UNIT-1

Introduction to DATA MINING

1 Introduction

This book is an introduction to the young and fast-growing field of *data mining* (also known as *knowledge discovery from data*, or *KDD* for short). The book focuses on fundamental data mining concepts and techniques for discovering interesting patterns from data in various applications. In particular, we emphasize prominent techniques for developing effective, efficient, and scalable data mining tools.

This chapter is organized as follows. In Section 1.1, you will learn why data mining is in high demand and how it is part of the natural evolution of information technology. Section 1.2 defines data mining with respect to the knowledge discovery process. Next, you will learn about data mining from many aspects, such as the kinds of data that can be mined (Section 1.3), the kinds of knowledge to be mined (Section 1.4), the kinds of technologies to be used (Section 1.5), and targeted applications (Section 1.6). In this way, you will gain a multidimensional view of data mining. Finally, Section 1.7 outlines major data mining research and development issues.

1.1 Why Data Mining?

Necessity, who is the mother of invention. – Plato

We live in a world where vast amounts of data are collected daily. Analyzing such data is an important need. Section 1.1.1 looks at how data mining can meet this need by providing tools to discover knowledge from data. In Section 1.1.2, we observe how data mining can be viewed as a result of the natural evolution of information technology.

1.1.1 Moving toward the Information Age

"*We are living in the information age*" is a popular saying; however, *we are actually living in the data age*. Terabytes or petabytes[1] of data pour into our computer networks, the World Wide Web (WWW), and various data storage devices every day from business,

[1] A petabyte is a unit of information or computer storage equal to 1 quadrillion bytes, or a thousand terabytes, or 1 million gigabytes.

society, science and engineering, medicine, and almost every other aspect of daily life. This explosive growth of available data volume is a result of the computerization of our society and the fast development of powerful data collection and storage tools. Businesses worldwide generate gigantic data sets, including sales transactions, stock trading records, product descriptions, sales promotions, company profiles and performance, and customer feedback. For example, large stores, such as Wal-Mart, handle hundreds of millions of transactions per week at thousands of branches around the world. Scientific and engineering practices generate high orders of petabytes of data in a continuous manner, from remote sensing, process measuring, scientific experiments, system performance, engineering observations, and environment surveillance.

Global backbone telecommunication networks carry tens of petabytes of data traffic every day. The medical and health industry generates tremendous amounts of data from medical records, patient monitoring, and medical imaging. Billions of Web searches supported by search engines process tens of petabytes of data daily. Communities and social media have become increasingly important data sources, producing digital pictures and videos, blogs, Web communities, and various kinds of social networks. The list of sources that generate huge amounts of data is endless.

This explosively growing, widely available, and gigantic body of data makes our time truly the data age. Powerful and versatile tools are badly needed to automatically uncover valuable information from the tremendous amounts of data and to transform such data into organized knowledge. This necessity has led to the birth of data mining. The field is young, dynamic, and promising. Data mining has and will continue to make great strides in our journey from the data age toward the coming information age.

Example 1.1 **Data mining turns a large collection of data into knowledge.** A search engine (e.g., Google) receives hundreds of millions of queries every day. Each query can be viewed as a transaction where the user describes her or his information need. What novel and useful knowledge can a search engine learn from such a huge collection of queries collected from users over time? Interestingly, some patterns found in user search queries can disclose invaluable knowledge that cannot be obtained by reading individual data items alone. For example, Google's *Flu Trends* uses specific search terms as indicators of flu activity. It found a close relationship between the number of people who search for flu-related information and the number of people who actually have flu symptoms. A pattern emerges when all of the search queries related to flu are aggregated. Using aggregated Google search data, *Flu Trends* can estimate flu activity up to two weeks faster than traditional systems can.[2] This example shows how data mining can turn a large collection of data into knowledge that can help meet a current global challenge. ■

1.1.2 Data Mining as the Evolution of Information Technology

Data mining can be viewed as a result of the natural evolution of information technology. The database and data management industry evolved in the development of

[2]This is reported in [GMP+09].

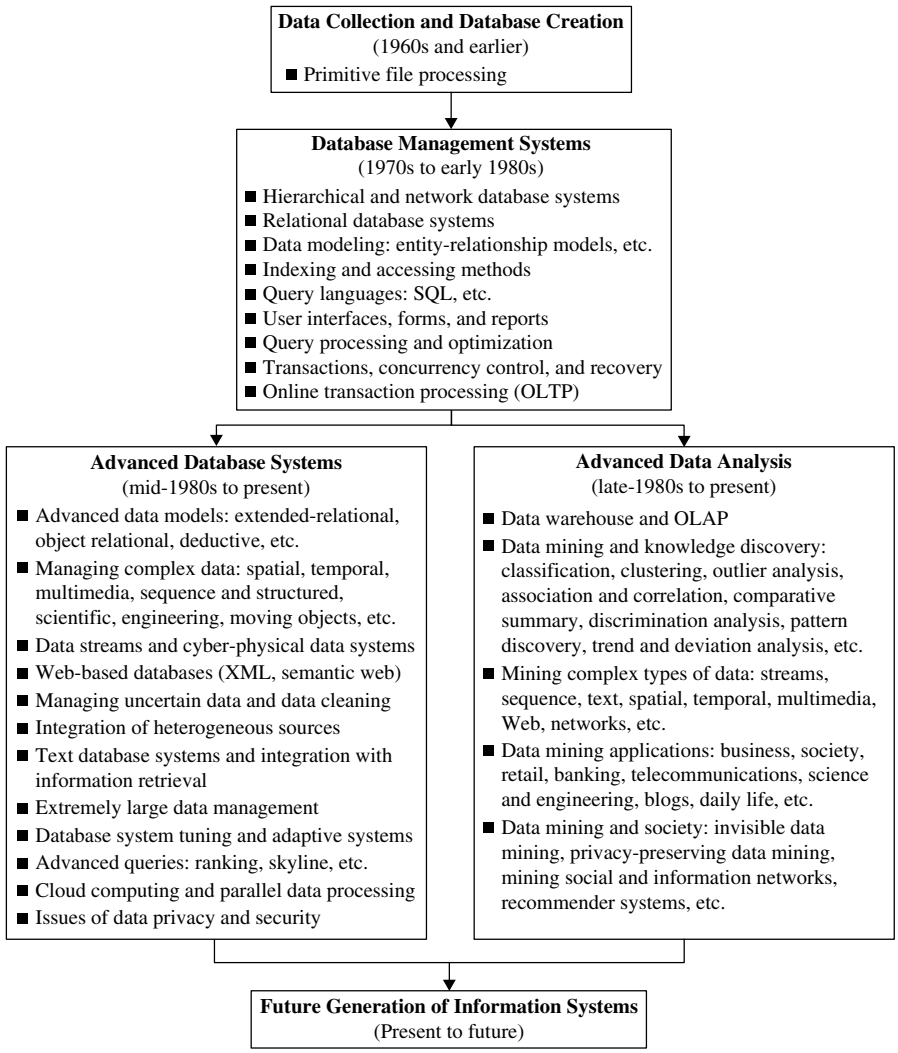


Figure 1.1 The evolution of database system technology.

several critical functionalities (Figure 1.1): *data collection and database creation, data management* (including data storage and retrieval and database transaction processing), and *advanced data analysis* (involving data warehousing and data mining). The early development of data collection and database creation mechanisms served as a prerequisite for the later development of effective mechanisms for data storage and retrieval, as well as query and transaction processing. Nowadays numerous database systems offer query and transaction processing as common practice. Advanced data analysis has naturally become the next step.

Since the 1960s, database and information technology has evolved systematically from primitive file processing systems to sophisticated and powerful database systems. The research and development in database systems since the 1970s progressed from early hierarchical and network database systems to relational database systems (where data are stored in relational table structures; see Section 1.3.1), data modeling tools, and indexing and accessing methods. In addition, users gained convenient and flexible data access through query languages, user interfaces, query optimization, and transaction management. Efficient methods for online transaction processing (OLTP), where a query is viewed as a read-only transaction, contributed substantially to the evolution and wide acceptance of relational technology as a major tool for efficient storage, retrieval, and management of large amounts of data.

After the establishment of database management systems, database technology moved toward the development of *advanced database systems*, *data warehousing*, and *data mining* for advanced data analysis and *web-based databases*. Advanced database systems, for example, resulted from an upsurge of research from the mid-1980s onward. These systems incorporate new and powerful data models such as extended-relational, object-oriented, object-relational, and deductive models. Application-oriented database systems have flourished, including spatial, temporal, multimedia, active, stream and sensor, scientific and engineering databases, knowledge bases, and office information bases. Issues related to the distribution, diversification, and sharing of data have been studied extensively.

Advanced data analysis sprang up from the late 1980s onward. The steady and dazzling progress of computer hardware technology in the past three decades led to large supplies of powerful and affordable computers, data collection equipment, and storage media. This technology provides a great boost to the database and information industry, and it enables a huge number of databases and information repositories to be available for transaction management, information retrieval, and data analysis. Data can now be stored in many different kinds of databases and information repositories.

One emerging data repository architecture is the **data warehouse** (Section 1.3.2). This is a repository of multiple heterogeneous data sources organized under a unified schema at a single site to facilitate management decision making. Data warehouse technology includes data cleaning, data integration, and online analytical processing (OLAP)—that is, analysis techniques with functionalities such as summarization, consolidation, and aggregation, as well as the ability to view information from different angles. Although OLAP tools support multidimensional analysis and decision making, additional data analysis tools are required for in-depth analysis—for example, data mining tools that provide data classification, clustering, outlier/anomaly detection, and the characterization of changes in data over time.

Huge volumes of data have been accumulated beyond databases and data warehouses. During the 1990s, the World Wide Web and web-based databases (e.g., XML databases) began to appear. Internet-based global information bases, such as the WWW and various kinds of interconnected, heterogeneous databases, have emerged and play a vital role in the information industry. The effective and efficient analysis of data from such different forms of data by integration of information retrieval, data mining, and information network analysis technologies is a challenging task.

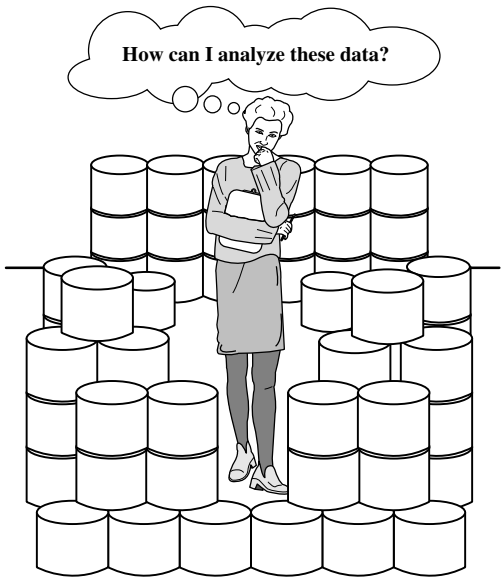


Figure 1.2 The world is data rich but information poor.

In summary, the abundance of data, coupled with the need for powerful data analysis tools, has been described as a *data rich but information poor* situation (Figure 1.2). The fast-growing, tremendous amount of data, collected and stored in large and numerous data repositories, has far exceeded our human ability for comprehension without powerful tools. As a result, data collected in large data repositories become "data tombs"—data archives that are seldom visited. Consequently, important decisions are often made based not on the information-rich data stored in data repositories but rather on a decision maker's intuition, simply because the decision maker does not have the tools to extract the valuable knowledge embedded in the vast amounts of data. Efforts have been made to develop expert system and knowledge-based technologies, which typically rely on users or domain experts to *manually* input knowledge into knowledge bases. Unfortunately, however, the manual knowledge input procedure is prone to biases and errors and is extremely costly and time consuming. The widening gap between data and information calls for the systematic development of *data mining tools* that can turn data tombs into "golden nuggets" of knowledge.

1.2 What Is Data Mining?

It is no surprise that data mining, as a truly interdisciplinary subject, can be defined in many different ways. Even the term *data mining* does not really present all the major components in the picture. To refer to the mining of gold from rocks or sand, we say *gold mining* instead of rock or sand mining. Analogously, data mining should have been more

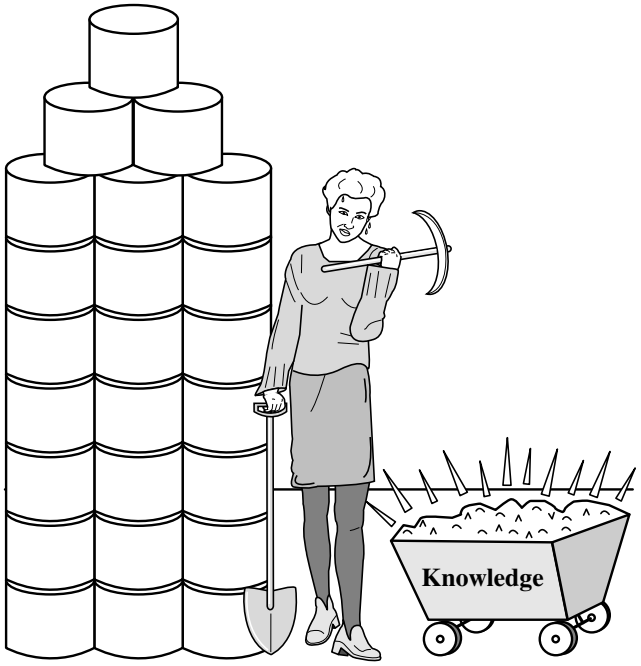


Figure 1.3 Data mining—searching for knowledge (interesting patterns) in data.

appropriately named "knowledge mining from data," which is unfortunately somewhat long. However, the shorter term, *knowledge mining* may not reflect the emphasis on mining from large amounts of data. Nevertheless, mining is a vivid term characterizing the process that finds a small set of precious nuggets from a great deal of raw material (Figure 1.3). Thus, such a misnomer carrying both "data" and "mining" became a popular choice. In addition, many other terms have a similar meaning to data mining—for example, *knowledge mining from data*, *knowledge extraction*, *data/pattern analysis*, *data archaeology*, and *data dredging*.

Many people treat data mining as a synonym for another popularly used term, **knowledge discovery from data**, or **KDD**, while others view data mining as merely an essential step in the process of knowledge discovery. The knowledge discovery process is shown in Figure 1.4 as an iterative sequence of the following steps:

1. **Data cleaning** (to remove noise and inconsistent data)
2. **Data integration** (where multiple data sources may be combined)[3]

[3]A popular trend in the information industry is to perform data cleaning and data integration as a preprocessing step, where the resulting data are stored in a data warehouse.

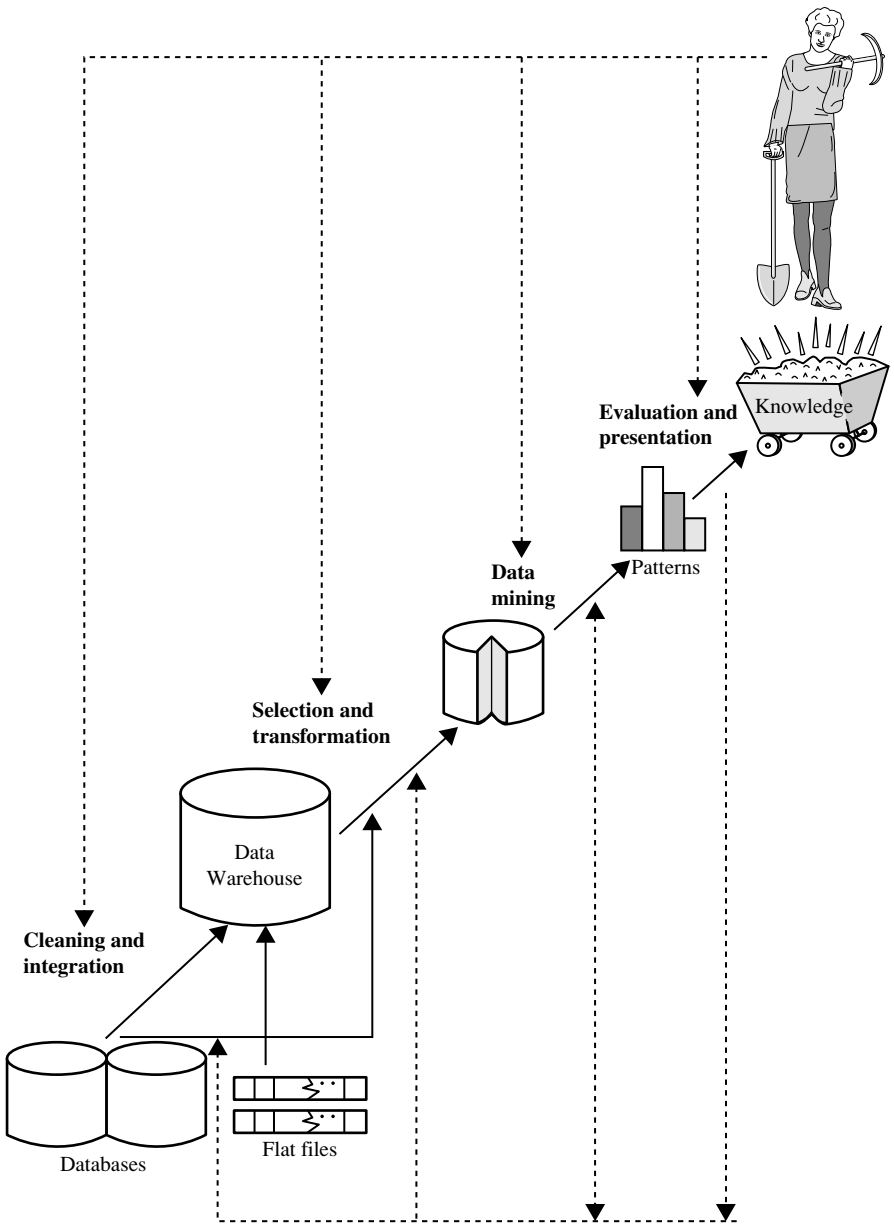


Figure 1.4 Data mining as a step in the process of knowledge discovery.

3. **Data selection** (where data relevant to the analysis task are retrieved from the database)
4. **Data transformation** (where data are transformed and consolidated into forms appropriate for mining by performing summary or aggregation operations)[4]
5. **Data mining** (an essential process where intelligent methods are applied to extract data patterns)
6. **Pattern evaluation** (to identify the truly interesting patterns representing knowledge based on *interestingness measures*—see Section 1.4.6)
7. **Knowledge presentation** (where visualization and knowledge representation techniques are used to present mined knowledge to users)

Steps 1 through 4 are different forms of data preprocessing, where data are prepared for mining. The data mining step may interact with the user or a knowledge base. The interesting patterns are presented to the user and may be stored as new knowledge in the knowledge base.

The preceding view shows data mining as one step in the knowledge discovery process, albeit an essential one because it uncovers hidden patterns for evaluation. However, in industry, in media, and in the research milieu, the term *data mining* is often used to refer to the entire knowledge discovery process (perhaps because the term is shorter than *knowledge discovery from data*). Therefore, we adopt a broad view of data mining functionality: **Data mining** is the *process* of discovering interesting patterns and knowledge from *large* amounts of data. The data sources can include databases, data warehouses, the Web, other information repositories, or data that are streamed into the system dynamically.

1.3 What Kinds of Data Can Be Mined?

As a general technology, data mining can be applied to any kind of data as long as the data are meaningful for a target application. The most basic forms of data for mining applications are database data (Section 1.3.1), data warehouse data (Section 1.3.2), and transactional data (Section 1.3.3). The concepts and techniques presented in this book focus on such data. Data mining can also be applied to other forms of data (e.g., data streams, ordered/sequence data, graph or networked data, spatial data, text data, multimedia data, and the WWW). We present an overview of such data in Section 1.3.4. Techniques for mining of these kinds of data are briefly introduced in Chapter 13. In-depth treatment is considered an advanced topic. Data mining will certainly continue to embrace new data types as they emerge.

[4]Sometimes data transformation and consolidation are performed before the data selection process, particularly in the case of data warehousing. *Data reduction* may also be performed to obtain a smaller representation of the original data without sacrificing its integrity.

1.3.1 Database Data

A database system, also called a **database management system** (**DBMS**), consists of a collection of interrelated data, known as a **database**, and a set of software programs to manage and access the data. The software programs provide mechanisms for defining database structures and data storage; for specifying and managing concurrent, shared, or distributed data access; and for ensuring consistency and security of the information stored despite system crashes or attempts at unauthorized access.

A **relational database** is a collection of **tables**, each of which is assigned a unique name. Each table consists of a set of **attributes** (*columns* or *fields*) and usually stores a large set of **tuples** (*records* or *rows*). Each tuple in a relational table represents an object identified by a unique *key* and described by a set of attribute values. A semantic data model, such as an **entity-relationship** (**ER**) data model, is often constructed for relational databases. An ER data model represents the database as a set of entities and their relationships.

Example 1.2 **A relational database for *AllElectronics*.** The fictitious *AllElectronics* store is used to illustrate concepts throughout this book. The company is described by the following relation tables: *customer, item, employee*, and *branch*. The headers of the tables described here are shown in Figure 1.5. (A header is also called the *schema* of a relation.)

- The relation *customer* consists of a set of attributes describing the customer information, including a unique customer identity number (*cust_ID*), customer name, address, age, occupation, annual income, credit information, and category.
- Similarly, each of the relations *item*, *employee*, and *branch* consists of a set of attributes describing the properties of these entities.
- Tables can also be used to represent the relationships between or among multiple entities. In our example, these include *purchases* (customer purchases items, creating a sales transaction handled by an employee), *items_sold* (lists items sold in a given transaction), and *works_at* (employee works at a branch of *AllElectronics*). ■

customer	(*cust_ID, name, address, age, occupation, annual_income, credit_information, category*, ...)
item	(*item_ID, brand, category, type, price, place_made, supplier, cost*, ...)
employee	(*empl_ID, name, category, group, salary, commission*, ...)
branch	(*branch_ID, name, address*, ...)
purchases	(*trans_ID, cust_ID, empl_ID, date, time, method_paid, amount*)
items_sold	(*trans_ID, item_ID, qty*)
works_at	(*empl_ID, branch_ID*)

Figure 1.5 Relational schema for a relational database, *AllElectronics*.

Relational data can be accessed by **database queries** written in a relational query language (e.g., SQL) or with the assistance of graphical user interfaces. A given query is transformed into a set of relational operations, such as join, selection, and projection, and is then optimized for efficient processing. A query allows retrieval of specified subsets of the data. Suppose that your job is to analyze the *AllElectronics* data. Through the use of relational queries, you can ask things like, "*Show me a list of all items that were sold in the last quarter.*" Relational languages also use aggregate functions such as sum, avg (average), count, max (maximum), and min (minimum). Using aggregates allows you to ask: "*Show me the total sales of the last month, grouped by branch,*" or "*How many sales transactions occurred in the month of December?*" or "*Which salesperson had the highest sales?*"

When **mining relational databases**, we can go further by *searching for trends* or *data patterns*. For example, data mining systems can analyze customer data to predict the credit risk of new customers based on their income, age, and previous credit information. Data mining systems may also detect deviations—that is, items with sales that are far from those expected in comparison with the previous year. Such deviations can then be further investigated. For example, data mining may discover that there has been a change in packaging of an item or a significant increase in price.

Relational databases are one of the most commonly available and richest information repositories, and thus they are a major data form in the study of data mining.

1.3.2 Data Warehouses

Suppose that *AllElectronics* is a successful international company with branches around the world. Each branch has its own set of databases. The president of *AllElectronics* has asked you to provide an analysis of the company's sales per item type per branch for the third quarter. This is a difficult task, particularly since the relevant data are spread out over several databases physically located at numerous sites.

If *AllElectronics* had a data warehouse, this task would be easy. A **data warehouse** is a repository of information collected from multiple sources, stored under a unified schema, and usually residing at a single site. Data warehouses are constructed via a process of data cleaning, data integration, data transformation, data loading, and periodic data refreshing. This process is discussed in Chapters 3 and 4. Figure 1.6 shows the typical framework for construction and use of a data warehouse for *AllElectronics*.

To facilitate decision making, the data in a data warehouse are organized around *major subjects* (e.g., customer, item, supplier, and activity). The data are stored to provide information from a *historical perspective*, such as in the past 6 to 12 months, and are typically *summarized*. For example, rather than storing the details of each sales transaction, the data warehouse may store a summary of the transactions per item type for each store or, summarized to a higher level, for each sales region.

A data warehouse is usually modeled by a multidimensional data structure, called a **data cube**, in which each **dimension** corresponds to an attribute or a set of attributes in the schema, and each **cell** stores the value of some aggregate measure such as *count*

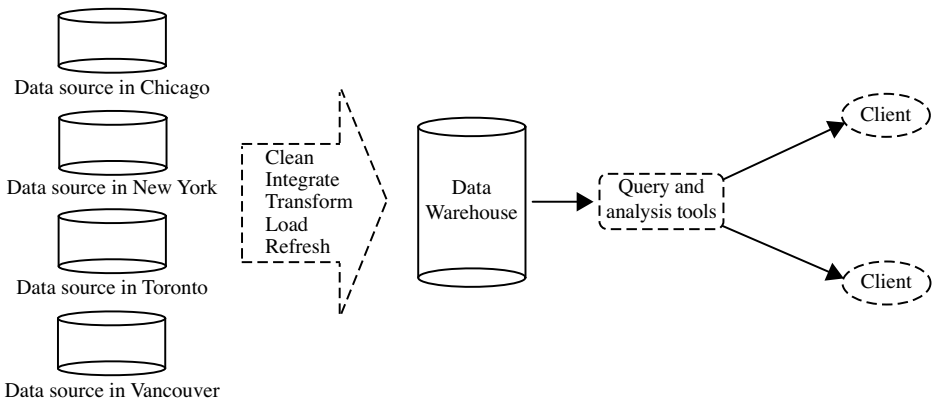


Figure 1.6 Typical framework of a data warehouse for *AllElectronics*.

or *sum*(*sales_amount*). A data cube provides a multidimensional view of data and allows the precomputation and fast access of summarized data.

Example 1.3 **A data cube for *AllElectronics*.** A data cube for summarized sales data of *AllElectronics* is presented in Figure 1.7(a). The cube has three dimensions: *address* (with city values *Chicago, New York, Toronto, Vancouver*), *time* (with quarter values *Q1, Q2, Q3, Q4*), and *item* (with item type values *home entertainment, computer, phone, security*). The aggregate value stored in each cell of the cube is *sales_amount* (in thousands). For example, the total sales for the first quarter, *Q1*, for the items related to security systems in Vancouver is $400,000, as stored in cell ⟨*Vancouver, Q1, security*⟩. Additional cubes may be used to store aggregate sums over each dimension, corresponding to the aggregate values obtained using different SQL group-bys (e.g., the total sales amount per city and quarter, or per city and item, or per quarter and item, or per each individual dimension). ■

By providing multidimensional data views and the precomputation of summarized data, data warehouse systems can provide inherent support for OLAP. Online analytical processing operations make use of background knowledge regarding the domain of the data being studied to allow the presentation of data at *different levels of abstraction*. Such operations accommodate different user viewpoints. Examples of OLAP operations include **drill-down** and **roll-up**, which allow the user to view the data at differing degrees of summarization, as illustrated in Figure 1.7(b). For instance, we can drill down on sales data summarized by *quarter* to see data summarized by *month*. Similarly, we can roll up on sales data summarized by *city* to view data summarized by *country*.

Although data warehouse tools help support data analysis, additional tools for data mining are often needed for in-depth analysis. **Multidimensional data mining** (also called **exploratory multidimensional data mining**) performs data mining in

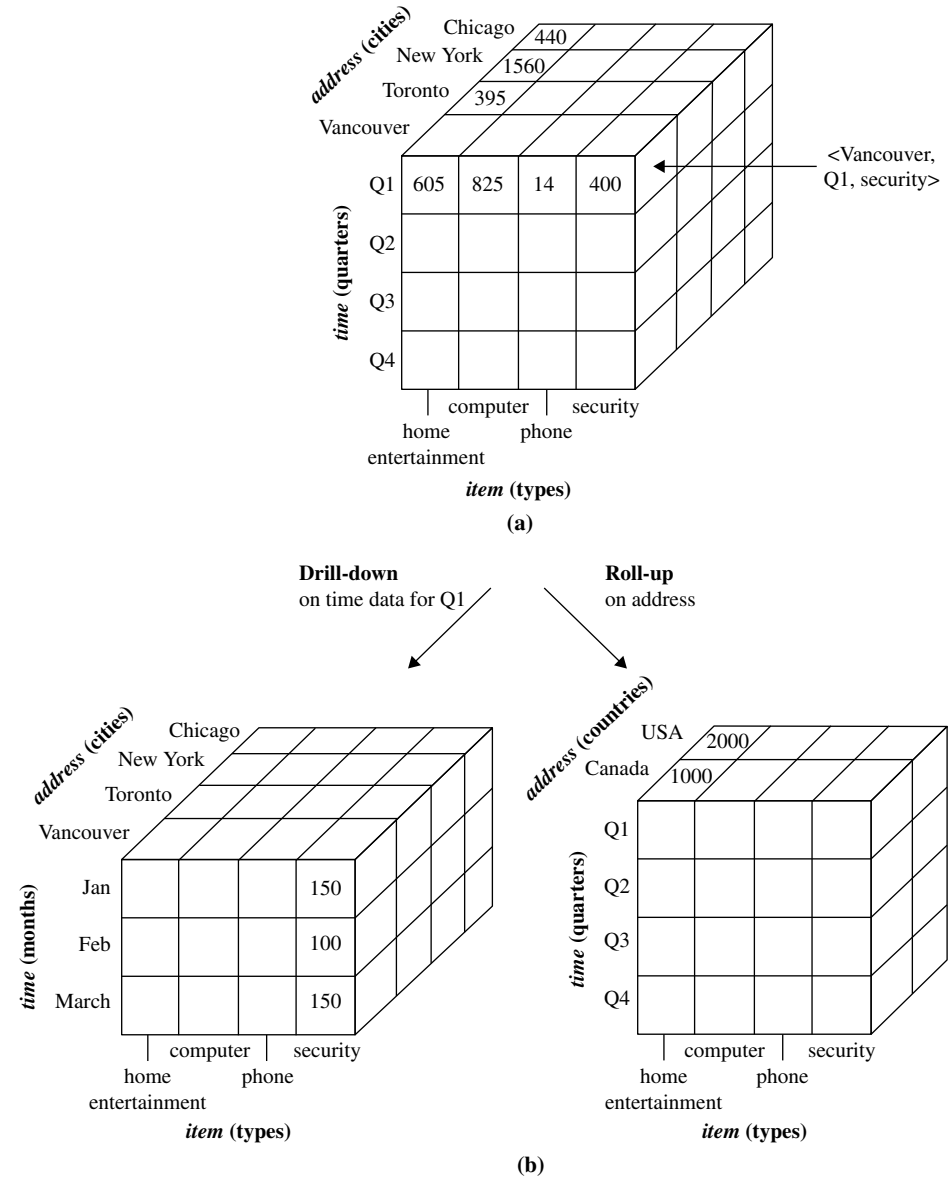


Figure 1.7 A multidimensional data cube, commonly used for data warehousing, (a) showing summarized data for *AllElectronics* and (b) showing summarized data resulting from drill-down and roll-up operations on the cube in (a). For improved readability, only some of the cube cell values are shown.

multidimensional space in an OLAP style. That is, it allows the exploration of multiple combinations of dimensions at varying levels of granularity in data mining, and thus has greater potential for discovering interesting patterns representing knowledge. An overview of data warehouse and OLAP technology is provided in Chapter 4. Advanced issues regarding data cube computation and multidimensional data mining are discussed in Chapter 5.

1.3.3 Transactional Data

In general, each record in a **transactional database** captures a transaction, such as a customer's purchase, a flight booking, or a user's clicks on a web page. A transaction typically includes a unique transaction identity number (*trans_ID*) and a list of the **items** making up the transaction, such as the items purchased in the transaction. A transactional database may have additional tables, which contain other information related to the transactions, such as item description, information about the salesperson or the branch, and so on.

Example 1.4 **A transactional database for *AllElectronics*.** Transactions can be stored in a table, with one record per transaction. A fragment of a transactional database for *AllElectronics* is shown in Figure 1.8. From the relational database point of view, the *sales* table in the figure is a nested relation because the attribute *list_of_item_IDs* contains a set of *items*. Because most relational database systems do not support nested relational structures, the transactional database is usually either stored in a flat file in a format similar to the table in Figure 1.8 or unfolded into a standard relation in a format similar to the *items_sold* table in Figure 1.5. ■

As an analyst of *AllElectronics*, you may ask, "*Which items sold well together?*" This kind of *market basket data analysis* would enable you to bundle groups of items together as a strategy for boosting sales. For example, given the knowledge that printers are commonly purchased together with computers, you could offer certain printers at a steep discount (or even for free) to customers buying selected computers, in the hopes of selling more computers (which are often more expensive than printers). A traditional database system is not able to perform market basket data analysis. Fortunately, data mining on transactional data can do so by mining *frequent itemsets*, that is, sets

trans_ID	*list_of_item_IDs*
T100	I1, I3, I8, I16
T200	I2, I8
...	...

Figure 1.8 Fragment of a transactional database for sales at *AllElectronics*.

of items that are frequently sold together. The mining of such frequent patterns from transactional data is discussed in Chapters 6 and 7.

1.3.4 Other Kinds of Data

Besides relational database data, data warehouse data, and transaction data, there are many other kinds of data that have versatile forms and structures and rather different semantic meanings. Such kinds of data can be seen in many applications: time-related or sequence data (e.g., historical records, stock exchange data, and time-series and biological sequence data), data streams (e.g., video surveillance and sensor data, which are continuously transmitted), spatial data (e.g., maps), engineering design data (e.g., the design of buildings, system components, or integrated circuits), hypertext and multimedia data (including text, image, video, and audio data), graph and networked data (e.g., social and information networks), and the Web (a huge, widely distributed information repository made available by the Internet). These applications bring about new challenges, like how to handle data carrying special structures (e.g., sequences, trees, graphs, and networks) and specific semantics (such as ordering, image, audio and video contents, and connectivity), and how to mine patterns that carry rich structures and semantics.

Various kinds of knowledge can be mined from these kinds of data. Here, we list just a few. Regarding temporal data, for instance, we can mine banking data for changing trends, which may aid in the scheduling of bank tellers according to the volume of customer traffic. Stock exchange data can be mined to uncover trends that could help you plan investment strategies (e.g., the best time to purchase *AllElectronics* stock). We could mine computer network data streams to detect intrusions based on the anomaly of message flows, which may be discovered by clustering, dynamic construction of stream models or by comparing the current frequent patterns with those at a previous time. With spatial data, we may look for patterns that describe changes in metropolitan poverty rates based on city distances from major highways. The relationships among a set of spatial objects can be examined in order to discover which subsets of objects are spatially autocorrelated or associated. By mining text data, such as literature on data mining from the past ten years, we can identify the evolution of hot topics in the field. By mining user comments on products (which are often submitted as short text messages), we can assess customer sentiments and understand how well a product is embraced by a market. From multimedia data, we can mine images to identify objects and classify them by assigning semantic labels or tags. By mining video data of a hockey game, we can detect video sequences corresponding to goals. *Web mining* can help us learn about the distribution of information on the WWW in general, characterize and classify web pages, and uncover web dynamics and the association and other relationships among different web pages, users, communities, and web-based activities.

It is important to keep in mind that, in many applications, multiple types of data are present. For example, in web mining, there often exist text data and multimedia data (e.g., pictures and videos) on web pages, graph data like web graphs, and map data on some web sites. In bioinformatics, genomic sequences, biological networks, and

3-D spatial structures of genomes may coexist for certain biological objects. Mining multiple data sources of complex data often leads to fruitful findings due to the mutual enhancement and consolidation of such multiple sources. On the other hand, it is also challenging because of the difficulties in data cleaning and data integration, as well as the complex interactions among the multiple sources of such data.

While such data require sophisticated facilities for efficient storage, retrieval, and updating, they also provide fertile ground and raise challenging research and implementation issues for data mining. Data mining on such data is an advanced topic. The methods involved are extensions of the basic techniques presented in this book.

1.4 What Kinds of Patterns Can Be Mined?

We have observed various types of data and information repositories on which data mining can be performed. Let us now examine the kinds of patterns that can be mined.

There are a number of ***data mining functionalities***. These include characterization and discrimination (Section 1.4.1); the mining of frequent patterns, associations, and correlations (Section 1.4.2); classification and regression (Section 1.4.3); clustering analysis (Section 1.4.4); and outlier analysis (Section 1.4.5). Data mining functionalities are used to specify the kinds of patterns to be found in data mining tasks. In general, such tasks can be classified into two categories: **descriptive** and **predictive**. Descriptive mining tasks characterize properties of the data in a target data set. Predictive mining tasks perform induction on the current data in order to make predictions.

Data mining functionalities, and the kinds of patterns they can discover, are described below. In addition, Section 1.4.6 looks at what makes a pattern interesting. Interesting patterns represent *knowledge*.

1.4.1 Class/Concept Description: Characterization and Discrimination

Data entries can be associated with classes or concepts. For example, in the *AllElectronics* store, classes of items for sale include *computers* and *printers*, and concepts of customers include *bigSpenders* and *budgetSpenders*. It can be useful to describe individual classes and concepts in summarized, concise, and yet precise terms. Such descriptions of a class or a concept are called **class/concept descriptions**. These descriptions can be derived using (1) *data characterization*, by summarizing the data of the class under study (often called the **target class**) in general terms, or (2) *data discrimination*, by comparison of the target class with one or a set of comparative classes (often called the **contrasting classes**), or (3) both data characterization and discrimination.

Data characterization is a summarization of the general characteristics or features of a target class of data. The data corresponding to the user-specified class are typically collected by a query. For example, to study the characteristics of software products with sales that increased by 10% in the previous year, the data related to such products can be collected by executing an SQL query on the sales database.

There are several methods for effective data summarization and characterization. Simple data summaries based on statistical measures and plots are described in Chapter 2. The data cube-based OLAP roll-up operation (Section 1.3.2) can be used to perform user-controlled data summarization along a specified dimension. This process is further detailed in Chapters 4 and 5, which discuss data warehousing. An *attribute-oriented induction* technique can be used to perform data generalization and characterization without step-by-step user interaction. This technique is also described in Chapter 4.

The output of data characterization can be presented in various forms. Examples include **pie charts**, **bar charts**, **curves**, **multidimensional data cubes**, and **multidimensional tables**, including crosstabs. The resulting descriptions can also be presented as **generalized relations** or in rule form (called **characteristic rules**).

Example 1.5 **Data characterization.** A customer relationship manager at *AllElectronics* may order the following data mining task: *Summarize the characteristics of customers who spend more than $5000 a year at AllElectronics.* The result is a general profile of these customers, such as that they are 40 to 50 years old, employed, and have excellent credit ratings. The data mining system should allow the customer relationship manager to drill down on any dimension, such as on *occupation* to view these customers according to their type of employment. ■

Data discrimination is a comparison of the general features of the target class data objects against the general features of objects from one or multiple contrasting classes. The target and contrasting classes can be specified by a user, and the corresponding data objects can be retrieved through database queries. For example, a user may want to compare the general features of software products with sales that increased by 10% last year against those with sales that decreased by at least 30% during the same period. The methods used for data discrimination are similar to those used for data characterization.

"*How are discrimination descriptions output?*" The forms of output presentation are similar to those for characteristic descriptions, although discrimination descriptions should include comparative measures that help to distinguish between the target and contrasting classes. Discrimination descriptions expressed in the form of rules are referred to as **discriminant rules**.

Example 1.6 **Data discrimination.** A customer relationship manager at *AllElectronics* may want to compare two groups of customers—those who shop for computer products regularly (e.g., more than twice a month) and those who rarely shop for such products (e.g., less than three times a year). The resulting description provides a general comparative profile of these customers, such as that 80% of the customers who frequently purchase computer products are between 20 and 40 years old and have a university education, whereas 60% of the customers who infrequently buy such products are either seniors or youths, and have no university degree. Drilling down on a dimension like *occupation*, or adding a new dimension like *income_level*, may help to find even more discriminative features between the two classes. ■

Concept description, including characterization and discrimination, is described in Chapter 4.

1.4.2 Mining Frequent Patterns, Associations, and Correlations

Frequent patterns, as the name suggests, are patterns that occur frequently in data. There are many kinds of frequent patterns, including frequent itemsets, frequent subsequences (also known as sequential patterns), and frequent substructures. A *frequent itemset* typically refers to a set of items that often appear together in a transactional data set—for example, milk and bread, which are frequently bought together in grocery stores by many customers. A frequently occurring subsequence, such as the pattern that customers, tend to purchase first a laptop, followed by a digital camera, and then a memory card, is a (*frequent*) *sequential pattern*. A substructure can refer to different structural forms (e.g., graphs, trees, or lattices) that may be combined with itemsets or subsequences. If a substructure occurs frequently, it is called a (*frequent*) *structured pattern*. Mining frequent patterns leads to the discovery of interesting associations and correlations within data.

Example 1.7 **Association analysis.** Suppose that, as a marketing manager at *AllElectronics*, you want to know which items are frequently purchased together (i.e., within the same transaction). An example of such a rule, mined from the *AllElectronics* transactional database, is

$$buys(X, \text{“}computer\text{”}) \Rightarrow buys(X, \text{“}software\text{”}) \; [support = 1\%, confidence = 50\%],$$

where X is a variable representing a customer. A **confidence**, or certainty, of 50% means that if a customer buys a computer, there is a 50% chance that she will buy software as well. A 1% **support** means that 1% of all the transactions under analysis show that computer and software are purchased together. This association rule involves a single attribute or predicate (i.e., *buys*) that repeats. Association rules that contain a single predicate are referred to as **single-dimensional association rules**. Dropping the predicate notation, the rule can be written simply as “*computer* $\Rightarrow$ *software* [1%, 50%].”

Suppose, instead, that we are given the *AllElectronics* relational database related to purchases. A data mining system may find association rules like

$$age(X, \text{“20..29”}) \wedge income(X, \text{“40K..49K”}) \Rightarrow buys(X, \text{“laptop”})$$
$$[support = 2\%, confidence = 60\%].$$

The rule indicates that of the *AllElectronics* customers under study, 2% are 20 to 29 years old with an income of $40,000 to $49,000 and have purchased a laptop (computer) at *AllElectronics*. There is a 60% probability that a customer in this age and income group will purchase a laptop. Note that this is an association involving more than one attribute or predicate (i.e., *age, income*, and *buys*). Adopting the terminology used in multidimensional databases, where each attribute is referred to as a dimension, the above rule can be referred to as a **multidimensional association rule**. ■

Typically, association rules are discarded as uninteresting if they do not satisfy both a **minimum support threshold** and a **minimum confidence threshold**. Additional analysis can be performed to uncover interesting statistical **correlations** between associated attribute–value pairs.

Frequent itemset mining is a fundamental form of frequent pattern mining. The mining of frequent patterns, associations, and correlations is discussed in Chapters 6 and 7, where particular emphasis is placed on efficient algorithms for frequent itemset mining. Sequential pattern mining and structured pattern mining are considered advanced topics.

1.4.3 Classification and Regression for Predictive Analysis

Classification is the process of finding a **model** (or function) that describes and distinguishes data classes or concepts. The model are derived based on the analysis of a set of **training data** (i.e., data objects for which the class labels are known). The model is used to predict the class label of objects for which the the class label is unknown.

"*How is the derived model presented?*" The derived model may be represented in various forms, such as *classification rules* (i.e., *IF-THEN rules*), *decision trees*, *mathematical formulae*, or *neural networks* (Figure 1.9). A **decision tree** is a flowchart-like tree structure, where each node denotes a test on an attribute value, each branch represents an outcome of the test, and tree leaves represent classes or class distributions. Decision trees can easily

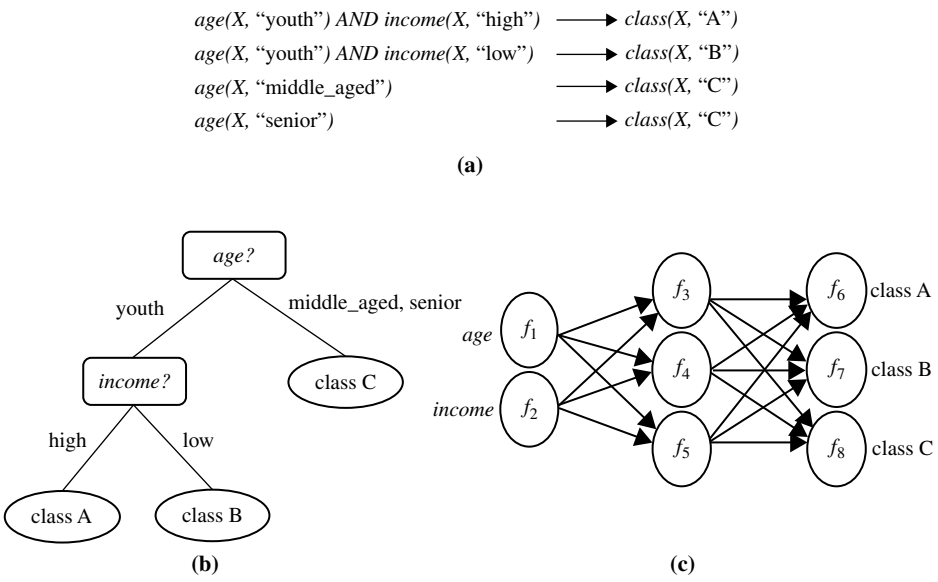


Figure 1.9 A classification model can be represented in various forms: (a) IF-THEN rules, (b) a decision tree, or (c) a neural network.

be converted to classification rules. A **neural network**, when used for classification, is typically a collection of neuron-like processing units with weighted connections between the units. There are many other methods for constructing classification models, such as naïve Bayesian classification, support vector machines, and *k*-nearest-neighbor classification.

Whereas classification predicts categorical (discrete, unordered) labels, **regression** models continuous-valued functions. That is, regression is used to predict missing or unavailable *numerical data values* rather than (discrete) class labels. The term *prediction* refers to both numeric prediction and class label prediction. **Regression analysis** is a statistical methodology that is most often used for numeric prediction, although other methods exist as well. Regression also encompasses the identification of distribution *trends* based on the available data.

Classification and regression may need to be preceded by **relevance analysis**, which attempts to identify attributes that are significantly relevant to the classification and regression process. Such attributes will be selected for the classification and regression process. Other attributes, which are irrelevant, can then be excluded from consideration.

Example 1.8 **Classification and regression.** Suppose as a sales manager of *AllElectronics* you want to classify a large set of items in the store, based on three kinds of responses to a sales campaign: *good response*, *mild response* and *no response*. You want to derive a model for each of these three classes based on the descriptive features of the items, such as *price*, *brand*, *place_made*, *type*, and *category*. The resulting classification should maximally distinguish each class from the others, presenting an organized picture of the data set.

Suppose that the resulting classification is expressed as a decision tree. The decision tree, for instance, may identify *price* as being the single factor that best distinguishes the three classes. The tree may reveal that, in addition to *price*, other features that help to further distinguish objects of each class from one another include *brand* and *place_made*. Such a decision tree may help you understand the impact of the given sales campaign and design a more effective campaign in the future.

Suppose instead, that rather than predicting categorical response labels for each store item, you would like to predict the amount of revenue that each item will generate during an upcoming sale at *AllElectronics*, based on the previous sales data. This is an example of regression analysis because the regression model constructed will predict a continuous function (or ordered value.) ■

Chapters 8 and 9 discuss classification in further detail. Regression analysis is beyond the scope of this book. Sources for further information are given in the bibliographic notes.

1.4.4 Cluster Analysis

Unlike classification and regression, which analyze class-labeled (training) data sets, **clustering** analyzes data objects without consulting class labels. In many cases, class-labeled data may simply not exist at the beginning. Clustering can be used to generate

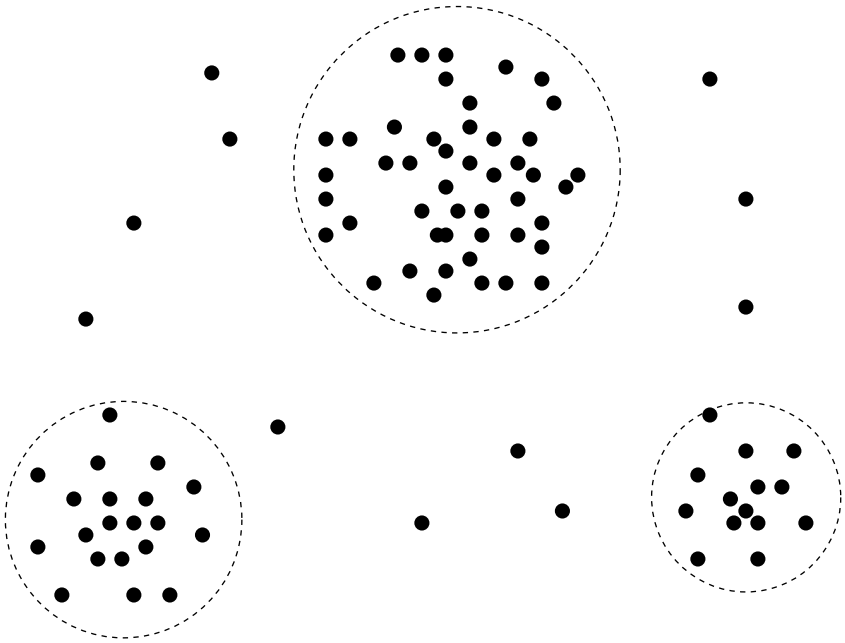


Figure 1.10 A 2-D plot of customer data with respect to customer locations in a city, showing three data clusters.

class labels for a group of data. The objects are clustered or grouped based on the principle of *maximizing the intraclass similarity and minimizing the interclass similarity*. That is, clusters of objects are formed so that objects within a cluster have high similarity in comparison to one another, but are rather dissimilar to objects in other clusters. Each cluster so formed can be viewed as a class of objects, from which rules can be derived. Clustering can also facilitate **taxonomy formation**, that is, the organization of observations into a hierarchy of classes that group similar events together.

Example 1.9 **Cluster analysis.** Cluster analysis can be performed on *AllElectronics* customer data to identify homogeneous subpopulations of customers. These clusters may represent individual target groups for marketing. Figure 1.10 shows a 2-D plot of customers with respect to customer locations in a city. Three clusters of data points are evident. ■

Cluster analysis forms the topic of Chapters 10 and 11.

1.4.5 Outlier Analysis

A data set may contain objects that do not comply with the general behavior or model of the data. These data objects are **outliers**. Many data mining methods discard outliers as noise or exceptions. However, in some applications (e.g., fraud detection) the rare

events can be more interesting than the more regularly occurring ones. The analysis of outlier data is referred to as **outlier analysis** or **anomaly mining**.

Outliers may be detected using statistical tests that assume a distribution or probability model for the data, or using distance measures where objects that are remote from any other cluster are considered outliers. Rather than using statistical or distance measures, density-based methods may identify outliers in a local region, although they look normal from a global statistical distribution view.

Example 1.10 **Outlier analysis.** Outlier analysis may uncover fraudulent usage of credit cards by detecting purchases of unusually large amounts for a given account number in comparison to regular charges incurred by the same account. Outlier values may also be detected with respect to the locations and types of purchase, or the purchase frequency. ■

Outlier analysis is discussed in Chapter 12.

1.4.6 Are All Patterns Interesting?

A data mining system has the potential to generate thousands or even millions of patterns, or rules.

You may ask, *"Are all of the patterns interesting?"* Typically, the answer is no—only a small fraction of the patterns potentially generated would actually be of interest to a given user.

This raises some serious questions for data mining. You may wonder, *"What makes a pattern interesting? Can a data mining system generate all of the interesting patterns? Or, Can the system generate only the interesting ones?"*

To answer the first question, a pattern is **interesting** if it is (1) *easily understood* by humans, (2) *valid* on new or test data with some degree of *certainty*, (3) potentially *useful*, and (4) *novel*. A pattern is also interesting if it validates a hypothesis that the user *sought to confirm*. An interesting pattern represents **knowledge**.

Several **objective measures of pattern interestingness** exist. These are based on the structure of discovered patterns and the statistics underlying them. An objective measure for association rules of the form $X \Rightarrow Y$ is rule **support**, representing the percentage of transactions from a transaction database that the given rule satisfies. This is taken to be the probability $P(X \cup Y)$, where $X \cup Y$ indicates that a transaction contains both X and Y, that is, the union of itemsets X and Y. Another objective measure for association rules is **confidence**, which assesses the degree of certainty of the detected association. This is taken to be the conditional probability $P(Y|X)$, that is, the probability that a transaction containing X also contains Y. More formally, support and confidence are defined as

$$support(X \Rightarrow Y) = P(X \cup Y),$$
$$confidence(X \Rightarrow Y) = P(Y|X).$$

In general, each interestingness measure is associated with a threshold, which may be controlled by the user. For example, rules that do not satisfy a confidence threshold of,

say, 50% can be considered uninteresting. Rules below the threshold likely reflect noise, exceptions, or minority cases and are probably of less value.

Other objective interestingness measures include *accuracy* and *coverage* for classification (IF-THEN) rules. In general terms, accuracy tells us the percentage of data that are correctly classified by a rule. Coverage is similar to support, in that it tells us the percentage of data to which a rule applies. Regarding understandability, we may use simple objective measures that assess the complexity or length in bits of the patterns mined.

Although objective measures help identify interesting patterns, they are often insufficient unless combined with subjective measures that reflect a particular user's needs and interests. For example, patterns describing the characteristics of customers who shop frequently at *AllElectronics* should be interesting to the marketing manager, but may be of little interest to other analysts studying the same database for patterns on employee performance. Furthermore, many patterns that are interesting by objective standards may represent common sense and, therefore, are actually uninteresting.

Subjective interestingness measures are based on user beliefs in the data. These measures find patterns interesting if the patterns are **unexpected** (contradicting a user's belief) or offer strategic information on which the user can act. In the latter case, such patterns are referred to as **actionable**. For example, patterns like "a large earthquake often follows a cluster of small quakes" may be highly actionable if users can act on the information to save lives. Patterns that are **expected** can be interesting if they confirm a hypothesis that the user wishes to validate or they resemble a user's hunch.

The second question—"*Can a data mining system generate* all *of the interesting patterns?*"—refers to the **completeness** of a data mining algorithm. It is often unrealistic and inefficient for data mining systems to generate all possible patterns. Instead, user-provided constraints and interestingness measures should be used to focus the search. For some mining tasks, such as association, this is often sufficient to ensure the completeness of the algorithm. Association rule mining is an example where the use of constraints and interestingness measures can ensure the completeness of mining. The methods involved are examined in detail in Chapter 6.

Finally, the third question—"*Can a data mining system generate only interesting patterns?*"—is an optimization problem in data mining. It is highly desirable for data mining systems to generate only interesting patterns. This would be efficient for users and data mining systems because neither would have to search through the patterns generated to identify the truly interesting ones. Progress has been made in this direction; however, such optimization remains a challenging issue in data mining.

Measures of pattern interestingness are essential for the efficient discovery of patterns by target users. Such measures can be used after the data mining step to rank the discovered patterns according to their interestingness, filtering out the uninteresting ones. More important, such measures can be used to guide and constrain the discovery process, improving the search efficiency by pruning away subsets of the pattern space that do not satisfy prespecified interestingness constraints. Examples of such a constraint-based mining process are described in Chapter 7 (with respect to pattern discovery) and Chapter 11 (with respect to clustering).

Methods to assess pattern interestingness, and their use to improve data mining efficiency, are discussed throughout the book with respect to each kind of pattern that can be mined.

1.5 Which Technologies Are Used?

As a highly application-driven domain, data mining has incorporated many techniques from other domains such as statistics, machine learning, pattern recognition, database and data warehouse systems, information retrieval, visualization, algorithms, high-performance computing, and many application domains (Figure 1.11). The interdisciplinary nature of data mining research and development contributes significantly to the success of data mining and its extensive applications. In this section, we give examples of several disciplines that strongly influence the development of data mining methods.

1.5.1 Statistics

Statistics studies the collection, analysis, interpretation or explanation, and presentation of data. Data mining has an inherent connection with statistics.

A **statistical model** is a set of mathematical functions that describe the behavior of the objects in a target class in terms of random variables and their associated probability distributions. Statistical models are widely used to model data and data classes. For example, in data mining tasks like data characterization and classification, statistical

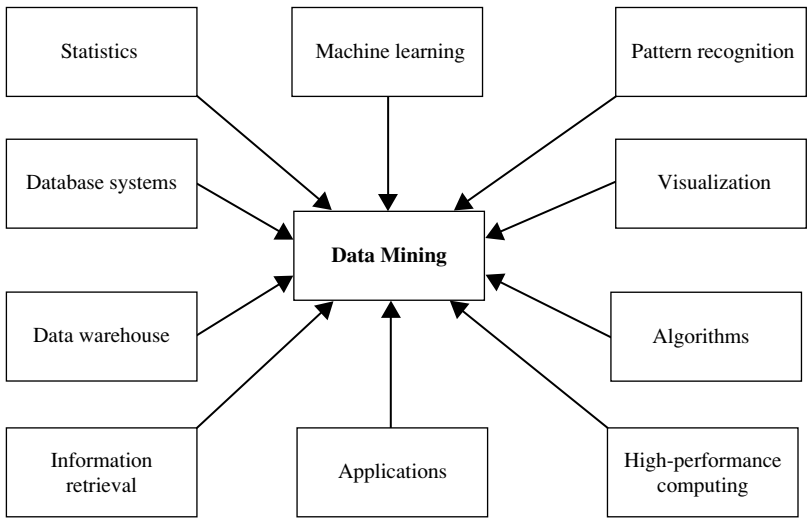


Figure 1.11 Data mining adopts techniques from many domains.

models of target classes can be built. In other words, such statistical models can be the outcome of a data mining task. Alternatively, data mining tasks can be built on top of statistical models. For example, we can use statistics to model noise and missing data values. Then, when mining patterns in a large data set, the data mining process can use the model to help identify and handle noisy or missing values in the data.

Statistics research develops tools for prediction and forecasting using data and statistical models. Statistical methods can be used to summarize or describe a collection of data. Basic **statistical descriptions** of data are introduced in Chapter 2. Statistics is useful for mining various patterns from data as well as for understanding the underlying mechanisms generating and affecting the patterns. **Inferential statistics** (or **predictive statistics**) models data in a way that accounts for randomness and uncertainty in the observations and is used to draw inferences about the process or population under investigation.

Statistical methods can also be used to verify data mining results. For example, after a classification or prediction model is mined, the model should be verified by statistical hypothesis testing. A **statistical hypothesis test** (sometimes called *confirmatory data analysis*) makes statistical decisions using experimental data. A result is called *statistically significant* if it is unlikely to have occurred by chance. If the classification or prediction model holds true, then the descriptive statistics of the model increases the soundness of the model.

Applying statistical methods in data mining is far from trivial. Often, a serious challenge is how to scale up a statistical method over a large data set. Many statistical methods have high complexity in computation. When such methods are applied on large data sets that are also distributed on multiple logical or physical sites, algorithms should be carefully designed and tuned to reduce the computational cost. This challenge becomes even tougher for online applications, such as online query suggestions in search engines, where data mining is required to continuously handle fast, real-time data streams.

1.5.2 Machine Learning

Machine learning investigates how computers can learn (or improve their performance) based on data. A main research area is for computer programs to *automatically* learn to recognize complex patterns and make intelligent decisions based on data. For example, a typical machine learning problem is to program a computer so that it can automatically recognize handwritten postal codes on mail after learning from a set of examples.

Machine learning is a fast-growing discipline. Here, we illustrate classic problems in machine learning that are highly related to data mining.

- **Supervised learning** is basically a synonym for classification. The supervision in the learning comes from the labeled examples in the training data set. For example, in the postal code recognition problem, a set of handwritten postal code images and their corresponding machine-readable translations are used as the training examples, which supervise the learning of the classification model.

- **Unsupervised learning** is essentially a synonym for clustering. The learning process is unsupervised since the input examples are not class labeled. Typically, we may use clustering to discover classes within the data. For example, an unsupervised learning method can take, as input, a set of images of handwritten digits. Suppose that it finds 10 clusters of data. These clusters may correspond to the 10 distinct digits of 0 to 9, respectively. However, since the training data are not labeled, the learned model cannot tell us the semantic meaning of the clusters found.
- **Semi-supervised learning** is a class of machine learning techniques that make use of both labeled and unlabeled examples when learning a model. In one approach, labeled examples are used to learn class models and unlabeled examples are used to refine the boundaries between classes. For a two-class problem, we can think of the set of examples belonging to one class as the *positive examples* and those belonging to the other class as the *negative examples*. In Figure 1.12, if we do not consider the unlabeled examples, the dashed line is the decision boundary that best partitions the positive examples from the negative examples. Using the unlabeled examples, we can refine the decision boundary to the solid line. Moreover, we can detect that the two positive examples at the top right corner, though labeled, are likely noise or outliers.
- **Active learning** is a machine learning approach that lets users play an active role in the learning process. An active learning approach can ask a user (e.g., a domain expert) to label an example, which may be from a set of unlabeled examples or synthesized by the learning program. The goal is to optimize the model quality by actively acquiring knowledge from human users, given a constraint on how many examples they can be asked to label.

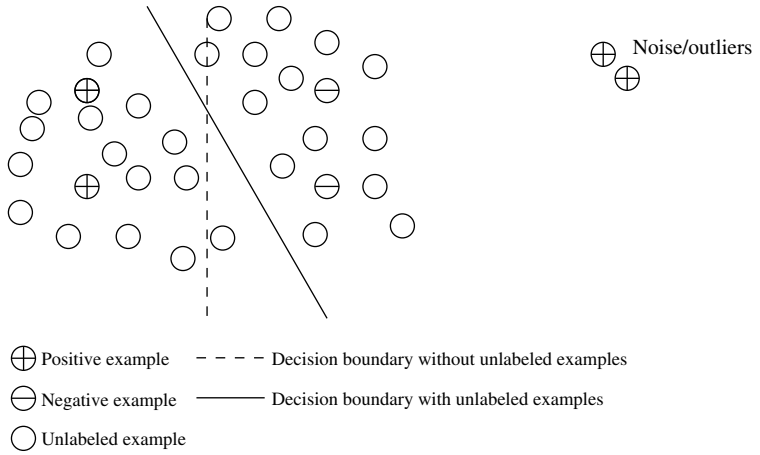


Figure 1.12 Semi-supervised learning.

You can see there are many similarities between data mining and machine learning. For classification and clustering tasks, machine learning research often focuses on the accuracy of the model. In addition to accuracy, data mining research places strong emphasis on the efficiency and scalability of mining methods on large data sets, as well as on ways to handle complex types of data and explore new, alternative methods.

1.5.3 Database Systems and Data Warehouses

Database systems research focuses on the creation, maintenance, and use of databases for organizations and end-users. Particularly, database systems researchers have established highly recognized principles in data models, query languages, query processing and optimization methods, data storage, and indexing and accessing methods. Database systems are often well known for their high scalability in processing very large, relatively structured data sets.

Many data mining tasks need to handle large data sets or even real-time, fast streaming data. Therefore, data mining can make good use of scalable database technologies to achieve high efficiency and scalability on large data sets. Moreover, data mining tasks can be used to extend the capability of existing database systems to satisfy advanced users' sophisticated data analysis requirements.

Recent database systems have built systematic data analysis capabilities on database data using data warehousing and data mining facilities. A **data warehouse** integrates data originating from multiple sources and various timeframes. It consolidates data in multidimensional space to form partially materialized data cubes. The data cube model not only facilitates OLAP in multidimensional databases but also promotes *multidimensional data mining* (see Section 1.3.2).

1.5.4 Information Retrieval

Information retrieval (**IR**) is the science of searching for documents or information in documents. Documents can be text or multimedia, and may reside on the Web. The differences between traditional information retrieval and database systems are twofold: Information retrieval assumes that (1) the data under search are unstructured; and (2) the queries are formed mainly by keywords, which do not have complex structures (unlike SQL queries in database systems).

The typical approaches in information retrieval adopt probabilistic models. For example, a text document can be regarded as a bag of words, that is, a multiset of words appearing in the document. The document's **language model** is the probability density function that generates the bag of words in the document. The similarity between two documents can be measured by the similarity between their corresponding language models.

Furthermore, a topic in a set of text documents can be modeled as a probability distribution over the vocabulary, which is called a **topic model**. A text document, which may involve one or multiple topics, can be regarded as a mixture of multiple topic models. By integrating information retrieval models and data mining techniques, we can find

the major topics in a collection of documents and, for each document in the collection, the major topics involved.

Increasingly large amounts of text and multimedia data have been accumulated and made available online due to the fast growth of the Web and applications such as digital libraries, digital governments, and health care information systems. Their effective search and analysis have raised many challenging issues in data mining. Therefore, text mining and multimedia data mining, integrated with information retrieval methods, have become increasingly important.

1.6 Which Kinds of Applications Are Targeted?

Where there are data, there are data mining applications

As a highly application-driven discipline, data mining has seen great successes in many applications. It is impossible to enumerate all applications where data mining plays a critical role. Presentations of data mining in knowledge-intensive application domains, such as bioinformatics and software engineering, require more in-depth treatment and are beyond the scope of this book. To demonstrate the importance of applications as a major dimension in data mining research and development, we briefly discuss two highly successful and popular application examples of data mining: *business intelligence* and *search engines*.

1.6.1 Business Intelligence

It is critical for businesses to acquire a better understanding of the commercial context of their organization, such as their customers, the market, supply and resources, and competitors. **Business intelligence** (**BI**) technologies provide historical, current, and predictive views of business operations. Examples include reporting, online analytical processing, business performance management, competitive intelligence, benchmarking, and predictive analytics.

"*How important is business intelligence?*" Without data mining, many businesses may not be able to perform effective market analysis, compare customer feedback on similar products, discover the strengths and weaknesses of their competitors, retain highly valuable customers, and make smart business decisions.

Clearly, data mining is the core of business intelligence. Online analytical processing tools in business intelligence rely on data warehousing and multidimensional data mining. Classification and prediction techniques are the core of predictive analytics in business intelligence, for which there are many applications in analyzing markets, supplies, and sales. Moreover, clustering plays a central role in customer relationship management, which groups customers based on their similarities. Using characterization mining techniques, we can better understand features of each customer group and develop customized customer reward programs.

1.6.2 Web Search Engines

A **Web search engine** is a specialized computer server that searches for information on the Web. The search results of a user query are often returned as a list (sometimes called *hits*). The hits may consist of web pages, images, and other types of files. Some search engines also search and return data available in public databases or open directories. Search engines differ from **web directories** in that web directories are maintained by human editors whereas search engines operate algorithmically or by a mixture of algorithmic and human input.

Web search engines are essentially very large data mining applications. Various data mining techniques are used in all aspects of search engines, ranging from *crawling*[5] (e.g., deciding which pages should be crawled and the crawling frequencies), indexing (e.g., selecting pages to be indexed and deciding to which extent the index should be constructed), and searching (e.g., deciding how pages should be ranked, which advertisements should be added, and how the search results can be personalized or made "context aware").

Search engines pose grand challenges to data mining. First, they have to handle a huge and ever-growing amount of data. Typically, such data cannot be processed using one or a few machines. Instead, search engines often need to use *computer clouds*, which consist of thousands or even hundreds of thousands of computers that collaboratively mine the huge amount of data. Scaling up data mining methods over computer clouds and large distributed data sets is an area for further research.

Second, Web search engines often have to deal with online data. A search engine may be able to afford constructing a model offline on huge data sets. To do this, it may construct a query classifier that assigns a search query to predefined categories based on the query topic (i.e., whether the search query "apple" is meant to retrieve information about a fruit or a brand of computers). Whether a model is constructed offline, the application of the model online must be fast enough to answer user queries in real time.

Another challenge is maintaining and incrementally updating a model on fast-growing data streams. For example, a query classifier may need to be incrementally maintained continuously since new queries keep emerging and predefined categories and the data distribution may change. Most of the existing model training methods are offline and static and thus cannot be used in such a scenario.

Third, Web search engines often have to deal with queries that are asked only a very small number of times. Suppose a search engine wants to provide *context-aware* query recommendations. That is, when a user poses a query, the search engine tries to infer the context of the query using the user's profile and his query history in order to return more customized answers within a small fraction of a second. However, although the total number of queries asked can be huge, most of the queries may be asked only once or a few times. Such severely skewed data are challenging for many data mining and machine learning methods.

[5] A Web crawler is a computer program that browses the Web in a methodical, automated manner.

1.7 Major Issues in Data Mining

Life is short but art is long. – Hippocrates

Data mining is a dynamic and fast-expanding field with great strengths. In this section, we briefly outline the major issues in data mining research, partitioning them into five groups: *mining methodology, user interaction, efficiency and scalability, diversity of data types*, and *data mining and society*. Many of these issues have been addressed in recent data mining research and development *to a certain extent* and are now considered *data mining requirements*; others are still at the research stage. The issues continue to stimulate further investigation and improvement in data mining.

1.7.1 Mining Methodology

Researchers have been vigorously developing new data mining methodologies. This involves the investigation of new kinds of knowledge, mining in multidimensional space, integrating methods from other disciplines, and the consideration of semantic ties among data objects. In addition, mining methodologies should consider issues such as data uncertainty, noise, and incompleteness. Some mining methods explore how user-specified measures can be used to assess the interestingness of discovered patterns as well as guide the discovery process. Let's have a look at these various aspects of mining methodology.

- *Mining various and new kinds of knowledge:* Data mining covers a wide spectrum of data analysis and knowledge discovery tasks, from data characterization and discrimination to association and correlation analysis, classification, regression, clustering, outlier analysis, sequence analysis, and trend and evolution analysis. These tasks may use the same database in different ways and require the development of numerous data mining techniques. Due to the diversity of applications, new mining tasks continue to emerge, making data mining a dynamic and fast-growing field. For example, for effective knowledge discovery in information networks, integrated clustering and ranking may lead to the discovery of high-quality clusters and object ranks in large networks.
- *Mining knowledge in multidimensional space:* When searching for knowledge in large data sets, we can explore the data in multidimensional space. That is, we can search for interesting patterns among combinations of dimensions (attributes) at varying levels of abstraction. Such mining is known as *(exploratory) multidimensional data mining*. In many cases, data can be aggregated or viewed as a multidimensional data cube. Mining knowledge in cube space can substantially enhance the power and flexibility of data mining.
- *Data mining—an interdisciplinary effort:* The power of data mining can be substantially enhanced by integrating new methods from multiple disciplines. For example,

to mine data with natural language text, it makes sense to fuse data mining methods with methods of information retrieval and natural language processing. As another example, consider the mining of software bugs in large programs. This form of mining, known as *bug mining*, benefits from the incorporation of software engineering knowledge into the data mining process.

- *Boosting the power of discovery in a networked environment:* Most data objects reside in a linked or interconnected environment, whether it be the Web, database relations, files, or documents. Semantic links across multiple data objects can be used to advantage in data mining. Knowledge derived in one set of objects can be used to boost the discovery of knowledge in a "related" or semantically linked set of objects.
- *Handling uncertainty, noise, or incompleteness of data:* Data often contain noise, errors, exceptions, or uncertainty, or are incomplete. Errors and noise may confuse the data mining process, leading to the derivation of erroneous patterns. Data cleaning, data preprocessing, outlier detection and removal, and uncertainty reasoning are examples of techniques that need to be integrated with the data mining process.
- *Pattern evaluation and pattern- or constraint-guided mining:* Not all the patterns generated by data mining processes are interesting. What makes a pattern interesting may vary from user to user. Therefore, techniques are needed to assess the interestingness of discovered patterns based on subjective measures. These estimate the value of patterns with respect to a given user class, based on user beliefs or expectations. Moreover, by using interestingness measures or user-specified constraints to *guide* the discovery process, we may generate more interesting patterns and reduce the search space.

1.7.2 User Interaction

The user plays an important role in the data mining process. Interesting areas of research include *how to interact with a data mining system*, *how to incorporate a user's background knowledge in mining*, and *how to visualize and comprehend data mining results*. We introduce each of these here.

- *Interactive mining:* The data mining process should be highly *interactive*. Thus, it is important to build flexible user interfaces and an exploratory mining environment, facilitating the user's interaction with the system. A user may like to first sample a set of data, explore general characteristics of the data, and estimate potential mining results. Interactive mining should allow users to dynamically change the focus of a search, to refine mining requests based on returned results, and to drill, dice, and pivot through the data and knowledge space interactively, dynamically exploring "cube space" while mining.
- *Incorporation of background knowledge:* Background knowledge, constraints, rules, and other information regarding the domain under study should be incorporated

into the knowledge discovery process. Such knowledge can be used for pattern evaluation as well as to guide the search toward interesting patterns.

- *Ad hoc data mining and data mining query languages:* Query languages (e.g., SQL) have played an important role in flexible searching because they allow users to pose ad hoc queries. Similarly, high-level data mining query languages or other high-level flexible user interfaces will give users the freedom to define ad hoc data mining tasks. This should facilitate specification of the relevant sets of data for analysis, the domain knowledge, the kinds of knowledge to be mined, and the conditions and constraints to be enforced on the discovered patterns. Optimization of the processing of such flexible mining requests is another promising area of study.
- *Presentation and visualization of data mining results:* How can a data mining system present data mining results, vividly and flexibly, so that the discovered knowledge can be easily understood and directly usable by humans? This is especially crucial if the data mining process is interactive. It requires the system to adopt expressive knowledge representations, user-friendly interfaces, and visualization techniques.

1.7.3 Efficiency and Scalability

Efficiency and scalability are always considered when comparing data mining algorithms. As data amounts continue to multiply, these two factors are especially critical.

- *Efficiency and scalability of data mining algorithms:* Data mining algorithms must be efficient and scalable in order to effectively extract information from huge amounts of data in many data repositories or in dynamic data streams. In other words, the running time of a data mining algorithm must be predictable, short, and acceptable by applications. *Efficiency, scalability, performance, optimization*, and the ability to execute in *real time* are key criteria that drive the development of many new data mining algorithms.
- *Parallel, distributed, and incremental mining algorithms:* The humongous size of many data sets, the wide distribution of data, and the computational complexity of some data mining methods are factors that motivate the development of **parallel and distributed data-intensive mining algorithms**. Such algorithms first partition the data into "pieces." Each piece is processed, in parallel, by searching for patterns. The parallel processes may interact with one another. The patterns from each partition are eventually merged.

  Cloud computing and *cluster computing*, which use computers in a distributed and collaborative way to tackle very large-scale computational tasks, are also active research themes in parallel data mining. In addition, the high cost of some data mining processes and the incremental nature of input promote **incremental** data mining, which incorporates new data updates without having to mine the entire data "from scratch." Such methods perform knowledge modification incrementally to amend and strengthen what was previously discovered.

1.7.4 Diversity of Database Types

The wide diversity of database types brings about challenges to data mining. These include

- *Handling complex types of data:* Diverse applications generate a wide spectrum of new data types, from structured data such as relational and data warehouse data to semi-structured and unstructured data; from stable data repositories to dynamic data streams; from simple data objects to temporal data, biological sequences, sensor data, spatial data, hypertext data, multimedia data, software program code, Web data, and social network data. It is unrealistic to expect one data mining system to mine all kinds of data, given the diversity of data types and the different goals of data mining. Domain- or application-dedicated data mining systems are being constructed for in-depth mining of specific kinds of data. The construction of effective and efficient data mining tools for diverse applications remains a challenging and active area of research.
- *Mining dynamic, networked, and global data repositories:* Multiple sources of data are connected by the Internet and various kinds of networks, forming gigantic, distributed, and heterogeneous global information systems and networks. The discovery of knowledge from different sources of structured, semi-structured, or unstructured yet interconnected data with diverse data semantics poses great challenges to data mining. Mining such gigantic, interconnected information networks may help disclose many more patterns and knowledge in heterogeneous data sets than can be discovered from a small set of isolated data repositories. Web mining, multisource data mining, and information network mining have become challenging and fast-evolving data mining fields.

1.7.5 Data Mining and Society

How does data mining impact society? What steps can data mining take to preserve the privacy of individuals? Do we use data mining in our daily lives without even knowing that we do? These questions raise the following issues:

- *Social impacts of data mining:* With data mining penetrating our everyday lives, it is important to study the impact of data mining on society. How can we use data mining technology to benefit society? How can we guard against its misuse? The improper disclosure or use of data and the potential violation of individual privacy and data protection rights are areas of concern that need to be addressed.
- *Privacy-preserving data mining:* Data mining will help scientific discovery, business management, economy recovery, and security protection (e.g., the real-time discovery of intruders and cyberattacks). However, it poses the risk of disclosing an individual's personal information. Studies on privacy-preserving data publishing and data mining are ongoing. The philosophy is to observe data sensitivity and preserve people's privacy while performing successful data mining.

- *Invisible data mining:* We cannot expect everyone in society to learn and master data mining techniques. More and more systems should have data mining functions built within so that people can perform data mining or use data mining results simply by mouse clicking, without any knowledge of data mining algorithms. Intelligent search engines and Internet-based stores perform such *invisible data mining* by incorporating data mining into their components to improve their functionality and performance. This is done often unbeknownst to the user. For example, when purchasing items online, users may be unaware that the store is likely collecting data on the buying patterns of its customers, which may be used to recommend other items for purchase in the future.

These issues and many additional ones relating to the research, development, and application of data mining are discussed throughout the book.

1.8 Summary

- *Necessity is the mother of invention.* With the mounting growth of data in every application, data mining meets the imminent need for effective, scalable, and flexible data analysis in our society. Data mining can be considered as a natural evolution of information technology and a confluence of several related disciplines and application domains.
- **Data mining** is the process of discovering interesting patterns from massive amounts of data. As a *knowledge discovery process*, it typically involves data cleaning, data integration, data selection, data transformation, pattern discovery, pattern evaluation, and knowledge presentation.
- A pattern is *interesting* if it is valid on test data with some degree of certainty, novel, potentially useful (e.g., can be acted on or validates a hunch about which the user was curious), and easily understood by humans. Interesting patterns represent **knowledge**. Measures of **pattern interestingness**, either *objective* or *subjective*, can be used to guide the discovery process.
- We present a **multidimensional view** of data mining. The major dimensions are **data**, **knowledge**, **technologies**, and **applications**.
- Data mining can be conducted on any kind of **data** as long as the data are meaningful for a target application, such as database data, data warehouse data, transactional data, and advanced data types. Advanced data types include time-related or sequence data, data streams, spatial and spatiotemporal data, text and multimedia data, graph and networked data, and Web data.
- A **data warehouse** is a repository for long-term storage of data from multiple sources, organized so as to facilitate management decision making. The data are stored under a unified schema and are typically summarized. Data warehouse systems provide multidimensional data analysis capabilities, collectively referred to as **online analytical processing**.

2 Getting to Know Your Data

It's tempting to jump straight into mining, but first, we need to get the data ready. This involves having a closer look at attributes and data values. Real-world data are typically noisy, enormous in volume (often several gigabytes or more), and may originate from a hodgepodge of heterogenous sources. This chapter is about getting familiar with your data. Knowledge about your data is useful for data preprocessing (see Chapter 3), the first major task of the data mining process. You will want to know the following: What are the types of *attributes* or fields that make up your data? What kind of values does each attribute have? Which attributes are discrete, and which are continuous-valued? What do the data *look like*? How are the values distributed? Are there ways we can visualize the data to get a better sense of it all? Can we spot any outliers? Can we measure the similarity of some data objects with respect to others? Gaining such insight into the data will help with the subsequent analysis.

"*So what can we learn about our data that's helpful in data preprocessing?*" We begin in Section 2.1 by studying the various attribute types. These include nominal attributes, binary attributes, ordinal attributes, and numeric attributes. Basic *statistical descriptions* can be used to learn more about each attribute's values, as described in Section 2.2. Given a *temperature* attribute, for example, we can determine its **mean** (average value), **median** (middle value), and **mode** (most common value). These are **measures of central tendency**, which give us an idea of the "middle" or center of distribution.

Knowing such basic statistics regarding each attribute makes it easier to fill in missing values, smooth noisy values, and spot outliers during data preprocessing. Knowledge of the attributes and attribute values can also help in fixing inconsistencies incurred during data integration. Plotting the measures of central tendency shows us if the data are symmetric or skewed. Quantile plots, histograms, and scatter plots are other graphic displays of basic statistical descriptions. These can all be useful during data preprocessing and can provide insight into areas for mining.

The field of data visualization provides many additional techniques for viewing data through graphical means. These can help identify relations, trends, and biases "hidden" in unstructured data sets. Techniques may be as simple as scatter-plot matrices (where

two attributes are mapped onto a 2-D grid) to more sophisticated methods such as treemaps (where a hierarchical partitioning of the screen is displayed based on the attribute values). Data visualization techniques are described in Section 2.3.

Finally, we may want to examine how similar (or dissimilar) data objects are. For example, suppose we have a database where the data objects are patients, described by their symptoms. We may want to find the similarity or dissimilarity between individual patients. Such information can allow us to find clusters of like patients within the data set. The similarity/dissimilarity between objects may also be used to detect outliers in the data, or to perform nearest-neighbor classification. (Clustering is the topic of Chapters 10 and 11, while nearest-neighbor classification is discussed in Chapter 9.) There are many measures for assessing similarity and dissimilarity. In general, such measures are referred to as proximity measures. Think of the proximity of two objects as a function of the *distance* between their attribute values, although proximity can also be calculated based on probabilities rather than actual distance. Measures of data proximity are described in Section 2.4.

In summary, by the end of this chapter, you will know the different attribute types and basic statistical measures to describe the central tendency and dispersion (spread) of attribute data. You will also know techniques to visualize attribute distributions and how to compute the similarity or dissimilarity between objects.

2.1 Data Objects and Attribute Types

Data sets are made up of data objects. A **data object** represents an entity—in a sales database, the objects may be customers, store items, and sales; in a medical database, the objects may be patients; in a university database, the objects may be students, professors, and courses. Data objects are typically described by attributes. Data objects can also be referred to as *samples, examples, instances, data points*, or *objects*. If the data objects are stored in a database, they are *data tuples*. That is, the rows of a database correspond to the data objects, and the columns correspond to the attributes. In this section, we define attributes and look at the various attribute types.

2.1.1 What Is an Attribute?

An **attribute** is a data field, representing a characteristic or feature of a data object. The nouns *attribute*, *dimension*, *feature*, and *variable* are often used interchangeably in the literature. The term *dimension* is commonly used in data warehousing. Machine learning literature tends to use the term *feature*, while statisticians prefer the term *variable*. Data mining and database professionals commonly use the term *attribute*, and we do here as well. Attributes describing a customer object can include, for example, *customer_ID*, *name*, and *address*. Observed values for a given attribute are known as *observations*. A set of attributes used to describe a given object is called an *attribute vector* (or *feature vector*). The distribution of data involving one attribute (or variable) is called *univariate*. A *bivariate* distribution involves two attributes, and so on.

The **type** of an attribute is determined by the set of possible values—nominal, binary, ordinal, or numeric—the attribute can have. In the following subsections, we introduce each type.

2.1.2 Nominal Attributes

Nominal means "relating to names." The values of a **nominal attribute** are symbols or *names of things*. Each value represents some kind of category, code, or state, and so nominal attributes are also referred to as **categorical**. The values do not have any meaningful order. In computer science, the values are also known as *enumerations*.

Example 2.1 **Nominal attributes.** Suppose that *hair_color* and *marital_status* are two attributes describing *person* objects. In our application, possible values for *hair_color* are *black*, *brown*, *blond*, *red*, *auburn*, *gray*, and *white*. The attribute *marital_status* can take on the values *single, married, divorced*, and *widowed*. Both *hair_color* and *marital_status* are nominal attributes. Another example of a nominal attribute is *occupation*, with the values *teacher, dentist, programmer, farmer*, and so on. ■

Although we said that the values of a nominal attribute are symbols or "names of things," it is possible to represent such symbols or "names" with numbers. With *hair_color*, for instance, we can assign a code of 0 for *black*, 1 for *brown*, and so on. Another example is *customor_ID*, with possible values that are all numeric. However, in such cases, the numbers are not intended to be used quantitatively. That is, mathematical operations on values of nominal attributes are not meaningful. It makes no sense to subtract one customer ID number from another, unlike, say, subtracting an age value from another (where *age* is a numeric attribute). Even though a nominal attribute may have integers as values, it is not considered a numeric attribute because the integers are not meant to be used quantitatively. We will say more on numeric attributes in Section 2.1.5.

Because nominal attribute values do not have any meaningful order about them and are not quantitative, it makes no sense to find the mean (average) value or median (middle) value for such an attribute, given a set of objects. One thing that is of interest, however, is the attribute's most commonly occurring value. This value, known as the *mode*, is one of the measures of central tendency. You will learn about measures of central tendency in Section 2.2.

2.1.3 Binary Attributes

A **binary attribute** is a nominal attribute with only two categories or states: 0 or 1, where 0 typically means that the attribute is absent, and 1 means that it is present. Binary attributes are referred to as **Boolean** if the two states correspond to *true* and *false*.

Example 2.2 **Binary attributes.** Given the attribute *smoker* describing a *patient* object, 1 indicates that the patient smokes, while 0 indicates that the patient does not. Similarly, suppose

the patient undergoes a medical test that has two possible outcomes. The attribute *medical_test* is binary, where a value of 1 means the result of the test for the patient is positive, while 0 means the result is negative. ■

A binary attribute is **symmetric** if both of its states are equally valuable and carry the same weight; that is, there is no preference on which outcome should be coded as 0 or 1. One such example could be the attribute *gender* having the states *male* and *female*.

A binary attribute is **asymmetric** if the outcomes of the states are not equally important, such as the *positive* and *negative* outcomes of a medical test for HIV. By convention, we code the most important outcome, which is usually the rarest one, by 1 (e.g., *HIV positive*) and the other by 0 (e.g., *HIV negative*).

2.1.4 Ordinal Attributes

An **ordinal attribute** is an attribute with possible values that have a meaningful order or *ranking* among them, but the magnitude between successive values is not known.

Example 2.3 **Ordinal attributes.** Suppose that *drink_size* corresponds to the size of drinks available at a fast-food restaurant. This nominal attribute has three possible values: *small, medium*, and *large*. The values have a meaningful sequence (which corresponds to increasing drink size); however, we cannot tell from the values *how much* bigger, say, a medium is than a large. Other examples of ordinal attributes include *grade* (e.g., *A+*, *A*, *A−*, *B+*, and so on) and *professional_rank*. Professional ranks can be enumerated in a sequential order: for example, *assistant*, *associate*, and *full* for professors, and *private, private first class, specialist, corporal, and sergeant* for army ranks.

Ordinal attributes are useful for registering subjective assessments of qualities that cannot be measured objectively; thus ordinal attributes are often used in surveys for ratings. In one survey, participants were asked to rate how satisfied they were as customers. Customer satisfaction had the following ordinal categories: *0: very dissatisfied, 1: somewhat dissatisfied, 2: neutral, 3: satisfied*, and *4: very satisfied.* ■

Ordinal attributes may also be obtained from the discretization of numeric quantities by splitting the value range into a finite number of ordered categories as described in Chapter 3 on data reduction.

The central tendency of an ordinal attribute can be represented by its mode and its median (the middle value in an ordered sequence), but the mean cannot be defined.

Note that nominal, binary, and ordinal attributes are *qualitative*. That is, they *describe* a feature of an object without giving an actual size or quantity. The values of such qualitative attributes are typically words representing categories. If integers are used, they represent computer codes for the categories, as opposed to measurable quantities (e.g., 0 for *small* drink size, 1 for *medium*, and 2 for *large*). In the following subsection we look at numeric attributes, which provide *quantitative* measurements of an object.

2.1.5 Numeric Attributes

A **numeric attribute** is *quantitative*; that is, it is a measurable quantity, represented in integer or real values. Numeric attributes can be *interval-scaled* or *ratio-scaled*.

Interval-Scaled Attributes

Interval-scaled attributes are measured on a scale of equal-size units. The values of interval-scaled attributes have order and can be positive, 0, or negative. Thus, in addition to providing a ranking of values, such attributes allow us to compare and quantify the *difference* between values.

Example 2.4 **Interval-scaled attributes.** A *temperature* attribute is interval-scaled. Suppose that we have the outdoor *temperature* value for a number of different days, where each day is an object. By ordering the values, we obtain a ranking of the objects with respect to *temperature*. In addition, we can quantify the difference between values. For example, a temperature of 20°C is five degrees higher than a temperature of 15°C. Calendar dates are another example. For instance, the years 2002 and 2010 are eight years apart. ■

Temperatures in Celsius and Fahrenheit do not have a true zero-point, that is, neither 0°C nor 0°F indicates "no temperature." (On the Celsius scale, for example, the unit of measurement is 1/100 of the difference between the melting temperature and the boiling temperature of water in atmospheric pressure.) Although we can compute the *difference* between temperature values, we cannot talk of one temperature value as being a *multiple* of another. Without a true zero, we cannot say, for instance, that 10°C is twice as warm as 5°C. That is, we cannot speak of the values in terms of ratios. Similarly, there is no true zero-point for calendar dates. (The year 0 does not correspond to the beginning of time.) This brings us to ratio-scaled attributes, for which a true zero-point exits.

Because interval-scaled attributes are numeric, we can compute their mean value, in addition to the median and mode measures of central tendency.

Ratio-Scaled Attributes

A **ratio-scaled attribute** is a numeric attribute with an inherent zero-point. That is, if a measurement is ratio-scaled, we can speak of a value as being a multiple (or ratio) of another value. In addition, the values are ordered, and we can also compute the difference between values, as well as the mean, median, and mode.

Example 2.5 **Ratio-scaled attributes.** Unlike temperatures in Celsius and Fahrenheit, the Kelvin (K) temperature scale has what is considered a true zero-point ($0°K = -273.15°C$): It is the point at which the particles that comprise matter have zero kinetic energy. Other examples of ratio-scaled attributes include *count* attributes such as *years_of_experience* (e.g., the objects are employees) and *number_of_words* (e.g., the objects are documents). Additional examples include attributes to measure weight, height, latitude and longitude

coordinates (e.g., when clustering houses), and monetary quantities (e.g., you are 100 times richer with $100 than with $1). ■

2.1.6 Discrete versus Continuous Attributes

In our presentation, we have organized attributes into nominal, binary, ordinal, and numeric types. There are many ways to organize attribute types. The types are not mutually exclusive.

Classification algorithms developed from the field of machine learning often talk of attributes as being either *discrete* or *continuous*. Each type may be processed differently. A **discrete attribute** has a finite or countably infinite set of values, which may or may not be represented as integers. The attributes *hair_color*, *smoker*, *medical_test*, and *drink_size* each have a finite number of values, and so are discrete. Note that discrete attributes may have numeric values, such as 0 and 1 for binary attributes or, the values 0 to 110 for the attribute *age*. An attribute is *countably infinite* if the set of possible values is infinite but the values can be put in a one-to-one correspondence with natural numbers. For example, the attribute *customer_ID* is countably infinite. The number of customers can grow to infinity, but in reality, the actual set of values is countable (where the values can be put in one-to-one correspondence with the set of integers). Zip codes are another example.

If an attribute is not discrete, it is **continuous**. The terms *numeric attribute* and *continuous attribute* are often used interchangeably in the literature. (This can be confusing because, in the classic sense, continuous values are real numbers, whereas numeric values can be either integers or real numbers.) In practice, real values are represented using a finite number of digits. Continuous attributes are typically represented as floating-point variables.

2.2 Basic Statistical Descriptions of Data

For data preprocessing to be successful, it is essential to have an overall picture of your data. Basic statistical descriptions can be used to identify properties of the data and highlight which data values should be treated as noise or outliers.

This section discusses three areas of basic statistical descriptions. We start with *measures of central tendency* (Section 2.2.1), which measure the location of the middle or center of a data distribution. Intuitively speaking, given an attribute, where do most of its values fall? In particular, we discuss the mean, median, mode, and midrange.

In addition to assessing the central tendency of our data set, we also would like to have an idea of the *dispersion of the data*. That is, how are the data spread out? The most common data dispersion measures are the *range*, *quartiles*, and *interquartile range*; the *five-number summary* and *boxplots*; and the *variance* and *standard deviation* of the data These measures are useful for identifying outliers and are described in Section 2.2.2.

Finally, we can use many graphic displays of basic statistical descriptions to visually inspect our data (Section 2.2.3). Most statistical or graphical data presentation software

packages include bar charts, pie charts, and line graphs. Other popular displays of data summaries and distributions include *quantile plots*, *quantile–quantile plots*, *histograms*, and *scatter plots*.

2.2.1 Measuring the Central Tendency: Mean, Median, and Mode

In this section, we look at various ways to measure the central tendency of data. Suppose that we have some attribute X, like *salary*, which has been recorded for a set of objects. Let $x_1, x_2, \ldots, x_N$ be the set of N observed values or *observations* for X. Here, these values may also be referred to as the data set (for X). If we were to plot the observations for *salary*, where would most of the values fall? This gives us an idea of the central tendency of the data. Measures of central tendency include the mean, median, mode, and midrange.

The most common and effective numeric measure of the "center" of a set of data is the *(arithmetic) mean*. Let $x_1, x_2, \ldots, x_N$ be a set of N values or *observations*, such as for some numeric attribute X, like *salary*. The **mean** of this set of values is

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N} = \frac{x_1 + x_2 + \cdots + x_N}{N}. \tag{2.1}$$

This corresponds to the built-in aggregate function, *average* (**avg()** in SQL), provided in relational database systems.

Example 2.6 **Mean.** Suppose we have the following values for *salary* (in thousands of dollars), shown in increasing order: 30, 36, 47, 50, 52, 52, 56, 60, 63, 70, 70, 110. Using Eq. (2.1), we have

$$\bar{x} = \frac{30 + 36 + 47 + 50 + 52 + 52 + 56 + 60 + 63 + 70 + 70 + 110}{12}$$
$$= \frac{696}{12} = 58.$$

Thus, the mean salary is $58,000. ■

Sometimes, each value x_i in a set may be associated with a weight w_i for $i = 1, \ldots, N$. The weights reflect the significance, importance, or occurrence frequency attached to their respective values. In this case, we can compute

$$\bar{x} = \frac{\sum_{i=1}^{N} w_i x_i}{\sum_{i=1}^{N} w_i} = \frac{w_1 x_1 + w_2 x_2 + \cdots + w_N x_N}{w_1 + w_2 + \cdots + w_N}. \tag{2.2}$$

This is called the **weighted arithmetic mean** or the **weighted average**.

Although the mean is the singlemost useful quantity for describing a data set, it is not always the best way of measuring the center of the data. A major problem with the mean is its sensitivity to extreme (e.g., outlier) values. Even a small number of extreme values can corrupt the mean. For example, the mean salary at a company may be substantially pushed up by that of a few highly paid managers. Similarly, the mean score of a class in an exam could be pulled down quite a bit by a few very low scores. To offset the effect caused by a small number of extreme values, we can instead use the **trimmed mean**, which is the mean obtained after chopping off values at the high and low extremes. For example, we can sort the values observed for *salary* and remove the top and bottom 2% before computing the mean. We should avoid trimming too large a portion (such as 20%) at both ends, as this can result in the loss of valuable information.

For skewed (asymmetric) data, a better measure of the center of data is the **median**, which is the middle value in a set of ordered data values. It is the value that separates the higher half of a data set from the lower half.

In probability and statistics, the median generally applies to numeric data; however, we may extend the concept to ordinal data. Suppose that a given data set of N values for an attribute X is sorted in increasing order. If N is odd, then the median is the *middle value* of the ordered set. If N is even, then the median is not unique; it is the two middlemost values and any value in between. If X is a numeric attribute in this case, by convention, the median is taken as the average of the two middlemost values.

Example 2.7 **Median.** Let's find the median of the data from Example 2.6. The data are already sorted in increasing order. There is an even number of observations (i.e., 12); therefore, the median is not unique. It can be any value within the two middlemost values of 52 and 56 (that is, within the sixth and seventh values in the list). By convention, we assign the average of the two middlemost values as the median; that is, $\frac{52+56}{2} = \frac{108}{2} = 54$. Thus, the median is \$54,000.

Suppose that we had only the first 11 values in the list. Given an odd number of values, the median is the middlemost value. This is the sixth value in this list, which has a value of \$52,000. ■

The median is expensive to compute when we have a large number of observations. For numeric attributes, however, we can easily *approximate* the value. Assume that data are grouped in intervals according to their x_i data values and that the frequency (i.e., number of data values) of each interval is known. For example, employees may be grouped according to their annual salary in intervals such as \$10–20,000, \$20–30,000, and so on. Let the interval that contains the median frequency be the *median interval*. We can approximate the median of the entire data set (e.g., the median salary) by interpolation using the formula

$$median = L_1 + \left(\frac{N/2 - \left(\sum freq\right)_l}{freq_{median}} \right) width, \tag{2.3}$$

where L_1 is the lower boundary of the median interval, N is the number of values in the entire data set, $\left(\sum freq\right)_l$ is the sum of the frequencies of all of the intervals that are

lower than the median interval, $freq_{median}$ is the frequency of the median interval, and *width* is the width of the median interval.

The *mode* is another measure of central tendency. The **mode** for a set of data is the value that occurs most frequently in the set. Therefore, it can be determined for qualitative and quantitative attributes. It is possible for the greatest frequency to correspond to several different values, which results in more than one mode. Data sets with one, two, or three modes are respectively called **unimodal**, **bimodal**, and **trimodal**. In general, a data set with two or more modes is **multimodal**. At the other extreme, if each data value occurs only once, then there is no mode.

Example 2.8 **Mode.** The data from Example 2.6 are bimodal. The two modes are \$52,000 and \$70,000. ■

For unimodal numeric data that are moderately skewed (asymmetrical), we have the following empirical relation:

$$mean - mode \approx 3 \times (mean - median). \tag{2.4}$$

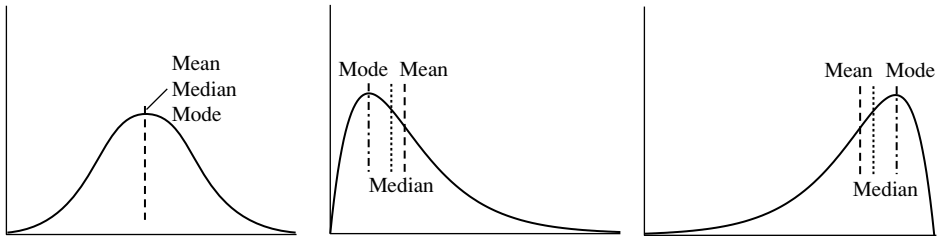
This implies that the mode for unimodal frequency curves that are moderately skewed can easily be approximated if the mean and median values are known.

The **midrange** can also be used to assess the central tendency of a numeric data set. It is the average of the largest and smallest values in the set. This measure is easy to compute using the SQL aggregate functions, max() and min().

Example 2.9 **Midrange.** The midrange of the data of Example 2.6 is $\frac{30{,}000+110{,}000}{2} = \$70{,}000$. ■

In a unimodal frequency curve with perfect **symmetric** data distribution, the mean, median, and mode are all at the same center value, as shown in Figure 2.1(a).

Data in most real applications are not symmetric. They may instead be either **positively skewed**, where the mode occurs at a value that is smaller than the median (Figure 2.1b), or **negatively skewed**, where the mode occurs at a value greater than the median (Figure 2.1c).



(a) Symmetric data **(b)** Positively skewed data **(c)** Negatively skewed data

Figure 2.1 Mean, median, and mode of symmetric versus positively and negatively skewed data.

2.2.2 Measuring the Dispersion of Data: Range, Quartiles, Variance, Standard Deviation, and Interquartile Range

We now look at measures to assess the dispersion or spread of numeric data. The measures include range, quantiles, quartiles, percentiles, and the interquartile range. The five-number summary, which can be displayed as a boxplot, is useful in identifying outliers. Variance and standard deviation also indicate the spread of a data distribution.

Range, Quartiles, and Interquartile Range

To start off, let's study the *range*, *quantiles*, *quartiles*, *percentiles*, and the *interquartile range* as measures of data dispersion.

Let $x_1, x_2, \ldots, x_N$ be a set of observations for some numeric attribute, X. The **range** of the set is the difference between the largest (max()) and smallest (min()) values.

Suppose that the data for attribute X are sorted in increasing numeric order. Imagine that we can pick certain data points so as to split the data distribution into equal-size consecutive sets, as in Figure 2.2. These data points are called *quantiles*. **Quantiles** are points taken at regular intervals of a data distribution, dividing it into essentially equal-size consecutive sets. (We say "essentially" because there may not be data values of X that divide the data into exactly equal-sized subsets. For readability, we will refer to them as equal.) The kth *q-quantile* for a given data distribution is the value x such that at most k/q of the data values are less than x and at most $(q-k)/q$ of the data values are more than x, where k is an integer such that $0 < k < q$. There are $q-1$ q-quantiles.

The 2-quantile is the data point dividing the lower and upper halves of the data distribution. It corresponds to the median. The 4-quantiles are the three data points that split the data distribution into four equal parts; each part represents one-fourth of the data distribution. They are more commonly referred to as **quartiles**. The 100-quantiles are more commonly referred to as **percentiles**; they divide the data distribution into 100 equal-sized consecutive sets. The median, quartiles, and percentiles are the most widely used forms of quantiles.

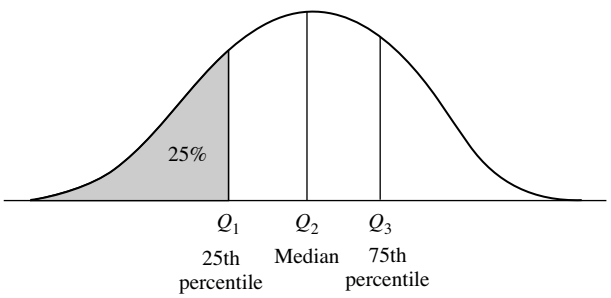


Figure 2.2 A plot of the data distribution for some attribute X. The quantiles plotted are quartiles. The three quartiles divide the distribution into four equal-size consecutive subsets. The second quartile corresponds to the median.

The quartiles give an indication of a distribution's center, spread, and shape. The **first quartile**, denoted by Q_1, is the 25th percentile. It cuts off the lowest 25% of the data. The **third quartile**, denoted by Q_3, is the 75th percentile—it cuts off the lowest 75% (or highest 25%) of the data. The second quartile is the 50th percentile. As the median, it gives the center of the data distribution.

The distance between the first and third quartiles is a simple measure of spread that gives the range covered by the middle half of the data. This distance is called the **interquartile range** (**IQR**) and is defined as

$$IQR = Q_3 - Q_1. \tag{2.5}$$

Example 2.10 **Interquartile range.** The quartiles are the three values that split the sorted data set into four equal parts. The data of Example 2.6 contain 12 observations, already sorted in increasing order. Thus, the quartiles for this data are the third, sixth, and ninth values, respectively, in the sorted list. Therefore, $Q_1 = \$47{,}000$ and Q_3 is $\$63{,}000$. Thus, the interquartile range is $IQR = 63 - 47 = \$16{,}000$. (Note that the sixth value is a median, $52,000, although this data set has two medians since the number of data values is even.) ■

Five-Number Summary, Boxplots, and Outliers

No single numeric measure of spread (e.g., *IQR*) is very useful for describing skewed distributions. Have a look at the symmetric and skewed data distributions of Figure 2.1. In the symmetric distribution, the median (and other measures of central tendency) splits the data into equal-size halves. This does not occur for skewed distributions. Therefore, it is more informative to also provide the two quartiles Q_1 and Q_3, along with the median. A common rule of thumb for identifying suspected **outliers** is to single out values falling at least $1.5 \times IQR$ above the third quartile or below the first quartile.

Because Q_1, the median, and Q_3 together contain no information about the endpoints (e.g., tails) of the data, a fuller summary of the shape of a distribution can be obtained by providing the lowest and highest data values as well. This is known as the *five-number summary*. The **five-number summary** of a distribution consists of the median (Q_2), the quartiles Q_1 and Q_3, and the smallest and largest individual observations, written in the order of *Minimum*, Q_1, *Median*, Q_3, *Maximum*.

Boxplots are a popular way of visualizing a distribution. A boxplot incorporates the five-number summary as follows:

- Typically, the ends of the box are at the quartiles so that the box length is the interquartile range.
- The median is marked by a line within the box.
- Two lines (called *whiskers*) outside the box extend to the smallest (*Minimum*) and largest (*Maximum*) observations.

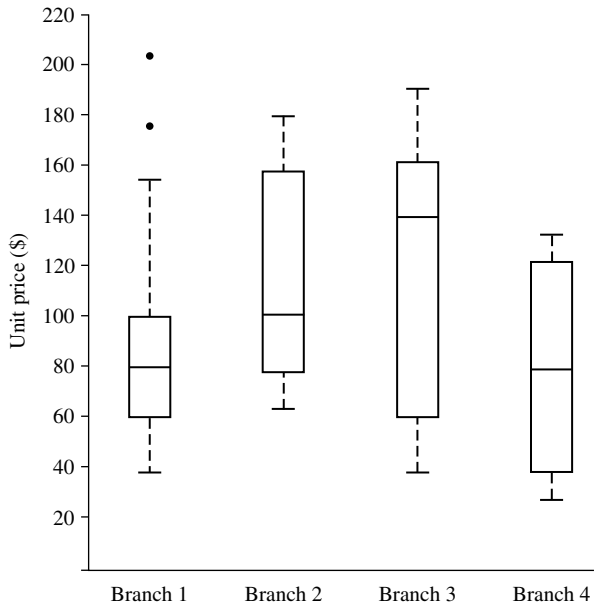


Figure 2.3 Boxplot for the unit price data for items sold at four branches of *AllElectronics* during a given time period.

When dealing with a moderate number of observations, it is worthwhile to plot potential outliers individually. To do this in a boxplot, the whiskers are extended to the extreme low and high observations *only if* these values are less than $1.5 \times IQR$ beyond the quartiles. Otherwise, the whiskers terminate at the most extreme observations occurring within $1.5 \times IQR$ of the quartiles. The remaining cases are plotted individually. Boxplots can be used in the comparisons of several sets of compatible data.

Example 2.11 **Boxplot.** Figure 2.3 shows boxplots for unit price data for items sold at four branches of *AllElectronics* during a given time period. For branch 1, we see that the median price of items sold is \$80, Q_1 is \$60, and Q_3 is \$100. Notice that two outlying observations for this branch were plotted individually, as their values of 175 and 202 are more than 1.5 times the IQR here of 40. ■

Boxplots can be computed in $O(n \log n)$ time. Approximate boxplots can be computed in linear or sublinear time depending on the quality guarantee required.

Variance and Standard Deviation

Variance and standard deviation are measures of data dispersion. They indicate how spread out a data distribution is. A low standard deviation means that the data observations tend to be very close to the mean, while a high standard deviation indicates that the data are spread out over a large range of values.

The **variance** of N observations, $x_1, x_2, \ldots, x_N$, for a numeric attribute X is

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2 = \left(\frac{1}{N}\sum_{i=1}^{N}x_i^2\right) - \bar{x}^2, \tag{2.6}$$

where $\bar{x}$ is the mean value of the observations, as defined in Eq. (2.1). The **standard deviation**, σ, of the observations is the square root of the variance, σ^2.

Example 2.12 **Variance and standard deviation.** In Example 2.6, we found $\bar{x} = \$58{,}000$ using Eq. (2.1) for the mean. To determine the variance and standard deviation of the data from that example, we set $N = 12$ and use Eq. (2.6) to obtain

$$\begin{aligned}\sigma^2 &= \frac{1}{12}(30^2 + 36^2 + 47^2 \ldots + 110^2) - 58^2 \\ &\approx 379.17 \\ \sigma &\approx \sqrt{379.17} \approx 19.47.\end{aligned}$$

■

The basic properties of the standard deviation, σ, as a measure of spread are as follows:

- σ measures spread about the mean and should be considered only when the mean is chosen as the measure of center.
- $\sigma = 0$ only when there is no spread, that is, when all observations have the same value. Otherwise, $\sigma > 0$.

Importantly, an observation is unlikely to be more than several standard deviations away from the mean. Mathematically, using Chebyshev's inequality, it can be shown that at least $\left(1 - \frac{1}{k^2}\right) \times 100\%$ of the observations are no more than k standard deviations from the mean. Therefore, the standard deviation is a good indicator of the spread of a data set.

The computation of the variance and standard deviation is scalable in large databases.

2.2.3 Graphic Displays of Basic Statistical Descriptions of Data

In this section, we study graphic displays of basic statistical descriptions. These include *quantile plots*, *quantile–quantile plots*, *histograms*, and *scatter plots*. Such graphs are helpful for the visual inspection of data, which is useful for data preprocessing. The first three of these show univariate distributions (i.e., data for one attribute), while scatter plots show bivariate distributions (i.e., involving two attributes).

Quantile Plot

In this and the following subsections, we cover common graphic displays of data distributions. A **quantile plot** is a simple and effective way to have a first look at a univariate data distribution. First, it displays all of the data for the given attribute (allowing the user

to assess both the overall behavior and unusual occurrences). Second, it plots quantile information (see Section 2.2.2). Let x_i, for $i = 1$ to N, be the data sorted in increasing order so that x_1 is the smallest observation and x_N is the largest for some ordinal or numeric attribute X. Each observation, x_i, is paired with a percentage, f_i, which indicates that approximately $f_i \times 100\%$ of the data are below the value, x_i. We say "approximately" because there may not be a value with exactly a fraction, f_i, of the data below x_i. Note that the 0.25 percentile corresponds to quartile Q_1, the 0.50 percentile is the median, and the 0.75 percentile is Q_3.

Let

$$f_i = \frac{i - 0.5}{N}. \tag{2.7}$$

These numbers increase in equal steps of $1/N$, ranging from $\frac{1}{2N}$ (which is slightly above 0) to $1 - \frac{1}{2N}$ (which is slightly below 1). On a quantile plot, x_i is graphed against f_i. This allows us to compare different distributions based on their quantiles. For example, given the quantile plots of sales data for two different time periods, we can compare their Q_1, median, Q_3, and other f_i values at a glance.

Example 2.13 **Quantile plot.** Figure 2.4 shows a quantile plot for the *unit price* data of Table 2.1. ■

Quantile–Quantile Plot

A **quantile–quantile plot**, or **q-q plot**, graphs the quantiles of one univariate distribution against the corresponding quantiles of another. It is a powerful visualization tool in that it allows the user to view whether there is a shift in going from one distribution to another.

Suppose that we have two sets of observations for the attribute or variable *unit price*, taken from two different branch locations. Let $x_1, \ldots, x_N$ be the data from the first branch, and $y_1, \ldots, y_M$ be the data from the second, where each data set is sorted in increasing order. If $M = N$ (i.e., the number of points in each set is the same), then we simply plot y_i against x_i, where y_i and x_i are both $(i - 0.5)/N$ quantiles of their respective data sets. If $M < N$ (i.e., the second branch has fewer observations than the first), there can be only M points on the q-q plot. Here, y_i is the $(i - 0.5)/M$ quantile of the y

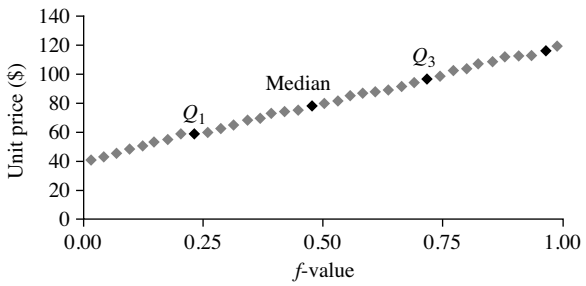


Figure 2.4 A quantile plot for the unit price data of Table 2.1.

Table 2.1 A Set of Unit Price Data for Items Sold at a Branch of *AllElectronics*

Unit price ($)	*Count of items sold*
40	275
43	300
47	250
—	—
74	360
75	515
78	540
—	—
115	320
117	270
120	350

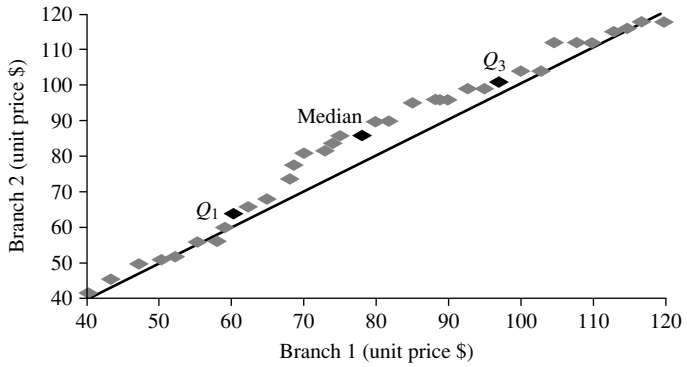


Figure 2.5 A q-q plot for unit price data from two *AllElectronics* branches.

data, which is plotted against the $(i - 0.5)/M$ quantile of the x data. This computation typically involves interpolation.

Example 2.14 **Quantile–quantile plot.** Figure 2.5 shows a quantile–quantile plot for *unit price* data of items sold at two branches of *AllElectronics* during a given time period. Each point corresponds to the same quantile for each data set and shows the unit price of items sold at branch 1 versus branch 2 for that quantile. (To aid in comparison, the straight line represents the case where, for each given quantile, the unit price at each branch is the same. The darker points correspond to the data for Q_1, the median, and Q_3, respectively.)

We see, for example, that at Q_1, the unit price of items sold at branch 1 was slightly less than that at branch 2. In other words, 25% of items sold at branch 1 were less than or

equal to \$60, while 25% of items sold at branch 2 were less than or equal to \$64. At the 50th percentile (marked by the median, which is also Q_2), we see that 50% of items sold at branch 1 were less than \$78, while 50% of items at branch 2 were less than \$85. In general, we note that there is a shift in the distribution of branch 1 with respect to branch 2 in that the unit prices of items sold at branch 1 tend to be lower than those at branch 2. ■

Histograms

Histograms (or **frequency histograms**) are at least a century old and are widely used. "Histos" means pole or mast, and "gram" means chart, so a histogram is a chart of poles. Plotting histograms is a graphical method for summarizing the distribution of a given attribute, X. If X is nominal, such as *automobile_model* or *item_type*, then a pole or vertical bar is drawn for each known value of X. The height of the bar indicates the frequency (i.e., count) of that X value. The resulting graph is more commonly known as a **bar chart**.

If X is numeric, the term *histogram* is preferred. The range of values for X is partitioned into disjoint consecutive subranges. The subranges, referred to as *buckets* or *bins*, are disjoint subsets of the data distribution for X. The range of a bucket is known as the **width**. Typically, the buckets are of equal width. For example, a *price* attribute with a value range of \$1 to \$200 (rounded up to the nearest dollar) can be partitioned into subranges 1 to 20, 21 to 40, 41 to 60, and so on. For each subrange, a bar is drawn with a height that represents the total count of items observed within the subrange. Histograms and partitioning rules are further discussed in Chapter 3 on data reduction.

Example 2.15 **Histogram.** Figure 2.6 shows a histogram for the data set of Table 2.1, where buckets (or bins) are defined by equal-width ranges representing \$20 increments and the frequency is the count of items sold. ■

Although histograms are widely used, they may not be as effective as the quantile plot, q-q plot, and boxplot methods in comparing groups of univariate observations.

Scatter Plots and Data Correlation

A **scatter plot** is one of the most effective graphical methods for determining if there appears to be a relationship, pattern, or trend between two numeric attributes. To construct a scatter plot, each pair of values is treated as a pair of coordinates in an algebraic sense and plotted as points in the plane. Figure 2.7 shows a scatter plot for the set of data in Table 2.1.

The scatter plot is a useful method for providing a first look at bivariate data to see clusters of points and outliers, or to explore the possibility of correlation relationships. Two attributes, X, and Y, are **correlated** if one attribute implies the other. Correlations can be positive, negative, or null (uncorrelated). Figure 2.8 shows examples of positive and negative correlations between two attributes. If the plotted points pattern slopes

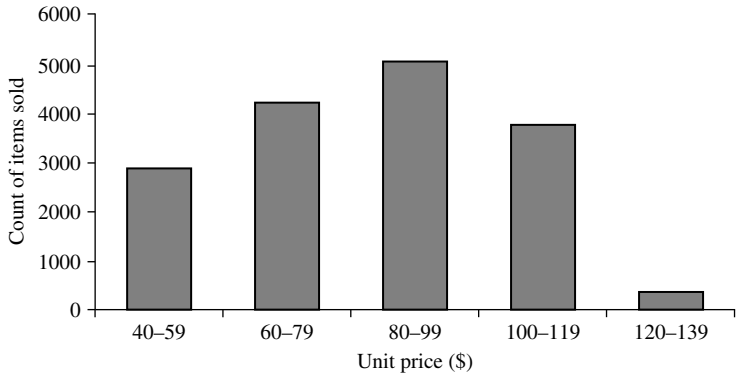


Figure 2.6 A histogram for the Table 2.1 data set.

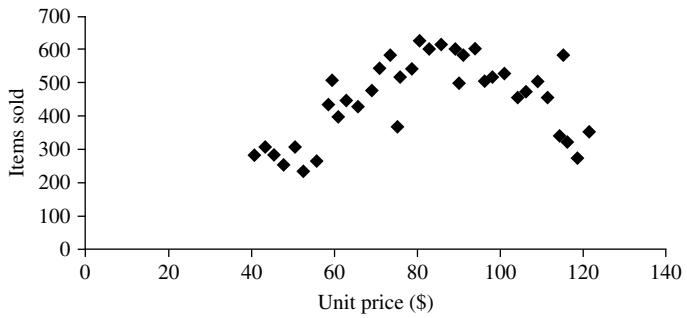


Figure 2.7 A scatter plot for the Table 2.1 data set.

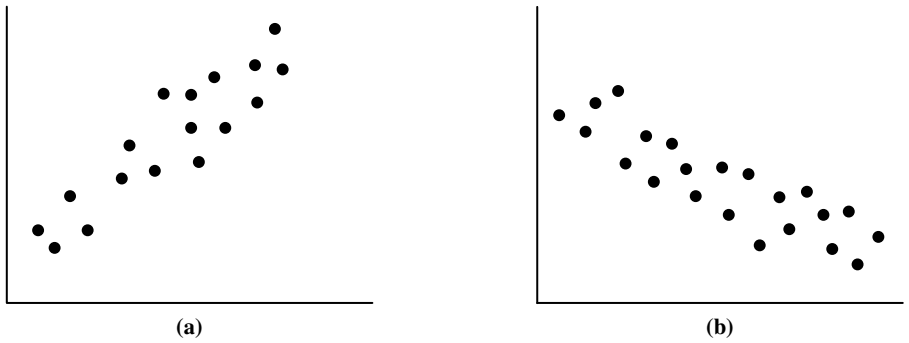


Figure 2.8 Scatter plots can be used to find (a) positive or (b) negative correlations between attributes.

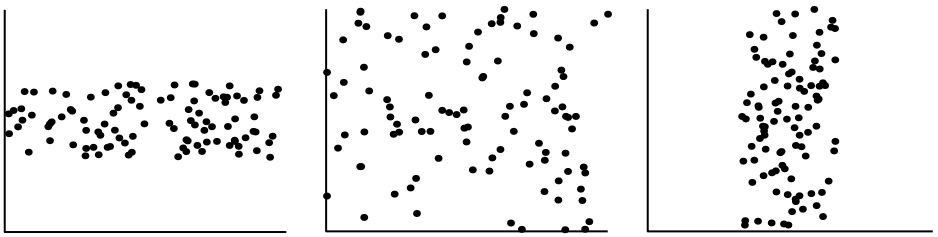


Figure 2.9 Three cases where there is no observed correlation between the two plotted attributes in each of the data sets.

from lower left to upper right, this means that the values of X increase as the values of Y increase, suggesting a *positive correlation* (Figure 2.8a). If the pattern of plotted points slopes from upper left to lower right, the values of X increase as the values of Y decrease, suggesting a *negative correlation* (Figure 2.8b). A line of best fit can be drawn to study the correlation between the variables. Statistical tests for correlation are given in Chapter 3 on data integration (Eq. (3.3)). Figure 2.9 shows three cases for which there is no correlation relationship between the two attributes in each of the given data sets. Section 2.3.2 shows how scatter plots can be extended to n attributes, resulting in a *scatter-plot matrix*.

In conclusion, basic data descriptions (e.g., measures of central tendency and measures of dispersion) and graphic statistical displays (e.g., quantile plots, histograms, and scatter plots) provide valuable insight into the overall behavior of your data. By helping to identify noise and outliers, they are especially useful for data cleaning.

2.3 Data Visualization

How can we convey data to users effectively? **Data visualization** aims to communicate data clearly and effectively through graphical representation. Data visualization has been used extensively in many applications—for example, at work for reporting, managing business operations, and tracking progress of tasks. More popularly, we can take advantage of visualization techniques to discover data relationships that are otherwise not easily observable by looking at the raw data. Nowadays, people also use data visualization to create fun and interesting graphics.

In this section, we briefly introduce the basic concepts of data visualization. We start with multidimensional data such as those stored in relational databases. We discuss several representative approaches, including pixel-oriented techniques, geometric projection techniques, icon-based techniques, and hierarchical and graph-based techniques. We then discuss the visualization of complex data and relations.

2.3.1 Pixel-Oriented Visualization Techniques

A simple way to visualize the value of a dimension is to use a pixel where the color of the pixel reflects the dimension's value. For a data set of m dimensions, **pixel-oriented techniques** create m windows on the screen, one for each dimension. The m dimension values of a record are mapped to m pixels at the corresponding positions in the windows. The colors of the pixels reflect the corresponding values.

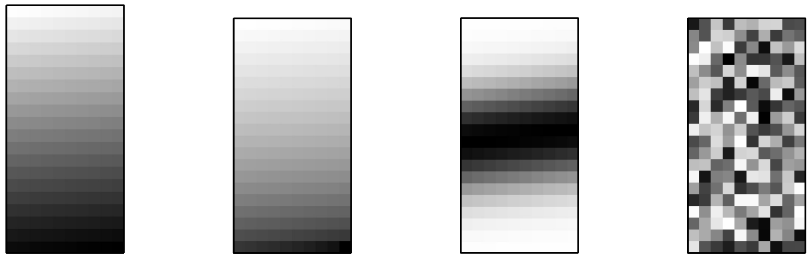
Inside a window, the data values are arranged in some global order shared by all windows. The global order may be obtained by sorting all data records in a way that's meaningful for the task at hand.

Example 2.16 **Pixel-oriented visualization.** *AllElectronics* maintains a customer information table, which consists of four dimensions: *income*, *credit_limit*, *transaction_volume*, and *age*. Can we analyze the correlation between *income* and the other attributes by visualization?

We can sort all customers in income-ascending order, and use this order to lay out the customer data in the four visualization windows, as shown in Figure 2.10. The pixel colors are chosen so that the smaller the value, the lighter the shading. Using pixel-based visualization, we can easily observe the following: *credit_limit* increases as *income* increases; customers whose income is in the middle range are more likely to purchase more from *AllElectronics*; there is no clear correlation between *income* and *age*. ■

In pixel-oriented techniques, data records can also be ordered in a query-dependent way. For example, given a point query, we can sort all records in descending order of similarity to the point query.

Filling a window by laying out the data records in a linear way may not work well for a wide window. The first pixel in a row is far away from the last pixel in the previous row, though they are next to each other in the global order. Moreover, a pixel is next to the one above it in the window, even though the two are not next to each other in the global order. To solve this problem, we can lay out the data records in a space-filling curve



(a) *income* **(b)** *credit_limit* **(c)** *transaction_volume* **(d)** *age*

Figure 2.10 Pixel-oriented visualization of four attributes by sorting all customers in *income* ascending order.

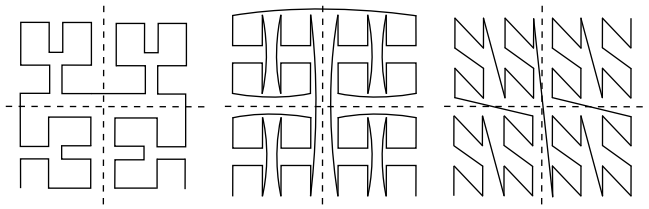


Figure 2.11 Some frequently used 2-D space-filling curves.

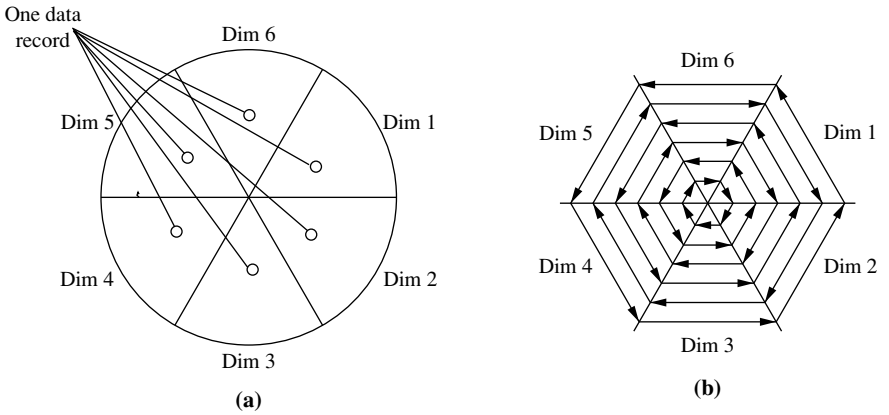


Figure 2.12 The circle segment technique. (a) Representing a data record in circle segments. (b) Laying out pixels in circle segments.

to fill the windows. A *space-filling curve* is a curve with a range that covers the entire n-dimensional unit hypercube. Since the visualization windows are 2-D, we can use any 2-D space-filling curve. Figure 2.11 shows some frequently used 2-D space-filling curves.

Note that the windows do not have to be rectangular. For example, the *circle segment technique* uses windows in the shape of segments of a circle, as illustrated in Figure 2.12. This technique can ease the comparison of dimensions because the dimension windows are located side by side and form a circle.

2.3.2 Geometric Projection Visualization Techniques

A drawback of pixel-oriented visualization techniques is that they cannot help us much in understanding the distribution of data in a multidimensional space. For example, they do not show whether there is a dense area in a multidimensional subspace. **Geometric**

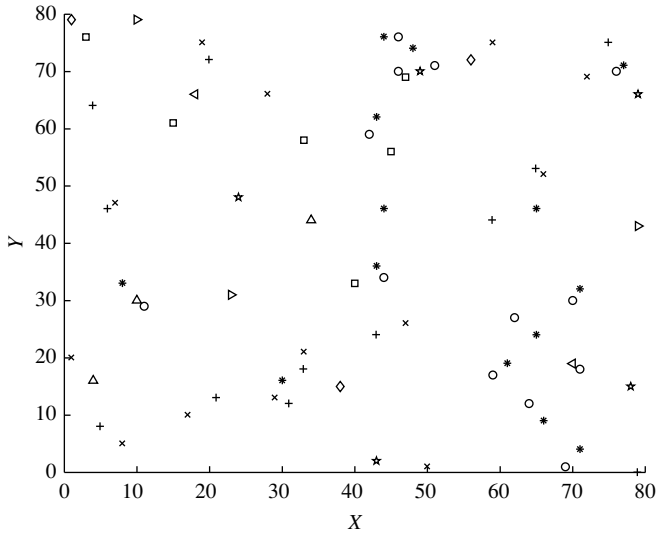


Figure 2.13 Visualization of a 2-D data set using a scatter plot. *Source: www.cs.sfu.ca/jpei/publications/rareevent-geoinformatica06.pdf*.

projection techniques help users find interesting projections of multidimensional data sets. The central challenge the geometric projection techniques try to address is how to visualize a high-dimensional space on a 2-D display.

A **scatter plot** displays 2-D data points using Cartesian coordinates. A third dimension can be added using different colors or shapes to represent different data points. Figure 2.13 shows an example, where X and Y are two spatial attributes and the third dimension is represented by different shapes. Through this visualization, we can see that points of types "+" and "×" tend to be colocated.

A 3-D scatter plot uses three axes in a Cartesian coordinate system. If it also uses color, it can display up to 4-D data points (Figure 2.14).

For data sets with more than four dimensions, scatter plots are usually ineffective. The **scatter-plot matrix** technique is a useful extension to the scatter plot. For an n-dimensional data set, a scatter-plot matrix is an $n \times n$ grid of 2-D scatter plots that provides a visualization of each dimension with every other dimension. Figure 2.15 shows an example, which visualizes the Iris data set. The data set consists of 450 samples from each of three species of Iris flowers. There are five dimensions in the data set: length and width of sepal and petal, and species.

The scatter-plot matrix becomes less effective as the dimensionality increases. Another popular technique, called parallel coordinates, can handle higher dimensionality. To visualize n-dimensional data points, the **parallel coordinates** technique draws n equally spaced axes, one for each dimension, parallel to one of the display axes.

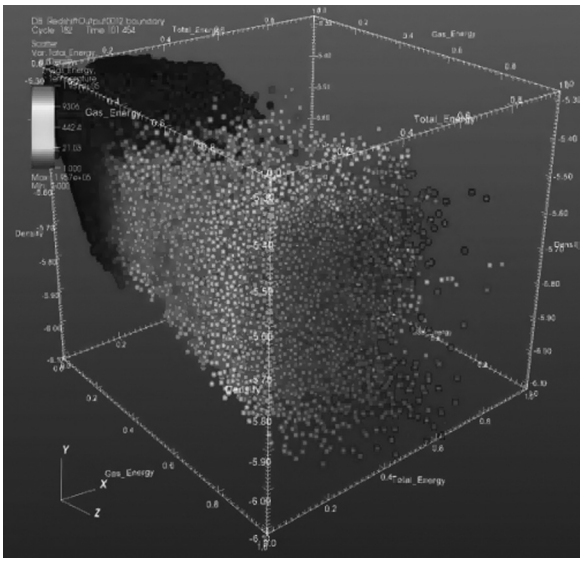


Figure 2.14 Visualization of a 3-D data set using a scatter plot. *Source: http://upload.wikimedia.org/wikipedia/commons/c/c4/Scatter_plot.jpg.*

A data record is represented by a polygonal line that intersects each axis at the point corresponding to the associated dimension value (Figure 2.16).

A major limitation of the parallel coordinates technique is that it cannot effectively show a data set of many records. Even for a data set of several thousand records, visual clutter and overlap often reduce the readability of the visualization and make the patterns hard to find.

2.3.3 Icon-Based Visualization Techniques

Icon-based visualization techniques use small icons to represent multidimensional data values. We look at two popular icon-based techniques: *Chernoff faces* and *stick figures*.

Chernoff faces were introduced in 1973 by statistician Herman Chernoff. They display multidimensional data of up to 18 variables (or dimensions) as a cartoon human face (Figure 2.17). Chernoff faces help reveal trends in the data. Components of the face, such as the eyes, ears, mouth, and nose, represent values of the dimensions by their shape, size, placement, and orientation. For example, dimensions can be mapped to the following facial characteristics: eye size, eye spacing, nose length, nose width, mouth curvature, mouth width, mouth openness, pupil size, eyebrow slant, eye eccentricity, and head eccentricity.

Chernoff faces make use of the ability of the human mind to recognize small differences in facial characteristics and to assimilate many facial characteristics at once.

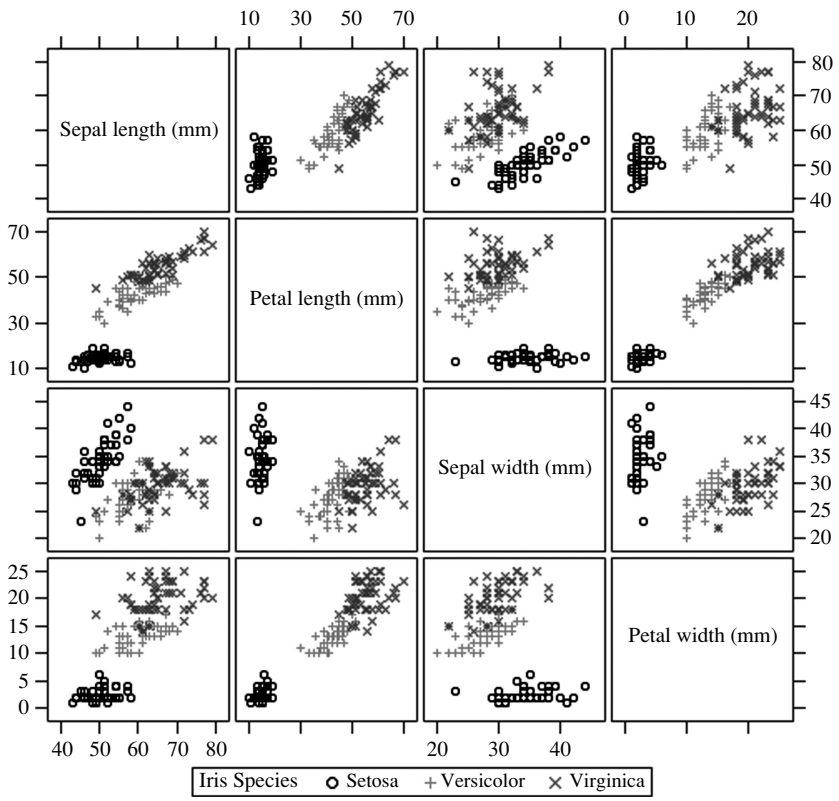


Figure 2.15 Visualization of the Iris data set using a scatter-plot matrix. *Source: http://support.sas.com/documentation/cdl/en/grstatproc/61948/HTML/default/images/gsgscmat.gif*.

Viewing large tables of data can be tedious. By condensing the data, Chernoff faces make the data easier for users to digest. In this way, they facilitate visualization of regularities and irregularities present in the data, although their power in relating multiple relationships is limited. Another limitation is that specific data values are not shown. Furthermore, facial features vary in perceived importance. This means that the similarity of two faces (representing two multidimensional data points) can vary depending on the order in which dimensions are assigned to facial characteristics. Therefore, this mapping should be carefully chosen. Eye size and eyebrow slant have been found to be important.

Asymmetrical Chernoff faces were proposed as an extension to the original technique. Since a face has vertical symmetry (along the y-axis), the left and right side of a face are identical, which wastes space. Asymmetrical Chernoff faces double the number of facial characteristics, thus allowing up to 36 dimensions to be displayed.

The **stick figure** visualization technique maps multidimensional data to five-piece stick figures, where each figure has four limbs and a body. Two dimensions are mapped to the display (x and y) axes and the remaining dimensions are mapped to the angle

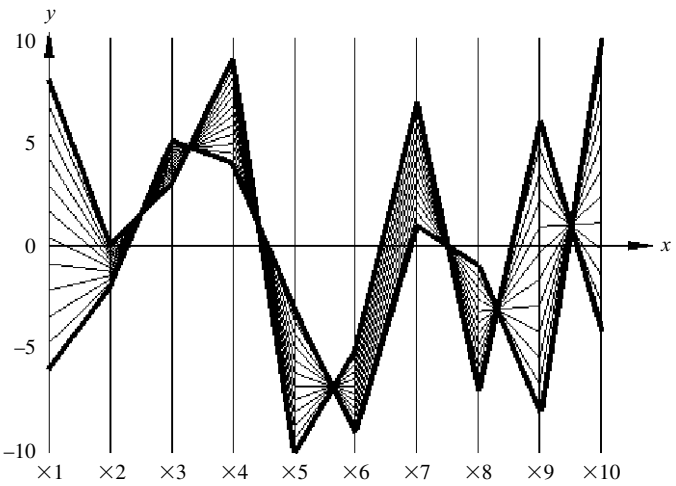


Figure 2.16 Here is a visualization that uses parallel coordinates. *Source: www.stat.columbia.edu/∼cook/movabletype/archives/2007/10/parallel_coordi.thml.*

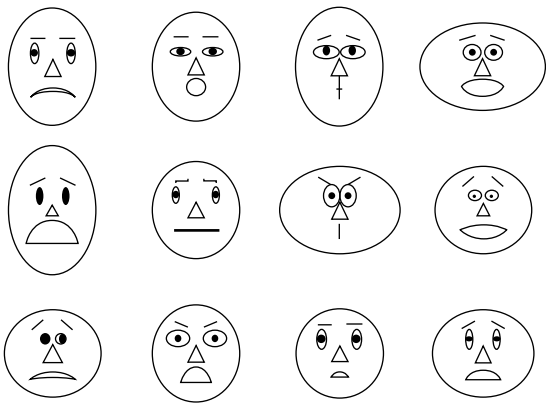


Figure 2.17 Chernoff faces. Each face represents an n-dimensional data point ($n \leq 18$).

and/or length of the limbs. Figure 2.18 shows census data, where *age* and *income* are mapped to the display axes, and the remaining dimensions (*gender, education*, and so on) are mapped to stick figures. If the data items are relatively dense with respect to the two display dimensions, the resulting visualization shows texture patterns, reflecting data trends.

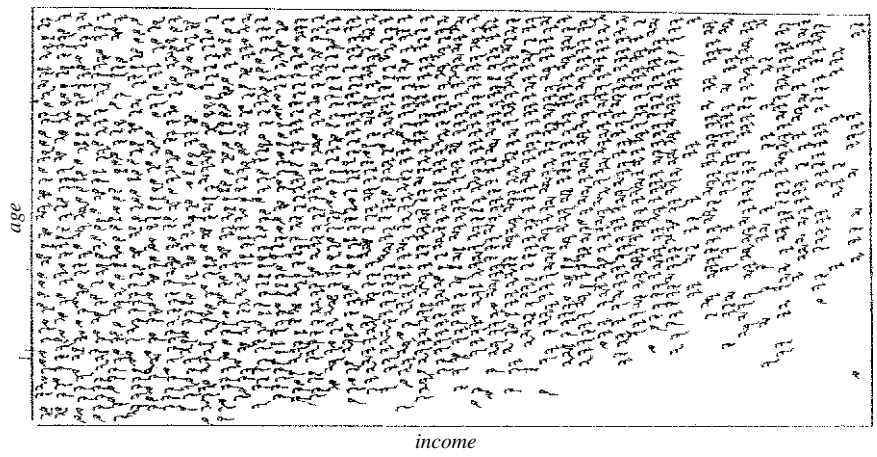


Figure 2.18 Census data represented using stick figures. *Source:* Professor G. Grinstein, Department of Computer Science, University of Massachusetts at Lowell.

2.3.4 Hierarchical Visualization Techniques

The visualization techniques discussed so far focus on visualizing multiple dimensions simultaneously. However, for a large data set of high dimensionality, it would be difficult to visualize all dimensions at the same time. **Hierarchical visualization techniques** partition all dimensions into subsets (i.e., subspaces). The subspaces are visualized in a hierarchical manner.

"Worlds-within-Worlds," also known as *n*-Vision, is a representative hierarchical visualization method. Suppose we want to visualize a 6-D data set, where the dimensions are $F, X_1, \ldots, X_5$. We want to observe how dimension F changes with respect to the other dimensions. We can first fix the values of dimensions X_3, X_4, X_5 to some selected values, say, c_3, c_4, c_5. We can then visualize F, X_1, X_2 using a 3-D plot, called a *world*, as shown in Figure 2.19. The position of the origin of the inner world is located at the point (c_3, c_4, c_5) in the outer world, which is another 3-D plot using dimensions X_3, X_4, X_5. A user can interactively change, in the outer world, the location of the origin of the inner world. The user then views the resulting changes of the inner world. Moreover, a user can vary the dimensions used in the inner world and the outer world. Given more dimensions, more levels of worlds can be used, which is why the method is called "worlds-within-worlds."

As another example of hierarchical visualization methods, **tree-maps** display hierarchical data as a set of nested rectangles. For example, Figure 2.20 shows a tree-map visualizing Google news stories. All news stories are organized into seven categories, each shown in a large rectangle of a unique color. Within each category (i.e., each rectangle at the top level), the news stories are further partitioned into smaller subcategories.

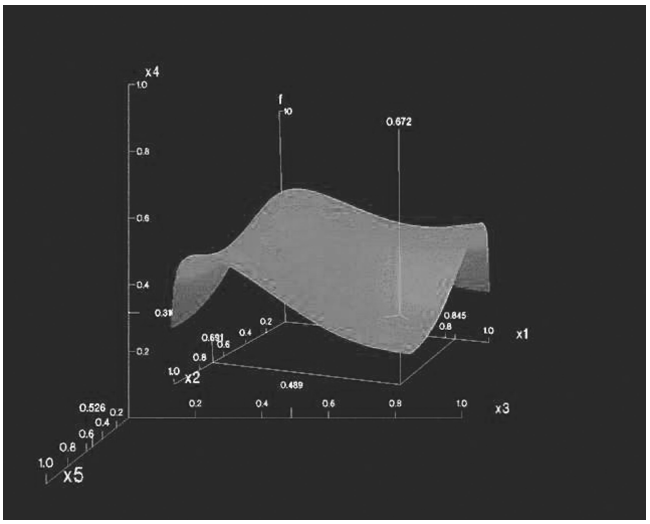


Figure 2.19 "Worlds-within-Worlds" (also known as *n*-Vision). *Source: http://graphics.cs.columbia.edu/projects/AutoVisual/images/1.dipstick.5.gif.*

2.3.5 Visualizing Complex Data and Relations

In early days, visualization techniques were mainly for numeric data. Recently, more and more non-numeric data, such as text and social networks, have become available. Visualizing and analyzing such data attracts a lot of interest.

There are many new visualization techniques dedicated to these kinds of data. For example, many people on the Web tag various objects such as pictures, blog entries, and product reviews. A **tag cloud** is a visualization of statistics of user-generated tags. Often, in a tag cloud, tags are listed alphabetically or in a user-preferred order. The importance of a tag is indicated by font size or color. Figure 2.21 shows a tag cloud for visualizing the popular tags used in a Web site.

Tag clouds are often used in two ways. First, in a tag cloud for a single item, we can use the size of a tag to represent the number of times that the tag is applied to this item by different users. Second, when visualizing the tag statistics on multiple items, we can use the size of a tag to represent the number of items that the tag has been applied to, that is, the popularity of the tag.

In addition to complex data, complex relations among data entries also raise challenges for visualization. For example, Figure 2.22 uses a disease influence graph to visualize the correlations between diseases. The nodes in the graph are diseases, and the size of each node is proportional to the prevalence of the corresponding disease. Two nodes are linked by an edge if the corresponding diseases have a strong correlation. The width of an edge is proportional to the strength of the correlation pattern of the two corresponding diseases.

Figure 2.20 Newsmap: Use of tree-maps to visualize Google news headline stories. *Source: www.cs.umd.edu/class/spring2005/cmsc838s/viz4all/ss/newsmap.png.*

In summary, visualization provides effective tools to explore data. We have introduced several popular methods and the essential ideas behind them. There are many existing tools and methods. Moreover, visualization can be used in data mining in various aspects. In addition to visualizing data, visualization can be used to represent the data mining process, the patterns obtained from a mining method, and user interaction with the data. Visual data mining is an important research and development direction.

2.4 Measuring Data Similarity and Dissimilarity

In data mining applications, such as clustering, outlier analysis, and nearest-neighbor classification, we need ways to assess how alike or unalike objects are in comparison to one another. For example, a store may want to search for clusters of *customer* objects, resulting in groups of customers with similar characteristics (e.g., similar income, area of residence, and age). Such information can then be used for marketing. A **cluster** is

animals architecture art asia australia autumn baby band barcelona beach berlin bike bird birds birthday black blackandwhite blue bw california canada canon car cat chicago china christmas church city clouds color concert cute dance day de dog england europe fall family fashion festival film florida flower flowers food football france friends fun garden geotagged germany girl girls graffiti green halloween hawaii holiday home house india iphone ireland island italia italy japan july kids la lake landscape light live london love macro me mexico model mountain mountains museum music nature new newyork newyorkcity night nikon nyc ocean old paris park party people photo photography photos portrait red river rock san sanfrancisco scotland sea seattle show sky snow spain spring street summer sun sunset taiwan texas thailand tokyo toronto tour travel tree trees trip uk urban usa vacation washington water wedding white winter yellow york zoo

Figure 2.21 Using a tag cloud to visualize popular Web site tags. *Source:* A snapshot of *www.flickr.com/photos/tags/*, January 23, 2010.

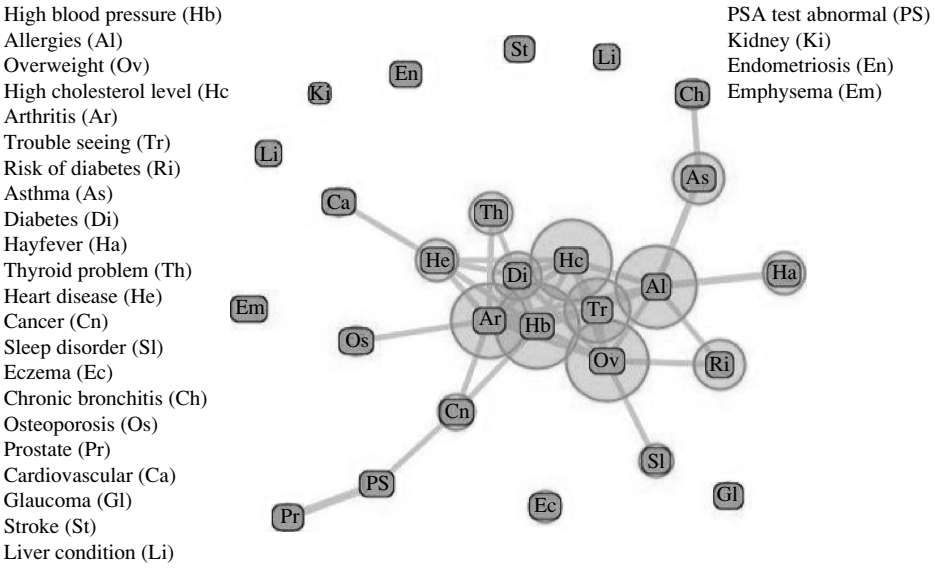


Figure 2.22 Disease influence graph of people at least 20 years old in the NHANES data set.

a collection of data objects such that the objects within a cluster are *similar* to one another and *dissimilar* to the objects in other clusters. Outlier analysis also employs clustering-based techniques to identify potential outliers as objects that are highly dissimilar to others. Knowledge of object similarities can also be used in nearest-neighbor classification schemes where a given object (e.g., a *patient*) is assigned a class label (relating to, say, a *diagnosis*) based on its similarity toward other objects in the model.

This section presents similarity and dissimilarity measures, which are referred to as measures of *proximity*. Similarity and dissimilarity are related. A similarity measure for two objects, i and j, will typically return the value 0 if the objects are unalike. The higher the similarity value, the greater the similarity between objects. (Typically, a value of 1 indicates complete similarity, that is, the objects are identical.) A dissimilarity measure works the opposite way. It returns a value of 0 if the objects are the same (and therefore, far from being dissimilar). The higher the dissimilarity value, the more dissimilar the two objects are.

In Section 2.4.1 we present two data structures that are commonly used in the above types of applications: the *data matrix* (used to store the data objects) and the *dissimilarity matrix* (used to store dissimilarity values for pairs of objects). We also switch to a different notation for data objects than previously used in this chapter since now we are dealing with objects described by more than one attribute. We then discuss how object dissimilarity can be computed for objects described by *nominal* attributes (Section 2.4.2), by *binary* attributes (Section 2.4.3), by *numeric* attributes (Section 2.4.4), by *ordinal* attributes (Section 2.4.5), or by combinations of these attribute types (Section 2.4.6). Section 2.4.7 provides similarity measures for very long and sparse data vectors, such as term-frequency vectors representing documents in information retrieval. Knowing how to compute dissimilarity is useful in studying attributes and will also be referenced in later topics on clustering (Chapters 10 and 11), outlier analysis (Chapter 12), and nearest-neighbor classification (Chapter 9).

2.4.1 Data Matrix versus Dissimilarity Matrix

In Section 2.2, we looked at ways of studying the central tendency, dispersion, and spread of observed values for some attribute X. Our objects there were one-dimensional, that is, described by a single attribute. In this section, we talk about objects described by *multiple* attributes. Therefore, we need a change in notation. Suppose that we have n objects (e.g., persons, items, or courses) described by p attributes (also called *measurements* or *features*, such as age, height, weight, or gender). The objects are $x_1 = (x_{11}, x_{12}, \ldots, x_{1p})$, $x_2 = (x_{21}, x_{22}, \ldots, x_{2p})$, and so on, where x_{ij} is the value for object x_i of the jth attribute. For brevity, we hereafter refer to object x_i as object i. The objects may be tuples in a relational database, and are also referred to as *data samples* or *feature vectors*.

Main memory-based clustering and nearest-neighbor algorithms typically operate on either of the following two data structures:

- **Data matrix** (or *object-by-attribute structure*): This structure stores the n data objects in the form of a relational table, or n-by-p matrix (n objects $\times p$ attributes):

$$\begin{bmatrix} x_{11} & \cdots & x_{1f} & \cdots & x_{1p} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{i1} & \cdots & x_{if} & \cdots & x_{ip} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{n1} & \cdots & x_{nf} & \cdots & x_{np} \end{bmatrix}. \tag{2.8}$$

Each row corresponds to an object. As part of our notation, we may use f to index through the p attributes.

- **Dissimilarity matrix** (or *object-by-object structure*): This structure stores a collection of proximities that are available for all pairs of n objects. It is often represented by an n-by-n table:

$$\begin{bmatrix} 0 & & & & \\ d(2,1) & 0 & & & \\ d(3,1) & d(3,2) & 0 & & \\ \vdots & \vdots & \vdots & & \\ d(n,1) & d(n,2) & \cdots & \cdots & 0 \end{bmatrix}, \tag{2.9}$$

where $d(i, j)$ is the measured **dissimilarity** or "difference" between objects i and j. In general, $d(i, j)$ is a non-negative number that is close to 0 when objects i and j are highly similar or "near" each other, and becomes larger the more they differ. Note that $d(i, i) = 0$; that is, the difference between an object and itself is 0. Furthermore, $d(i, j) = d(j, i)$. (For readability, we do not show the $d(j, i)$ entries; the matrix is symmetric.) Measures of dissimilarity are discussed throughout the remainder of this chapter.

Measures of similarity can often be expressed as a function of measures of dissimilarity. For example, for nominal data,

$$sim(i, j) = 1 - d(i, j), \tag{2.10}$$

where $sim(i, j)$ is the similarity between objects i and j. Throughout the rest of this chapter, we will also comment on measures of similarity.

A data matrix is made up of two entities or "things," namely rows (for objects) and columns (for attributes). Therefore, the data matrix is often called a **two-mode** matrix. The dissimilarity matrix contains one kind of entity (dissimilarities) and so is called a **one-mode** matrix. Many clustering and nearest-neighbor algorithms operate on a dissimilarity matrix. Data in the form of a data matrix can be transformed into a dissimilarity matrix before applying such algorithms.

2.4.2 Proximity Measures for Nominal Attributes

A nominal attribute can take on two or more states (Section 2.1.2). For example, *map_color* is a nominal attribute that may have, say, five states: *red, yellow, green, pink*, and *blue*.

Let the number of states of a nominal attribute be M. The states can be denoted by letters, symbols, or a set of integers, such as $1, 2, \ldots, M$. Notice that such integers are used just for data handling and do not represent any specific ordering.

"How is dissimilarity computed between objects described by nominal attributes?" The dissimilarity between two objects i and j can be computed based on the ratio of mismatches:

$$d(i, j) = \frac{p - m}{p}, \tag{2.11}$$

where m is the number of *matches* (i.e., the number of attributes for which i and j are in the same state), and p is the total number of attributes describing the objects. Weights can be assigned to increase the effect of m or to assign greater weight to the matches in attributes having a larger number of states.

Example 2.17 **Dissimilarity between nominal attributes.** Suppose that we have the sample data of Table 2.2, except that only the *object-identifier* and the attribute *test-1* are available, where *test-1* is nominal. (We will use *test-2* and *test-3* in later examples.) Let's compute the dissimilarity matrix (Eq. 2.9), that is,

$$\begin{bmatrix} 0 & & & \\ d(2,1) & 0 & & \\ d(3,1) & d(3,2) & 0 & \\ d(4,1) & d(4,2) & d(4,3) & 0 \end{bmatrix}.$$

Since here we have one nominal attribute, *test-1*, we set $p = 1$ in Eq. (2.11) so that $d(i, j)$ evaluates to 0 if objects i and j match, and 1 if the objects differ. Thus, we get

$$\begin{bmatrix} 0 & & & \\ 1 & 0 & & \\ 1 & 1 & 0 & \\ 0 & 1 & 1 & 0 \end{bmatrix}.$$

From this, we see that all objects are dissimilar except objects 1 and 4 (i.e., $d(4,1) = 0$). ■

Table 2.2 A Sample Data Table Containing Attributes of Mixed Type

Object Identifier	*test-1 (nominal)*	*test-2 (ordinal)*	*test-3 (numeric)*
1	code A	excellent	45
2	code B	fair	22
3	code C	good	64
4	code A	excellent	28

Alternatively, similarity can be computed as

$$sim(i, j) = 1 - d(i, j) = \frac{m}{p}. \tag{2.12}$$

Proximity between objects described by nominal attributes can be computed using an alternative encoding scheme. Nominal attributes can be encoded using asymmetric binary attributes by creating a new binary attribute for each of the M states. For an object with a given state value, the binary attribute representing that state is set to 1, while the remaining binary attributes are set to 0. For example, to encode the nominal attribute *map_color*, a binary attribute can be created for each of the five colors previously listed. For an object having the color *yellow*, the *yellow* attribute is set to 1, while the remaining four attributes are set to 0. Proximity measures for this form of encoding can be calculated using the methods discussed in the next subsection.

2.4.3 Proximity Measures for Binary Attributes

Let's look at dissimilarity and similarity measures for objects described by either *symmetric* or *asymmetric binary attributes*.

Recall that a binary attribute has only one of two states: 0 and 1, where 0 means that the attribute is absent, and 1 means that it is present (Section 2.1.3). Given the attribute *smoker* describing a patient, for instance, 1 indicates that the patient smokes, while 0 indicates that the patient does not. Treating binary attributes as if they are numeric can be misleading. Therefore, methods specific to binary data are necessary for computing dissimilarity.

"*So, how can we compute the dissimilarity between two binary attributes?*" One approach involves computing a dissimilarity matrix from the given binary data. If all binary attributes are thought of as having the same weight, we have the 2×2 contingency table of Table 2.3, where q is the number of attributes that equal 1 for both objects i and j, r is the number of attributes that equal 1 for object i but equal 0 for object j, s is the number of attributes that equal 0 for object i but equal 1 for object j, and t is the number of attributes that equal 0 for both objects i and j. The total number of attributes is p, where $p = q + r + s + t$.

Recall that for symmetric binary attributes, each state is equally valuable. Dissimilarity that is based on symmetric binary attributes is called **symmetric binary dissimilarity**. If objects i and j are described by symmetric binary attributes, then the

Table 2.3 Contingency Table for Binary Attributes

		Object j		
		1	0	sum
Object i	1	q	r	$q+r$
	0	s	t	$s+t$
	sum	$q+s$	$r+t$	p

dissimilarity between i and j is

$$d(i, j) = \frac{r + s}{q + r + s + t}. \tag{2.13}$$

For asymmetric binary attributes, the two states are not equally important, such as the *positive* (1) and *negative* (0) outcomes of a disease test. Given two asymmetric binary attributes, the agreement of two 1s (a positive match) is then considered more significant than that of two 0s (a negative match). Therefore, such binary attributes are often considered "monary" (having one state). The dissimilarity based on these attributes is called **asymmetric binary dissimilarity**, where the number of negative matches, t, is considered unimportant and is thus ignored in the following computation:

$$d(i, j) = \frac{r + s}{q + r + s}. \tag{2.14}$$

Complementarily, we can measure the difference between two binary attributes based on the notion of similarity instead of dissimilarity. For example, the **asymmetric binary similarity** between the objects i and j can be computed as

$$sim(i, j) = \frac{q}{q + r + s} = 1 - d(i, j). \tag{2.15}$$

The coefficient $sim(i, j)$ of Eq. (2.15) is called the **Jaccard coefficient** and is popularly referenced in the literature.

When both symmetric and asymmetric binary attributes occur in the same data set, the mixed attributes approach described in Section 2.4.6 can be applied.

Example 2.18 **Dissimilarity between binary attributes.** Suppose that a patient record table (Table 2.4) contains the attributes *name, gender, fever, cough, test-1, test-2, test-3*, and *test-4*, where *name* is an object identifier, *gender* is a symmetric attribute, and the remaining attributes are asymmetric binary.

For asymmetric attribute values, let the values Y (*yes*) and P (*positive*) be set to 1, and the value N (*no* or *negative*) be set to 0. Suppose that the distance between objects

Table 2.4 Relational Table Where Patients Are Described by Binary Attributes

name	*gender*	*fever*	*cough*	*test-1*	*test-2*	*test-3*	*test-4*
Jack	M	Y	N	P	N	N	N
Jim	M	Y	Y	N	N	N	N
Mary	F	Y	N	P	N	P	N
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

(patients) is computed based only on the asymmetric attributes. According to Eq. (2.14), the distance between each pair of the three patients—Jack, Mary, and Jim—is

$$d(Jack, Jim) = \frac{1+1}{1+1+1} = 0.67,$$

$$d(Jack, Mary) = \frac{0+1}{2+0+1} = 0.33,$$

$$d(Jim, Mary) = \frac{1+2}{1+1+2} = 0.75.$$

These measurements suggest that Jim and Mary are unlikely to have a similar disease because they have the highest dissimilarity value among the three pairs. Of the three patients, Jack and Mary are the most likely to have a similar disease. ■

2.4.4 Dissimilarity of Numeric Data: Minkowski Distance

In this section, we describe distance measures that are commonly used for computing the dissimilarity of objects described by numeric attributes. These measures include the *Euclidean, Manhattan*, and *Minkowski distances.*

In some cases, the data are normalized before applying distance calculations. This involves transforming the data to fall within a smaller or common range, such as $[-1, 1]$ or $[0.0, 1.0]$. Consider a *height* attribute, for example, which could be measured in either meters or inches. In general, expressing an attribute in smaller units will lead to a larger range for that attribute, and thus tend to give such attributes greater effect or "weight." Normalizing the data attempts to give all attributes an equal weight. It may or may not be useful in a particular application. Methods for normalizing data are discussed in detail in Chapter 3 on data preprocessing.

The most popular distance measure is **Euclidean distance** (i.e., straight line or "as the crow flies"). Let $i = (x_{i1}, x_{i2}, \ldots, x_{ip})$ and $j = (x_{j1}, x_{j2}, \ldots, x_{jp})$ be two objects described by p numeric attributes. The Euclidean distance between objects i and j is defined as

$$d(i, j) = \sqrt{(x_{i1} - x_{j1})^2 + (x_{i2} - x_{j2})^2 + \cdots + (x_{ip} - x_{jp})^2}. \tag{2.16}$$

Another well-known measure is the **Manhattan (or city block) distance**, named so because it is the distance in blocks between any two points in a city (such as 2 blocks down and 3 blocks over for a total of 5 blocks). It is defined as

$$d(i, j) = |x_{i1} - x_{j1}| + |x_{i2} - x_{j2}| + \cdots + |x_{ip} - x_{jp}|. \tag{2.17}$$

Both the Euclidean and the Manhattan distance satisfy the following mathematical properties:

Non-negativity: $d(i, j) \geq 0$: Distance is a non-negative number.

Identity of indiscernibles: $d(i, i) = 0$: The distance of an object to itself is 0.

Symmetry: $d(i, j) = d(j, i)$: Distance is a symmetric function.

Triangle inequality: $d(i, j) \leq d(i, k) + d(k, j)$: Going directly from object i to object j in space is no more than making a detour over any other object k.

A measure that satisfies these conditions is known as **metric**. Please note that the non-negativity property is implied by the other three properties.

Example 2.19 Euclidean distance and Manhattan distance. Let $\boldsymbol{x_1} = (1, 2)$ and $\boldsymbol{x_2} = (3, 5)$ represent two objects as shown in Figure 2.23. The Euclidean distance between the two is $\sqrt{2^2 + 3^2} = 3.61$. The Manhattan distance between the two is $2 + 3 = 5$. ■

Minkowski distance is a generalization of the Euclidean and Manhattan distances. It is defined as

$$d(i, j) = \sqrt[h]{|x_{i1} - x_{j1}|^h + |x_{i2} - x_{j2}|^h + \cdots + |x_{ip} - x_{jp}|^h}, \tag{2.18}$$

where h is a real number such that $h \geq 1$. (Such a distance is also called L_p **norm** in some literature, where the symbol p refers to our notation of h. We have kept p as the number of attributes to be consistent with the rest of this chapter.) It represents the Manhattan distance when $h = 1$ (i.e., L_1 norm) and Euclidean distance when $h = 2$ (i.e., L_2 norm).

The **supremum distance** (also referred to as $\boldsymbol{L_{max}}$, $\boldsymbol{L_\infty}$ **norm** and as the **Chebyshev distance**) is a generalization of the Minkowski distance for $h \to \infty$. To compute it, we find the attribute f that gives the maximum difference in values between the two objects. This difference is the supremum distance, defined more formally as:

$$d(i, j) = \lim_{h \to \infty} \left(\sum_{f=1}^{p} |x_{if} - x_{jf}|^h \right)^{\frac{1}{h}} = \max_{f}^{p} |x_{if} - x_{jf}|. \tag{2.19}$$

The L^∞ norm is also known as the *uniform norm*.

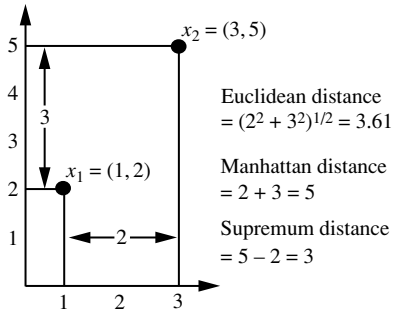


Figure 2.23 Euclidean, Manhattan, and supremum distances between two objects.

Example 2.20 **Supremum distance.** Let's use the same two objects, $\boldsymbol{x_1} = (1, 2)$ and $\boldsymbol{x_2} = (3, 5)$, as in Figure 2.23. The second attribute gives the greatest difference between values for the objects, which is $5 - 2 = 3$. This is the supremum distance between both objects. ■

If each attribute is assigned a weight according to its perceived importance, the **weighted Euclidean distance** can be computed as

$$d(i, j) = \sqrt{w_1|x_{i1} - x_{j1}|^2 + w_2|x_{i2} - x_{j2}|^2 + \cdots + w_m|x_{ip} - x_{jp}|^2}. \tag{2.20}$$

Weighting can also be applied to other distance measures as well.

2.4.5 Proximity Measures for Ordinal Attributes

The values of an ordinal attribute have a meaningful order or ranking about them, yet the magnitude between successive values is unknown (Section 2.1.4). An example includes the sequence *small*, *medium*, *large* for a *size* attribute. Ordinal attributes may also be obtained from the discretization of numeric attributes by splitting the value range into a finite number of categories. These categories are organized into ranks. That is, the range of a numeric attribute can be mapped to an ordinal attribute f having M_f states. For example, the range of the interval-scaled attribute *temperature* (in Celsius) can be organized into the following states: -30 to -10, -10 to 10, 10 to 30, representing the categories *cold temperature*, *moderate temperature*, and *warm temperature*, respectively. Let M represent the number of possible states that an ordinal attribute can have. These ordered states define the ranking $1, \ldots, M_f$.

"*How are ordinal attributes handled?*" The treatment of ordinal attributes is quite similar to that of numeric attributes when computing dissimilarity between objects. Suppose that f is an attribute from a set of ordinal attributes describing n objects. The dissimilarity computation with respect to f involves the following steps:

1. The value of f for the ith object is x_{if}, and f has M_f ordered states, representing the ranking $1, \ldots, M_f$. Replace each x_{if} by its corresponding rank, $r_{if} \in \{1, \ldots, M_f\}$.
2. Since each ordinal attribute can have a different number of states, it is often necessary to map the range of each attribute onto [0.0, 1.0] so that each attribute has equal weight. We perform such data normalization by replacing the rank r_{if} of the ith object in the fth attribute by

$$z_{if} = \frac{r_{if} - 1}{M_f - 1}. \tag{2.21}$$

3. Dissimilarity can then be computed using any of the distance measures described in Section 2.4.4 for numeric attributes, using z_{if} to represent the f value for the ith object.

Example 2.21 **Dissimilarity between ordinal attributes.** Suppose that we have the sample data shown earlier in Table 2.2, except that this time only the *object-identifier* and the continuous ordinal attribute, *test-2*, are available. There are three states for *test-2*: *fair*, *good*, and *excellent*, that is, $M_f = 3$. For step 1, if we replace each value for *test-2* by its rank, the four objects are assigned the ranks 3, 1, 2, and 3, respectively. Step 2 normalizes the ranking by mapping rank 1 to 0.0, rank 2 to 0.5, and rank 3 to 1.0. For step 3, we can use, say, the Euclidean distance (Eq. 2.16), which results in the following dissimilarity matrix:

$$\begin{bmatrix} 0 & & & \\ 1.0 & 0 & & \\ 0.5 & 0.5 & 0 & \\ 0 & 1.0 & 0.5 & 0 \end{bmatrix}.$$

Therefore, objects 1 and 2 are the most dissimilar, as are objects 2 and 4 (i.e., $d(2,1) = 1.0$ and $d(4,2) = 1.0$). This makes intuitive sense since objects 1 and 4 are both *excellent*. Object 2 is *fair*, which is at the opposite end of the range of values for *test-2*. ■

Similarity values for ordinal attributes can be interpreted from dissimilarity as $sim(i,j) = 1 - d(i,j)$.

2.4.6 Dissimilarity for Attributes of Mixed Types

Sections 2.4.2 through 2.4.5 discussed how to compute the dissimilarity between objects described by attributes of the same type, where these types may be either *nominal*, *symmetric binary*, *asymmetric binary*, *numeric*, or *ordinal*. However, in many real databases, objects are described by a *mixture* of attribute types. In general, a database can contain all of these attribute types.

"So, how can we compute the dissimilarity between objects of mixed attribute types?" One approach is to group each type of attribute together, performing separate data mining (e.g., clustering) analysis for each type. This is feasible if these analyses derive compatible results. However, in real applications, it is unlikely that a separate analysis per attribute type will generate compatible results.

A more preferable approach is to process all attribute types together, performing a single analysis. One such technique combines the different attributes into a single dissimilarity matrix, bringing all of the meaningful attributes onto a common scale of the interval [0.0, 1.0].

Suppose that the data set contains p attributes of mixed type. The dissimilarity $d(i, j)$ between objects i and j is defined as

$$d(i,j) = \frac{\sum_{f=1}^{p} \delta_{ij}^{(f)} d_{ij}^{(f)}}{\sum_{f=1}^{p} \delta_{ij}^{(f)}}, \tag{2.22}$$

where the indicator $\delta_{ij}^{(f)} = 0$ if either (1) x_{if} or x_{jf} is missing (i.e., there is no measurement of attribute f for object i or object j), or (2) $x_{if} = x_{jf} = 0$ and attribute f is asymmetric binary; otherwise, $\delta_{ij}^{(f)} = 1$. The contribution of attribute f to the dissimilarity between i and j (i.e., $d_{ij}^{(f)}$) is computed dependent on its type:

- If f is numeric: $d_{ij}^{(f)} = \frac{|x_{if} - x_{jf}|}{max_h x_{hf} - min_h x_{hf}}$, where h runs over all nonmissing objects for attribute f.
- If f is nominal or binary: $d_{ij}^{(f)} = 0$ if $x_{if} = x_{jf}$; otherwise, $d_{ij}^{(f)} = 1$.
- If f is ordinal: compute the ranks r_{if} and $z_{if} = \frac{r_{if} - 1}{M_f - 1}$, and treat z_{if} as numeric.

These steps are identical to what we have already seen for each of the individual attribute types. The only difference is for numeric attributes, where we normalize so that the values map to the interval [0.0, 1.0]. Thus, the dissimilarity between objects can be computed even when the attributes describing the objects are of different types.

Example 2.22 **Dissimilarity between attributes of mixed type.** Let's compute a dissimilarity matrix for the objects in Table 2.2. Now we will consider *all* of the attributes, which are of different types. In Examples 2.17 and 2.21, we worked out the dissimilarity matrices for each of the individual attributes. The procedures we followed for *test-1* (which is nominal) and *test-2* (which is ordinal) are the same as outlined earlier for processing attributes of mixed types. Therefore, we can use the dissimilarity matrices obtained for *test-1* and *test-2* later when we compute Eq. (2.22). First, however, we need to compute the dissimilarity matrix for the third attribute, *test-3* (which is numeric). That is, we must compute $d_{ij}^{(3)}$. Following the case for numeric attributes, we let $max_h x_h = 64$ and $min_h x_h = 22$. The difference between the two is used in Eq. (2.22) to normalize the values of the dissimilarity matrix. The resulting dissimilarity matrix for *test-3* is

$$\begin{bmatrix} 0 & & & \\ 0.55 & 0 & & \\ 0.45 & 1.00 & 0 & \\ 0.40 & 0.14 & 0.86 & 0 \end{bmatrix}.$$

We can now use the dissimilarity matrices for the three attributes in our computation of Eq. (2.22). The indicator $\delta_{ij}^{(f)} = 1$ for each of the three attributes, f. We get, for example, $d(3, 1) = \frac{1(1) + 1(0.50) + 1(0.45)}{3} = 0.65$. The resulting dissimilarity matrix obtained for the

data described by the three attributes of mixed types is:

$$\begin{bmatrix} 0 & & & \\ 0.85 & 0 & & \\ 0.65 & 0.83 & 0 & \\ 0.13 & 0.71 & 0.79 & 0 \end{bmatrix}.$$

From Table 2.2, we can intuitively guess that objects 1 and 4 are the most similar, based on their values for *test*-1 and *test*-2. This is confirmed by the dissimilarity matrix, where $d(4, 1)$ is the lowest value for any pair of different objects. Similarly, the matrix indicates that objects 1 and 2 are the least similar. ■

2.4.7 Cosine Similarity

A document can be represented by thousands of attributes, each recording the frequency of a particular word (such as a keyword) or phrase in the document. Thus, each document is an object represented by what is called a *term-frequency vector*. For example, in Table 2.5, we see that *Document1* contains five instances of the word *team*, while *hockey* occurs three times. The word *coach* is absent from the entire document, as indicated by a count value of 0. Such data can be highly asymmetric.

Term-frequency vectors are typically very long and **sparse** (i.e., they have many 0 values). Applications using such structures include information retrieval, text document clustering, biological taxonomy, and gene feature mapping. The traditional distance measures that we have studied in this chapter do not work well for such sparse numeric data. For example, two term-frequency vectors may have many 0 values in common, meaning that the corresponding documents do not share many words, but this does not make them similar. We need a measure that will focus on the words that the two documents *do* have in common, and the occurrence frequency of such words. In other words, we need a measure for numeric data that ignores zero-matches.

Cosine similarity is a measure of similarity that can be used to compare documents or, say, give a ranking of documents with respect to a given vector of query words. Let $\boldsymbol{x}$ and $\boldsymbol{y}$ be two vectors for comparison. Using the cosine measure as a

Table 2.5 Document Vector or Term-Frequency Vector

Document	*team*	*coach*	*hockey*	*baseball*	*soccer*	*penalty*	*score*	*win*	*loss*	*season*
Document1	5	0	3	0	2	0	0	2	0	0
Document2	3	0	2	0	1	1	0	1	0	1
Document3	0	7	0	2	1	0	0	3	0	0
Document4	0	1	0	0	1	2	2	0	3	0

similarity function, we have

$$sim(\boldsymbol{x}, \boldsymbol{y}) = \frac{\boldsymbol{x} \cdot \boldsymbol{y}}{||\boldsymbol{x}|| ||\boldsymbol{y}||}, \tag{2.23}$$

where $||\boldsymbol{x}||$ is the Euclidean norm of vector $\boldsymbol{x} = (x_1, x_2, \ldots, x_p)$, defined as $\sqrt{x_1^2 + x_2^2 + \cdots + x_p^2}$. Conceptually, it is the length of the vector. Similarly, $||\boldsymbol{y}||$ is the Euclidean norm of vector $\boldsymbol{y}$. The measure computes the cosine of the angle between vectors $\boldsymbol{x}$ and $\boldsymbol{y}$. A cosine value of 0 means that the two vectors are at 90 degrees to each other (orthogonal) and have no match. The closer the cosine value to 1, the smaller the angle and the greater the match between vectors. Note that because the cosine similarity measure does not obey all of the properties of Section 2.4.4 defining metric measures, it is referred to as a *nonmetric measure*.

Example 2.23 **Cosine similarity between two term-frequency vectors.** Suppose that $\boldsymbol{x}$ and $\boldsymbol{y}$ are the first two term-frequency vectors in Table 2.5. That is, $\boldsymbol{x} = (5, 0, 3, 0, 2, 0, 0, 2, 0, 0)$ and $\boldsymbol{y} = (3, 0, 2, 0, 1, 1, 0, 1, 0, 1)$. How similar are $\boldsymbol{x}$ and $\boldsymbol{y}$? Using Eq. (2.23) to compute the cosine similarity between the two vectors, we get:

$$\begin{aligned}
\boldsymbol{x}^t \cdot \boldsymbol{y} &= 5 \times 3 + 0 \times 0 + 3 \times 2 + 0 \times 0 + 2 \times 1 + 0 \times 1 + 0 \times 0 + 2 \times 1 \\
&\quad + 0 \times 0 + 0 \times 1 = 25 \\
||\boldsymbol{x}|| &= \sqrt{5^2 + 0^2 + 3^2 + 0^2 + 2^2 + 0^2 + 0^2 + 2^2 + 0^2 + 0^2} = 6.48 \\
||\boldsymbol{y}|| &= \sqrt{3^2 + 0^2 + 2^2 + 0^2 + 1^2 + 1^2 + 0^2 + 1^2 + 0^2 + 1^2} = 4.12 \\
sim(\boldsymbol{x}, \boldsymbol{y}) &= 0.94
\end{aligned}$$

Therefore, if we were using the cosine similarity measure to compare these documents, they would be considered quite similar. ■

When attributes are binary-valued, the cosine similarity function can be interpreted in terms of shared features or attributes. Suppose an object $\boldsymbol{x}$ possesses the ith attribute if $x_i = 1$. Then $\boldsymbol{x}^t \cdot \boldsymbol{y}$ is the number of attributes possessed (i.e., shared) by both $\boldsymbol{x}$ and $\boldsymbol{y}$, and $|\boldsymbol{x}||\boldsymbol{y}|$ is the *geometric mean* of the number of attributes possessed by $\boldsymbol{x}$ and the number possessed by $\boldsymbol{y}$. Thus, $sim(\boldsymbol{x}, \boldsymbol{y})$ is a measure of relative possession of common attributes.

A simple variation of cosine similarity for the preceding scenario is

$$sim(\boldsymbol{x}, \boldsymbol{y}) = \frac{\boldsymbol{x} \cdot \boldsymbol{y}}{\boldsymbol{x} \cdot \boldsymbol{x} + \boldsymbol{y} \cdot \boldsymbol{y} - \boldsymbol{x} \cdot \boldsymbol{y}}, \tag{2.24}$$

which is the ratio of the number of attributes shared by $\boldsymbol{x}$ and $\boldsymbol{y}$ to the number of attributes possessed by $\boldsymbol{x}$ or $\boldsymbol{y}$. This function, known as the **Tanimoto coefficient** or **Tanimoto distance**, is frequently used in information retrieval and biology taxonomy.