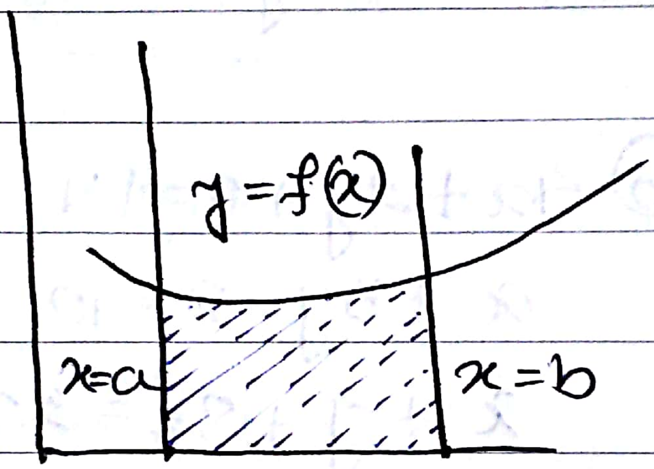
Numerical Integration

$$\int_a^b f(x)\,dx$$

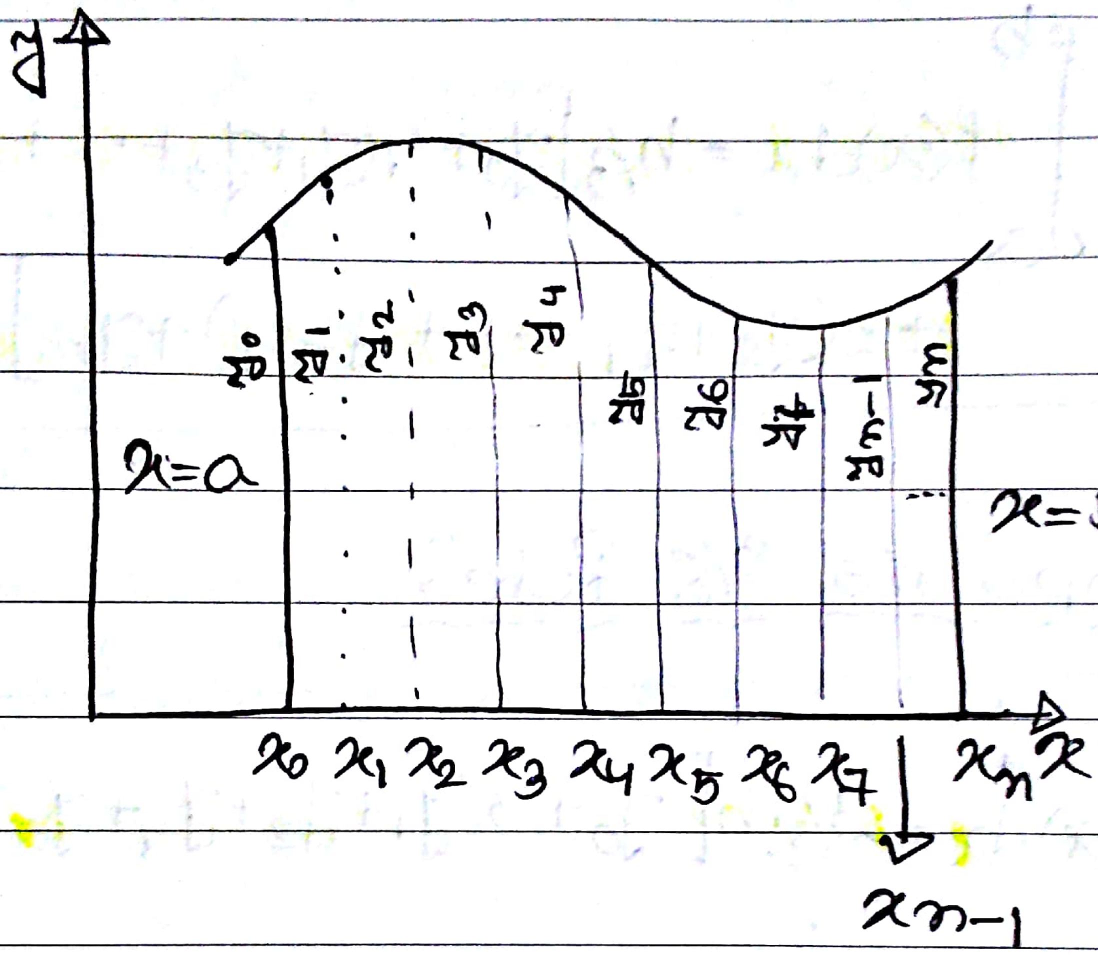


methods:

Theory H.W

1. Trapizoidal Rule
2. Simpson's 1/3 Rule
3. Simpson's 3/8 Rule

Trapizoidal Rule:



$$\int_a^b f(x)\,dx = h/2\left[y_0 + 2\left(y_1 + y_2 + y_3 + \cdots + y_{n-1}\right) + y_n\right]$$

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Simpson's 1/3 Rule:

$$\int_a^b f(x)\,dx = h/3\left[y_0 + 4(y_1 + y_3 + \cdots + y_{n-1}) + 2(y_2 + y_4 + \cdots + y_{n-2}) + y_n\right]$$

Simpson's 3/8 Rule:

$$\int_a^b f(x)\,dx = 3/8\,h\left[y_0 + 3(y_1 + y_2 + y_4 + y_5 + \cdots + y_{n-2} + y_{n-1}) + 2(y_3 + y_6 + y_9 + \cdots + y_{n-3}) + y_n\right]$$

⇒ evaluate $\int_0^6 dx/1+x^2$ by using

(a) Trapezoidal Rule

(b) Simpson's 1/3 Rule

(c) Simpson's 3/8 Rule

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Soln: Divide the range of integration into 6 equal parts. Each of width $h=(6-0)/6$. The value of $y=1/(1+x^2)$ for each point of subdivision are given below:

x	0	1	2	3	4	5	6
$y=1/(1+x^2)$	1 y_0	0.5 y_1	0.2 y_2	0.1 y_3	0.0588235 y_4	0.0384615 y_5	0.0270270 y_6

∴ from trapizoidal Rule:

$$\int_0^6 y\,dx = h/2\left[y_0+2(y_1+y_2+y_3+y_4+y_5)+y_6\right]$$

$$= 1/2\left[1+2(.5+.2+.1+.0588235+.0384615)+.0270270\right]$$

$$= 1.4107985 \text{ (Ans)}$$

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(b) Simpson's 1/3 Rule:

$$\int_0^6 f(x)dx = h/3[y_0 + 4(y_1+y_3+y_5)+2(y_2+y_4) + y_6]$$

$$= 1/3[1+4(.5+.1+.0384615) + 2(.2+.0588235)+.0270270]$$

$$= 1.3661734 \text{ (Ans)}$$

(c) Simpson's 3/8 Rule:

$$\int_0^6 y\,dx = 3/8\,h[y_0+3(y_1+y_2+y_4+y_5)+2y_3 + y_6]$$

$$= 1.3570806 \text{ (Ans)}$$

assignment: 04

(1) evaluate $\int_0^1 dx/1+x$

by (1) Trapizoidal Rule

(2) Simpson's 1/3 Rule

(3) Simpson's 3/8 Rule

Interpolation:

Year	1891	1901	1911	1921	1931
Population	98,752	1,32,285	1,68,076	1,95,690	2,46,050

Forward Difference Table:

Year (x)	Population (y)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1891	98,752	33533	2258	-10435	41358
1901	1,32,285	35791	-8177	30923	
1911	1,68,076	27614	22746		
1921	1,95,690	50360			
1931	2,46,050				

$$\Delta y = y_{n+1} - y_n$$

$$\Delta^2 y = \Delta y_{n+1} - \Delta y_n$$

$$\Delta^3 y = \Delta^2 y_{n+1} - \Delta^2 y_n$$

$$\Delta^4 y = \Delta^3 y_{n+1} - \Delta^3 y_n$$

By Newton's Forward Interpolation Formula:

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$$+ \cdots$$

$$\boxed{u = \frac{x - x_0}{h}} \qquad = \frac{1905 - 1891}{10} = 1.4$$

$$\therefore y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y$$

$$+ \cdots$$

$$= 98.752 + (1.4) \times 33.533 + \frac{(1.4) \times (1.4-1)}{2!} \times 2.258$$

$$+ \frac{1.4 \times (1.4-1)(1.4-2)}{3!} \times (-10.435)$$

$$+ \frac{1.4(1.4-1)(1.4-2)(1.4-3)}{4!} \times 4.1358$$

$= 1,47,841$ (approximately)

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⊛ যেদিকে Data বেশি থাকবে সেদিকের formula apply করতে হবে। যেমন, 1895 এর জন্য বের করতে চাইলে আমাদের দিকে Data বেশি। তাই forward interpolation method. আবার, 1925 এর জন্য পিছনের দিকে Data বেশি, তাই Backward interpolation method. আর মাঝামাঝি থাকলে যেকোনো একটা method.

for 1925:

year(x)	Population(y)	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1891	98,752				
1901	1,32,285	33533			
1911	1,68,076	35791	2258		
1921	1,95,690	27614	-8177	-10435	
1931	2,46,050	50360	22746	30923	41,358

By Newton's Backward Interpola-
-tion Formula:

$$y(x) = y_n + u\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}$$

$$\nabla^3 y_n + \frac{u(u+1)(u+2)(u+3)}{4!}\nabla^4 y_4 + \ldots$$

$$= 2099.82 \text{ (approximately)}$$

assignment: 05

Find the area of a circle of diameter 52. Given that the area 'A' of a circle of diameter 'd' are as follows:

d	50	55	60	65	70
A	1963	2376	2827	3318	3848

Derive Lagrange's interpolation Formula:

Soln: let, $y=f(x)$ is a polynomial of n'th degree, which takes the values $x_0, x_1, x_2, \ldots x_n$ of the argument x. This polynomial may be written as-

$$f(x) = a_0(x-x_1)(x-x_2)\ldots(x-x_n) + a(x-x_0)(x-x_2)\ldots(x-x_n) + a_2(x-x_0)(x-x_1)(x-x_3)\ldots(x-x_n) + \ldots + a_n(x-x_0)(x-x_1)(x-x_2)\ldots(x-x_{n-1}) \quad \text{--- (1)}$$

লক্ষ্য করিঃ a_i থাকলে $(x-x_i)$ থাকবে না।

where a's are the constant.

To find the values of a's, we put $x = x_0, x_1, x_2, \ldots, x_n$ in (1) successively then we get, S M T W T F S

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$$f(x_0) = a_0(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)$$

$$\therefore a_0 = \frac{f(x_0)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)}$$

$$\therefore a_1 = \frac{f(x_1)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)}$$

$$\therefore a_2 = \frac{f(x_2)}{(x_2-x_0)(x_2-x_1)\ldots(x_2-x_n)}$$

$$\therefore a_n = \frac{f(x_n)}{(x_n-x_0)(x_n-x_1)\ldots(x_n-x_{n-1})}$$

$$f(x) = \frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_2)} f(x_0)$$

$$+ \frac{(x-x_0)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)} f(x_1)$$

$$\ldots + \frac{(x-x_0)(x-x_1)\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots(x_n-x_{n-1})} f(x_n)$$

This is the derived formula.

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⇒ Using Lagrange's interpolation formula, find the form of $f(x)$ from the following table:

x_0	0	1	3	4
$f(x)$	-12	0	12	24

Soln: here, $x_0=0$, $x_1=1$; $x_2=3$, $x_3=4$; $f(x_0)=-12$, $f(x_1)=0$, $f(x_2)=12$, $f(x_3)=24$

$$\therefore f(x)=\frac{(x-1)(x-3)(x-4)}{(-1)(-3)(-4)}\times(-12)$$

$$+\frac{x(x-3)(x-4)}{1\times(-2)\times(-3)}\times 0$$

$$+\frac{x(x-1)(x-4)}{3\times 2\times(-1)}\times 12$$

$$+\frac{x(x-1)(x-3)}{4\times 3\times 1}\times 24$$

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$= x^3 - 6x^2 + 17x - 12$; which is the required formula form of $f(x)$

⇒ The following table gives certain corresponding values of x and $\log_{10} x$. Compute the value of $\log_{10} 323.5$ using Lagrange's interpolation formula:

x	321.0	322.8	324.2	325.0
$\log_{10} x$	2.50651	2.50893	2.51081	2.51188

$\therefore f(323.5) = \dfrac{(323.5 - 322.8)(323.5 - 324.2)(323.5}{}$

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$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} f(x_0)$$

$$+ \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} f(x_1)$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} f(x_2)$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} f(x_3)$$

$$\therefore f(323.5) = \frac{(323.5-322.8)(323.5-324.2)(323.5-325)}{(321-322.8)(321-324.2)(321-325)}$$

$$\times f(321)$$

$$+ \frac{(323.5-321)(323.5-324.2)(323.5-325)}{(322.8-321)(322.8-324.2)(322.8-325)}$$

$$\times f(322.8)$$

$$+ \frac{(323.5-321)(323.5-322.8)(323.5-325)}{(324.2-321)(324.2-322.8)(324.2-325)}$$

$$\times f(324.2)$$

$$+ \frac{(323.5-321)(323.5-322.8)(323.5-324.2)}{(325-321)(325-322.8)(325-324.2)}$$

$$\times f(325)$$

$$= 2.50987$$

thus, $\log_{10} 323.5 = 2.50987$ (Ans)

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