1. $a = 25$

$b = 37$

a)

$f(n) = n^5 + 7n^3 - 1$

$g(n) = n^7 + n^3 + 5n$

$$\lim_{n\to\infty} \frac{n^5+7n^3-1}{n^7+n^3+5n} = \left[\frac{\infty}{\infty}\right] = \lim_{n\to\infty} \left(\frac{\cancel{n^7}\cdot\left(\frac{1}{n^2}+\frac{7}{n^4}-\frac{1}{n^7}\right)}{\cancel{n^7}\cdot\left(1+\frac{1}{n^4}+\frac{5}{n^6}\right)} \right) =$$

$$= \lim_{n\to\infty} \left(\frac{\frac{1}{n^2}+\frac{7}{n^4}-\frac{1}{n^7}}{1+\frac{1}{n^4}+\frac{5}{n^6}} \right) = 0$$

Atb.: $f(n) = O(g(n))$ and $g(n) = \Omega(f(n))$

b)

$f(n) = \log_2^2(n^2)$

$g(n) = \sqrt{n^3}\log_3(n)$

$$\lim_{n\to\infty} \frac{\log_2^2(n^2)}{\sqrt{n^3}\log_3(n)} = 4\lim_{n\to\infty} \frac{\frac{1}{(\log n^2)^2}}{\frac{\sqrt{n^3}}{\log n^3}} =$$

$$= 4\frac{\ln(3)}{\ln(2)} \lim_{n\to\infty} \frac{\ln(n)}{\sqrt{n^3}} = \left[\frac{\infty}{\infty}\right] = 4\frac{2\ln(3)}{3\ln(2)} \lim_{n\to\infty} \frac{1}{x^{\frac{3}{2}}} =$$

$$= \left[\frac{1}{\infty}\right] = 0$$

Atb.: $f(n) = O(g(n))$ and $g(n) = \Omega f(n)$

2 nr = 14

$O((\ln(n^2))^2 + \sqrt[4]{n^3})$

$f(n) = (\ln(n^2))^2$

$g(n) = \sqrt[4]{n^3}$

$$\lim_{n \to \infty} \frac{(\ln(n^2))^2}{\sqrt[4]{n^3}} = \lim_{n \to \infty} \left(\frac{\frac{d}{dx}(\ln(n^2))^2}{\frac{d}{dx}(\sqrt[4]{n^3})} \right) =$$

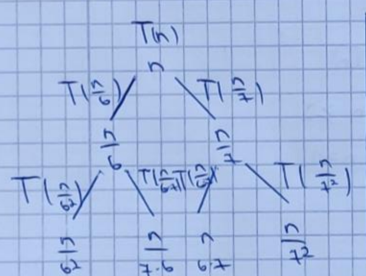
$$= \lim_{n \to \infty} \frac{\frac{8(\ln(x))}{x}}{6x^2} = 0$$

Deci: $f(n) = O(g(n))$ daca $g(n) = \Omega(f(n))$

$O((\ln(n^2))^2 + \sqrt[4]{n^3}) = O(n)$

5. $T(n) = \begin{cases} 1, & \text{daca } n = 1 \\ T(\frac{n}{6}) + T(\frac{n}{7}) + n, & \text{daca } n \geq 2 \end{cases}$

nr=14.



n

$n(\frac{1}{6} + \frac{1}{7})$

$n(\frac{1}{6^2} + \frac{2}{(6 \cdot 7)} + \frac{1}{7^2})$ $\quad h = \log_6 n$

$$n\left(\frac{1}{6^2} + \frac{2}{6 \cdot 7} + \frac{1}{7^2}\right) = n\left(\frac{1}{6} + \frac{1}{7}\right)^2$$

$$T(n) = \sum_{i=0}^{h} \left(\frac{13}{42}\right)^i n < \sum_{i=0}^{\infty} \left(\frac{13}{42}\right)^i n = \frac{n}{1 - \frac{13}{42}} = \frac{42n}{29}$$

$$T(n) = \sum_{i=0}^{h} \left(\frac{13}{42}\right)^i n \geq \sum_{i=0}^{\log_7 n} \left(\frac{13}{42}\right)^i n = n \frac{\left(\frac{13}{42}\right)^{\log_7 n + 1} - 1}{\frac{13}{42} - 1} =$$

$$= n \frac{\left(\frac{13}{42}\right)^{\log_7 n + 1} - 1}{-\frac{29}{42}} \geq n$$

Deci: $T(n) = \Theta(n)$

3. $m = 13$

$$\sum_{i=1}^{n} (i^6 + 10i^3 + 8)$$

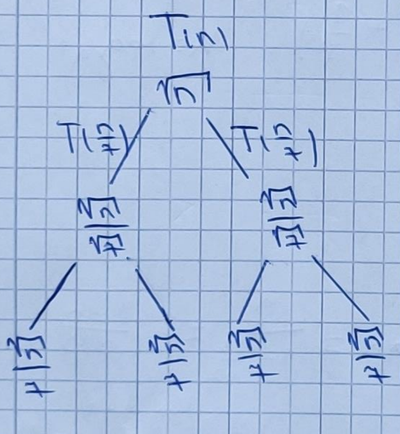
$$\int_0^n (x^6 + 10x^3 + 8)\,dx \le \sum_{i=1}^{n} (i^6 + 10i^3 + 8) \le \int_1^{n+1} (x^6 + 10x^3 - 8)\,dx$$

$$\left(\frac{x^7}{7} + \frac{5x^4}{2} + 8x\right)\Big|_0^n \le \sum_{i=1}^{n} (i^6 + 10i^3 + 8) \le \left(\frac{x^7}{7} + \frac{5x^4}{2} + 8x\right)\Big|_1^{n+1}$$

$$\frac{n^7}{7} + \frac{5n^4}{2} + 8n \le \sum_{i=1}^{n} (i^6 + 10i^3 + 8) \le \frac{(n+1)^7}{7} + \frac{5(n+1)^4}{2} + 8n - \frac{37}{14}$$

4. $m = 14$ $\quad T(n) = 4T\left(\frac{n}{7}\right) + \sqrt{n}$

$$T(n) = \begin{cases} 4T\left(\frac{n}{7}\right) + \sqrt{n}, & \text{daca } n \ge 2 \\ 1, & \text{daca } n = 1 \end{cases}$$



$n^{\frac{1}{2}}$

$2 \cdot \frac{n^{\frac{1}{2}}}{7^{\frac{1}{2}}}$

$4 \cdot \frac{n^{\frac{1}{2}}}{7}$ $\qquad h = \log_2 n$

$$T(n) = \sum_{i=0}^{n} \left(\frac{2}{\sqrt{7}}\right)^i \cdot \sqrt{n} < \sum_{i=0}^{\infty} \left(\frac{2}{\sqrt{7}}\right)^i \sqrt{n} = \frac{\sqrt{7} \cdot \sqrt{n}}{\sqrt{7} - 1}$$

$$T(n) = \sum_{i=0}^{n} \left(\frac{2}{\sqrt{7}}\right)^i \cdot \sqrt{n} \geq \sum_{i=0}^{\log_7 n} \left(\frac{2}{\sqrt{7}}\right)^i \cdot \sqrt{n} = \sqrt{n}\,\frac{\left(\frac{2}{\sqrt{7}}\right)^{\log_7(n)-1} + 1}{\frac{2}{\sqrt{7}} - 1}$$

$\sqrt{n} \cdot \frac{\left(\frac{2}{\sqrt{7}}\right)^{\log_7(n)-1} + 1}{\frac{2}{\sqrt{7}} - 1} \geq n$, retiuna $\quad$ Ath: $T(n) = O(\sqrt{n})$

6. $T(n) = 4T\left(\frac{n}{7}\right) + \sqrt{n}$ $\quad a = 4 \quad b = 7 \quad f(n) = \sqrt{n}$

$$f(n) = O(n^{\log_b a - \varepsilon})$$

$$\lim_{n \to \infty} \frac{n^{\frac{1}{2}}}{n^{\log_7 4}} = \lim_{n \to \infty} n^{\frac{-4\log_7(2)+1}{2}} = 0, \text{ kai } 1-\varepsilon > 0$$

Ats.: $T(n) = \Theta(n^{\log_7 4}) = \Theta(n)$

7.

$$T(n) = T\left(\frac{n}{6}\right) + T\left(\frac{n}{7}\right) + n$$

$$T(n) = O(n^2)$$

$$T(n) = cn^2$$

$$T\left(\frac{n}{6}\right) < c\left(\frac{n}{6}\right)^2$$

$$T\left(\frac{n}{7}\right) < c\left(\frac{n}{7}\right)^2$$

Teisinga, nes galima pasirinkti kažkokią konstantą n_0, su kuria $n > n_0$

$$T(n) = c \cdot \frac{n^2}{36} + c \cdot \frac{n^2}{49} + n < cn^2$$

$$cn^2\left(\frac{1}{36} + \frac{1}{49}\right) + n < cn^2$$

$$cn^2 \cdot \frac{85}{1764} - cn^2 < -n \quad |:n$$

$$c > \frac{1764}{1679n}$$