MF 803 Homework 4

Due: Wednesday, October 16th, by 6:30pm

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a) Zero coupon bonds' prices

| Maturity | YTM | Price |
|----------|-------|---------|
| 1 | 0.025 | 97.561 |
| 2 | 0.026 | 94.996 |
| 3 | 0.027 | 92.3185 |
| 5 | 0.03 | 86.2609 |
| 10 | 0.035 | 70.8919 |
| 30 | 0.04 | 30.8319 |

The zero coupon bond with the shortest maturity has the highest price. This is reasonable because the bond with the shortest maturity has the smallest risk.

b) Duration of zero coupon bonds

| Maturity | YTM | Duration |
|----------|-------|----------|
| 1 | 0.025 | 0.975702 |
| 2 | 0.026 | 1.94969 |
| 3 | 0.027 | 2.92205 |
| 5 | 0.03 | 4.85757 |
| 10 | 0.035 | 9.68168 |
| 30 | 0.04 | 29.2894 |

The higher the yield is, the lower the bond price is. The price is more sensitive to higher yield. Besides, the duration of zero coupon bonds equal with its maturity.

c) Prices of coupon bonds

| Maturity | YTM | Price |
|----------|-------|---------|
| 1 | 0.025 | 100.488 |
| 2 | 0.026 | 100.77 |
| 3 | 0.027 | 100.854 |
| 5 | 0.03 | 100 |
| 10 | 0.035 | 95.8417 |
| 30 | 0.04 | 82.708 |

The bonds with YTM greater than 0.03 are below 100 dollars. The others are above 100 dollars. This is because, when the yields of the bonds are smaller than coupon rate, the price would be greater than face value. When the yields of the

bonds are greater than coupon rate, the price would be smaller than face value.

d) Duration of coupon bonds

| Maturity | YTM | Duration |
|----------|-------|----------|
| 1 | 0.025 | 0.975633 |
| 2 | 0.026 | 1.92112 |
| 3 | 0.027 | 2.83748 |
| 5 | 0.03 | 4.58043 |
| 10 | 0.035 | 8.46279 |
| 30 | 0.04 | 18.423 |

Zero coupon bonds have higher duration than coupon bonds. Since there is no cash flow during the whole period except at maturity for zero coupon bonds, it requires longer time to give all its cash flow than coupon bonds.

e) Convexity

| Maturity | YTM | Zero-coupon | Coupon |
|----------|-------|-------------|---------|
| 1 | 0.025 | 1.87683 | 1.82217 |
| 2 | 0.026 | 5.94315 | 5.90546 |
| 3 | 0.027 | 11.074 | 10.5908 |
| 5 | 0.03 | 27.595 | 25.6348 |
| 10 | 0.035 | 101.593 | 85.0172 |
| 30 | 0.04 | 856.429 | 458.642 |

The convexities of the bonds are all positive. This is because yield curve is convex in nature, due to quadratic nature of equations we use in compounding and discounting

f) Initial value of the portfolio

The initial value of the portfolio is: -0.112503

g) Duration and convexity of the portfolio

The duration of the portfolio is following: 48.86

The convexity of the portfolio is following: -678.15

Duration is greater. However, in terms of absolute quantity, convexity is much bigger.

h) Initial value of portfolio when rates sell off by 100 basis points

In this case, the initial value of portfolio is -0.0500793

i) Initial value of portfolio when rates rally by 100 basis points

In this case, the initial value of portfolio is -0.159592

Considering that the decrease of portfolio value due to rise in interest rate is

greater than the increase of portfolio value due to fall in interest rate, convexity contribute to hedging the risk of interest rate going up. Thus, this portfolio may be good to hold.

j) Cash flow of amortizing bond

| Year | Cash flow |
|------|-----------|
| 1 | 23 |
| 2 | 23 |
| 3 | 23 |
| 4 | 23 |
| 5 | 23 |

k) Price and Duration of amortizing bond

The 5-year amortizing bond price is following: 105.916

The 5-year amortizing duration is following: 2.85857

The price of this amortized bond is greater than its zero coupon and coupon equivalents. That is because more cash flow is prepaid in advance, which can also be reflected by its smaller duration.