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## Worksheet 8 - copied from Martha Lewis

### Q1

$X$  and  $Y$  are two (binary) random variables. If  $X$  and  $Y$  are independent, then:

$$P(X, Y) = P(X)P(Y) \quad (1)$$

1. Give an example of two random variables that are independent.
2. Complete the probability table below in such a way that the variables  $X$  and  $Y$  are independent.

	$X = 0$	$X = 1$
$Y = 0$	?	?
$Y = 1$	?	?

3. Determine the missing entries  $(a, b)$  of the joint distribution in such a way that the variables  $X$  and  $Y$  are again independent:

$$P(Y = 0, X = 0) = 0.1$$

$$P(Y = 0, X = 1) = 0.3$$

$$P(Y = 1, X = 0) = a$$

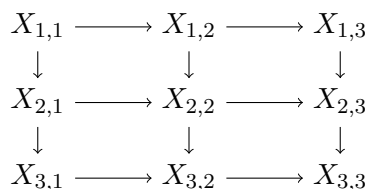
$$P(Y = 1, X = 1) = b$$

### Q2

Exactly a fifth of the people in a town have lycanthropy. There are two tests for lycanthropy, ASSAY1 and ASSAY2. When a person goes to a doctor to test for lycanthropy, with probability  $2/3$  the doctor conducts ASSAY1 on them and with probability  $1/3$  the doctor conducts ASSAY2 on them. When ASSAY1 is done on a person, the outcome is as follows: If the person has the disease, the result is positive with probability  $3/4$ . If the person does not have the disease, the result is positive with probability  $1/4$ . When ASSAY2 is done on a person, the outcome is as follows: If the person has the disease, the result is positive with probability 1. If the person does not have the disease, the result is positive with probability  $1/2$ . A person is picked uniformly at random from the town and is sent to a doctor to test for lycanthropy. The result comes out positive. What is the probability that the person has the disease?<sup>1</sup>

### Q3

Consider the following Bayesian network:



- (a) Which random variables are independent of  $X_{3,1}$ ?
- (b) Which random variables are independent of  $X_{3,1}$  given  $X_{1,1}$ ?

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<sup>1</sup>Taken from [allendowney.blogspot.com/2017/02/a-nice-bayes-theorem-problem-medical.html](http://allendowney.blogspot.com/2017/02/a-nice-bayes-theorem-problem-medical.html)