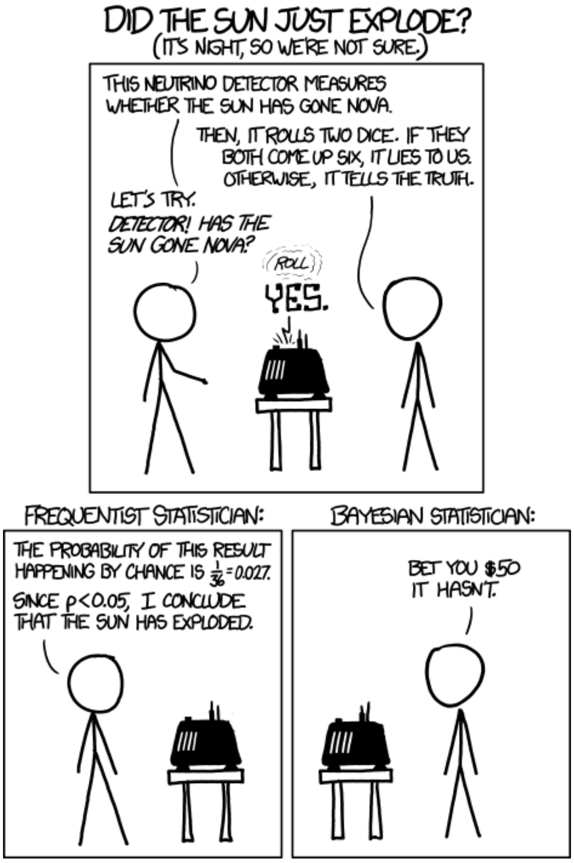
Worksheet 7



`https://xkcd.com/1132/`. It is also worth reading Randall Monroe's comment on the response to this cartoon:

> Hey! I was kinda blindsided by the response to this comic.
>
> I'm in the middle of reading a series of books about forecasting errors (including N*** S***'s book, which I really enjoyed), and again and again kept hitting examples of mistakes caused by blind application of the textbook confidence interval approach.
>
> Someone asked me to explain it in simple terms, but I realized that in the common examples used to illustrate this sort of error, like the cancer screening/drug test false positive ones, the correct result is surprising or unintuitive. So I came up with the sun-explosion example, to illustrate a case where naïve application of that significance test can give a result that's obviously nonsense.
>
> I seem to have stepped on a hornet's nest, though, by adding 'Frequentist' and 'Bayesian' titles to the panels. This came as a surprise to me, in part because I actually added them as an afterthought, along with the final punchline. (I originally had the guy on the right making some other cross-panel comment, but I thought the 'bet' thing was cuter.)
>
> The truth is, I genuinely didn't realize Frequentists and Bayesians were actual camps of people, all of whom are now emailing me. I thought they were loosely-

applied labels, perhaps just labels appropriated by the books I had happened to read recently, for the standard textbook approach we learned in science class versus an approach which more carefully incorporates the ideas of prior probabilities.

I meant this as a jab at the kind of shoddy misapplications of statistics I keep running into in things like cancer screening (which is an emotionally wrenching subject full of poorly-applied probability) and political forecasting. I wasn't intending to characterize the merits of the two sides of what turns out to be a much more involved and ongoing academic debate than I realized.

A sincere thank you for the gentle corrections; I've taken them to heart, and you can be confident I will avoid such mischaracterizations in the future!

At least, 95.45% confident.

Useful facts

- The **conditional probability** of event R given C:
$$P(R|C) = \frac{P(R \cap C)}{P(C)} \tag{1}$$
This is the probability of getting an outcome in event R if we know the outcome is in event C.

- **Independence**: two events A and B are **independent** iff
$$P(A \cap B) = P(A)P(B) \tag{2}$$
They are **conditionally independent**, given C, iff
$$P(A \cap B|C) = P(A|C)P(B|C) \tag{3}$$

- **Bayes's rule**
$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \tag{4}$$

- Here I use set notation for events, this is one approach, sometimes people prefer logic notation, even when the set notation is strictly correct! **Set notation**:
  - The bar '|' in sets should be read as 'such that', so $A = \{x|\text{some stuff}\}$ should be read as A is the set of x **such that** 'some stuff' is true and $A = \{x \in \mathbf{Z}|x > 3 \text{ and } x < 10\}$ is the set $A = \{4, 5, 6, 7, 8, 9\}$. $\mathbf{Z}$ by the way is the set of integers.
  - $A \cup B$ is the union so $A \cup B = \{x|x \in A \text{ or } x \in B\}$. If $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$ then $A \cup B = \{1, 2, 3, 4, 5\}$
  - $A \cap B$ is the intersection so $A \cap B = \{x|x \in A \text{ and } x \in B\}$. If $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$ then $A \cup B = \{3\}$
  - $A \setminus B$ is the set minus so $A \setminus B = \{x|x \in A \text{ and } x \notin B\}$. If $A = \{1, 2, 3, 4\}$ and $B = \{1, 3, 5\}$ then $A \setminus B = \{2, 4\}$
  - If C is a subset, the complement of C, that is the set of all the elements not in C, is written $\bar{C}$. If $X = \{1, 2, 3, 4\}$ and $C = \{1, 2\}$ then $\bar{C} = \{3, 4\}$.

  For events, $A \cup B$ is the event of A or B happening, $A \cap B$ is the event of A and B happening, $A \setminus B$ is the event of A happening but B not happening and $\bar{C}$ is the event of C not happening.

Questions

These are the questions you should make sure you work on in the workshop.

1. This problem was also on the last problem sheet. Two events A and B have probabilities $P(A) = 0.2$, $P(B) = 0.3$ and $P(A \cup B) = 0.4$. Find
   a) Find $P(A \cap B)$.
   b) Find $P(\bar{A} \cap \bar{B})$.
   c) Find $P(A|B)$.

2. In a library where all books have blue or yellow spines, four fifths of books with yellow spines are about mathematics but only a fifth of books with blue spines are about mathematics. There are the same number of yellow and blue spined books, you come upom a book open on a table; the book is about mathematics. What is the chance it has a yellow spine?

3. You want to go for a walk. However, when you wake up the day is cloudy and half of all raining days start off cloudy. On the other hand, two days in five start off cloudy and it's been rather dry recently with only rain only on one day in ten. What is the chance it will rain?

4. One night in a bar in Las Vegas you meet a dodgy character who tells you that there are two types of slot machine in the Topicana, one that pays out 10% of the time, the other 20%. One sort of machine is blue, the other red. Unfortunately the dodgy character is too drunk to remember which is which. The next day you randomly select red to try, you find a red machine and put in a coin. You lose. Assuming the dodgy character was telling the truth, what is the chance the red machine is the one that pays out more. If you had won instead of losing, what would the chance be?[1]

5. In the xkcd cartoon above, what is the chance the Bayesian will win his or her bet if the chance the sun has exploded is one in a million? In reality the chance is, of course, much less than one in a million! Show the answer to six decimal places.

[1] I stole this problem from `courses.smp.uq.edu.au/MATH3104/`