NANO106 Handout 8 - Einstein Notation

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1 Introduction

This document provides a summary of the Einstein notation.

2 General Principle

The Einstein notation, or Einstein summation convention, is a convetion that implies summation over repeated indices. It allows us to write many relationships in a much more succinct manner. For the purposes of crystallography, we will mainly be working with the range of indices over the set {1, 2, 3}. Therefore,

$$y = c_i a_i = \sum_{i=1}^{3} c_i a_i = c_1 a_1 + c_2 a_2 + c_3 a_3$$

To illustrate the power of this notation, we will now write many expressions in linear algebra in Einstein notation.

2.1 Scalar or dot product of two column vectors

$$\mathbf{x} \cdot \mathbf{y} = x_i y_i$$

2.2 Product of a matrix with column vector

$$\mathbf{y} = \mathbf{A}\mathbf{x} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

In Einstein notation, this is written as:

$$y_i = a_{ij} x_j$$

It is important to note that non-repeated indices are "dummy" indices and have no special meaning. Only *repeated* indices on the same side of the equation implies summation.

2.3 Product of row vector with matrix

$$\mathbf{y}^T = \mathbf{x}^T \mathbf{A} \quad = \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

In Einstein notation, this is written as:

$$y_j = a_{ij} x_i$$

Be very careful about the indices! Note the difference in the order of the indices in this expression compared to that of the product of the matrix with a column vector.