\right)\Big|_0^n \le \sum_{i=1}^{n} (i^5 + 9i^4 + 10) \le \left(\frac{x^6}{6} + \frac{9x^5}{5} + 10x\right)\Big|_0^{n+1}$$

$$\frac{n^6}{6} + \frac{9}{5}n^5 + 10n \le \sum_{i=1}^{n} (i^5 + 9i^4 + 10) \le \frac{(n+1)^6}{6} + \frac{9}{5}(n+1)^5 + 10n - \frac{1}{6} - \frac{9}{5}$$

$$\frac{5(n+1)^6 + 54(n+1)^5 + \cancel{300n}\, 300n}{30} \le \sum_{i=1}^{n} (i^5 + 9i^4 + 10) \le$$

Ats.: $\le \frac{5(n+1)^6 + 54(n+1)^5 + 300n - 59}{30}$

$$\frac{(\ldots(x))^2}{(\ldots(3))^2} =$$

$$] = \frac{(\ln(3))^2}{4} \lim_{n\to\infty} \frac{3x^2}{\frac{4\ln(x)}{x}} =$$

$$\lim_{n\to\infty} 3x^3 = \infty$$

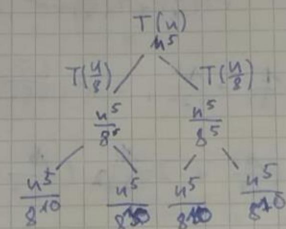
$g(n) = O(f(n))$

$O(n^3)$



21. $T(n) = 2T(\frac{n}{8}) + n^5$

$$T(n) \begin{cases} 2T(\frac{n}{8}) + n^5 & \text{, ако } n \geq 2 \\ 1 & \text{, ако } n = 1 \end{cases}$$



$n^5$

$2\frac{n^5}{8^5}$

$4\frac{n^5}{8^{10}}$ $\quad h = \log_8 n$

$$T(n) = \sum_{i=0}^{h} \left(\frac{2}{8}\right)^i n^5 < \sum_{i=0}^{\infty} \left(\frac{2}{8}\right)^i n^5 = \frac{n^5}{1-\frac{2}{8}} = \frac{4n^5}{3}$$

$$T(n) = \sum_{i=0}^{h} \left(\frac{2}{8}\right)^i n^5 \geq \sum_{i=0}^{\log_8 n} \left(\frac{2}{8}\right)^i n^5 = n^5 \frac{\left(\frac{2}{8}\right)^{\log_8 n + 1} - 1}{\frac{2}{8} - 1} =$$

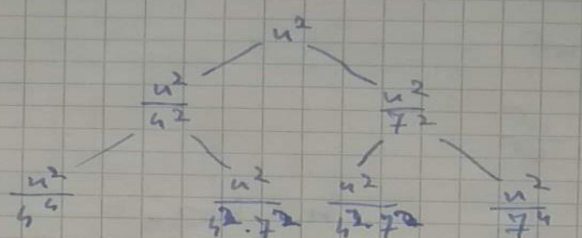
$$= n^5 \frac{\left(\frac{1}{4}\right)^{\log_8 n + 1} - 1}{-\frac{3}{4}} > n^5$$

~~$T(n) \geq \Omega(n^5)$~~ Отг.: $T(n) = \Theta(n^5)$



18. $T(n) = T(\frac{n}{4}) + T(\frac{n}{7}) + n^2$

$$T(n) \begin{cases} 1 & \text{, ако } n = 1 \\ T(\frac{n}{4}) + T(\frac{n}{7}) + n^2 & \text{, ако } n \geq 2 \end{cases}$$



$n^2$

$$n^2\left(\frac{1}{4^2} + \frac{1}{7^2}\right) = n^2\left(\frac{65}{784}\right)$$

$$n^2\left(\frac{1}{4^4} + 2\frac{1}{4^2 \cdot 7^2} + \frac{1}{7^4}\right) = n^2\left(\frac{1}{4^2} + \frac{1}{7^2}\right)^2$$

$h = \log_7 n$

$$T(n) = n^2 \sum_{i=0}^{h} \left(\frac{65}{784}\right)^i < n^2 \sum^{\infty} \left(\frac{65}{784}\right)^i = \frac{n^2}{1-\frac{65}{784}} = \frac{784}{719} n^2$$

$$T(n) = n^2 \sum_{i=0}^{h} \left(\frac{65}{784}\right)^i \geq n^2 \sum_{i=0}^{\log_7 n} \left(\frac{65}{784}\right) = n^2 \frac{\left(\frac{65}{784}\right)^{\log_7 n + 1} - 1}{\frac{65}{784} - 1} =$$

$$= n^2 \frac{\left(\frac{65}{784}\right)^{\log_7 n + 1} - 1}{-\frac{719}{784}} > n^2$$

Отг.: $T(n) = \Theta(n^2)$



21. $T(n) = 2T(\frac{n}{8}) + n^5$ $\quad a = 2 \quad b = 8 \quad f(n) = n^5$

$f(n) = \Theta(n^{\log_8 2})$

$$\lim_{n\to\infty} \frac{n^5}{n^{\log_8 2}} = \lim_{n\to\infty} n^{5-\frac{1}{3}} = \lim_{n\to\infty} n^{\frac{14}{3}} = \infty$$

Отг.: $T(n) = \Theta(n^{\log_8 2} \log_2 n)$

18. $T(n) = T\left(\frac{n}{4}\right) + T\left(\frac{n}{7}\right) + n^2$

$$T(n) = O(n^2)$$

$$T(n) \leq cn^2$$

$$T\left(\frac{n}{7}\right) \leq c\left(\frac{n}{7}\right)^2$$

$$T\left(\frac{n}{4}\right) \leq c\left(\frac{n}{4}\right)^2$$

$$T(n) = c\frac{n^2}{16} + c\frac{n^2}{49} + n^2 < cn^2$$

$$\cancel{n^2\left(c\left(\frac{1}{16} + \frac{1}{49}\right) + 1\right) < cn^2}$$

$$cn^2 \cdot \left(\frac{1}{16} + \frac{1}{49}\right) + n^2 < cn^2$$

$$cn^2 \frac{65}{784} + n^2 < cn^2$$

$$cn^2 \frac{65}{784} - cn^2 < -n^2$$

$$-c\frac{719}{784} < -1$$

$$\frac{65}{784} > \frac{1}{c} \qquad c > \frac{719}{784}$$

$$\frac{784}{65} > c$$

Tenkina, nes ~~galim~~ egzistuoja konstanta $n_0$, su kuria $n > n_0$