# Martynas Kemežys IF-8/1

**(C8230)**

1. Palyginti

| | |
|---|---|
| 0. | $f(n) = a, g(n) = \frac{100}{\ln n}, a = const$ |

$$\lim_{n\to\infty} \frac{a}{\frac{100}{\ln n}} = \frac{a}{100} \lim_{n\to\infty} \ln n = \frac{a}{100} \cdot \infty = \infty$$

Ats: $a = \Omega\left(\frac{100}{\ln n}\right)$

2. Suprastinti

| | |
|---|---|
| 3. | $O\left(n\ln^2 n + \sqrt[6]{n^4}\right)$ |

$$\lim_{n\to\infty} \frac{n \ln^2 n}{\sqrt[6]{n^4}} = \lim_{n\to\infty} \frac{n \cdot \ln^2 \cdot n}{n^{\frac{4}{6}}} = \lim_{n\to\infty} \frac{n \cdot \ln^2 \cdot n}{n^{\frac{2}{3}}} =$$

$$= \lim_{n\to\infty} n^{\frac{1}{3}} \ln^2 n = \lim_{n\to\infty} n^{\frac{1}{3}} \cdot \lim_{n\to\infty} \ln^2 n =$$

$=$

$$\lim_{n\to\infty} n^{\frac{1}{3}} = \infty$$

$$= \infty \cdot \infty = \infty$$

$$\lim_{n\to\infty} \ln^2 n = \infty$$

Ats: $O\left(n \ln^2 n + \sqrt[6]{n^4}\right) = O\left(n \ln^2 n\right)$

3. Įvertinti sumą

| | |
|---|---|
| 2. | $\sum_{i=1}^{m} \ln^3 i$ |

$$\int_0^m \ln^3 x\,dx = x\ln^3 x - \int x\,d\ln^3 x = x\ln^3 x - 3\int \frac{x \cdot \ln^2 x}{x}\,dx =$$

$$= x\ln^3 x - 3\int \ln^2 x\,dx = x\ln^3 x - 3x\ln^2 x + 3\int x\,d\ln^2 x =$$

$$= x\ln^3 x - 3x\ln^2 x + 6\int \ln x\,dx = x\ln^3 x - 3x\ln^2 x + 6x\ln x -$$

$$-6\int x\,d\ln x \Big|_0^m =$$

$-6x$

Ats.:

$$x\ln^3 x - 3x\ln^2 x + 6x\ln x - 6\int x\,d\ln x \le \sum_{i=0}^{m+1} \ln^3 i \le$$

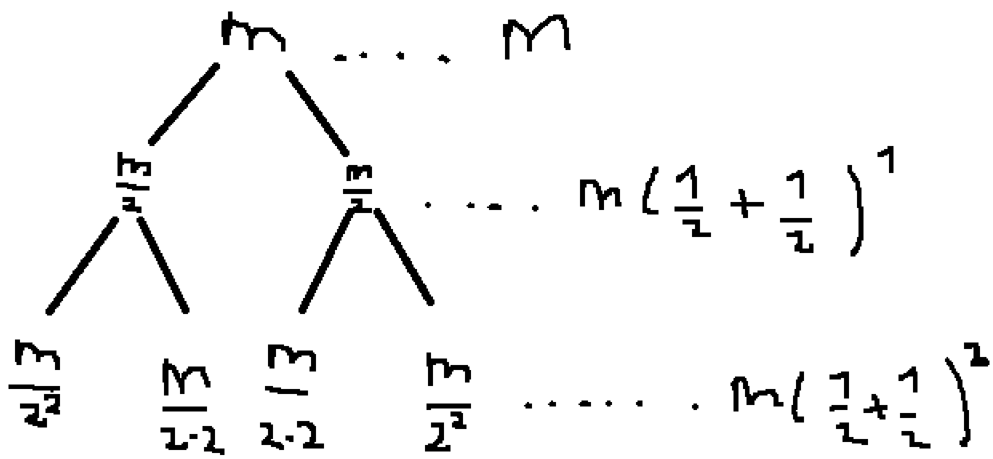
$$\le x\ln^3 x - 3x\ln^2 x + 6x\ln x - 6\int x\,d\ln x$$

4. Surasti lygties $T(n) = aT\ (\ n\ b\ ) + f(n)$ sprendinį, taikant medžio metodą:

| | |
|---|---|
| 0. | $T(n) = T(\sqrt{n}) + T\left(\sqrt{n-1}\right) + \log_6 n$ |

Keitinys: $n = 6^m$

$$S(m) = S\left(\frac{m}{2}\right) + S\left(\frac{m}{2}\right) + m$$




$$S(m) = m \sum_{i=0}^{h} \left(\frac{1}{2} + \frac{1}{2}\right)^i$$

$h = \log_2 m$

$$S(m) < 2S\left(\frac{m}{2}\right) + m$$

$$S(m) \perp 2S\left(\frac{m}{2}\right) + m$$

5. Surasti lygties $T(n) = aT\left(\frac{n}{b}\right) + f(n)$ sprendinį, taikant pagrindinę teoremą:

| | |
|---|---|
| 3. | $T(n) = 3T\left(\frac{n}{3}\right) + \frac{\ln n}{n}$ |

$a = 3$

$b = 3$

$f(n) = \frac{\ln n}{n}$

$$\lim_{n\to\infty} \frac{\ln n}{n^{\log_3 3 + \varepsilon}} = \lim_{n\to\infty} \frac{\ln n}{n^{1+\varepsilon}} =$$

$$= n^{1+\varepsilon} = \infty, \text{ kai } 0 < \varepsilon < 1$$

$$3 \ln \frac{n}{3} < c \ln n$$

$$\ln n < c \ln n$$

$$\Downarrow$$

$$1 < c < 1$$

Ats.: $T(n) = \Theta(n^{\log_3 3})$

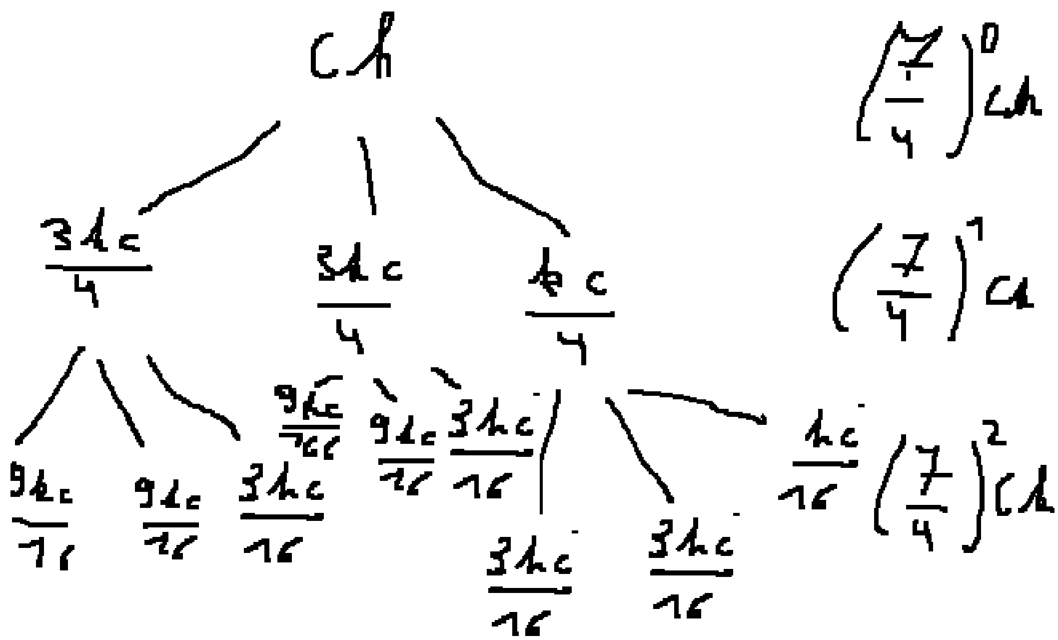
6. Įvertinti sudėtingumą:

2.

```
public void AA(int[] C, int m, int n)
{
        int s = 0;
        int p = BB(n - m + 1);
        if (p < n - m) {
                AA(C, m, n - p);
                AA(C, m, n - p);
                for (int i = m; i <= n; i++)
                {
                C(i) = C(i) + 1;
                }
                AA(C, n - p + 1, n);
                AA(C, n - p + 1, n);
        }
}
public int BB(int k)
{
        int s = 0;
        for (int i = 1; i <= k; i++)
        {
                for (int j = 1; j <= k; j++)
                {
                s = s + 1;
                }
        }
        return Sqrt(s) / 2;
}
```

| 2. | public void AA(int[] C, int m, int n) | kaina | kiekis |
|---|---|---|---|
| | { | | |
| | int s = 0; | c | 1 |
| | int p = BB(n - m + 1); | c | 1 |
| | if (p < n - m) { | c | 1 |
| | AA(C, m, n - p); | AA(m, n - p) | 1 |
| | AA(C, m, n - p); | AA(m, n - p) | 1 |
| | for (int i = m; i <= n; i++) | C | n - m + 2 |
| | { | | |
| | C(i) = C(i) + 1; | c | n - m + 1 |
| | } | | |
| | AA(C, n - p + 1, n); | AA(n - p + 1, n) | 1 |
| | AA(C, n - p + 1, n); | AA(m, n - p) | 1 |
| | } | | |
| | } | | |
| | public int BB(int k) | | |
| | { | | |
| | int s = 0; | c | 1 |
| | for (int i = 1; i <= k; i++) | c | k + 1 |
| | { | | |
| | for (int j = 1; j <= k; j++) | c | k |
| | { | | |
| | s = s + 1; | c | k - 1 |
| | } | | |
| | } | | |
| | return Sqrt(s) / 2; | c | 1 |
| | } | | |

$$T(m,n) = 3C + 3\,TAA(m, n-p) +$$
$$+ C(n-m+1) + C(n-m+2) +$$
$$+ TAA(n-p+1, n) = 3\,TAA(m, n-p) +$$
$$+ TAA(n-p+1, n) + 2C(n-m+1{,}5) + 3C$$
$$T(m,n) = T(k)$$
$$k = n-m+1$$
$$p = \frac{n-m+1}{4} = \frac{k}{4}$$
$$T(k) = 3T(n-m-p+1) + T(p-1+1) +$$
$$+ \underbrace{2C(n-m+1{,}5) + 3C}_{Ck}$$
$$T(k) = 3T\left(\frac{3k}{4}\right) + T\left(\frac{k}{4}\right) + Ck$$




$$\log_4 n \le h \le \log_{\frac{4}{3}} n$$

$$T(n) = cn\sum_{i=0}^{h}\left(\frac{7}{4}\right)^i = \frac{\left(\frac{7}{4}\right)^{h+1}-1}{\frac{7}{4}-1} = \frac{4\left(\frac{7}{4}\right)^{h+1}-4}{3}$$

$$4\left(\frac{7}{3}\right)^{\log_4 n+1}-4 \le T(n) \le \frac{4\left(\frac{7}{4}\right)^{\log_{\frac{4}{3}} n+1}-4}{3}$$

$$T(n) = \Theta(c^{\log n})$$

$$T_{AA}(m,n) = \Theta(c^{\log(n-m)})$$