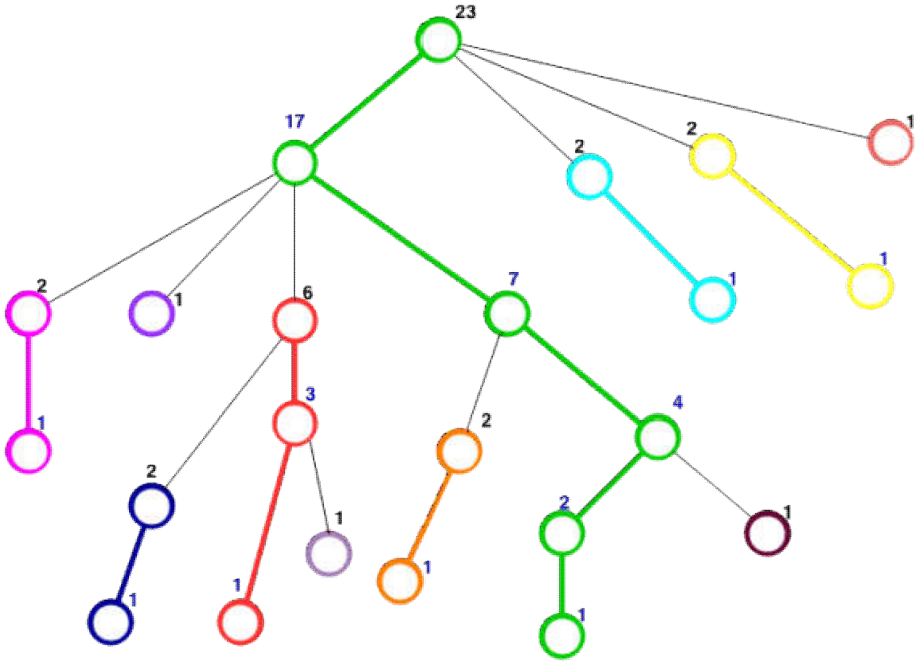
Template [New]:

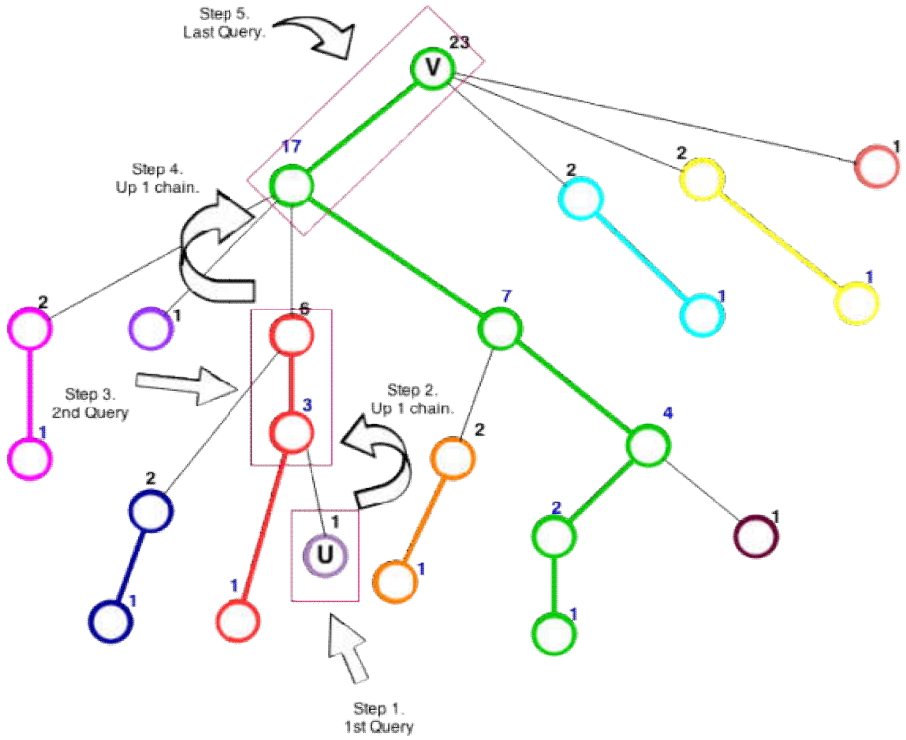
```
/**
 *    @author     : Maruf Tuhin
 *    @College    : CUET CSE 11
 *    @Topcoder   : the_redback
 *    @CodeForces : the_redback
 *    @UVA        : the_redback
 *    @link       : http://www.fb.com/maruf.2hin
 */

#include <bits/stdc++.h>
using namespace std;

typedef long long          ll;
typedef unsigned long long llu;

#define ft         first
#define sd         second
#define mp         make_pair
#define pb(x)      push_back(x)
#define all(x)     x.begin(),x.end()
#define allr(x)    x.rbegin(),x.rend()
#define mem(a,b)   memset(a,b,sizeof(a))
#define repv(i,a)  for(i=0;i<(ll)a.size();i++)
#define revv(i,a)  for(i=(ll)a.size()-1;i>=0;i--)
#define rep(i,a,b) for(i=a;i<=b;i++)
#define rev(i,a,b) for(i=a;i>=b;i--)
#define sf(a)      scanf("%lld",&a)
#define sf2(a,b)   scanf("%lld %lld",&a,&b)
#define sf3(a,b,c) scanf("%lld %lld %lld",&a,&b,&c)
#define inf        1e9
#define eps        1e-9
#define mod        1000000007
#define NN         100010

#ifdef  redback
#define bug printf("line=%d\n",__LINE__);
#define debug(args...) {cout<<":: "; dbg,args; cerr<<endl;}
struct  debugger{template<typename T>debugger& operator ,(const T& v){cerr<<v<<"
";return *this;}}dbg;
#else
#define bug
#define debug(args...)
#endif  //debugging macros



int main()
{
    //ios_base::sync_with_stdio(0); cin.tie(0);
    #ifdef redback
        freopen("C:\\Users\\Maruf\\Desktop\\in.txt","r",stdin);
    #endif

    ll t=1,tc;
    //sf(tc);
    ll l,m,n;
    while(~sf(n)) {
        ll i,j,k;
    }
    return 0;
}
```

HLD [Initialization]:



Each Chain is represented with different color.
Thin Black lines represent the connecting edges. They connect 2 chains.

HLD [Query]:



Consider the path from U to V.
Step 1 : Query for chain 1 in image.
Step 2 : Move up to chain 2.
Step 3 : Query for chain 2 in image. Update Answer.
Step 4 : Move up to chain 3.
Step 5 : Query for chain 3 in image. Update answer.

HLD- with comments [Loj-1348]:

```
/**
Problem Description:
you are given a tree (a connected graph with no cycles) with n nodes,
nodes represent places, edges represent roads. In each node,
initially there are an arbitrary number of genies.
But the numbers of genies change in time.
So, you are given a tree, the number of genies in each node and several queries of two
types. They are:

1)      0 i j, it means that you have to find the total number of genies in the nodes
that occur in path from node i to j (0 <= i, j < n).
2)      1 i v, it means that number of genies in node i is changed to v (0 <= i < n, 0
<= v <= 1000).
*/

#include <bits/stdc++.h>

using namespace std;
typedef long long          ll;

#define NN        50010
#define read(a)   scanf("%lld",&a)
#define root 0
#define LN 16

vector <ll> adj[NN];
ll baseArray[NN], ptr, value[NN];
ll chainNo, chainInd[NN], chainHead[NN], posInBase[NN];
ll depth[NN], par[NN][LN], subsize[NN];
ll seg[NN*4];

/*
 * make_tree:
 * Used to construct the segment tree. It uses the baseArray for construction
 */
void make_tree(ll node, ll low, ll high)
{
    if(low == high)
    {
        seg[node] = baseArray[low];
        return;
    }
    ll left = node<<1;
    ll right = left | 1;
    ll mid = (low + high)>>1;

    make_tree(left, low, mid);
    make_tree(right, mid+1, high);
    seg[node]=seg[left]+seg[right];
    return;
}
```

```
/*
 * update_tree:
 * Point update. Update a single element of the segment tree.
 */
void update_tree(ll node, ll low, ll high, ll ind, ll val)
{

    if(low == ind && low == high)
    {
        seg[node] = val;
        return;
    }
    ll left = node<<1;
    ll right = left | 1;
    ll mid = (low + high)>>1;

    if(ind<=mid)
        update_tree(left, low, mid, ind, val);
    else
        update_tree(right, mid + 1, high, ind, val);

    seg[node]=seg[left]+seg[right];
    return ;
}

/*
 * query_tree:
 * Given S and E, it will return the maximum value in the range [S,E)
 */
ll query_tree(ll node, ll low, ll high, ll rlow, ll rhigh)
{

    if(low>= rlow && high <= rhigh)
    {
        return seg[node];
    }
    ll left = node<<1;
    ll right = left | 1;
    ll mid = (low + high)>>1;


    if(rhigh<=mid)
        return query_tree(left, low, mid, rlow, rhigh);
    else if(rlow>mid)
        return query_tree(right, mid + 1, high, rlow, rhigh);
    else
    {
        ll L = query_tree(left, low, mid, rlow, mid);
        ll R = query_tree(right, mid + 1, high, mid + 1, rhigh);
        return L+R;
    }
}

/*
 * query_up:
 * It takes two nodes u and v, condition is that v is an ancestor of u
 * We query the chain in which u is present till chain head, then move to next chain up
 * We do that way till u and v are in the same chain, we query for that part of chain
and break
 */
```

```
ll query_up(ll u, ll v)
{
    ll uchain, vchain = chainInd[v], ans = 0;
    // uchain and vchain are chain numbers of u and v
    while(1)
    {
        uchain = chainInd[u];
        if(uchain == vchain)
        {
            // Both u and v are in the same chain, so we need to query from u to v,
update answer and break.
            // We break because we came from u up till v, we are done
            //if(u==v) break;
            ans+=query_tree(1, 1, ptr-1, posInBase[v], posInBase[u]);
            // Above is call to segment tree query function
            break;
        }
        ans+=query_tree(1, 1, ptr-1, posInBase[chainHead[uchain]], posInBase[u]);
        // Above is call to segment tree query function. We do from chainHead of u
till u. That is the whole chain from
        // start till head. We then update the answer
        u = chainHead[uchain]; // move u to u's chainHead
        u = par[u][0]; //Then move to its parent, that means we changed chains
    }
    return ans;
}

/*
 * LCA:
 * Takes two nodes u, v and returns Lowest Common Ancestor of u, v
 */
ll LCA(ll u, ll v)
{
    if(depth[u] < depth[v])
        swap(u,v);
    ll diff = depth[u] - depth[v];
    for(ll i=0; i<LN; i++)
        if( (diff>>i)&1 )
            u = par[u][i];
    if(u == v)
        return u;
    for(ll i=LN-1; i>=0; i--)
        if(par[u][i] != par[v][i])
        {
            u = par[u][i];
            v = par[v][i];
        }
    return par[u][0];
}


ll query(ll u, ll v)
{
    /*
     * We have a query from u to v, we break it into two queries, u to LCA(u,v) and
LCA(u,v) to v
     */
    ll lca = LCA(u, v);
    ll ans = query_up(u, lca); // One part of path
    ll ans2 = query_up(v, lca); // another part of path
    return ans+ans2-query_up(lca,lca); // take the maximum of both paths
}
```

```
/*
 * change:
 * We just need to find its position in segment tree and update it
 */
void change(ll u, ll val)
{
    //ll u = otherEnd[i];
    update_tree(1, 1, ptr-1, posInBase[u], val);
}

/*
 * Actual HL-Decomposition part
 * Initially all entries of chainHead[] are set to -1.
 * So when ever a new chain is started, chain head is correctly assigned.
 * As we add a new node to chain, we will note its position in the baseArray.
 * In the first for loop we find the child node which has maximum sub-tree size.
 * The following if condition is failed for leaf nodes.
 * When the if condition passes, we expand the chain to special child.
 * In the second for loop we recursively call the function on all normal nodes.
 * chainNo++ ensures that we are creating a new chain for each normal child.
 */
void HLD(ll curNode, ll prev)
{
    if(chainHead[chainNo] == -1)
    {
        chainHead[chainNo] = curNode; // Assign chain head
    }
    chainInd[curNode] = chainNo;
    posInBase[curNode] = ptr; // Position of this node in baseArray which we will use in
Segtree
    baseArray[ptr++] = value[curNode];

    ll sc = -1, ncost;
    // Loop to find special child
    for(ll i=0; i<adj[curNode].size(); i++)
        if(adj[curNode][i] != prev)
        {
            if(sc == -1 || subsize[sc] < subsize[adj[curNode][i]])
            {
                sc = adj[curNode][i];
            }
        }

    if(sc != -1)
    {
        // Expand the chain
        HLD(sc, curNode);
    }

    for(ll i=0; i<adj[curNode].size(); i++)
        if(adj[curNode][i] != prev)
        {
            if(sc != adj[curNode][i])
            {
                // New chains at each normal node
                chainNo++;
                HLD(adj[curNode][i], curNode);
            }
        }
}
```

```
/*
 * dfs used to set parent of a node, depth of a node, subtree size of a node
 */

void dfs(ll cur, ll prev, ll _depth=0)
{
    par[cur][0] = prev;
    depth[cur] = _depth;
    subsize[cur] = 1;
    for(ll i=0; i<adj[cur].size(); i++)
        if(adj[cur][i] != prev)
        {
            dfs(adj[cur][i], cur, _depth+1);
            subsize[cur] += subsize[adj[cur][i]];
        }
}

int main()
{
    ll tc,t=1;
    scanf("%lld ", &tc);
    while(tc--)
    {
        ptr = 1;
        ll n;
        scanf("%lld", &n);
        // Cleaning step, new test case
        for(ll i=0; i<=n; i++)
        {
            adj[i].clear();
            chainHead[i] = -1;
            for(ll j=0; j<LN; j++) par[i][j] = -1;
        }

        for(ll i=0; i<n; i++)
        {
            read(value[i]);
        }

        for(ll i=1; i<n; i++)
        {
            ll u, v, c;
            scanf("%lld %lld", &u, &v);
            adj[u].push_back(v);
            adj[v].push_back(u);
        }

        chainNo = 0;
        dfs(root, -1); // We set up subsize, depth and parent for each node
        HLD(root, -1); // We decomposed the tree and created baseArray
        make_tree(1, 1, ptr-1); // We use baseArray and construct the needed segment tree

        // Below Dynamic programming code is for LCA.
        for(ll lev = 1; lev <= LN-1; lev++)
        {
            for(ll i = 0; i < n; i++)
            {
                if(par[i][lev - 1] != -1)
                    par[i][lev] = par[par[i][lev - 1]][lev - 1];
            }
        }
```

```
        ll q;
        scanf("%lld",&q);
        printf("Case %lld:\n",t++);

        while(q--)
        {
            ll tp;
            scanf("%lld", &tp);

            ll a, b;

            scanf("%lld %lld", &a, &b);
            if(tp==0)
            {
                ll ans=query(a, b);
                printf("%lld\n",ans);
            }
            else
            {
                change(a, b);
            }
        }
    }
    return 0;
}

/*

Sample Input
1

4
10 20 30 40
0 1
1 2
1 3
3
0 2 3
1 1 100
0 2 3


Output for Sample Input
Case 1:
90
170
*/
```

HLD- without comments [Loj-1348]:

```
#include <bits/stdc++.h>

using namespace std;
typedef long long          ll;

#define NN        50010
#define read(a)   scanf("%lld",&a)
#define root 0
#define LN 16

vector <ll> adj[NN];
ll baseArray[NN], ptr, value[NN];
ll chainNo, chainInd[NN], chainHead[NN], posInBase[NN];
ll depth[NN], par[NN][LN], subsize[NN];
ll seg[NN*4];

void make_tree(ll node, ll low, ll high)
{
    if(low == high)
    {
        seg[node] = baseArray[low];
        return;
    }
    ll left = node<<1;
    ll right = left | 1;
    ll mid = (low + high)>>1;

    make_tree(left, low, mid);
    make_tree(right, mid+1, high);
    seg[node]=seg[left]+seg[right];
    return;
}

void update_tree(ll node, ll low, ll high, ll ind, ll val)
{

    if(low == ind && low == high)
    {
        seg[node] = val;
        return;
    }
    ll left = node<<1;
    ll right = left | 1;
    ll mid = (low + high)>>1;

    if(ind<=mid)
        update_tree(left, low, mid, ind, val);
    else
        update_tree(right, mid + 1, high, ind, val);

    seg[node]=seg[left]+seg[right];
    return ;
}
```

```
ll query_tree(ll node, ll low, ll high, ll rlow, ll rhigh){

    if(low>= rlow && high <= rhigh)
    {
        return seg[node];
    }
    ll left = node<<1;
    ll right = left | 1;
    ll mid = (low + high)>>1;

    if(rhigh<=mid)
        return query_tree(left, low, mid, rlow, rhigh);
    else if(rlow>mid)
        return query_tree(right, mid + 1, high, rlow, rhigh);
    else
    {
        ll L = query_tree(left, low, mid, rlow, mid);
        ll R = query_tree(right, mid + 1, high, mid + 1, rhigh);
        return L+R;
    }
}

ll query_up(ll u, ll v) //v is an ancestor of u
{
    ll uchain, vchain = chainInd[v], ans = 0;
    // uchain and vchain are chain numbers of u and v
    while(1)
    {
        uchain = chainInd[u];
        if(uchain == vchain)
        {
            ans+=query_tree(1, 1, ptr-1, posInBase[v], posInBase[u]);
            break;
        }
        ans+=query_tree(1, 1, ptr-1, posInBase[chainHead[uchain]], posInBase[u]);
        u = chainHead[uchain]; // move u to u's chainHead
        u = par[u][0]; //Then move to its parent, that means we changed chains
    }
    return ans;
}

ll LCA(ll u, ll v)
{
    if(depth[u] < depth[v])
        swap(u,v);
    ll diff = depth[u] - depth[v];
    for(ll i=0; i<LN; i++)
        if( (diff>>i)&1 )
            u = par[u][i];
    if(u == v)
        return u;
    for(ll i=LN-1; i>=0; i--)
        if(par[u][i] != par[v][i])
        {
            u = par[u][i];
            v = par[v][i];
        }
    return par[u][0];
}
```

```
ll query(ll u, ll v){
    ll lca = LCA(u, v);
    ll ans = query_up(u, lca); // One part of path
    ll ans2 = query_up(v, lca); // another part of path
    return ans+ans2-query_up(lca,lca); // take the maximum of both paths
}

void change(ll u, ll val){
    update_tree(1, 1, ptr-1, posInBase[u], val);
}


void HLD(ll curNode, ll prev)
{
    if(chainHead[chainNo] == -1)
    {
        chainHead[chainNo] = curNode; // Assign chain head
    }
    chainInd[curNode] = chainNo;
    posInBase[curNode] = ptr;
    baseArray[ptr++] = value[curNode];

    ll sc = -1, ncost;
    // Loop to find special child
    for(ll i=0; i<adj[curNode].size(); i++)
        if(adj[curNode][i] != prev)
        {
            if(sc == -1 || subsize[sc] < subsize[adj[curNode][i]])
            {
                sc = adj[curNode][i];
            }
        }

    if(sc != -1)
    {
        HLD(sc, curNode); // Expand the chain
    }

    for(ll i=0; i<adj[curNode].size(); i++)
        if(adj[curNode][i] != prev)
        {
            if(sc != adj[curNode][i])
            {
                // New chains at each normal node
                chainNo++;
                HLD(adj[curNode][i], curNode);
            }
        }
}

void dfs(ll cur, ll prev, ll _depth=0)
{
    par[cur][0] = prev;
    depth[cur] = _depth;
    subsize[cur] = 1;
    for(ll i=0; i<adj[cur].size(); i++)
        if(adj[cur][i] != prev)
        {
            dfs(adj[cur][i], cur, _depth+1);
            subsize[cur] += subsize[adj[cur][i]];
        }
}
```

```
int main()  {
    ll tc,t=1;
    scanf("%lld ", &tc);
    while(tc--)  {
        ptr = 1;
        ll n;
        scanf("%lld", &n);
        for(ll i=0; i<=n; i++) {
            adj[i].clear();
            chainHead[i] = -1;
            for(ll j=0; j<LN; j++) par[i][j] = -1;
        }

        for(ll i=0; i<n; i++) {
            read(value[i]);
        }

        for(ll i=1; i<n; i++) {
            ll u, v, c;
            scanf("%lld %lld", &u, &v);
            adj[u].push_back(v);
            adj[v].push_back(u);
        }

        chainNo = 0;
        dfs(root, -1); // We set up subsize, depth and parent for each node
        HLD(root, -1); // We decomposed the tree and created baseArray
        make_tree(1, 1, ptr-1); //We use baseArray and construct the needed segment tree

        // Below Dynamic programming code is for LCA.
        for(ll lev = 1; lev <= LN-1; lev++) {
            for(ll i = 0; i < n; i++)
            {
                if(par[i][lev - 1] != -1)
                    par[i][lev] = par[par[i][lev - 1]][lev - 1];
            }
        }

        ll q;
        scanf("%lld",&q);
        printf("Case %lld:\n",t++);
        while(q--) {
            ll tp;
            scanf("%lld", &tp);

            ll a, b;

            scanf("%lld %lld", &a, &b);
            if(tp==0)
            {
                ll ans=query(a, b);
                printf("%lld\n",ans);
            }
            else
            {
                change(a, b);
            }
        }
    }
    return 0;
}
```

HLD [Anudeep] Spoj-QTREE:

```
/*
You are given a tree (an acyclic undirected connected graph) with N nodes,
and edges numbered 1, 2, 3...N-1.
We will ask you to perfrom some instructions of the following form:

1.CHANGE i ti : change the cost of the i-th edge to ti
2.QUERY a b : ask for the maximum edge cost on the path from node a to node b
*/

#include <cstdio>
#include <vector>
using namespace std;

#define root 0
#define N 10100
#define LN 14

vector <int> adj[N], costs[N], indexx[N];
int baseArray[N], ptr;
int chainNo, chainInd[N], chainHead[N], posInBase[N];
int depth[N], pa[LN][N], otherEnd[N], subsize[N];
int st[N*6], qt[N*6];

void make_tree(int cur, int s, int e) {
      if(s == e-1) {
            st[cur] = baseArray[s];
            return;
      }
      int c1 = (cur<<1), c2 = c1 | 1, m = (s+e)>>1;
      make_tree(c1, s, m);
      make_tree(c2, m, e);
      st[cur] = st[c1] > st[c2] ? st[c1] : st[c2];
}

void update_tree(int cur, int s, int e, int x, int val) {
      if(s > x || e <= x) return;
      if(s == x && s == e-1) {
            st[cur] = val;
            return;
      }
      int c1 = (cur<<1), c2 = c1 | 1, m = (s+e)>>1;
      update_tree(c1, s, m, x, val);
      update_tree(c2, m, e, x, val);
      st[cur] = st[c1] > st[c2] ? st[c1] : st[c2];
}

void query_tree(int cur, int s, int e, int S, int E) {
      if(s >= E || e <= S) {
            qt[cur] = -1;
            return;
      }
      if(s >= S && e <= E) {
            qt[cur] = st[cur];
            return;
      }
      int c1 = (cur<<1), c2 = c1 | 1, m = (s+e)>>1;
      query_tree(c1, s, m, S, E);
      query_tree(c2, m, e, S, E);
      qt[cur] = qt[c1] > qt[c2] ? qt[c1] : qt[c2];
}
```

```
int query_up(int u, int v)
 {
      if(u == v) return 0; // Trivial
      int uchain, vchain = chainInd[v], ans = -1;
                                  // uchain and vchain are chain numbers of u and v
      while(1)
      {
            uchain = chainInd[u];
            if(uchain == vchain)
            {
                  // Both u and v are in the same chain, so we need to query from u to v,
                  // update answer and break.
                  // We break because we came from u up till v, we are done
                  if(u==v)
                        break;
                  query_tree(1, 0, ptr, posInBase[v]+1, posInBase[u]+1);
                  // Above is call to segment tree query function
                  if(qt[1] > ans)
                         ans = qt[1]; // Update answer
                  break;
            }
            query_tree(1, 0, ptr, posInBase[chainHead[uchain]], posInBase[u]+1);
            // Above is call to segment tree query function. We do from chainHead of u
            // till u. That is the whole chain from
            // start till head. We then update the answer
            if(qt[1] > ans)
                   ans = qt[1];
            u = chainHead[uchain]; // move u to u's chainHead
            u = pa[0][u]; //Then move to its parent, that means we changed chains
      }
      return ans;
}

int LCA(int u, int v)
{
      if(depth[u] < depth[v]) swap(u,v);
      int diff = depth[u] - depth[v];
      for(int i=0; i<LN; i++) if( (diff>>i)&1 ) u = pa[i][u];
      if(u == v) return u;
      for(int i=LN-1; i>=0; i--)
          if(pa[i][u] != pa[i][v])
          {
              u = pa[i][u];
              v = pa[i][v];
          }
      return pa[0][u];
}

void query(int u, int v)
{
      int lca = LCA(u, v);
      int ans = query_up(u, lca); // One part of path
      int temp = query_up(v, lca); // another part of path
      if(temp > ans) ans = temp; // take the maximum of both paths
      printf("%d\n", ans);
}

void change(int i, int val)
{
      int u = otherEnd[i];
      update_tree(1, 0, ptr, posInBase[u], val);
}
```

```
void HLD(int curNode, int cost, int prev) {
	if(chainHead[chainNo] == -1) {
		chainHead[chainNo] = curNode; // Assign chain head
	}
	chainInd[curNode] = chainNo;
	posInBase[curNode] = ptr; // Position of this node in baseArray which we will use
				  // in Segtree
	baseArray[ptr++] = cost;

	int sc = -1, ncost;
	// Loop to find special child
	for(int i=0; i<adj[curNode].size(); i++) if(adj[curNode][i] != prev) {
		if(sc == -1 || subsize[sc] < subsize[adj[curNode][i]]) {
			sc = adj[curNode][i];
			ncost = costs[curNode][i];
		}
	}

	if(sc != -1) {
		HLD(sc, ncost, curNode); // Expand the chain
	}

	for(int i=0; i<adj[curNode].size(); i++) if(adj[curNode][i] != prev) {
		if(sc != adj[curNode][i]) {
			// New chains at each normal node
			chainNo++;
			HLD(adj[curNode][i], costs[curNode][i], curNode);
		}
	}
}

void dfs(int cur, int prev, int _depth=0) {
	pa[0][cur] = prev;
	depth[cur] = _depth;
	subsize[cur] = 1;
	for(int i=0; i<adj[cur].size(); i++)
		if(adj[cur][i] != prev) {
			otherEnd[indexx[cur][i]] = adj[cur][i];
			dfs(adj[cur][i], cur, _depth+1);
			subsize[cur] += subsize[adj[cur][i]];
		}
}

int main() {
	int t;
	scanf("%d ", &t);
	while(t--) {
		ptr = 0;
		int n;
		scanf("%d", &n);
		// Cleaning step, new test case
		for(int i=0; i<n; i++) {
			adj[i].clear();
			costs[i].clear();
			indexx[i].clear();
			chainHead[i] = -1;
			for(int j=0; j<LN; j++) pa[j][i] = -1;
		}
```

```
            for(int i=1; i<n; i++) {
                  int u, v, c;
                  scanf("%d %d %d", &u, &v, &c);
                  u--; v--;
                  adj[u].push_back(v);
                  costs[u].push_back(c);
                  indexx[u].push_back(i-1);
                  adj[v].push_back(u);
                  costs[v].push_back(c);
                  indexx[v].push_back(i-1);
            }

            chainNo = 0;
            dfs(root, -1); // We set up subsize, depth and parent for each node
            HLD(root, -1, -1); // We decomposed the tree and created baseArray
            make_tree(1, 0, ptr);//We use baseArray and construct the needed segment tree

            // Below Dynamic programming code is for LCA.
            for(int i=1; i<LN; i++)
                  for(int j=0; j<n; j++)
                        if(pa[i-1][j] != -1)
                              pa[i][j] = pa[i-1][pa[i-1][j]];

            while(1) {
                  char s[100];
                  scanf("%s", s);
                  if(s[0]=='D') {
                        break;
                  }
                  int a, b;
                  scanf("%d %d", &a, &b);
                  if(s[0]=='Q') {
                        query(a-1, b-1);
                  } else {
                        change(a-1, b);
                  }
            }
      }
}
```

```
Input:
1

3
1 2 1
2 3 2
QUERY 1 2
CHANGE 1 3
QUERY 1 2
DONE

Output:
1
3
```

Segmented Sieve [Prime] :

```
#define MAX 46656
#define LMT 216
#define LEN 4830
#define RNG 100032

unsigned base[MAX/64], segment[RNG/64],
primes[LEN];

#define sq(x) ((x)*(x))
#define mset(x,v) memset(x,v,sizeof(x))
#define chkC(x,n)
(x[n>>6]&(1<<((n>>1)&31)))
#define setC(x,n)
(x[n>>6]|=(1<<((n>>1)&31)))

/* Generates all the necessary prime
    numbers and marks them in base[]*/
void sieve()
{
    unsigned i, j, k;
    for(i=3; i<LMT; i+=2)
        if(!chkC(base, i))
            for(j=i*i,k=i<<1; j<MAX; j+=k)
                setC(base, j);
    for(i=3, j=0; i<MAX; i+=2)
        if(!chkC(base, i))
            primes[j++] = i;
}

/* Returns the prime-count within range
    [a,b] and marks them in segment[]
*/
int segmented_sieve(int a, int b)
{
    unsigned i, j, k;
    unsigned cnt=(a<=2 && 2<=b)? 1 : 0;
    if(b<2) return 0;
    if(a<3) a = 3;
    if(a%2==0) a++;
    mset(segment,0);
    for(i=0; sq(primes[i])<=b; i++)
    {
        j = primes[i] *
          ((a+primes[i]-1) / primes[i]);
        if(j%2==0) j += primes[i];
        for(k=primes[i]<<1; j<=b; j+=k)
            if(j!=primes[i])
                setC(segment, (j-a));
    }
    for(i=0; i<=b-a; i+=2)
        if(!chkC(segment, i))
            cnt++;
    return cnt;
}
```

Catalan Number :

** Formula:

$$C_0 = 1 \quad \text{and} \quad C_{n+1} = \sum_{i=0}^{n} C_i C_{n-i} \quad \text{for } n \geq 0$$

Recursive:

```
#include<iostream>
using namespace std;

unsigned long catalan(unsigned int n)
{
    if (n <= 1) return 1;

    unsigned long int res = 0;
    for (int i=0; i<n; i++)
       res += catalan(i)*catalan(n-i-1);

    return res;
}

// Driver program to test above function
int main()
{
    for (int i=0; i<10; i++)
        cout << catalan(i) << " ";
    return 0;
}
```

DP:

```
#include<iostream>
using namespace std;

unsigned long catalanDP(unsigned int n)
{
    unsigned long int catalan[n+1];
    catalan[0] = catalan[1] = 1;

    for (int i=2; i<=n; i++)
    {
        catalan[i] = 0;
        for (int j=0; j<i; j++)
          catalan[i] +=
            catalan[j] * catalan[i-j-1];
    }
    return catalan[n];
}

int main()
{
    for (int i = 0; i < 10; i++)
        cout << catalanDP(i) << " ";
    return 0;
}
```

Output: 1 1 2 5 14 42 132 429 1430 4862

Complexity: $O(n^2)$

Segment Tree-Computing Fast Average [Loj-1183]:

```
/*
Problem Description:
Given an array of integers (0 indexed),
you have to perform two types of queries in the array.
1. 1 i j v - change the value of the elements from ith index to jth index to v.
2. 2 i j - find the average value of the integers from ith index to jth index.
You can assume that initially all the values in the array are 0.
*/

#include <bits/stdc++.h>
using namespace std;
typedef long long          ll;
#define read(a)   scanf("%lld",&a)

struct data {
    ll sum , xtra;
}tree[300010];

void init(ll node, ll low, ll high) {
    if(low==high) {
        tree[node].sum=0;
        tree[node].xtra=-1;
        return;
    }
    ll left = node*2;
    ll right = left + 1;
    ll mid = (low + high)/2;

    init(left, low, mid);
    init(right, mid + 1, high);
    tree[node].sum = tree[left].sum+tree[right].sum;
    tree[node].xtra=-1;
    return;
}

void update(ll node, ll low, ll high, ll rlow, ll rhigh, ll value) {
    if(low>=rlow && high<=rhigh) {
        tree[node].sum = (high-low+1)*value;
        tree[node].xtra = value;
        return;
    }
    ll left = node*2;
    ll right = left+1;
    ll mid = (low+high)/2;

    if(tree[node].xtra!=-1) {
        tree[left].xtra=tree[node].xtra;
        tree[right].xtra=tree[node].xtra;
        tree[left].sum=(mid-low+1)*tree[left].xtra;
        tree[right].sum=(high-mid)*tree[right].xtra;
        tree[node].xtra=-1;
    }

    if(rhigh <= mid)     update(left, low, mid, rlow, rhigh, value);
    else if(rlow > mid)  update(right, mid+1, high, rlow, rhigh, value);
    else {
        update(left, low, mid, rlow, mid, value);
        update(right, mid+1, high, mid+1, rhigh, value);
    }
    tree[node].sum = tree[left].sum+ tree[right].sum;
}
```

```
ll query(ll node, ll low, ll high, ll rlow, ll rhigh, ll carry)
{
    if(carry!=-1) {
        return (rhigh-rlow+1)*carry;
    }

    if(low>=rlow && high<=rhigh) {
        return tree[node].sum;
    }

    ll left = node*2;
    ll right = left + 1;
    ll mid = (low + high)/2;
    ll p1=0, p2=0;

    if((high-low+1)*tree[node].xtra == tree[node].sum )
        carry=tree[node].xtra;

    if(rhigh<=mid)      p1=query(left, low, mid, rlow, rhigh, carry);
    else if(rlow>mid)   p2=query(right, mid+1, high, rlow, rhigh, carry);
    else {
        p1=query(left, low, mid, rlow, mid, carry);
        p2=query(right, mid+1, high, mid+1, rhigh, carry);
    }
    return p1+p2;
}

main()
{
    ll tc, t=1;
    cin>>tc;
    while(tc--)
    {
        ll n, q;
        cin>>n>>q;
        printf("Case %d:\n", t++);
        init(1,1,n);
        while(q--)
        {
            ll i, j, k, l;
            cin>>i;
            if(i==1)
            {
                cin>>j>>k>>l;
                update(1, 1, n, j+1, k+1, l);
            }
            else if(i==2)
            {
                cin>>j>>k;
                ll ans=query(1, 1, n, j+1, k+1, -1);
                ll res=(k-j+1);
                ll gcd=__gcd(res,ans);
                if(res/gcd>1)
                    printf("%lld/%lld\n", ans/gcd,res/gcd );
                else
                    printf("%lld\n", ans/gcd);
            }
        }
    }
    return 0;
}
```

```
Sample Input:
1
10 6
1 0 6 6
2 0 1
1 1 1 2
2 0 5
1 0 3 7
2 0 1

Output for Sample Input:
Case 1:
6
16/3
7
```

Sum of Series:

Computing the sums for $p = 1, \ldots, 10$ gives

$$\sum_{k=1}^{n} k = \frac{1}{2}\left(n^2 + n\right)$$

$$\sum_{k=1}^{n} k^2 = \frac{1}{6}\left(2n^3 + 3n^2 + n\right)$$

$$\sum_{k=1}^{n} k^3 = \frac{1}{4}\left(n^4 + 2n^3 + n^2\right)$$

$$\sum_{k=1}^{n} k^4 = \frac{1}{30}\left(6n^5 + 15n^4 + 10n^3 - n\right)$$

$$\sum_{k=1}^{n} k^5 = \frac{1}{12}\left(2n^6 + 6n^5 + 5n^4 - n^2\right)$$

$$\sum_{k=1}^{n} k^6 = \frac{1}{42}\left(6n^7 + 21n^6 + 21n^5 - 7n^3 + n\right)$$

$$\sum_{k=1}^{n} k^7 = \frac{1}{24}\left(3n^8 + 12n^7 + 14n^6 - 7n^4 + 2n^2\right)$$

$$\sum_{k=1}^{n} k^8 = \frac{1}{90}\left(10n^9 + 45n^8 + 60n^7 - 42n^5 + 20n^3 - 3n\right)$$

$$\sum_{k=1}^{n} k^9 = \frac{1}{20}\left(2n^{10} + 10n^9 + 15n^8 - 14n^6 + 10n^4 - 3n^2\right)$$

$$\sum_{k=1}^{n} k^{10} = \frac{1}{66}\left(6n^{11} + 33n^{10} + 55n^9 - 66n^7 + 66n^5 - 33n^3 + 5n\right)$$

or in factored form,

$$\sum_{k=1}^{n} k = \frac{1}{2} n (n+1)$$

$$\sum_{k=1}^{n} k^2 = \frac{1}{6} n (n+1)(2n+1)$$

$$\sum_{k=1}^{n} k^3 = \frac{1}{4} n^2 (n+1)^2$$

$$\sum_{k=1}^{n} k^4 = \frac{1}{30} n (n+1)(2n+1)\left(3n^2 + 3n - 1\right)$$

$$\sum_{k=1}^{n} k^5 = \frac{1}{12} n^2 (n+1)^2 \left(2n^2 + 2n - 1\right)$$

$$\sum_{k=1}^{n} k^6 = \frac{1}{42} n (n+1)(2n+1)\left(3n^4 + 6n^3 - 3n + 1\right)$$

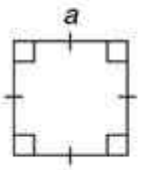
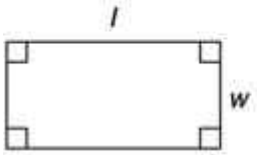
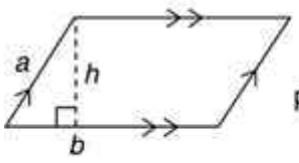
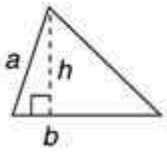
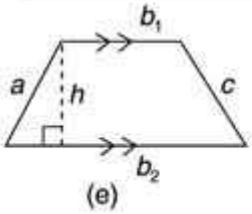
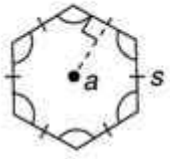
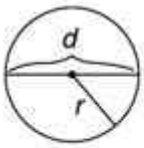
$$\sum_{k=1}^{n} k^7 = \frac{1}{24} n^2 (n+1)^2 \left(3n^4 + 6n^3 - n^2 - 4n + 2\right)$$

$$\sum_{k=1}^{n} k^8 = \frac{1}{90} n (n+1)(2n+1)\left(5n^6 + 15n^5 + 5n^4 - 15n^3 - n^2 + 9n - 3\right)$$

$$\sum_{k=1}^{n} k^9 = \frac{1}{20} n^2 (n+1)^2 \left(n^2 + n - 1\right)\left(2n^4 + 4n^3 - n^2 - 3n + 3\right)$$

$$\sum_{k=1}^{n} k^{10} = \frac{1}{66} n (n+1)(2n+1)\left(n^2 + n - 1\right)\left(3n^6 + 9n^5 + 2n^4 - 11n^3 + 3n^2 + 10n - 5\right)$$

Geometry Areas:

Figure	Name	Perimeter/ Circumference	Area
 (a)	square	$4a$	a^2
 (b)	rectangle	$2l + 2w$ or $2(l + w)$	lw
 (c)	parallelogram	$2a + 2b$ or $2(a + b)$	bh
 (d)	triangle	$a + b + c$	$1/2bh$
 (e)	trapezoid	$a + b_1 + c + b_2$	$1/2(b_1+b_2)h$
 (f)	regular polygon	ns n = number of sides	$1/2ap$ p = perimeter a = apothem
 (g)	circle	πd or $2\pi r$	πr^2