Third Year B.S. (Honors) 2022-2023

Department of Applied Mathematics, University of Dhaka

Course Title: Math Lab III (Matlab), Course No.: AMTH 350

Assignment 04 (A)

Name: Aminul Islam Manuf **Roll:** SH-123-071 **Group:** B

Use Matlab to solve each of the following problems.

1. For a flow, the velocity potential is $\phi = 2logR$ and stream function is $\psi = 2\theta$. Sketch the streamlines using solid line, and equipotential lines using dashed line, and hence identify the type of flow.
2. If there are sources at (3, 0) and (-3, 0) and sinks at (0, 3) and (0, -3) all of equal strengths 1 then show that the circle through these four points is a streamline. Also indicate the direction of flows in any quadrant.
3. Determine the velocity potential for the velocity field $u = 2(x^2 - y^2)$, $v = -4xy$, $w = 0$, and plot it.
4. For doublet flow, the complex potential is defined as, $F(z) = \frac{\mu}{z}$. Find the stream function for this flow, and hence plot it.

University of Dhaka

Department of Applied Mathematics

Third Year B.S. (Honors), Academic Session: 2022-2023

Course Title: Math Lab III (MATLAB), Course Code: AMTH 350

Assignment No.: 4B

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Instruction: Write an appropriate programming code using **MATLAB** software to get the output of each problem and sketch them properly.

1. A complex function $f(z)=u(x,y)+iv(x,y)$ has a complex derivative $f'(z)$ if and only if its real and imaginary parts are continuously differentiable and satisfy the Cauchy – Riemann equations $u_x = v_y$, $u_y = -v_x$. In this case, the complex derivative of $f(z)$ is equal to any of the following expressions:

$$f'(z)=u_x+iv_x=v_y-iu_y$$

For the function $f(z)=z^2+z^3$, write a MATLAB code to check the differentiability and hence find the differentiation of the function. Also, compare with the differentiation of the function with usual rules.

2. Consider the complex potential $F(z=x+iy)=\phi(x,y)+i\psi(x,y)$ with respect to the velocity field $\vec{V}=\langle \alpha(x^2-y^2), 2\alpha xy\rangle$, where $\alpha>0$. Check whether the stream function and the velocity potential can be determined by the velocity field. If so, plot the streamlines and the velocity potential to interpret the complex potential.

3. Consider the line integral $\oint \frac{e^z}{z}dz$. Now using MATLAB,
   (i). Define the integrand with an anonymous function.
   (ii). Integrate using contour as a unit circle.
   (iii). Integrate along the contours

(a) $C=[2+i \quad 2+2i \quad 1+2i]$ from $(1+i, 1+i)$.

(b) $C=[1+i \quad -1+i \quad -1-i \quad 1-i]$ from $(1,1)$.

Which of these contours enclose singularities? Do the results support the Cauchy's Integral Theorem?

4. Consider the Bessel's differential equation $t^2 \dfrac{d^2y}{dt^2}+t\dfrac{dy}{dt}+\left(t^2-\dfrac{1}{4}\right)y=0$. Now, write a MATLAB code to determine Bessel functions of order $\dfrac{1}{2}$ and sketch it.

5. Find the Laplace transform of the following functions using MATLAB:

(i). $f(t)=t^2 \sinh 7t$.

(ii). $f(t)=5e^{-3t}\sin(t-45^0)$.

(iii). $f(t)=5t^4\cos(3t+60^0)$

6. Determine the inverse Laplace transform of the following functions using MATLAB.

(i). $F(s)=\dfrac{s}{s(s+2)(s+6)}$

(ii). $F(s)=\dfrac{1}{s^2(s+5)}$.

(iii). $F(s)=\dfrac{3s+1}{s^2+2s+9}$.

(iv). $F(s)=\dfrac{s-25}{s(s^2+3s+20)}$.

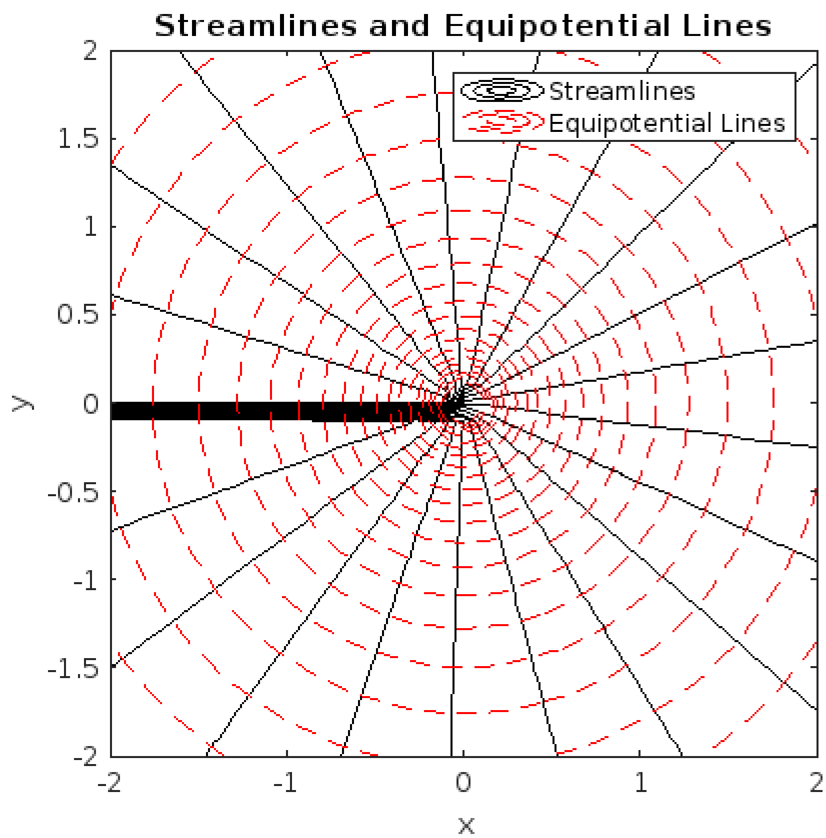
7. Expand the following functions into partial functions using MATLAB. Hence, determine the inverse Laplace transform of each function.

(i). $F(s)=\dfrac{1}{s^4+5s^3+7s^2}$.

(ii). $F(s)=\dfrac{5s^2+3s+6}{s^4+3s^3+7s^2+9s+12}$.

```
% Define grid
[x, y] = meshgrid(-2:0.1:2, -2:0.1:2);
% Calculate velocity potential and stream function
phi = 2*log(sqrt(x.^2 + y.^2));
psi = 2*atan2(y, x);

% Plot streamlines
contour(x, y, psi, 20, 'k'); % solid black lines for streamlines
hold on
% Plot equipotential lines
contour(x, y, phi, 20, '--r'); % dashed red lines for equipotential lines
xlabel('x');
ylabel('y');
title('Streamlines and Equipotential Lines');
legend('Streamlines', 'Equipotential Lines');
axis equal;
```



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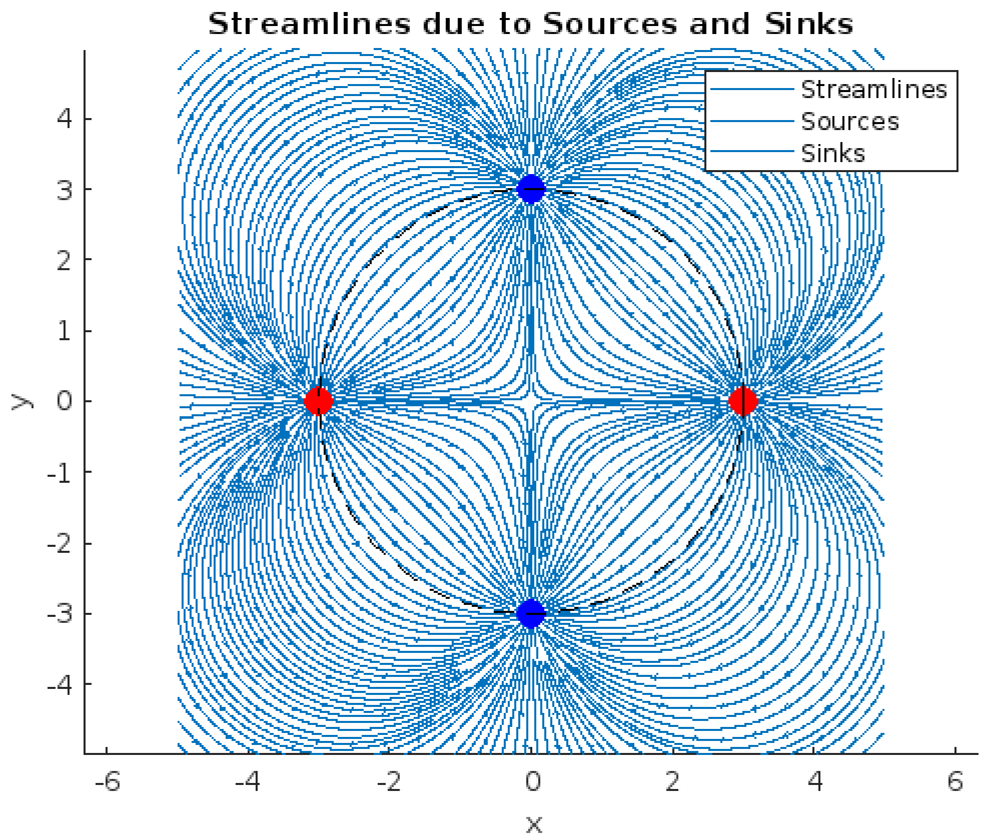
```
% Define the strengths of the sources and sinks
strength = 1;

% Define the positions of the sources and sinks
sources = [3, 0; -3, 0];
sinks = [0, 3; 0, -3];

% Define the grid
[x, y] = meshgrid(-5:0.1:5, -5:0.1:5);

% Initialize velocity components
u = zeros(size(x));
v = zeros(size(y));

% Calculate the velocity components due to the sources
for i = 1:size(sources, 1)
    u = u + strength ./ (2 * pi) .* (x - sources(i, 1)) ./ ((x - sources(i,
1)).^2 + (y - sources(i, 2)).^2);
    v = v + strength ./ (2 * pi) .* (y - sources(i, 2)) ./ ((x - sources(i,
1)).^2 + (y - sources(i, 2)).^2);
end

% Calculate the velocity components due to the sinks
for i = 1:size(sinks, 1)
    u = u - strength ./ (2 * pi) .* (x - sinks(i, 1)) ./ ((x - sinks(i,
1)).^2 + (y - sinks(i, 2)).^2);
    v = v - strength ./ (2 * pi) .* (y - sinks(i, 2)) ./ ((x - sinks(i,
1)).^2 + (y - sinks(i, 2)).^2);
end

% Plot the streamlines
figure;
streamslice(x, y, u, v, 10);
hold on;

theta = linspace(0, 2*pi, 100);
x_circle = 3 * cos(theta);
y_circle = 3 * sin(theta);

% Plot the sources and sinks
plot(sources(:, 1), sources(:, 2), 'ro', 'MarkerSize', 10, 'MarkerFaceColor',
'r');
plot(sinks(:, 1), sinks(:, 2), 'bo', 'MarkerSize', 10, 'MarkerFaceColor',
'b');
plot(x_circle, y_circle, 'k--', 'MarkerSize', 20, 'MarkerFaceColor', 'y');

axis equal;
title('Streamlines due to Sources and Sinks');
xlabel('x');
ylabel('y');
legend('Streamlines', 'Sources', 'Sinks');
```



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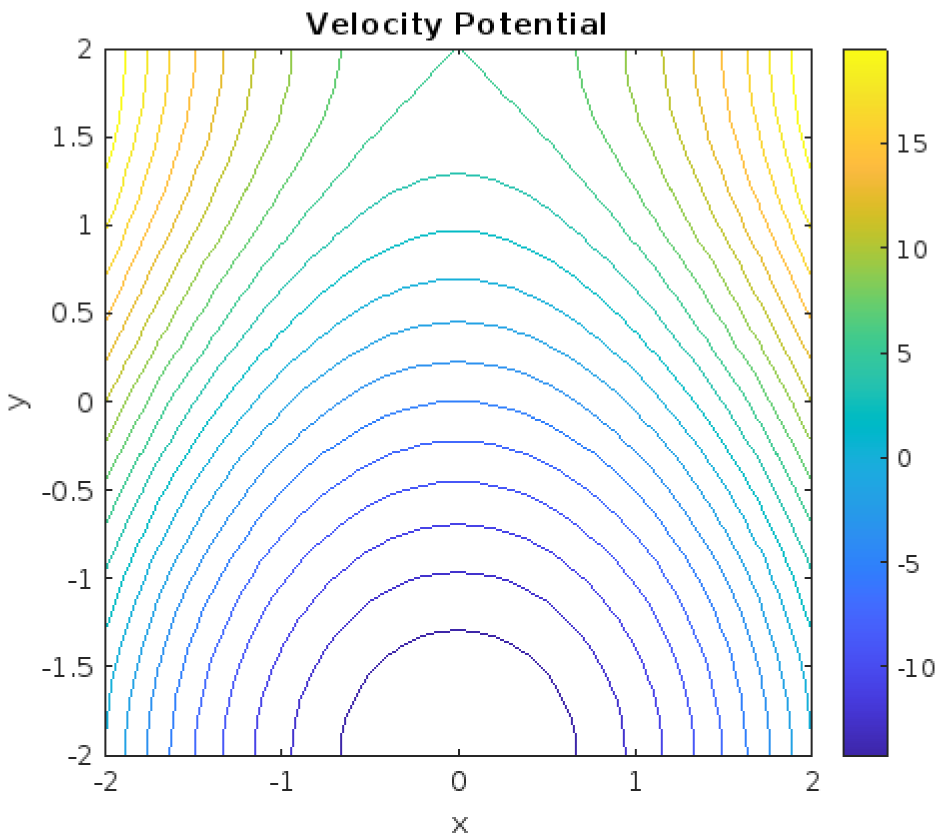
```
% Define grid
[x, y] = meshgrid(-2:0.1:2, -2:0.1:2);

% Calculate velocity components
u = 2 * (x.^2 - y.^2);
v = -4 * x .* y;

% Integrate to find velocity potential
phi = cumtrapz(x(1,:), u, 1) + cumtrapz(y(:,1), v, 2);

% Plot velocity potential
contour(x, y, phi, 20);
xlabel('x');
ylabel('y');
title('Velocity Potential');
colorbar;
axis equal;
```



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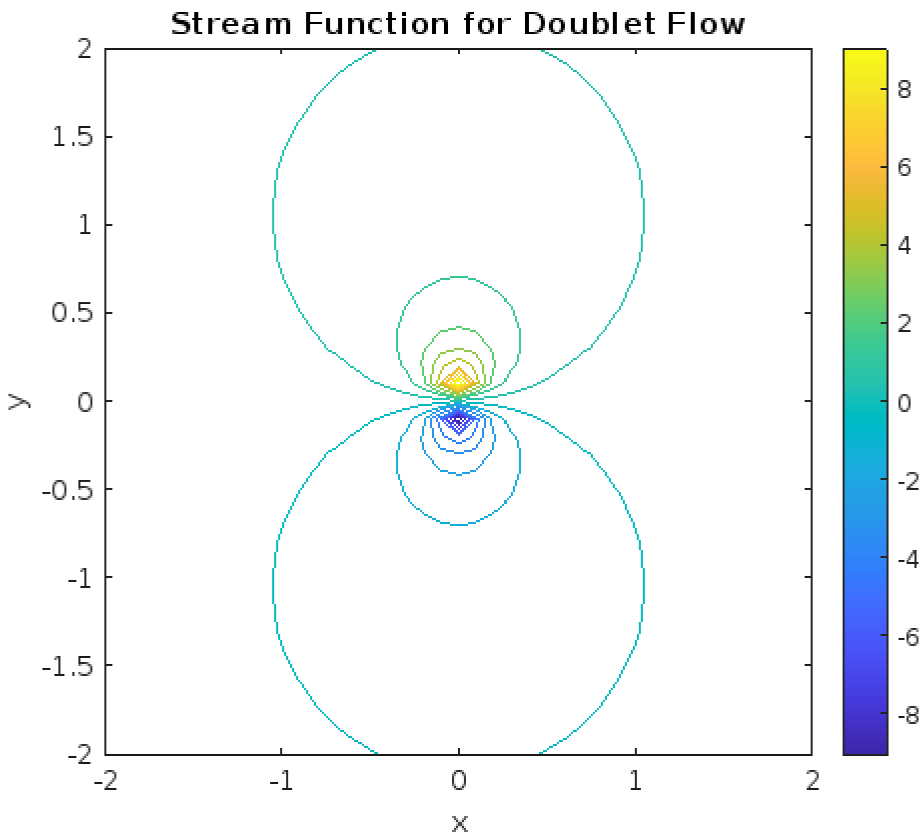
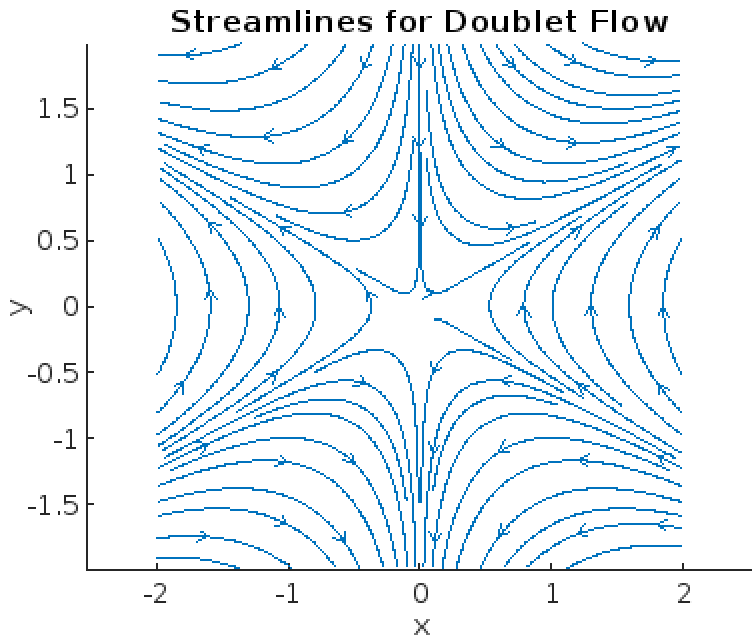
```
% Define grid
[x, y] = meshgrid(-2:0.1:2, -2:0.1:2);

% Given strength of the doublet
mu = 1;

% Calculate stream function
psi = -imag(mu./(x + 1i*y));

% Calculate velocity components from stream function
u = -imag(mu./(x + 1i*y).^2);
v = real(mu./(x + 1i*y).^2);

% Plot stream function contour
contour(x, y, psi, 20);
xlabel('x');
ylabel('y');
title('Stream Function for Doublet Flow');
colorbar;
axis equal;

% Additional visualization with streamslice
figure;
streamslice(x, y, u, v);
xlabel('x');
ylabel('y');
title('Streamlines for Doublet Flow');
axis equal;
```





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```
%A4bq1
clc;
syms x y real
z=x+1i*y;
f=z^2+z^3;
u=real(f)
v=imag(f)
dux = diff(u, x);
duy = diff(u, y);
dvx = diff(v, x);
dvy = diff(v, y);
% Check if the Cauchy-Riemann equations hold
if (dux - dvy) == 0 && (duy + dvx) == 0
 disp('The function is differentiable point.');
else
 disp('The function is not differentiable.');
end
df1= dux + 1i*dvx;
df2= dvy -1i*duy;
if df1==df2
 disp('verified')
else
 disp('failed')
end
```

```
u =

x*(x^2 - y^2) - 2*x*y^2 + x^2 - y^2


v =

y*(x^2 - y^2) + 2*x*y + 2*x^2*y

The function is differentiable point.
verified
```

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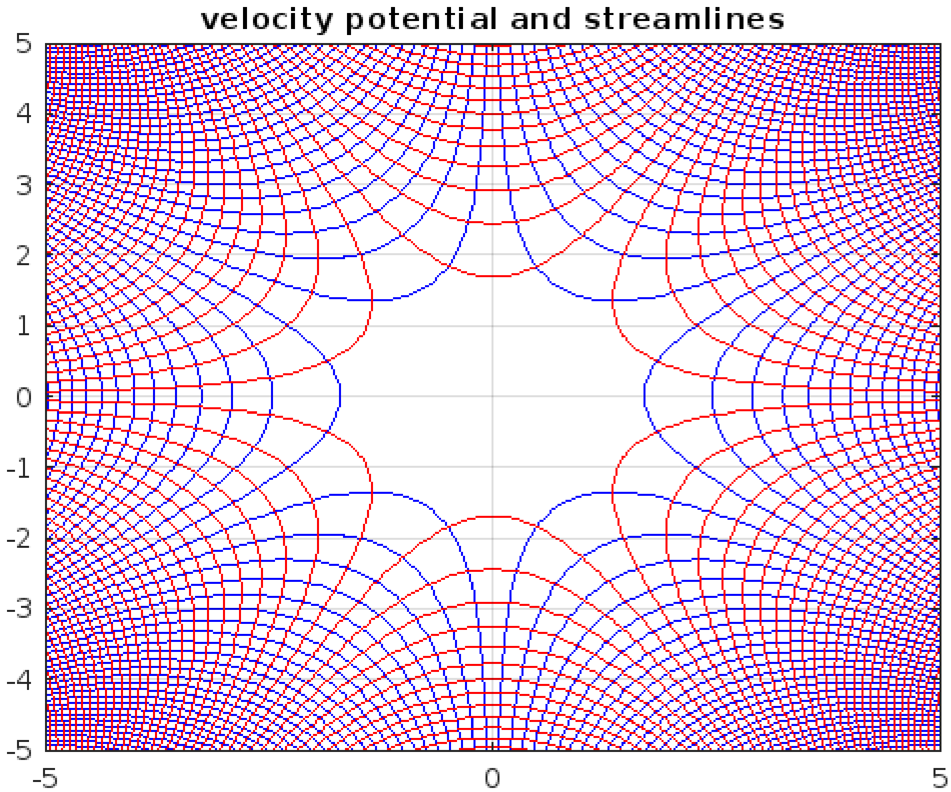
```
%A4bq2
clc,clear all;
a=2;
syms x y phi shi
u=a*(x^2-y^2);
v=-2*a*x*y;
phi(x,y)=int(u,x);
shi(x,y)=int(u,y);
[x,y]=meshgrid(linspace(-5,5,100));
% Plot the streamline
contour(x, y,phi(x,y),50,'b');
title('Streamlines');
xlabel('x');
ylabel('y');
% Plot the velocity potential
contour(x, y, shi(x,y), 50,'r');
title('Velocity Potential');
xlabel('x');
ylabel('y');
contour(x, y,phi(x,y),50,'b');
hold on
contour(x, y, shi(x,y), 50,'r');
hold off
title('velocity potential and streamlines')
grid on
```



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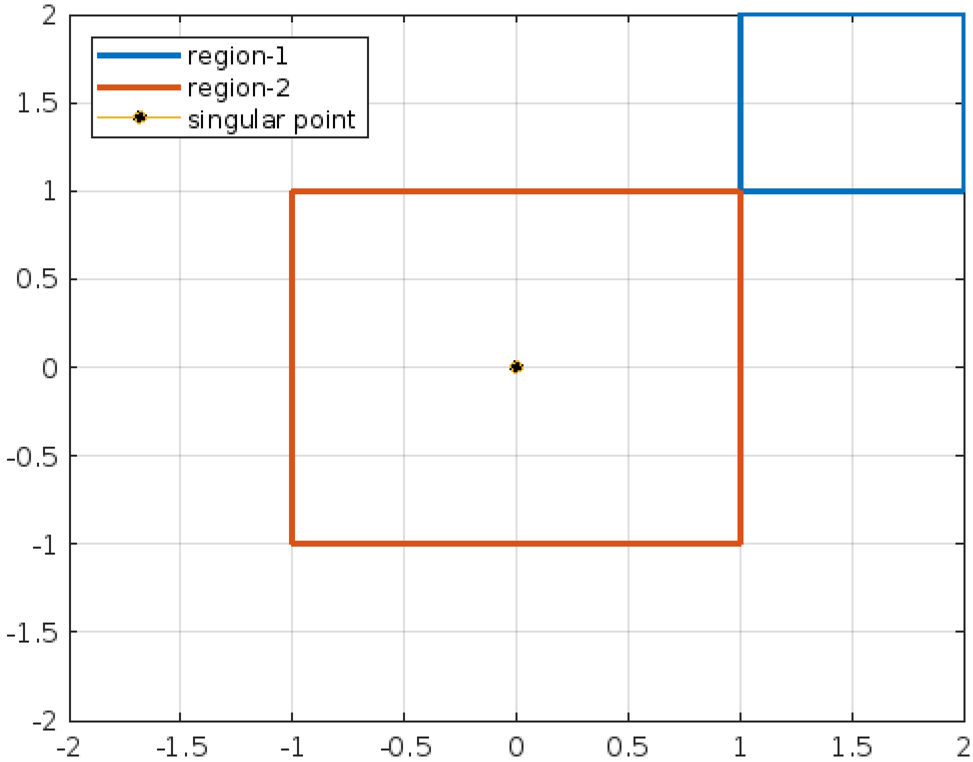
```
%A4bq3
clc,clear all
syms t real
z=exp(1i*t);
f=exp(z)/z;
dz= diff(z,t);
integrand= f*dz;
along_c=int(@(t) integrand,t,0,2*pi);
fprintf('The result along the unit circle is %fi',along_c/1i)
integrand=matlabFunction(integrand);
c1=integral(@(p) exp(p)./p,1+1i,1+1i,'waypoints',[2+1i,2+2i,1+2i]);
c2=integral(@(p) exp(p)./p,1,1,'waypoints',[1+1i,-1+1i,-1-1i,1-1i]);
fprintf('The integral along contour 1 is %f + %fi',real(c1),imag(c1))
fprintf('The integral along contour 1 is %f + %fi',real(c2),imag(c2))
%plot the region
plot([1 2 2 1 1],[1 1 2 2 1],[1 1 -1 -1 1 1],[1 1 1 -1 -1 1],linewidth=2)
hold on
plot([0],[0],'Marker','o','MarkerSize',5,'MarkerFaceColor','black')
hold off
legend('region-1','region-2','singular point','Location','northwest')
grid on
xlim([-2 2]);ylim([-2 2])
```

The result along the unit circle is 6.283185iThe integral along contour 1 is 0.000000 + -0.000000iThe integral along contour 1 is -0.000000 + 6.283185i



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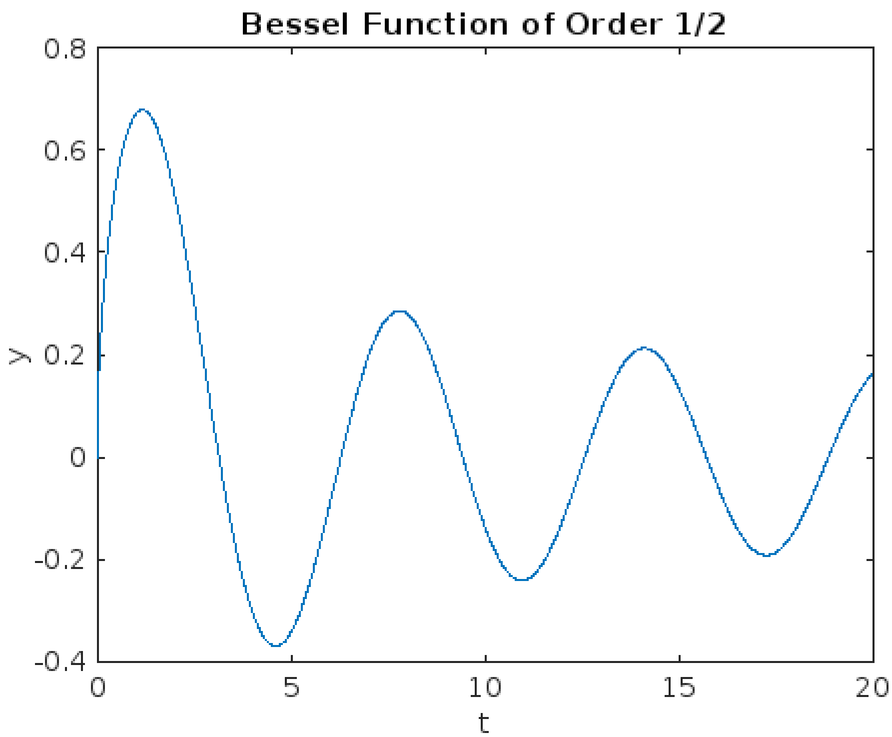
```
%A4bq4
clc,clear all
n = 0.5;
t = 0:0.01:20;
y = besselj(n, t);
figure;
plot(t, y);
title('Bessel Function of Order 1/2');
xlabel('t');
ylabel('y');
```



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```
%A4bq5
clc,clear all
syms t
f1=t^2*sinh(7*t);
f2=5*exp(-3*t)*sind(t-45);
f3=5*t^4*cosd(3*t+ 60 );
%Taking laplace transform
F1=laplace(f1);
F2=laplace(f2);
F3=laplace(f3);
fprintf('The laplace transfrom of f1 : %s\n',F1)
fprintf('The laplace transfrom of f2 : %s\n',F2)
fprintf('The laplace transfrom of f3 : %s\n',F3)
```

```
The laplace transfrom of f1 : (56*s^2)/(s^2 - 49)^3 - 14/(s^2 - 49)^2
The laplace transfrom of f2 : (2^(1/2)*pi)/(72*((s + 3)^2 + pi^2/32400)) -
(5*2^(1/2)*(s + 3))/(2*((s + 3)^2 + pi^2/32400))
The laplace transfrom of f3 : (300*s)/(pi^2/3600 + s^2)^3 -
(5*3^(1/2)*((2*pi)/(5*(pi^2/3600 + s^2)^3) - (24*s^2*pi)/(5*(pi^2/3600 +
s^2)^4) + (32*s^4*pi)/(5*(pi^2/3600 + s^2)^5)))/2 - (1200*s^3)/(pi^2/3600 +
s^2)^4 + (960*s^5)/(pi^2/3600 + s^2)^5
```

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```
%A4bq6
clc,clear all
syms s
F1=s/(s*(s+2)*(s+6));
F2=1/(s.^2*(s+5));
F3=(3*s+1)/(s.^2+2*s+9);
%Taking inverse laplace transform
f1=ilaplace(F1)
```

f1 =

$$\frac{e^{-2t}}{4} - \frac{e^{-6t}}{4}$$

```
f2=ilaplace(F2)
```

f2 =

$$\frac{t}{5} + \frac{e^{-5t}}{25} - \frac{1}{25}$$

```
f3=ilaplace(F3)
```

f3 =

$$3\,e^{-t}\left(\cos\left(2\,\sqrt{2}\,t\right) - \frac{\sqrt{2}\,\sin\left(2\,\sqrt{2}\,t\right)}{6}\right)$$

```
fprintf('F1 : %s',f1)
```

```
F1 : exp(-2*t)/4 - exp(-6*t)/4
```

```
fprintf('F2 : %s',f2)
```

```
F2 : t/5 + exp(-5*t)/25 - 1/25
```

```
fprintf('F3 : %s',f3)
```

```
F3 : 3*exp(-t)*(cos(2*2^(1/2)*t) - (2^(1/2)*sin(2*2^(1/2)*t))/6)
```

```
clc,clear all
syms s
F1 = 1/(s^4+5*s^3+7*s^2);
F2= (5*s^2+3*s+6)/(s^4+3*s^3+7*s^2+9*s+12);
%partial decompostion
F1=partfrac(F1);
F2=partfrac(F2);
%inverse laplace transform
f1=ilaplace(F1)
```

f1 =

$$\frac{t}{7}+\frac{5\,\mathrm{e}^{-\frac{5\,t}{2}}\left(\cos\left(\frac{\sqrt{3}\,t}{2}\right)+\frac{11\,\sqrt{3}\,\sin\left(\frac{\sqrt{3}\,t}{2}\right)}{15}\right)}{49}-\frac{5}{49}$$

```
f2=ilaplace(F2)
```

f2 =

$$\frac{15\cos\left(\sqrt{3}\,t\right)}{14}+\frac{3\,\sqrt{3}\,\sin\left(\sqrt{3}\,t\right)}{14}-\frac{15\,\mathrm{e}^{-\frac{3\,t}{2}}\left(\cos\left(\frac{\sqrt{7}\,t}{2}\right)-\frac{11\,\sqrt{7}\,\sin\left(\frac{\sqrt{7}\,t}{2}\right)}{15}\right)}{14}$$