Worksheet 8 - copied from Martha Lewis

Q1

X and Y are two (binary) random variables. If X and Y are independent, then:

$$P(X, Y) = P(X)P(Y) \tag{1}$$

1. Give an example of two random variables that are independent.
2. Complete the probability table below in such a way that the variables X and Y are independent.

	$X = 0$	$X = 1$
$Y = 0$	?	?
$Y = 1$	?	?

3. Determine the missing entries (a, b) of the joint distribution in such a way that the variables X and Y are again independent:

$$P(Y = 0, X = 0) = 0.1$$
$$P(Y = 0, X = 1) = 0.3$$
$$P(Y = 1, X = 0) = a$$
$$P(Y = 1, X = 1) = b$$

Q2

Exactly a fifth of the people in a town have lycanthropy. There are two tests for lycanthropy, ASSAY1 and ASSAY2. When a person goes to a doctor to test for lycanthropy, with probability 2/3 the doctor conducts ASSAY1 on them and with probability 1/3 the doctor conducts ASSAY2 on them. When ASSAY1 is done on a person, the outcome is as follows: If the person has the disease, the result is positive with probability 3/4. If the person does not have the disease, the result is positive with probability 1/4. When ASSAY2 is done on a person, the outcome is as follows: If the person has the disease, the result is positive with probability 1. If the person does not have the disease, the result is positive with probability 1/2. A person is picked uniformly at random from the town and is sent to a doctor to test for lycanthropy. The result comes out positive. What is the probability that the person has the disease?[1]

Q3

Consider the following Bayesian network:

$$\begin{tikzcd}
X_{1,1} \arrow[r] \arrow[d] & X_{1,2} \arrow[r] \arrow[d] & X_{1,3} \arrow[d] \\
X_{2,1} \arrow[r] \arrow[d] & X_{2,2} \arrow[r] \arrow[d] & X_{2,3} \arrow[d] \\
X_{3,1} \arrow[r] & X_{3,2} \arrow[r] & X_{3,3}
\end{tikzcd}$$

(a) Which random variables are independent of $X_{3,1}$?
(b) Which random variables are independent of $X_{3,1}$ given $X_{1,1}$?

[1] Taken from allendowney.blogspot.com/2017/02/a-nice-bayes-theorem-problem-medical.html