MF 803 Homework 6

Due: Wednesday, December 11th, by 6:30pm

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1. Covariance Matrix Decomposition

(a) Download historical price data of ETFs

The head and tail of the price dataframe is as followed:

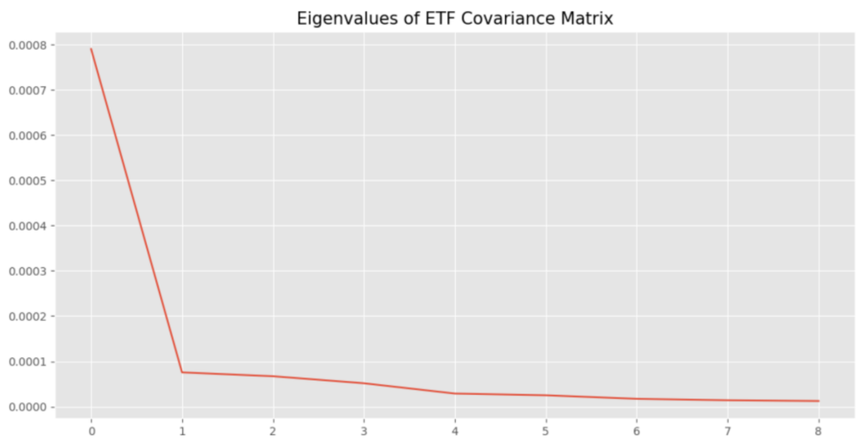
	XLB	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY
Date									
2010-01-04	27.40	46.58	7.77	23.23	19.84	20.51	21.76	26.82	26.05
2010-01-05	27.49	46.96	7.91	23.31	19.81	20.51	21.50	26.56	26.15
2010-01-06	27.96	47.52	7.92	23.36	19.59	20.50	21.62	26.83	26.18
2010-01-07	27.74	47.45	8.09	23.61	19.51	20.50	21.53	26.92	26.40
2010-01-08	28.13	47.76	8.05	23.99	19.64	20.43	21.51	26.97	26.39
...	...	...	...	...	...	...	...	...	...
2019-11-15	60.36	60.08	29.75	82.31	87.24	61.33	62.85	97.46	121.37
2019-11-18	60.21	59.35	29.81	82.00	87.50	61.65	62.98	97.13	121.79
2019-11-19	60.08	58.47	29.87	81.93	87.67	61.57	62.82	97.79	120.45
2019-11-20	59.33	59.06	29.73	81.28	87.14	61.71	63.20	97.66	120.17
2019-11-21	59.23	60.03	29.70	81.25	86.70	61.40	62.98	97.91	119.49

(b) Covariance matrix

The Covariance matrix is as followed:

	XLB	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY
XLB	0.000147	0.000132	0.000121	0.000116	0.000100	0.000057	0.000045	0.000080	0.000100
XLE	0.000132	0.000187	0.000121	0.000113	0.000097	0.000055	0.000047	0.000078	0.000096
XLF	0.000121	0.000121	0.000185	0.000117	0.000101	0.000059	0.000046	0.000084	0.000104
XLI	0.000116	0.000113	0.000117	0.000120	0.000097	0.000056	0.000044	0.000078	0.000097
XLK	0.000100	0.000097	0.000101	0.000097	0.000116	0.000052	0.000039	0.000076	0.000095
XLP	0.000057	0.000055	0.000059	0.000056	0.000052	0.000056	0.000042	0.000049	0.000054
XLU	0.000045	0.000047	0.000046	0.000044	0.000039	0.000042	0.000077	0.000039	0.000040
XLV	0.000080	0.000078	0.000084	0.000078	0.000076	0.000049	0.000039	0.000088	0.000074
XLY	0.000100	0.000096	0.000104	0.000097	0.000095	0.000054	0.000040	0.000074	0.000106

(c) Eigenvalue decomposition



As we can see from the plot above, all of the eigenvalues are positive. None of them is negative or zero. We can see there are 2 eigenvalues that are significantly different from 0.

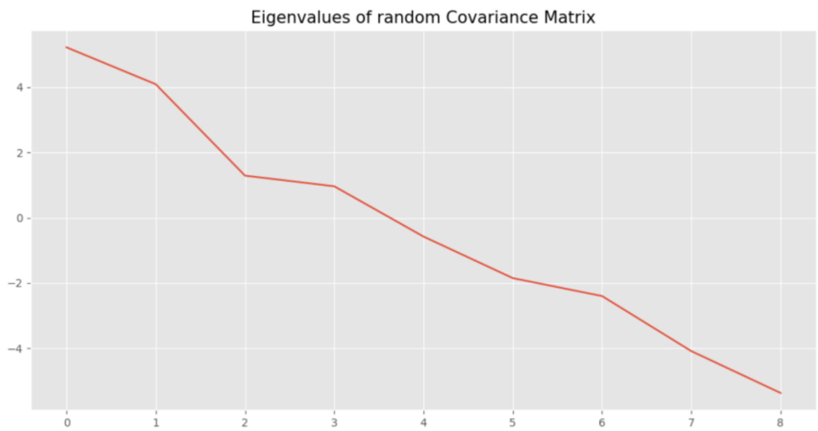
Since a covariance matrix is positive semi-definite, all of its eigenvalues should be expected to be positive.

(d) Random Covariance Matrix

	0	1	2	3	4	5	6	7	8
0	0.209820	-0.533690	1.801846	-0.125553	-0.320965	-0.404059	-0.067424	-2.921811	-1.340665
1	-0.533690	-1.273240	1.573817	-0.192612	-0.034345	1.128094	0.739820	-0.531338	1.716874
2	1.801846	1.573817	-0.568337	-0.319981	1.746758	0.493580	0.066468	-0.754004	-0.414595
3	-0.125553	-0.192612	-0.319981	0.539958	-0.769897	-1.145582	-1.861586	0.392540	1.740488
4	-0.320965	-0.034345	1.746758	-0.769897	-1.119308	0.057545	-0.979630	-0.005368	-0.840278
5	-0.404059	1.128094	0.493580	-1.145582	0.057545	-0.346674	0.357816	-1.383297	1.088073
6	-0.067424	0.739820	0.066468	-1.861586	-0.979630	0.357816	-0.783964	-1.700725	-0.508603
7	-2.921811	-0.531338	-0.754004	0.392540	-0.005368	-1.383297	-1.700725	0.371062	-0.213339
8	-1.340665	1.716874	-0.414595	1.740488	-0.840278	1.088073	-0.508603	-0.213339	1.650148

The random covariance matrix is generated as above. Since this is a covariance matrix, it must be symmetric.

(e) Eigenvalue decomposition of Random Covariance Matrix



The eigenvalues of random covariance matrix is plotted as above. 44% of eigenvalues are positive. 55% of eigenvalues are negative. 0% of eigenvalues are zero. Since the data is generated randomly, we can find negative eigenvalues, which is not the same case as the real covariance matrix.

2. Portfolio Optimization

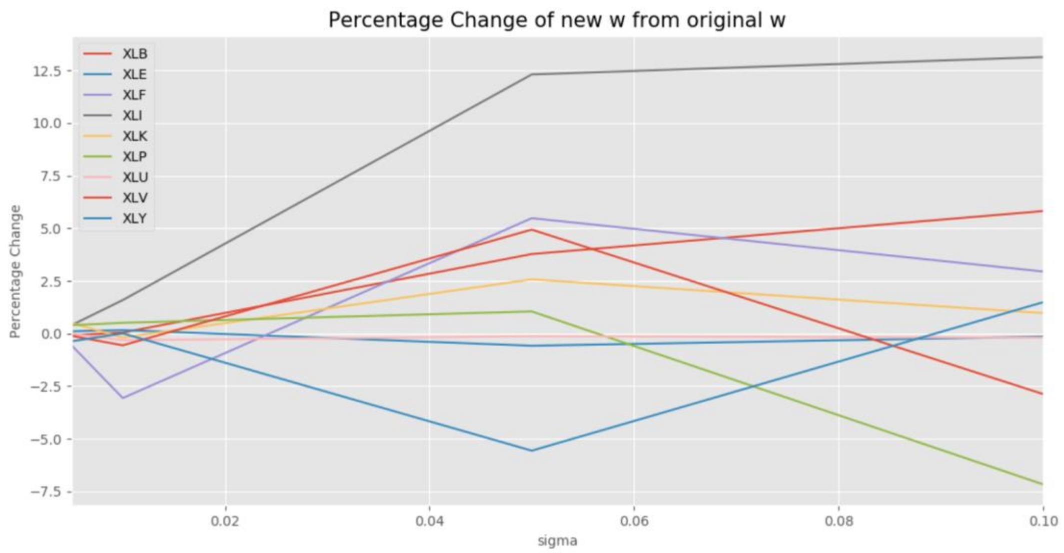
(a) Annualized returns of ETFs

	XLB	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY
ann_return	0.078049	0.025683	0.135758	0.126768	0.149312	0.111015	0.107598	0.131103	0.154219

(b) Unconstrained mean-variance optimal portfolio

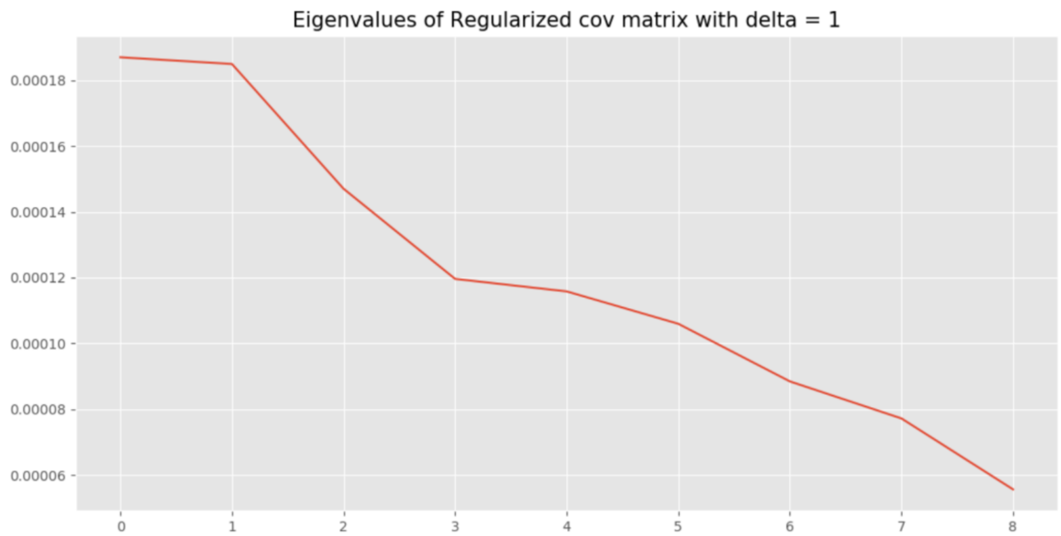
	XLB	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY
weight	-557.202974	-544.887176	49.458829	279.636736	297.895166	305.514597	352.251567	274.153461	694.003756

(c) Stability of portfolio weights



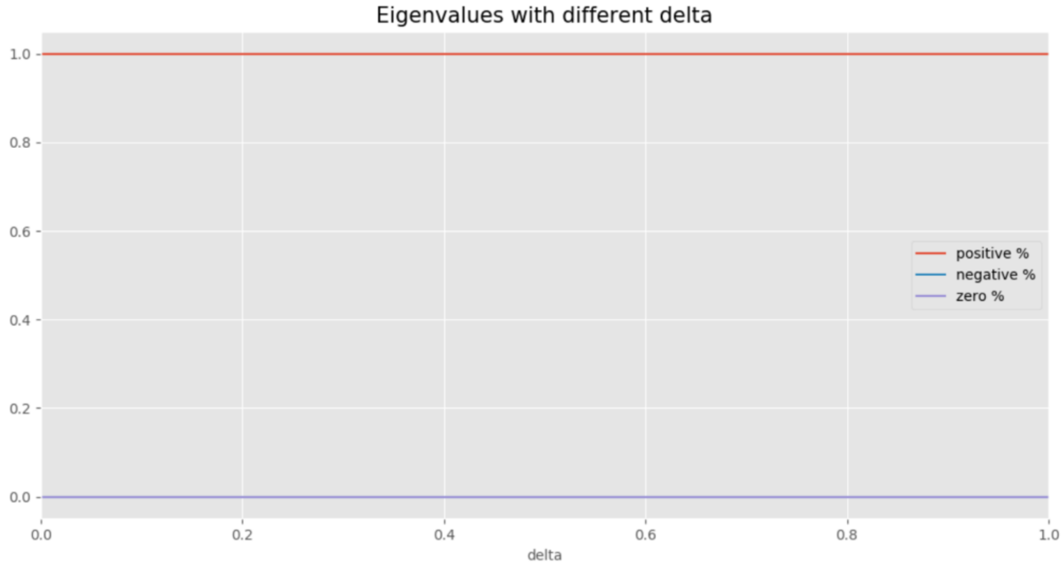
Here, I use the percentage change of the new weights from original weights to measure the instability of the weights. As we can see from the plot above, the weights become more instable as sigma become larger.

(e) Regularized covariance matrix when δ =1



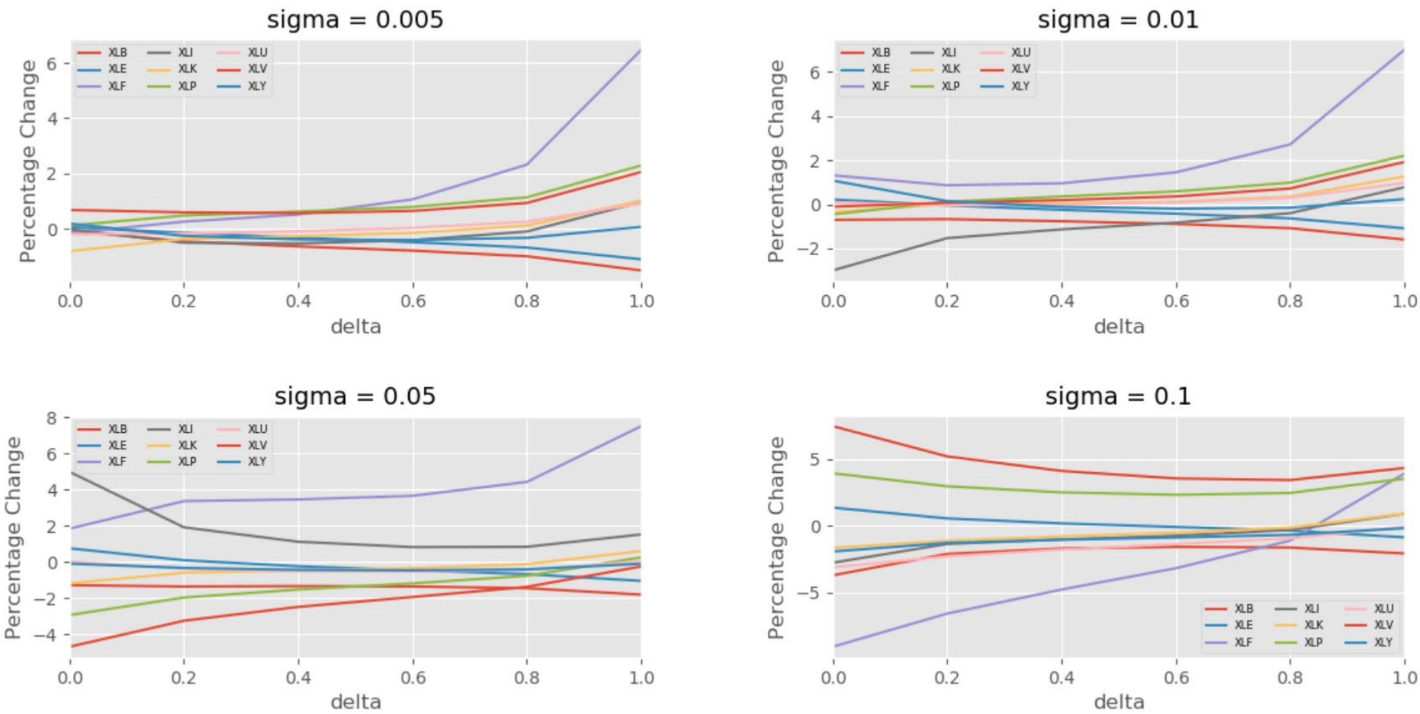
As we can see form the plot above, all the eigenvalues are positive. The rank of the matrix is 9 (full rank).

(f) Regularized covariance matrixes with different δs



We can see that all the eigenvalues are positive no matter what value delta is, since the matrixes are positive-definite.

(g) Stability of portfolio weights with different parameters



From the plots above, we can see that the percentage change of portfolio weights become much less stable as sigma becomes larger. Also, when sigma is held constant, there is also a clear tendency that weights become less stable as delta becomes larger.