A Separation Principle for Dynamic Positioning of Ships: Theoretical and Experimental Results

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***Abstract*—This paper presents a globally asymptotically stabilizing (GAS) controller for regulation and dynamic positioning of ships, using only position measurements. It is assumed that these are corrupted with white noise hence a passive observer which reconstructs the rest of the states is applied. The observer produces noise-free estimates of the position, the slowly varying environmental disturbances and the velocity which are used in a proportional-derivative (PD)-type control law. The stability proof is based on a separation principle which holds for the nonlinear ship model in this paper. This separation principle is theoretically supported by recent results on cascaded nonlinear systems and standard Lyapunov theory, and it is validated in practice by experimentation with a model ship scale 1 : 70.**

***Index Terms*—Cascade systems, global positioning system, marine vehicles, mechanical systems, output feedback, separation principle.**

I. INTRODUCTION

THIS PAPER contributes to the design of nonlinear globally asymptotically stabilizing (GAS) *output* feedback controllers for dynamic positioning (DP) of ships. A DP system is defined as a ship control system where the ship position is controlled exclusively by means of thrusters and main propellers aft of the ship. Different type of thruster devices are available like tunnel thrusters which can produce thrust in the sideway direction and azimuth thrusters which are usually mounted under the hull of the ship. The azimuth thrusters can be rotated 360° and therefore produce thrust in any direction on the xy-plane. This is convenient since the environmental loads due to wind, waves, and currents change over time both in direction and magnitude, see [4] for details.

Since, only position and attitude sensors are available for most commercial ships emphasis is placed on output feedback control where the velocities and disturbances are estimated by using a nonlinear state observer. The passive nonlinear ship observer of [6] is applied for this purpose. The main reason for using a passive observer instead of a conventional extended Kalman filter (as it is commonly used), is that passivity arguments simplify the tuning of the observer. In addition, concerning the control system performance, *global* exponential stability (GES) has been proven for the passive observer while only local exponential stability (LES) has been proven for the extended Kalman filter [3], [8].

This problem has also been addressed in [5] where a vectorial output-feedback backstepping controller was applied, based on the assumption that the slowly varying disturbances due to wind, waves, and currents, could be neglected. However, the latter is very restrictive in a practical system and therefore, the environmental disturbances must be compensated; a common practice is to add an integral action to the controller. As far as we know, PI control was started by N. Minorsky in marine applications and dates back to 1922. A recent contribution to the dynamic ship positioning using integral action is presented in [1].

In this paper we show both, theoretically and experimentally, that the integral action is *not* needed. Instead, a globally converging observer and a simple PD-type control law plus a nonlinear term of observer bias *estimates* is used to compensate for slowly varying and constant environmental disturbances. We first design the control law as if the whole state were available for measurement and free of noise. Then, we implement the control law with the state *estimates*. The stability proof of this approach requires that the *separation principle* hold for nonlinear systems. Some work in this direction has been done recently for local stabilization of input–output linearizable systems [2] and for the case of nonaffine systems in [9]. In this paper we show for the ship nonlinear model that a separation principle also holds and moreover, *global* asymptotic stability can be achieved.

It is clear that our main goal in this paper is to solve the dynamic ship positioning problem and not to enunciate a new separation principle for nonlinear systems. However, in contrast to the approaches of [2] and [9], due to the passivity properties of the ship model neither high gains nor bounded feedbacks are required. Particularly, the high-gain is undesirable for practical applications. From a theoretical point of view we rely on recent Lyapunov theorems on stability of cascaded time-varying systems [11] to prove our claims.

Organization of This Paper: The rest of Section I is devoted to the problem statement and the definition of the ship model. In Section II we provide a simple global asymptotic stability proof for a state-feedback controller. In Section III we briefly recall the main features of the observer design proposed in [6]. In Section IV we show that the ship model in closed-loop with an output feedback controller and the observer has a convenient *cascaded* structure, this allows to apply a recent theorem on Lyapunov stability to show that a separation principle holds for this system. The output-feedback control law and the nonlinear observer have been implemented on a model ship (*CyberShip I*) scale 1 : 70 and tested out in a model basin at the Department of

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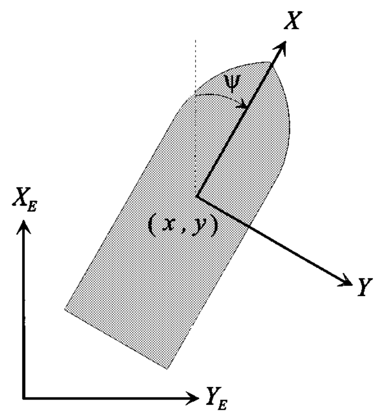

Fig. 1. Earth-fixed and vessel-fixed coordinate frames.

Engineering Cybernetics, NTNU. The results of this experiment are summarized in Section V and some concluding remarks are given at the end of the paper.

A. A Lagrangian Ship Model

Let the position (x, y) and heading ψ of the vessel relative to an Earth-fixed frame be expressed in vector form by $\eta = [x, y, \psi]^\top$, and let the velocities decomposed in a vessel-fixed reference frame be $\nu = [u, v, r]^\top$. These three modes are referred to as the *surge*, *sway*, and *yaw* modes of a ship. The origin of the vessel-fixed frame is located at the vessel center line in a distance x_G from the center of gravity.

All ships are *metacentric stable* which implies that there exist restoring moments in roll and pitch. Since the roll and pitch angles are zero in mean, it is a common assumption to assume that the motions induced by the rolling and pitching of the ship is negligible when considering DP applications. Therefore the transformation between the vessel-fixed and the Earth-fixed velocity vectors (see Fig. 1) is

$$\dot{\eta} = J(\psi)\nu \tag{1}$$

where the yaw angle rotation matrix is defined as

$$J(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

which is orthogonal, that is $J^{-1}(\psi) = J^\top(\psi)$.

With these considerations, the low-frequency (LF) motion of a large class of surface ships can be described by the following Lagrangian model [4]:

$$M\dot{\nu} + D\nu = \tau + J^\top(\psi)b \tag{3}$$
$$\tau = B_u u. \tag{4}$$

Here $\tau \in \mathbb{R}^3$ is a control vector of forces and moment provided by the propulsion system, that is main propellers aft of the ship and thrusters which can produce surge and sway forces as well as a yaw moment. The control inputs are denoted by $u \in \mathbb{R}^r$ $(r \geq 3)$ and $B_u \in \mathbb{R}^{3\times r}$ is a constant matrix describing the actuator configuration.

For DP applications a small *Froude number* can be assumed. Hence, the inertia matrix $M \in \mathbb{R}^{3\times 3}$ which includes hydrodynamic added inertia, is positive definite. For control applications which are restricted to low-frequency motions such as dynamic positioning, wave frequency independence of added inertia can be assumed. This implies that $\dot{M} \equiv 0$.

For conventional ships, the eigenvalues of the *damping* matrix $D \in \mathbb{R}^{3\times 3}$ are all strictly positive due to the dissipative structure of wave drift damping and laminar skin friction. In general the damping forces will be nonlinear. However, for DP and cruising at constant speed linear damping is a good assumption. For more details the reader is suggested to consult [4].

B. Environmental Disturbances

Unmodeled external forces and moment due to wind, currents, and waves are lumped together into an Earth-fixed constant (or slowly varying) bias term $b \in \mathbb{R}^3$. It is assumed that the bias forces in surge and sway, and the yaw moment are slowly varying. A frequently used bias model for marine control applications is the *first-order Markov process*

$$\dot{b} = -T^{-1}b + \Psi n \tag{5}$$

where

$b \in \mathbb{R}^3$	vector of bias forces and moment;
$n \in \mathbb{R}^3$	vector of zero-mean Gaussian white noise;
$T \in \mathbb{R}^{3\times 3}$	diagonal matrix of positive bias time constants;
$\Psi \in \mathbb{R}^{3\times 3}$	diagonal matrix scaling the amplitude of n.

This model can be used to describe slowly varying environmental forces and moments due to second-order wave drift, ocean currents, wind, and unmodeled dynamics [4].

A linear wave frequency (WF) model of order p can in general be expressed as

$$\dot{\xi} = \Omega\xi + \Sigma w \tag{6}$$

$$\eta_w = \Gamma\xi \tag{7}$$

where $\xi \in \mathbb{R}^{3\times p}$, $w \in \mathbb{R}^3$, and Ω, Σ, and Γ are constant matrices of appropriate dimensions. The WF response of the ship is generated by using the principle of linear superposition, that is the total ship motion is the sum of the LF-motion components and the WF-motion components. It is assumed that the WF model excitations w in (6) are zero-mean Gaussian white noise, see Fig. 2.

The wave frequency (WF) motion is modeled as a second-order system which is the most frequently used approximation for the linear wave spectrum. For each of the three degrees of freedom we have in the Laplace domain

$$\xi_i(s) = \left\{ \frac{\sigma_i s}{s^2 + 2\zeta_i\omega_{0i}s + \omega_{0i}^2} \right\} w(s) \tag{8}$$

where

ω_{0i} $(i = 1 \cdots 3)$	dominating wave frequency;
ζ_i $(i = 1 \cdots 3)$	relative damping ratio;
σ_i $(i = 1 \cdots 3)$	parameter related to the wave intensity.

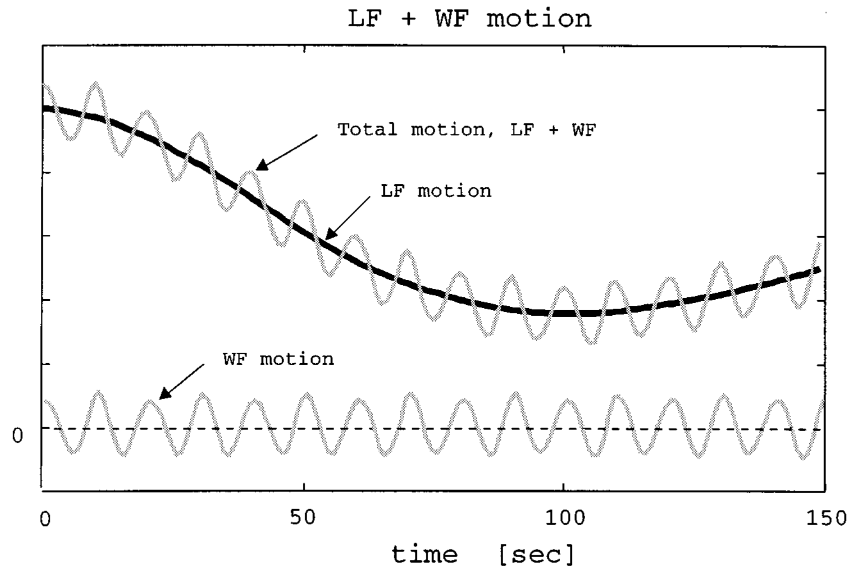

Fig. 2. The total ship motion as the sum of the LF-motion and the WF-motion. Notice that the first-order WF motion is oscillatory.

C. Problem Formulation

For conventional ships only position and heading measurements are available. Several position measurement systems are commercially available, such as hydro-acoustic positioning reference (HPR) systems and differential global positioning systems (DGPS). The two commercial available satellite systems are the American Navstar GPS and the Russian GLONASS. Unfortunately the accuracy of the GPS satellite navigation systems is degraded for civilian users and this results in root-mean-squares errors of about 50 m for the horizontal positions while DGPS has an accuracy of 1 m. Conventional dynamic positioning (DP) systems for ships and offshore rigs therefore use DGPS since meter accuracy is necessary during offshore operations and station-keeping. This is also the case for ship way-point tracking and maneuvering systems, see Fossen [4]. Ship navigation systems are also discussed in Genrich and Minster [7].

The main reason that the DGPS solution is preferred to the Kinematic GPS (KGPS) solution is reliability. This is reflected through the requirements of the classification societies Lloyds, Det norske Veritas (DnV), ABS, etc., where DGPS is a requirement to obtain a so-called "DP Class" for offshore operation of the vessel. However, there is a great need for more accurate solutions like KGPS since centimeter accuracy is needed for automatic berthing of ships and for safe maneuvering in shallow and confined waters. This is also the case for high-speed craft operating in narrow fjords. These solutions will, however, not been used offshore until safety and reliability have been improved and documented such that problems due to phase slip, reflections, etc., are solved.

The heading of the vessel is usually measured by using a gyro compass where gyro drift can be compensated for by using a magnetic compass. The accuracy of a gyro compass is typically in the magnitude of 0.1°.

Thus, the *only* measurable state variables are the position and heading which are contaminated with noise, that is,

$$y = \eta + \eta_w + v \tag{9}$$

where

η_w vessel's WF motion due to first-order wave-induced disturbances;

$v \in \mathbb{R}^3$ zero-mean Gaussian white measurement noise, see Fig. 2.

It is therefore important that the observer be tuned in such a way that the its gains reflect the differences in the noise levels of the measured signals.

Definition 1—Wave Filtering: Wave filtering can be defined as the reconstruction of the LF motion components η from the noisy measurement $y = \eta + \eta_w + v$ by means of an observer (state estimator). In addition to this, a noise-free estimate of the LF velocity ν should be produced from y. This is crucial in ship motion control systems since the oscillatory motion η_w due to first-order wave-induced disturbances will, if it enters the feedback loop, cause wear and tear of the actuators and increase the fuel consumption. □

For purposes of the stability analysis of the closed-loop dynamics, the following assumptions are made.

A1) The inertia matrix $M = M^\top > 0$ and $\dot{M} \equiv 0$ holds. This is an unrestrictive assumption for most low-speed applications [4].

A2) $n = w = 0$. These terms are omitted in the observer stability analysis since the estimator states are driven by the estimation error instead. Zero mean Gaussian white measurement noise is not included in the analysis ($v = 0$) since this term is negligible compared to the first-order wave disturbances η_w [6].

A3) $J(\psi) = J(y_3)$ where $y_3 = \psi + \psi_w$. This assumption is not restrictive since the magnitude of the wave-induced yaw disturbance ψ_w is typically less than 5° in extreme weather situations (sea state codes 5–10), and less than 1° during normal operation of the ship (sea state codes 1–5).

Definition 2—Dynamic Ship Positioning: Under the assumptions formulated above, consider the nonlinear system

$$M\dot{\nu} = -D\nu + J^\top(y_3)b + \tau \tag{10}$$

$$\dot{\eta} = J(y_3)\nu \tag{11}$$

$$\dot{\xi} = \Omega\xi \tag{12}$$

$$\dot{b} = -T^{-1}b \tag{13}$$

$$y = \eta + \Gamma\xi. \tag{14}$$

and design an *output* feedback controller[1] $\tau = \tau(\hat{\nu}, \hat{\eta}, \hat{\xi}, \hat{b})$ where $(\hat{\nu}, \hat{\eta}, \hat{\xi}, \hat{b})$ is the estimate of the true state (ν, η, ξ, b), such that

$$\lim_{t\to\infty} \eta(t) = \eta_d \tag{15}$$

where η_d is the desired *constant* position vector. □

In order to solve the above stated problem we proceed in a constructive and simple manner. Our approach consists of designing the observer and the controller *separately*. Then we prove that a *separation principle* holds for *the nonlinear* system (10)–(14) and invoke some recent results on cascaded nonlinear systems to claim GAS.

II. State Feedback Control

In this section we prove that a state feedback PD-type control law with bias compensation globally asymptotically stabilizes the ship model (10)–(14) at the origin.

Proposition 1—State Feedback: Consider the ship nonlinear model (10)–(14) in closed loop with the control law

$$\tau^* = -J^\top(\psi)b - K_d\nu - J^\top(\psi)K_p e_\eta \tag{16}$$

where K_d, K_p are positive definite, then the system (10)(14) and (16) is GAS. Furthermore, define $e \triangleq \mathrm{col}[\nu, e_\eta, \xi, b]$, the matrix $K_p'(\psi) \triangleq J(\psi)^\top K_p + K_p J(\psi)$ and the open set

$$B_\delta \triangleq \left\{e \in \mathbb{R}^{12}\colon K_p'(\psi) > 0\right\} \tag{17}$$

and let $1 >> \varepsilon > 0$, then for all $e \in B_\delta$ such that

$$k_{pm} > \varepsilon^2 m_M \tag{18}$$

$$k_{dm} \geq 2\varepsilon m_M \tag{19}$$

$$k'_{pm} \geq \frac{\varepsilon^2(k_{d_M} + d_M)^2}{k_{dm} - 2\varepsilon m_M} \tag{20}$$

the system (10)–(16) is exponentially stable. □

Proof: Since the desired references $\xi_d = \nu_d = b_d = 0$ the tracking error dynamics (10)–(16) is given by

$$M\dot{\nu} = -\tfrac{1}{2}\left(D + D^\top\right)\nu - K_d\nu - J^\top(\psi)K_p e_\eta \tag{21}$$

and (11)–(13). Consider now the function

$$V_c(e) = \tfrac{1}{2}\left(\nu^\top M\nu + \|b\|^2 + \|\xi\|^2 + e_\eta^\top K_p e_\eta\right) \tag{22}$$

which is clearly positive definite since M and K_p are positive definite matrices, hence $V_c(e)$ qualifies as a Lyapunov function candidate for the closed-loop system (10)–(16). Furthermore, it is easy to verify that the time derivative of V_c along the trajectories of the closed-loop system is

$$\dot{V}_c(e) \leq -\nu^\top K_d\nu - \tfrac{1}{2}\left(b^\top\left(T^{-1}T^{-\top}\right)b + \xi^\top\left(\Omega + \Omega^\top\right)\xi\right) \tag{23}$$

where we used that $(D + D^\top) > 0$. GAS of the origin $e = 0$ follows by invoking Krasovskii–LaSalle's invariance principle: notice that $\dot{V}_c \equiv 0$ is equivalent to $\nu \equiv \xi \equiv b \equiv 0$. Then, since $J(\psi)$ has full rank for all $\psi \in \mathbb{R}$ we obtain from (21) that $e_\eta \equiv 0$.

Second the proof of exponential stability for all $e \in B_\delta$ follows by using the Lyapunov function candidate

$$V_{c_2}(e) = \tfrac{1}{2}\left(\nu^\top M\nu + \|b\|^2 + \|\xi\|^2 + e_\eta^\top K_p e_\eta\right) + \varepsilon\nu^\top M e_\eta \tag{24}$$

which is positive definite if (18) holds. The time derivative of $V_{c_2}(e)$ along the closed-loop trajectories is then bounded by

$$\begin{aligned}\dot{V}_{c_2}(e) \leq &-\tfrac{1}{2}\left(b^\top\left(T^{-1}T^{-\top}\right)b + \xi^\top\left(\Omega + \Omega^\top\right)\xi\right)\\ &-(k_{dm} - 2\varepsilon m_M)\|\nu\|^2 + \varepsilon d_M\|\nu\|\,\|e_\eta\|\\ &+\varepsilon K_d M\|e_\eta\|\,\|\nu\| - \tfrac{1}{2}k'_{p_m}(\psi)\|e_\eta\|^2\end{aligned} \tag{25}$$

where d_M is the largest eigenvalue of the positive definite matrix $D + D^\top$. It is clear that the function $\dot{V}_{c_2}(e)$ is negative definite for all $e \in B_\delta$ if (18)–(20) hold hence exponential stability of the origin $e = 0$ follows from standard Lyapunov theory. ■

III. Observer Design

In [6] the first GES observer for ships was proposed by exploiting the *passivity* property of the ship dynamics. For the sake of completeness and further development, we briefly present in this section, the main contribution of [6]. See also [10] on passivity-based observer design for Lagrangian systems.

By copying the dynamics (10)–(14) the following nonlinear observer was proposed:

$$\dot{\hat{\xi}} = \Omega\hat{\xi} + K_1\tilde{y} \tag{26}$$

$$\dot{\hat{\eta}} = J(y_3)\hat{\nu} + K_2\tilde{y} \tag{27}$$

[1]Notice that τ is not required to depend on the measurable output y, this is done on purpose since the control law shall not use the *noisy* position measurements y but the filtered estimates of the state vector.

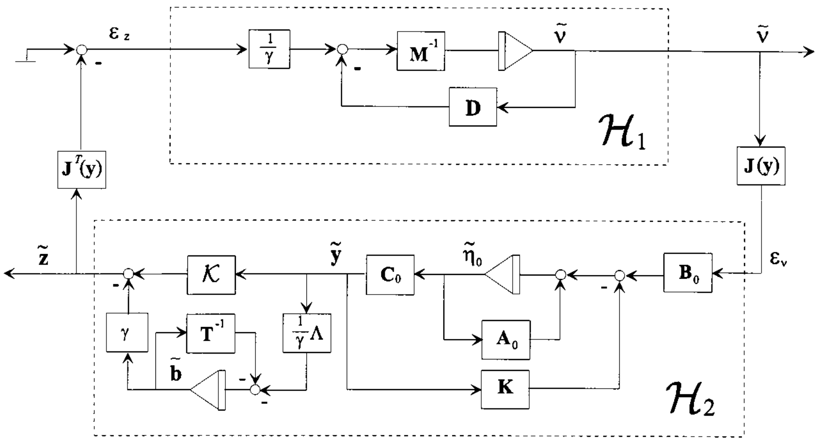

Fig. 3. Block diagram showing the dynamics of the position/bias and velocity estimation errors.

$$\dot{\hat{b}} = -T^{-1}\hat{b} + \frac{1}{\gamma}\Lambda\tilde{y} \tag{28}$$

$$M\dot{\hat{\nu}} = -D\hat{\nu} + J^\top(y_3)\hat{b} + \tau + \frac{1}{\gamma}J^\top(y_3)\mathcal{K}\tilde{y} \tag{29}$$

$$\hat{y} = \hat{\eta} + \Gamma\hat{\xi} \tag{30}$$

where

$\tilde{y} = y - \hat{y}$	estimation error;
$K_1 \in \mathbb{R}^{6\times 3}$, $K_2 \in \mathbb{R}^{3\times 3}$, $\mathcal{K} \in \mathbb{R}^{3\times 3}$, and $\Lambda \in \mathbb{R}^{3\times 3}$	observer gain matrices to be defined later;
$\gamma > 0$	additional scalar tuning parameter motivated by the Lyapunov stability analysis.

A. Observer Error Dynamics

The estimation errors are defined as $\tilde{\nu} = \nu - \hat{\nu}$, $\tilde{b} = b - \hat{b}$ $\tilde{\eta} = \eta - \hat{\eta}$, and $\tilde{\xi} = \xi - \hat{\xi}$. Further, in order to compact the notation let $\eta_0 = \mathrm{col}[\eta, \xi]$ $A_0 \triangleq$ block-diag$\{\Omega, 0\}$, $B_0 \triangleq$ block-diag$\{0, I\}$, and $C_0 \triangleq$ block-diag$\{\Gamma, I\}$. Then the estimation error dynamics can be written as

$$\dot{\tilde{\eta}}_0 = (A_0 - KC_0)\tilde{\eta}_0 + B_0 J(y_3)\tilde{\nu} \tag{31}$$

$$\dot{\tilde{b}} = -T^{-1}\tilde{b} - \frac{1}{\gamma}\Lambda\tilde{y} \tag{32}$$

$$M\dot{\tilde{\nu}} = -D\tilde{\nu} - \frac{1}{\gamma}J^\top(y_3)\tilde{z} \tag{33}$$

$$\tilde{z} \triangleq \mathcal{K}\tilde{y} - \gamma\tilde{b}. \tag{34}$$

It is important to remark that using the orthogonality and boundedness of the rotation matrix $J(y_3)$, the estimation error system (31)–(34) can be regarded as an interconnection of two *passive* blocks $\mathcal{H}_1$ and $\mathcal{H}_2$ with linear dynamics (cf. Fig. 3). This is a crucial property that was exploited in [6] to design the observer and realize the convergence analysis of the estimation error. More precisely, the following result was established in that reference.

Proposition 2—Globally Exponentially Stable Nonlinear Observer: Suppose that Assumptions A1)–A4) hold and the observer gains are chosen to $\gamma > 0$

$$K_1 = \begin{bmatrix} k_{11} & 0 & 0 \\ 0 & k_{12} & 0 \\ 0 & 0 & k_{13} \\ k_{21} & 0 & 0 \\ 0 & k_{22} & 0 \\ 0 & 0 & k_{23} \end{bmatrix}, \quad \mathcal{K} = \begin{bmatrix} \kappa_1 & 0 & 0 \\ 0 & \kappa_2 & 0 \\ 0 & 0 & \kappa_3 \end{bmatrix} \tag{35}$$

$$K_2 = \begin{bmatrix} k_{31} & 0 & 0 \\ 0 & k_{32} & 0 \\ 0 & 0 & k_{33} \end{bmatrix}, \quad \Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \tag{36}$$

with

$$k_{1i} = -2\omega_{ci}(\zeta_{ni} - \zeta_i)\frac{1}{\omega_{oi}} \tag{37}$$

$$k_{2i} = 2\omega_{oi}(\zeta_{ni} - \zeta_i) \tag{38}$$

$$k_{3i} = \omega_{ci}. \tag{39}$$

$$1/T_i \ll \lambda_i/\kappa_i < \omega_{oi} < \omega_{ci} \qquad (i = 1\cdots 3). \tag{40}$$

Then the mapping $\mathcal{H}_2$: $J(y_3)\tilde{\nu} \mapsto \tilde{z}$ is strictly passive, that is, its dynamics satisfies the Kalman–Yakubovich–Popov Lemma. Consequently, the nonlinear observer (26)–(30) in closed-loop with the vessel (10)–(14) is GES. □

IV. Output Feedback Control via a Separation Principle

We have proven so far that a *passive* observer can be designed for the plant (10)–(14) which ensures an exponentially fast estimation error convergence. Also, in Proposition 1 we presented a simple *state* feedback controller that ensures GAS of the closed-loop system.

The most important question to solve is: "What happens when the observer-error system (31)–(34) is coupled with the nonlinear plant (10)–(14) and the control law (16) by using the *estimated* state instead of the true one?" We answer this question below.

Proposition 3—GAS by Output Feedback: The system (10)–(14) in closed loop with (26)–(29) and

$$\tau = -J^\top(y_3)\hat{b} - K_d\hat{\nu} - J^\top(y_3)K_p\hat{e}_\eta \tag{41}$$

is GAS and LES under the conditions of Propositions 1 and 2.□

Organization of the Proof: The proof is constructive. We have already designed a GES observer and a GAS state feedback controller. In Section IV-A we prove that a separation principle holds for the overall system, for this we show that the estimation and tracking error dynamics can be *decoupled* yielding a *cascaded system*. Finally in Section IV-A2 we invoke a recent result from [11] on Lyapunov stability of cascaded nonlinear systems, see also the Appendix. ■

A. A Separation Principle

To this point, we find it convenient to present the observation error-dynamics (31)–(34) in the state-space form

$$\begin{bmatrix} \dot{\tilde{\nu}} \\ \dot{\tilde{e}}_\eta \\ \dot{\tilde{\xi}} \\ \dot{\tilde{b}} \end{bmatrix} = A_0 \cdot \begin{bmatrix} \tilde{\nu} \\ \tilde{e}_\eta \\ \tilde{\xi} \\ \tilde{b} \end{bmatrix}$$

where

$$A_0 = \begin{bmatrix} -M^{-1} & -\frac{1}{\gamma}M^{-1}J^\top\mathcal{K} & -\frac{1}{\gamma}J^\top\mathcal{K}\Gamma & M^{-1}J^\top \\ J & -K_2 & -K_2\Gamma & 0 \\ 0 & -K_1 & \Omega - K_1\Gamma & 0 \\ 0 & -\frac{1}{\gamma}\Lambda & 0 & -T^{-1} \end{bmatrix} \tag{42}$$

and for simplicity we dropped the argument of $J(y_3)$. Define the observation and tracking error, respectively, as $x_2 \triangleq \mathrm{col}[\tilde{\nu}, \tilde{e}_\eta, \tilde{\xi}, \tilde{b}]$ and $x_1 \triangleq \mathrm{col}[\nu, e_\eta, \xi, b]$, then we write the estimation error (42) and tracking dynamics (11)–(13) and (21), respectively, as $\dot{x}_2 \triangleq A_o(x_1)x_2$ and $\dot{x}_1 \triangleq A_c(x_1)x_1$, that is shown in (43) at the bottom of the page. Now, notice that the control input (41) can be written as $\tau = \tau^* + g(x_1)x_2$ where we defined

$$g(x_1)x_2 \triangleq \tilde{J}^\top\tilde{b} + K_d\tilde{\nu} + \tilde{J}^\top K_p\tilde{e}_\eta \tag{44}$$

then, the system (10)–(14) in closed loop with (26), (29), and (41) can be written in the form

$$\Sigma_1'\colon \dot{x}_1 = A_c(x_1)x_1 + g(x_1)x_2 \tag{45}$$

$$\Sigma_2'\colon \dot{x}_2 = A_o(x_1)x_2 \tag{46}$$

which is similar to the cascaded form (56) and (57) however, notice that the dynamics of Σ_2' is not decoupled from that of Σ_1' due to the presence of $J(y_3)$ in the matrix $A_o(x_1)$. Therefore, in order to consider systems Σ_1', Σ_2' as a *cascade* we must prove that the system (45) and (46) is *complete* that is, that the solutions $(x_1(t), x_2(t))$ can be continued for all $t \geq 0$. This allows us to consider $A_o(x_1)$ as a *time-varying* matrix $\tilde{A}_0(t) \triangleq A_0((x_1(t))$, that is we regard the ship-observer system as a time-varying system dependent on the position error trajectories $x_1(t)$. Notice that $A_0(x_1)$ depends only on the heading angle ψ. With an abuse of notation in the sequel we will write $A_0(t)$ instead of $A_0((x_1(t))$.

1) Completeness of the Closed-Loop System: To prove that the system (45) and (46) is complete consider the Lyapunov function candidate

$$V(x) = \tfrac{1}{2}x_1^\top P_c x_1 + \tfrac{1}{2}x_2^\top P_o x_2 \tag{47}$$

where

P_o positive definite;[2]

P_c $\triangleq$ block-diag$\{M, K_pI, I\}$.

From Propositions 1 and 2 the derivative of $V(x)$ along the closed-loop system trajectories is

$$\dot{V} = -x_1^\top Q_o x_1 - x_2^\top Q_c x_2 + x_1^\top P_c g(x_1)x_2 \tag{48}$$

where Q_o is positive definite and we used (23) to define $Q_c \triangleq$ block-diag$\{D + D^\top + K_d, 0, (1/2)(T^{-1} + T^\top), (1/2)(\Omega + \Omega^\top)\}$. Next notice that there exists $\delta > 0$ such that $\|g(x_1)\| \leq \delta$ for all x_1 hence, since the first two terms on the right-hand side of (48) are negative we can write

$$\dot{V}(x) \leq p_{c_M}\delta\|x_1\|\,\|x_2\| \tag{49}$$

where $p_{c_M} < \infty$ is the largest eigenvalue of P_c. Using the Shwartz inequality we have that

$$\dot{V}(x) \leq p_{c_M}\delta\|x_1\|\,\|x_2\| \leq 2p_{c_M}\delta\left(\|x_1\|^2 + \|x_2\|^2\right).$$

From this it is clear that there also exists a constant c_1 such that $\dot{V}(x(t)) \leq c_1V(x(t))$. It follows using the comparison equations method that there exist constants c_2, c_3 such that $V(t) \leq$

[2]This fact follows from Proposition 2; for more details see [6].

$$\begin{bmatrix} \dot{\nu} \\ \dot{e}_\eta \\ \dot{\xi} \\ \dot{b} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}M^{-1}\left(D + D^\top + K_d\right) & -M^{-1}J^\top K_p & 0 & 0 \\ J & 0 & 0 & 0 \\ 0 & 0 & \Omega + \Omega^\top & 0 \\ 0 & 0 & 0 & -\left(T^{-1} + T^\top\right) \end{bmatrix} \begin{bmatrix} \nu \\ e_\eta \\ \xi \\ b \end{bmatrix}. \tag{43}$$

$c_1 V(0) e^{c_2 t}$ that is, $V(t)$ is bounded for all bounded t and since $V(x)$ is positive definite we obtain that the solutions $x(t)$ exist and can be continued for all $t \geq 0$.

2) Proof of GAS: A key observation to this point is that the observer error dynamics is globally exponentially stable, uniformly in the tracking error trajectories $x_1(t)$. That is, even though system dynamics Σ_2' depends also on x_1 (specifically on the yaw angle ψ) the estimation error trajectories $x_2(t)$ tend to zero exponentially for any trajectory $x_1(t)$. Second, we have proven that the solutions of the closed-loop system can be continued for all $t \geq 0$ thus the system Σ_2' can be written as

$$\Sigma_2'\colon \dot{x}_2 = A_o(x_1(t))x_2$$

or simply

$$\Sigma_2'\colon \dot{x}_2 = A_o(t)x_2 \tag{50}$$

which under the conditions of Proposition 2, is GES since $A_o(t)$ is Hurwitz for all $x_1(t)$, that is for all $t \geq 0$.

The stability proof finishes by observing that the system (45), (50) has a *cascaded* structure, hence we can invoke Theorem 1 from the Appendix and show that the cascaded closed-loop system (45)–(50) satisfies the conditions of this theorem. This result will therefore guarantee that the estimation and position error dynamics can be analyzed *separately* and therefore, that the observer and controller can be tuned independently of one another.

Assumption A4: A constant c_1 that satisfies (59) is easily computed noticing that

$$\left\| \frac{\partial V}{\partial x_1} \right\| \|x_1\| \leq \max\left\{m_M,\, K'_{pM},\, 1\right\} \|x_1\|^2 \tag{51}$$

then it is easy to see that if c_1 satisfies

$$c_1 \geq \frac{\max\{m_M,\, k'_{pM},\, 1\}}{\min\{m_m,\, k'_{pm}, 1\}}$$

the inequality (59) holds. Also from (51) it is clear that the inequality (61) is satisfied with $c_2 = \max\{m_M,\, k'_{pM},\, 1\}\mu$.

Assumption A5: On the growth rate on the tracking error x_1 is satisfied by choosing $\theta_1 \equiv \mathrm{const}$, and $\theta_2 \equiv 0$.

Assumption A6: Holds since the observer-error dynamics is globally *exponentially* stable, uniformly in the tracking error $x_1(t)$, that is there exist positive constants λ_1 and λ_2 which depend on the initial conditions $x_1(t_0)$, such that $\|x_2(t)\| \leq \lambda_1 \|x_2(t_0)\| e^{-\lambda_2(t-t_0)}$ and therefore we can take $\phi(\|x_2(t_0)\|) = (\lambda_1/\lambda_2)\|x_2(t_0)\|$.

Thus, invoking Theorem 1 global asymptotic stability of system (10)–(13) in closed loop with (26), (27), and (16) follows. ■

V. Experimental Results

The experiments were performed at the *Guidance, Navigation, and Control (GNC) Laboratory* at the Department of Engineering Cybernetics, NTNU. The laboratory consists of two model ships, *CyberShip I* and *CyberShip II*, and a model basin.

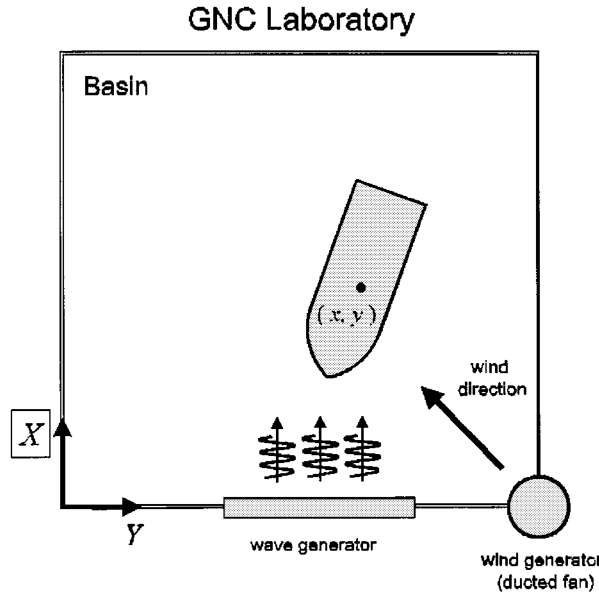

Fig. 4. Experimental setup.

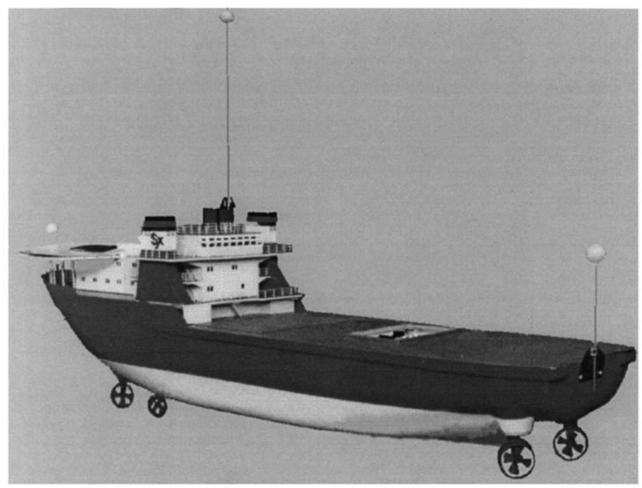

Fig. 5. *CyberShip I*: scale 1 : 70 of a supply vessel.

The size of the model basin is 6 × 10 m and the depth is limited to 0.3 m. Water currents are generated by using a water pump while a wave generator (flap mounted at one of the ends of the basin) is used to produce waves. The flap can move at different frequencies which is necessary in order to produce different wave amplitudes. Wind is generated by using a ducted fan which can be moved to the location of the ship during station-keeping. The experimental setup is shown in Fig. 4.

We have used the *CyberShip I* which is a model of an offshore supply vessel scale 1 : 70, see Fig. 5. The model ship has a mass of 17.6 kg and a length of 1.19 m. By assuming that the *Froude number*

$$\mathrm{Fn} = \frac{V}{\sqrt{Lg}} = \text{constant} \tag{52}$$

where

- V velocity of the ship;
- L length of the ship;
- g acceleration of gravity.

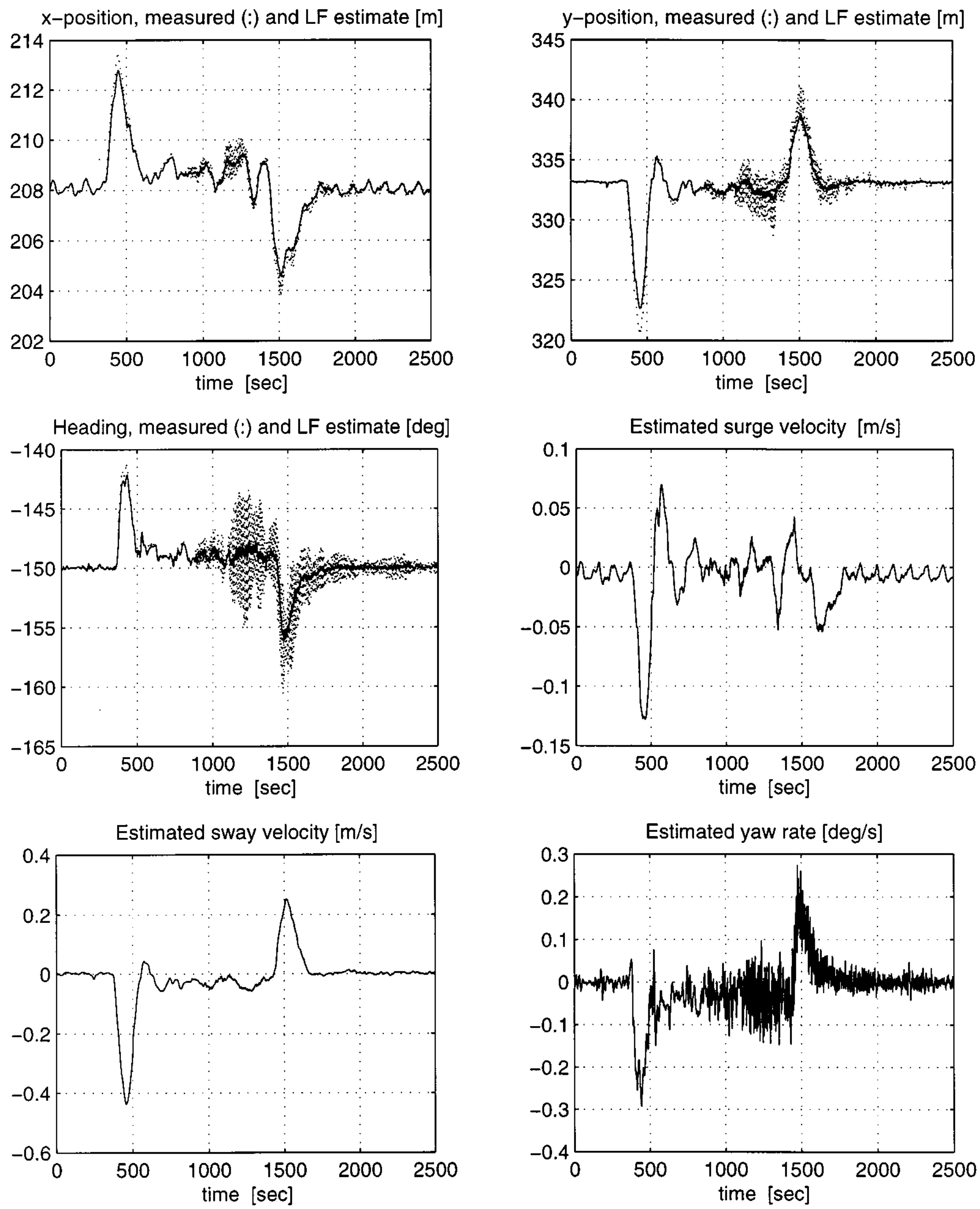

Fig. 6. Plots 1–3 show the components of the measurement vector $y = [x + x_w,\ y + y_w,\ \psi + \psi_w]$ and the wave filtered (LF-estimate) $\hat{\eta} = [\hat{x},\ \hat{y},\ \hat{\psi}]$ during DP. Plots 4–6 show the estimated LF velocity components $\hat{\nu} = [\hat{u},\ \hat{v},\ \hat{r}]$ versus time.

The following scalings are obtained (Bis-scaling):

position	L_s/L_m
linear velocity	$\sqrt{L_s/L_m}$
angular velocity	$\sqrt{L_m/L_s}$
linear acceleration	1
angular acceleration	L_m/L_s
force	m_s/m_m
moment	$(m_s/m_m)(L_s/L_m)$
time	$\sqrt{L_s/L_m}$

where

m mass;

subscript M model ship;

subscript S full-scale ship.

For a supply ship with length $L_s = 76$ m, we obtain the speed scaling $V_s \approx 8.37 V_m$ (m/s).

A. Discussion of the Experimental Results

In the experiments the desired position and heading of the ship during DP were chosen as

$$x_d = 208\ \text{(m)} \tag{53}$$

$$y_d = 334\ \text{(m)} \tag{54}$$

$$\psi_d = -150^\circ. \tag{55}$$

The numerical values for the observer were chosen as

$$\gamma = 0.5$$

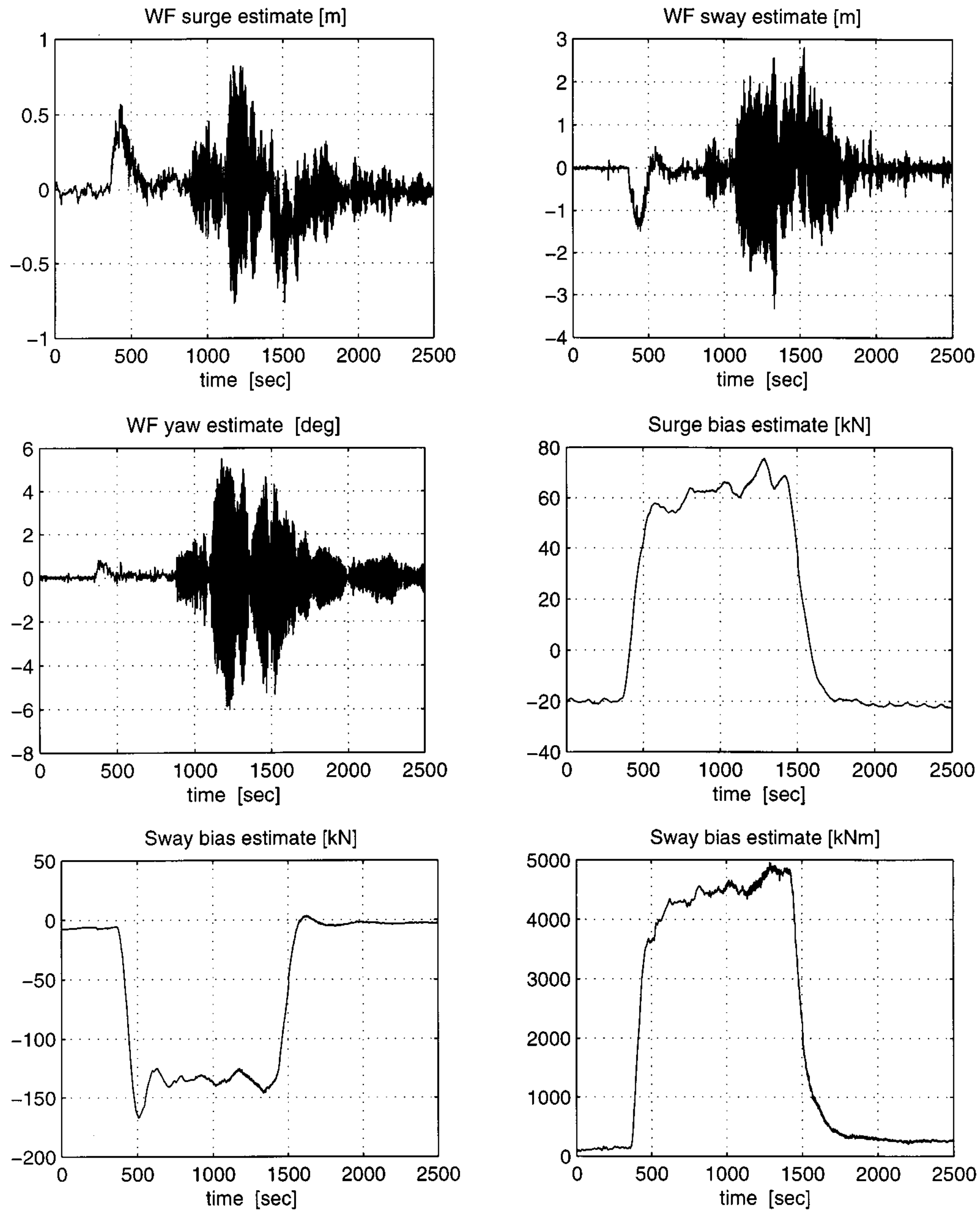

Fig. 7. Plots 1–3 show the estimated wave frequency (WF) motion components $\hat{\eta}_w = [\hat{x}_w, \hat{y}_w, \hat{\psi}_w]$ while plots 4–6 show the bias estimates $\hat{b} = [\hat{b}_1, \hat{b}_2, \hat{b}_3]$ versus time.

and

$$\mathcal{K} = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 10 \end{bmatrix} \qquad \Lambda = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}$$

with bias time constants

$$T = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 100 \end{bmatrix}.$$

The matrices K_1 and K_2 were computed according to Proposition 4 by choosing

$$\omega_{oi} = 7.0 \text{ rad/s}$$

and

$$\omega_{ci} = 10.0 \text{ rad/s}.$$

The PD-control law where tuned to yield a critical damped closed-loop system with natural frequency

$$\omega_n = 0.8 \text{ rad/s}.$$

In the first three plots of Fig. 6 we show the controlled and estimated Cartesian position and heading of the ship. The estimated velocities are shown in the last three plots of the same figure. The control efforts of the thrusters are shown in Fig. 9. We remind the reader that the experiments have been realized on a ship model scale 1 : 70 however, the scaling corresponding to meters [m], on the abscissa of the plots, shall be considered for the *full-size* vessel. The development of the experiments is as follows.

1) During the first 350 s there are no environmental loads perturbing the ship.

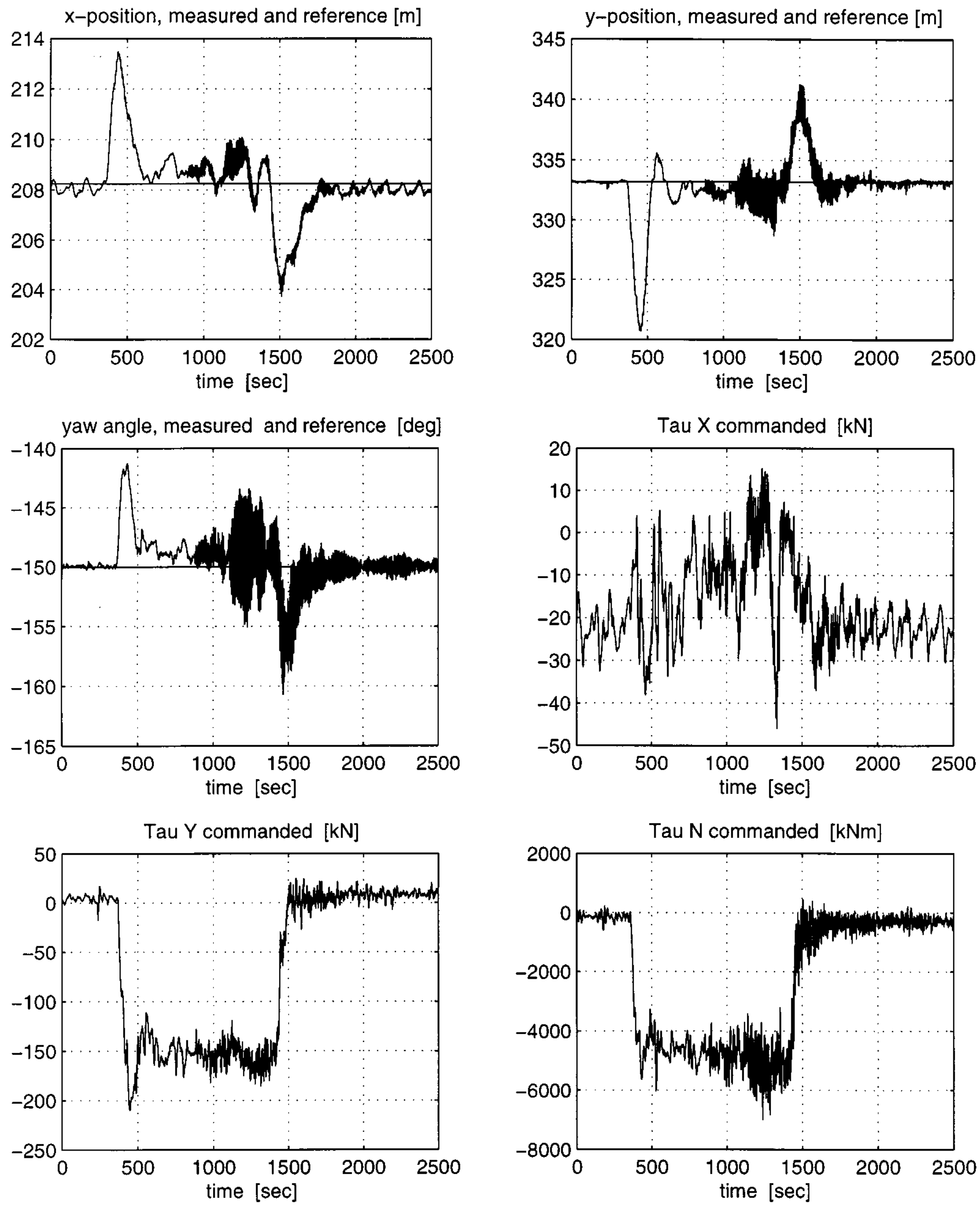

Fig. 8. Plots 1–3 show the components of the measured position $y = [x + x_w,\ y + y_w,\ \psi + \psi_w]$ together with the desired position $\eta_d = [x_d,\ y_d,\ \psi_d]$ while plots 4–6 are the control inputs $\tau = [\tau_1,\ \tau_2,\ \tau_3]$ versus time.

Comments: From Fig. 7 we see that the bias estimate $\hat{b}$ and the WF estimate $\hat{\eta}_w$ both are approximately zero as expected the first 350 s. The nonzero values of $\hat{b}$ are due to the water motion generated by the propellers. We also see that the regulation and estimation errors are very small during this phase, see upper plots in Figs. 6 and 8.

2) After 350 s wind loads are generated by using a ducted fan directed approximately 30° at the port side of the bow of the ship.

Comments: When turned on, the fan produces a step input disturbance to the system, notice the peaks in Figs. 6 and 8. This step is an unrealistic situation (in full-scale applications, no abrupt changes in the bias occur) however, it can be generated in the laboratory to show the performance of our observer-based controller. The bias estimates $\hat{b}$ from the observer are integrated up to constant off-sets reselecting the magnitude of the wind force, see Fig. 7. These *estimated* values are used in the output feedback control law to obtain a perfect regulation, which validates the separation principle, see Fig. 8. It is seen that most of the wind disturbance is compensated by the control input τ and therefore the regulation errors converge to zero in 100–150 s, see the first three plots of Fig. 6. However, since the wind disturbance is a step, the observer needs some time for the bias estimate to converge to its true value, after which the controller compensates for the bias, hence keeping the boat almost still.

3) After 800 s the wave generator is turned on.

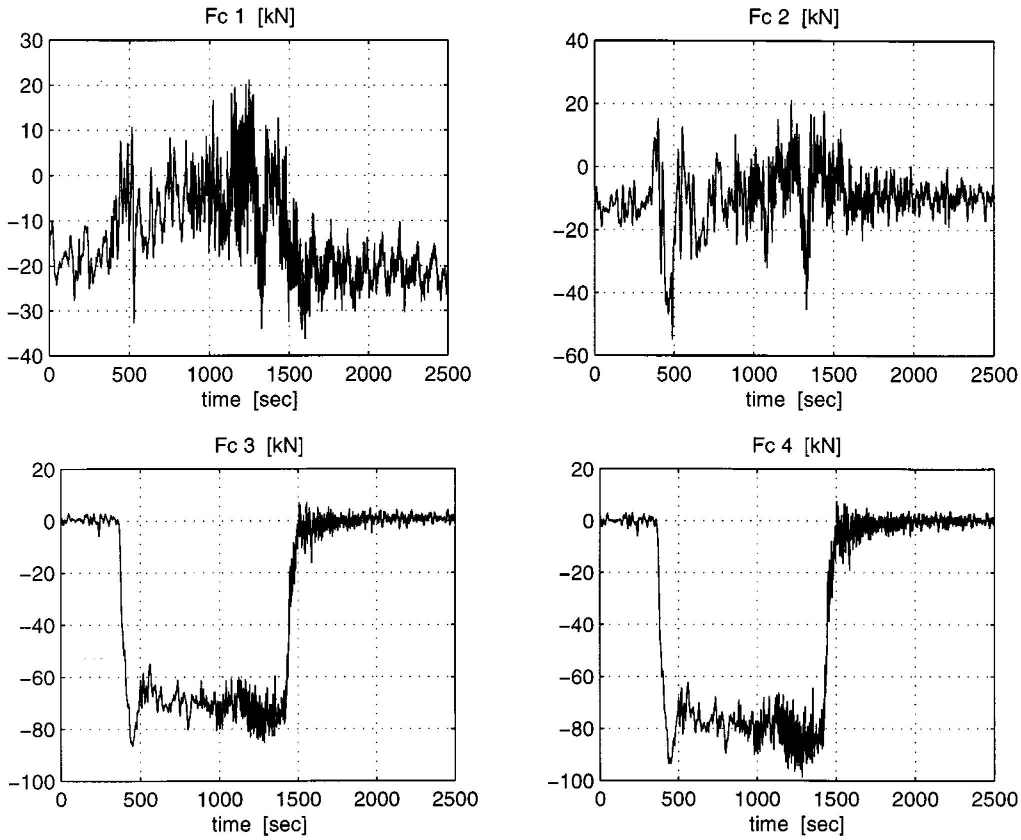

Fig. 9. Plots 1–4 show the four thruster forces u_i $(i = 1 \cdots 4)$ corresponding to the three control inputs τ_i $(i = 1 \cdots 3)$ versus time. The thruster forces are computed by using a pseudoinverse.

Comments: This results in an oscillatory wave frequency motion η_w which is building up over time. The estimated wave frequency motion $\hat{\eta}_w$ is shown in the upper plots of Fig. 7. Their effect in the position measurements is shown in the upper plots of Fig. 6. In order to avoid $\hat{\eta}_w$ to enter the feedback loop this signal is filtered out from the position measurement. This wave filtering results in more smooth controls, see bottom plots of Fig. 7. The low frequency estimates is clearly shown in the upper plots of Fig. 6.

4) After 1700 s both the wind and wave generators are turned off.

Comments: Turning off the fan, produces a second step input disturbance while the wave-induced motion decays more slowly. We see from Fig. 7 that the bias estimates drop to approximately their initial values in 100–150 s while the amplitudes of the WF motion estimates drop quite slowly. Again, almost perfect regulation to zero is obtained as soon as the bias estimates have converged to their true values. This clearly demonstrates the separation principle. In a full-scale implementation also the wind force will build up quite slow. Hence, the step inputs do not constitute a problem.

VI. Concluding Remarks

We have solved the problem of dynamic positioning of nonlinear ships with only *noisy* position measurements. An observer which filters out the noise from the measurements can be designed using passivity arguments. Our approach is simple to implement since it is based on the key property that for the nonlinear Lagrangian model, a separation principle holds. This allows then to design and tune a simple PD like controller with bias compensation, as if the true state measurements were available and free of noise. We have demonstrated the performance of our approach in experiments.

The theoretical approach undertaken here is based on recent cascaded systems results. A challenging theoretical problem of unquestionable practical interest is to find conditions under which a separation principle can be applied to a more general class of nonlinear systems, for instance Euler–Lagrange systems with nonconservative forces.

Appendix
Background on Cascaded Systems

In [11], we considered the stability *analysis* problem for the time varying system

$$\Sigma_1\colon \dot{x} + 1 = f_1(t, x_1) + g(t, x)x_2 \tag{56}$$

$$\Sigma_2\colon \dot{x}_2 = f_2(t, x_2) \tag{57}$$

where $x_1 \in \mathbb{R}^n$, $x_2 \in \mathbb{R}^m$, $x \triangleq \mathrm{col}[x_1, x_2]$. The function $f_1(t, x_1)$ is continuously differentiable in (t, x_1) and $f_2(t, x_2)$, $g(t, x)$ are continuous in their arguments, and locally Lipschitz.

Among other results, in [11] we gave sufficient conditions to guarantee that a globally uniformly stable nonlinear time-varying system

$$\dot{x}_1 = f_1(t,\, x_1) \tag{58}$$

remains GUAS when it is perturbed by the output of a GUAS system like Σ_2, under the assumption that the (perturbing) output $x_2(t)$ is absolutely integrable. For the sake of completeness and for the purposes of this paper we reformulate [11, Th. 1 and 2] below.

Theorem 1: If Assumptions A4–A6 below are satisfied then the cascaded system (56), (57) is globally uniformly asymptotically stable.

A4) The system $\dot{x}_1 = f_1(t,\, x_1)$ is globally uniformly asymptotically stable with a Lyapunov function $V(t,\, x_1)$, $V\colon\ \mathbb{R}_{\geq 0} \times \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ positive definite [that is $V(t,\, 0) = 0$ and $V(t,\, x_1) > 0$ for all $x_1 \neq 0$] and proper (that is, radially unbounded) which satisfies

$$\left\| \frac{\partial V}{\partial x_1} \right\| \|x_1\| \leq c_1 V(t,\, x_1) \qquad \forall \|x_1\| \geq \mu \qquad (59)\ (60)$$

where $c_1,\, \mu > 0$. We also assume that $(\partial V/\partial x_1)(t,\, x_1)$ is bounded uniformly in t for all $\|x_1\| \leq \mu$, that is, there exists a constant $c_2 > 0$ such that for all $t \geq t_0 \geq 0$

$$\left\| \frac{\partial V}{\partial x_1} \right\| \leq c_2 \qquad \forall \|x_1\| \leq \mu \qquad (61)\ (62)$$

A5) The function $g(t,\, x)$ satisfies

$$\|g(t,\, x)\| \leq \theta_1(\|x_2\|) + \theta_2(\|x_2\|)\, \|x_1\| \tag{63}$$

where $\theta_1,\, \theta_2\colon \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ are continuous.

A6) Equation $\dot{x}_2 = f_2(t,\, x_2)$ is globally uniformly asymptotically stable and for all $t_0 \geq 0$,

$$\int_{t_0}^{\infty} \|x_2(t,\, t_0,\, x_2(t_0))\|\, dt \leq \phi(\|x_2(t_0)\|) \tag{64}$$

where function $\phi(\cdot)$ is a class $\mathcal{K}$ function.

References

[1] M. F. Aarset, J. P. Strand, and T. I. Fossen, "Nonlinear vectorial observer backstepping with integral action and wave filtering for ships," in *Proc. IFAC Conf. Control Applicat. Marine Syst. (CAMS'98)*, Fukuoka, Japan, Oct. 1998.

[2] A. N. Atassi and H. Khalil, "A separation principle for the stabilization of a class of nonlinear systems," presented at the Proc. 4th. European Contr. Conf., Bruxelles, Belgium, 1997, CD-ROM Paper 952.

[3] J. G. Balchen, N. A. Jenssen, and S. Saelid, "A dynamic ship positioning system based on Kalman filtering and optimal control," *Modeling, Identification, and Control*, vol. 1, no. 3, pp. 135–163, 1980.

[4] T. I. Fossen, *Guidance and Control of Ocean Vehicles*. New York: Wiley, 1994.

[5] T. I. Fossen and Å. Grøvlen, "Nonlinear output feedback control of dynamically positioned ships using vectorial observer backstepping," *IEEE Trans. Contr. Syst. Technol.*, vol. 6, pp. 121–128, Jan. 1998.

[6] T. I. Fossen and J. P. Strand, "Passive nonlinear observer design for ships using Lyapunov methods: Full-scale experiments with a supply vessel," *Automatica*, vol. 35, no. 1, pp. 3–16, 1999.

[7] J. F. Genrich and J. B. Minster, "Smooth ship navigation for geophysical applications from Kalman-filtered gps fixes," *Geophysics*, vol. 56, pp. 1961–1970, 1991.

[8] M. J. Grimble, R. J. Patton, and D. A. Wise, "Use of Kalman filtering techniques in dynamic ship positioning systems," *Proc. Inst. Elect. Eng.*, vol. 127, no. 3, pp. 93–102, 1980.

[9] W. Lin, "Bounded smooth state feedback and a global separation principle for nonaffine nonlinear systems," *Syst. Contr. Lett.*, vol. 26, pp. 41–53, 1995.

[10] R. Ortega, A. Loría, P. J. Nicklasson, and H. Sira-Ramírez, *Passivity-Based Control of Euler–Lagrange Systems: Mechanical, Electrical and Electromechanical Applications*. London, U.K.: Springer-Verlag, 1998.

[11] E. Panteley and A. Loria, "On global uniform asymptotic stability of non linear time-varying non autonomous systems in cascade," *Syst. Contr. Lett.*, vol. 33, no. 2, pp. 131–138, 1998.

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