MF 803 Homework 1

Due: Wednesday, September 18th, by 6:30pm

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1. Historical Analysis of Sector ETFs

a)   Data Downloading and Processing

Download ETF Data from Yahoo Finance using package yfinance. The time range is from January 1st 2010 to September 13th 2019. Head of price data is shown as follows:

Date	SPY	XLB	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY
2010-01-04	93.68	27.54	47.00	7.70	23.35	19.90	20.63	21.93	26.93	26.14
2010-01-05	93.92	27.63	47.39	7.84	23.43	19.87	20.64	21.66	26.67	26.24
2010-01-06	93.99	28.10	47.96	7.85	23.48	19.65	20.62	21.79	26.94	26.27
2010-01-07	94.39	27.88	47.88	8.02	23.74	19.57	20.62	21.69	27.03	26.49
2010-01-08	94.70	28.27	48.20	7.97	24.12	19.70	20.55	21.67	27.08	26.48

b)   The annualized return and standard deviation of each ETF

	SPY	XLB	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY
Ann_Return	1.29	0.85	0.29	1.45	1.36	1.56	1.20	1.17	1.36	1.72
Standard_Deviation	61.51	10.23	9.12	7.25	16.76	17.65	11.67	11.19	21.88	27.29

c)   Covariance and Correlation

The covariance matrix of daily returns:

	SPY	XLB	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY
SPY	0.000088	0.000100	0.000102	0.000106	0.000096	0.000093	0.000054	0.000044	0.000076	0.000090
XLB	0.000100	0.000149	0.000133	0.000123	0.000117	0.000101	0.000058	0.000046	0.000081	0.000101
XLE	0.000102	0.000133	0.000187	0.000122	0.000114	0.000098	0.000056	0.000048	0.000079	0.000098
XLF	0.000106	0.000123	0.000122	0.000188	0.000119	0.000102	0.000060	0.000046	0.000085	0.000105
XLI	0.000096	0.000117	0.000114	0.000119	0.000121	0.000098	0.000057	0.000045	0.000079	0.000098
XLK	0.000093	0.000101	0.000098	0.000102	0.000098	0.000117	0.000053	0.000040	0.000077	0.000096
XLP	0.000054	0.000058	0.000056	0.000060	0.000057	0.000053	0.000056	0.000043	0.000050	0.000055
XLU	0.000044	0.000046	0.000048	0.000046	0.000045	0.000040	0.000043	0.000078	0.000039	0.000041
XLV	0.000076	0.000081	0.000079	0.000085	0.000079	0.000077	0.000050	0.000039	0.000089	0.000075
XLY	0.000090	0.000101	0.000098	0.000105	0.000098	0.000096	0.000055	0.000041	0.000075	0.000107

The covariance matrix of monthly returns:

	SPY	XLB	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY
SPY	0.001299	0.001609	0.001665	0.001545	0.001552	0.001356	0.000775	0.000328	0.001030	0.001422
XLB	0.001609	0.002590	0.002368	0.001935	0.002064	0.001595	0.000765	0.000178	0.001214	0.001755
XLE	0.001665	0.002368	0.003527	0.002104	0.002087	0.001514	0.000748	0.000225	0.001069	0.001751
XLF	0.001545	0.001935	0.002104	0.002834	0.001923	0.001498	0.000695	0.000139	0.001132	0.001610
XLI	0.001552	0.002064	0.002087	0.001923	0.002124	0.001518	0.000838	0.000288	0.001183	0.001687
XLK	0.001356	0.001595	0.001514	0.001498	0.001518	0.001747	0.000752	0.000290	0.000936	0.001488
XLP	0.000775	0.000765	0.000748	0.000695	0.000838	0.000752	0.000969	0.000608	0.000724	0.000834
XLU	0.000328	0.000178	0.000225	0.000139	0.000288	0.000290	0.000608	0.001139	0.000356	0.000251
XLV	0.001030	0.001214	0.001069	0.001132	0.001183	0.000936	0.000724	0.000356	0.001309	0.001093
XLY	0.001422	0.001755	0.001751	0.001610	0.001687	0.001488	0.000834	0.000251	0.001093	0.001795

The correlation matrix of Daily Returns:

	SPY	XLB	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY
SPY	1.00	0.88	0.80	0.82	0.93	0.92	0.78	0.53	0.86	0.93
XLB	0.88	1.00	0.80	0.74	0.88	0.76	0.64	0.43	0.70	0.80
XLE	0.80	0.80	1.00	0.65	0.76	0.66	0.54	0.40	0.61	0.69
XLF	0.82	0.74	0.65	1.00	0.79	0.69	0.58	0.38	0.66	0.74
XLI	0.93	0.88	0.76	0.79	1.00	0.82	0.70	0.47	0.76	0.87
XLK	0.92	0.76	0.66	0.69	0.82	1.00	0.65	0.42	0.75	0.86
XLP	0.78	0.64	0.54	0.58	0.70	0.65	1.00	0.65	0.70	0.71
XLU	0.53	0.43	0.40	0.38	0.47	0.42	0.65	1.00	0.47	0.45
XLV	0.86	0.70	0.61	0.66	0.76	0.75	0.70	0.47	1.00	0.77
XLY	0.93	0.80	0.69	0.74	0.87	0.86	0.71	0.45	0.77	1.00

The correlation matrix of Monthly Returns:

	SPY	XLB	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY
SPY	1.00	0.88	0.78	0.81	0.93	0.90	0.69	0.27	0.79	0.93
XLB	0.88	1.00	0.78	0.71	0.88	0.75	0.48	0.10	0.66	0.81
XLE	0.78	0.78	1.00	0.67	0.76	0.61	0.40	0.11	0.50	0.70
XLF	0.81	0.71	0.67	1.00	0.78	0.67	0.42	0.08	0.59	0.71
XLI	0.93	0.88	0.76	0.78	1.00	0.79	0.58	0.18	0.71	0.86
XLK	0.90	0.75	0.61	0.67	0.79	1.00	0.58	0.21	0.62	0.84
XLP	0.69	0.48	0.40	0.42	0.58	0.58	1.00	0.58	0.64	0.63
XLU	0.27	0.10	0.11	0.08	0.18	0.21	0.58	1.00	0.29	0.18
XLV	0.79	0.66	0.50	0.59	0.71	0.62	0.64	0.29	1.00	0.71
XLY	0.93	0.81	0.70	0.71	0.86	0.84	0.63	0.18	0.71	1.00

Difference of Monthly and Daily Correlation Matrix:

	SPY	XLB	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY
SPY	0.00	0.00	0.02	0.01	0.00	0.02	0.09	0.26	0.07	0.00
XLB	0.00	0.00	0.02	0.03	0.00	0.01	0.16	0.33	0.04	-0.01
XLE	0.02	0.02	0.00	-0.02	0.00	0.05	0.14	0.29	0.11	-0.01
XLF	0.01	0.03	-0.02	0.00	0.01	0.02	0.16	0.30	0.07	0.03
XLI	0.00	0.00	0.00	0.01	0.00	0.03	0.12	0.29	0.05	0.01
XLK	0.02	0.01	0.05	0.02	0.03	0.00	0.07	0.21	0.13	0.02
XLP	0.09	0.16	0.14	0.16	0.12	0.07	0.00	0.07	0.06	0.08
XLU	0.26	0.33	0.29	0.30	0.29	0.21	0.07	0.00	0.18	0.27
XLV	0.07	0.04	0.11	0.07	0.05	0.13	0.06	0.18	0.00	0.06
XLY	0.00	-0.01	-0.01	0.03	0.01	0.02	0.08	0.27	0.06	0.00

From the matrixes above, we can see that the difference of monthly and daily correlations of return is mostly tiny. Specifically, the difference is bigger in the coefficients of symbol XLU, which means Utility sector's correlation with market and other sectors is more sensitive to the period of return.

d) Rolling correlation

Rolling 90-day correlation of each sector ETF with the S&P index:

Rolling Correlation Coefficient with SPY (window: 90)
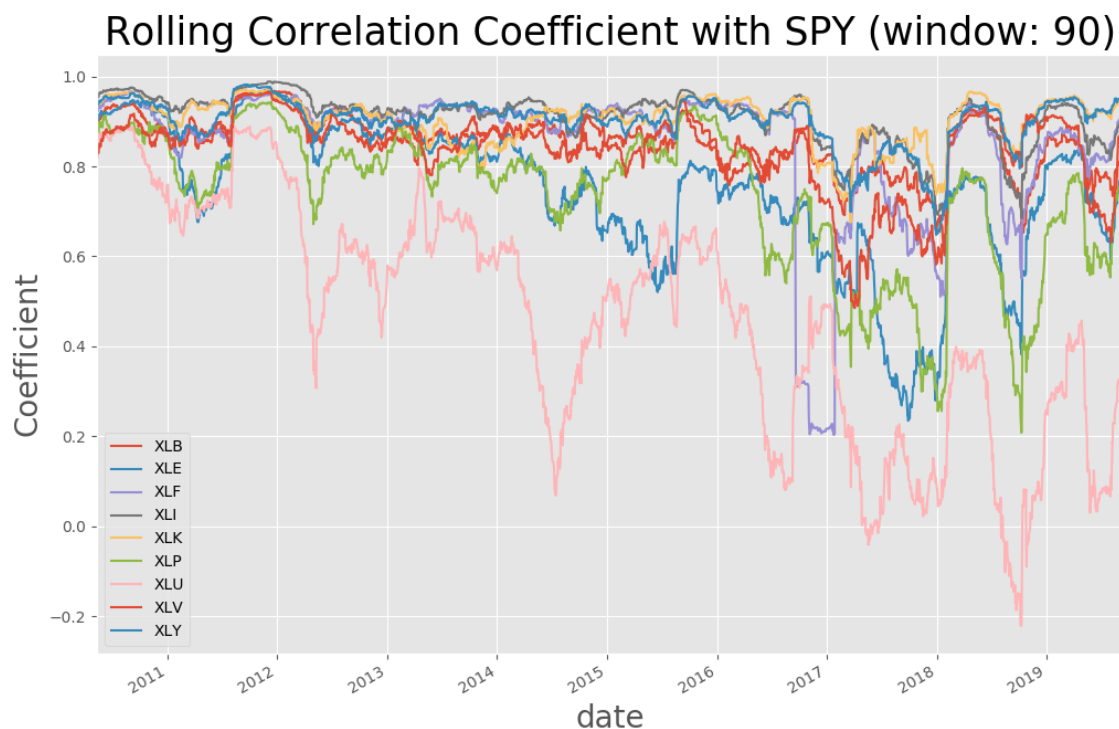
From the figure above, we can see the rolling coefficients of most sectors are stable before 2016. However, they suffered a great variation after 2016, possibly due to the great global market crush in 2015. Besides, the rolling correlation of Utility sector is highly unstable across all time. This again reflects that Utility sector has a relatively low and variant correlation with whole market.

e) Beta

Whole-period beta calculated by CAPM:

	XLB	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY
beta	0.68	0.55	0.56	0.8	0.8	0.97	0.57	0.85	0.84

Rolling beta with window of 90:
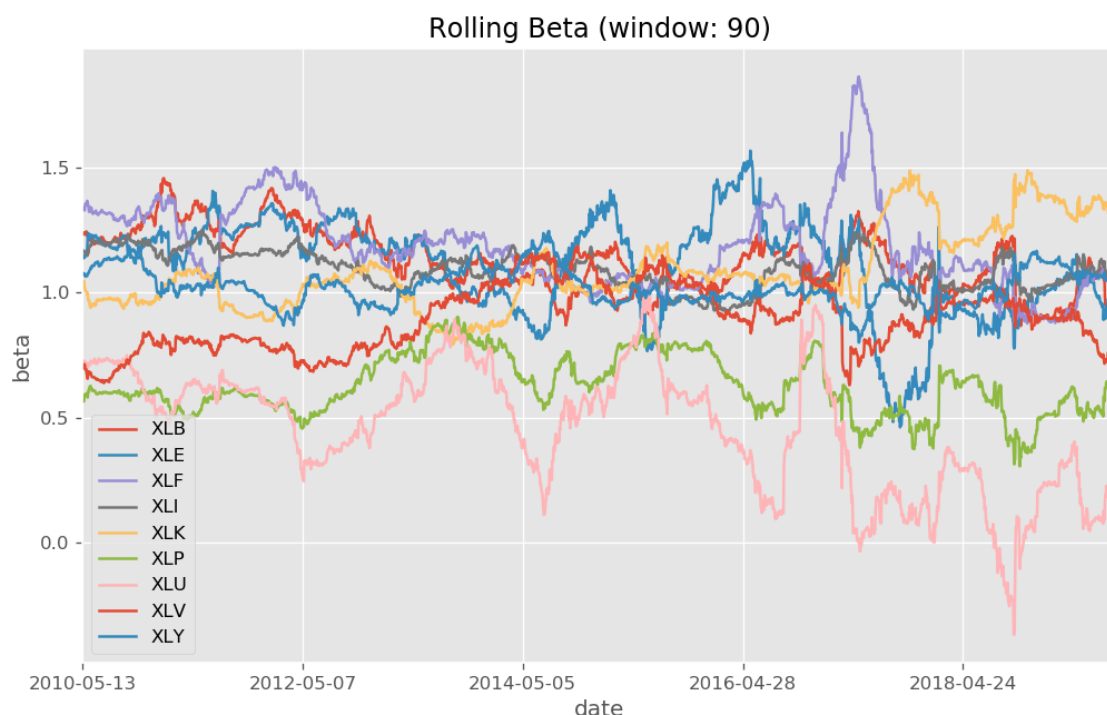
From the figure above, we can conclude that the rolling betas of these sectors are mostly stable. Again, Utility sector is more volatile compared with other sectors.
Comparing the results of rolling beta with rolling correlation, we can easily see that they have a similar trend. That is because the nature of linear regression model used in our problem is to find the linear relationship of sectors and market, which is just what correlation coefficient reflects.

f) auto-correlation of each ETF

|	SPY	XLB	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY
AR_coefficient	-0.039	-0.013	-0.001	-0.076	-0.009	-0.030	-0.040	-0.033	-0.030	-0.019
AR_pvalue	0.054	0.507	0.960	0.000	0.652	0.143	0.048	0.108	0.145	0.347

From the result above, we can conclude that the absolute value of all the auto regressive coefficients are less than 0.1. This could be considered as a good evidence that shows

there doesn't exist auto-correlation in these return data, though some regression results fail to pass the hypothesis tests. The result comforts to our common sense, because the return series of financial market is always considered to be random walk, generally with no significant relation with the past return data.

2. Exotic Option Pricing via Simulation

a) Generate simulated paths of asset price

Set simulated time as 1000, step as 250. The plot of the simulated paths is as follows:

Simulated Paths of Security Price
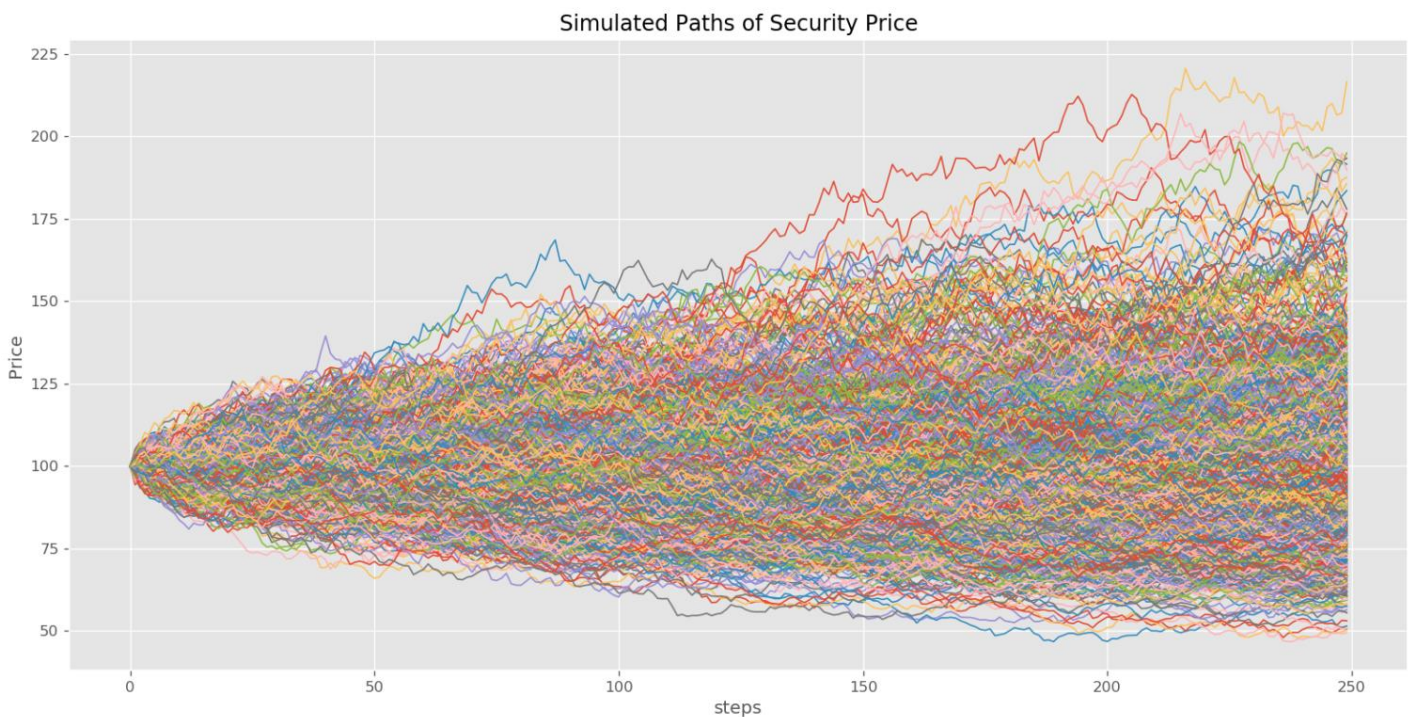
The mean of the terminal value of these paths is: 101.98
The variance of the terminal value of these paths is: 680.06
The simulated paths are consistent with the underlying dynamic, since they all have a obvious tendency that the price of the next day depends on the price of last day, with a plus of random walk.

b) Calculate the payoff of a European put option

the mean of the payoffs: 9.35
the standard deviation of the payoffs: 12.28

The histogram of payoff of European put option is shown as follows:

Distribution of Payoff of European option
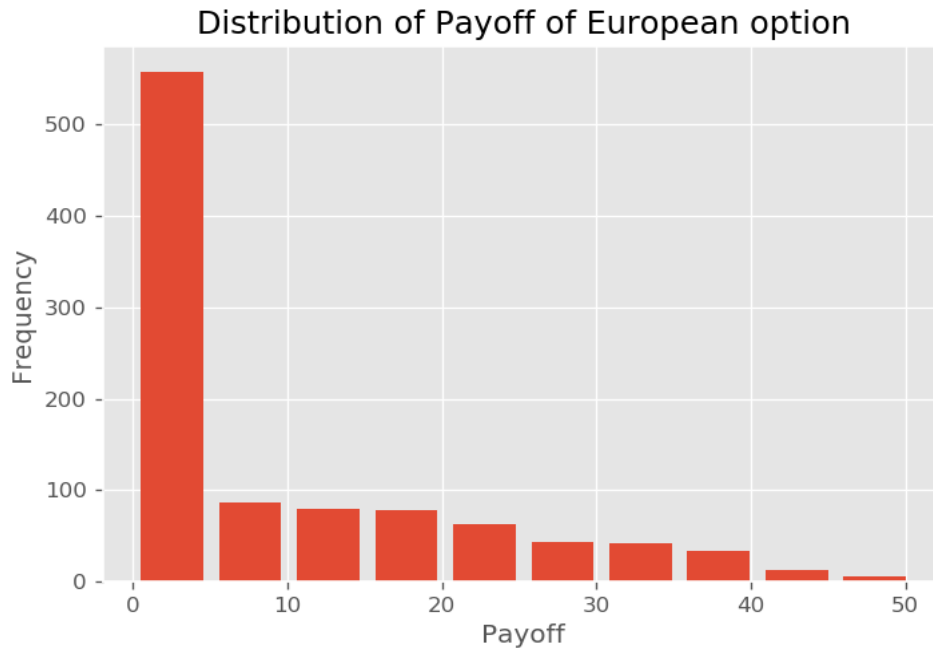
c) Simulated price of European put option

Simulated approximation to the price of European put option is: 9.35
(Since the discounted rate is 0 here, the price is just same as the mean payoff)

d) Compare the simulated price of European Option with BSM price

Price of European Option calculated by BSM is: 9.91
Difference of the two types of prices: 0.56
As we can see, the difference generated by two types is tiny. Actually, with the simulation times increasing, the difference becomes smaller, which means that two prices would eventually converge.

e) Calculate the simulated payoff of fixed strike lookback put option

The histogram of payoff of lookback put option is shown as follows:

Distribution of Payoff of Lookback put option
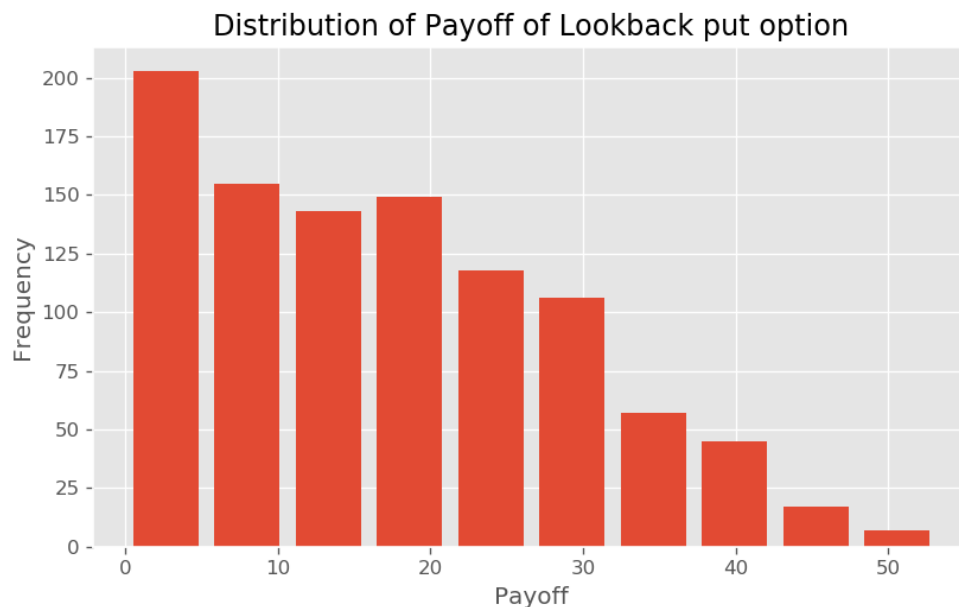
The simulated payoff of fixed strike lookback put option is: 17.18
The simulated price of fixed strike lookback put option is: 17.18
Compared with European put option, lookback put option has a higher percentage of non-0 values.

f) Calculate the premium for the extra optionality embedded in the lookback option

The premium is: 7.83

g) Price of options with different volatility

By using different volatility from 0.1 to 0.4 with step of 0.01, I get 30 different pairs of prices. The results are shown as follows:
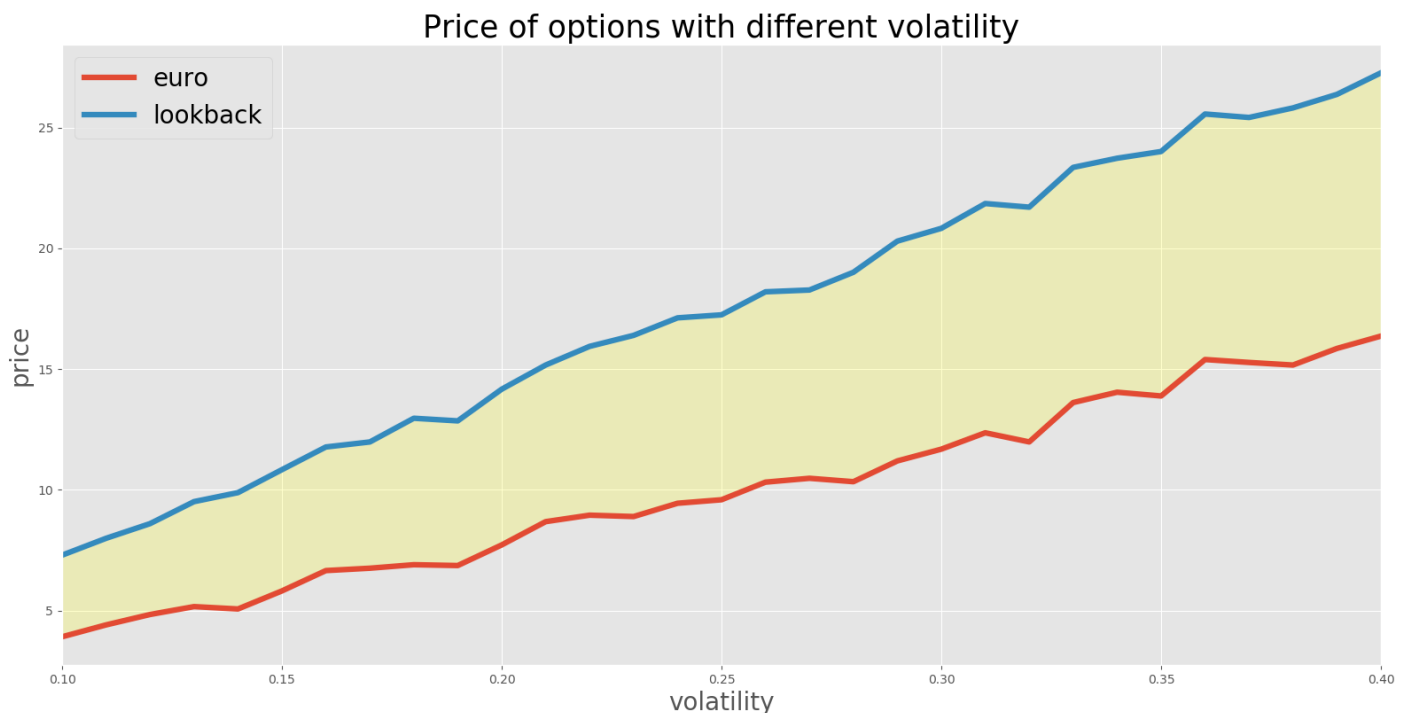
volatility	euro	lookback	premium
0.10	3.902	7.246	3.344
0.11	4.031	7.862	3.831
0.12	4.718	8.674	3.956
0.13	5.447	9.657	4.211
0.14	5.400	10.074	4.674
0.15	5.833	10.936	5.104
0.16	6.464	11.665	5.201
0.17	6.627	12.071	5.444
0.18	7.411	13.192	5.781
0.19	7.534	13.885	6.352

volatility	euro	lookback	premium
0.20	7.944	14.052	6.108
0.21	8.378	15.137	6.759
0.22	8.756	15.570	6.814
0.23	9.470	16.600	7.130
0.24	9.817	17.039	7.223
0.25	10.261	17.962	7.701
0.26	10.477	18.244	7.766
0.27	10.151	18.478	8.327
0.28	10.953	20.091	9.138
0.29	11.286	20.028	8.743

volatility	euro	lookback	premium
0.30	12.533	21.251	8.718
0.31	12.233	21.390	9.157
0.32	12.211	22.067	9.856
0.33	13.550	22.965	9.414
0.34	12.787	22.765	9.978
0.35	13.983	23.967	9.983
0.36	14.591	24.841	10.250
0.37	14.321	24.343	10.022
0.38	14.571	25.168	10.597
0.39	14.768	25.300	10.532
0.40	16.880	27.665	10.785

From the results, we can conclude that the prices of both european option and lookback option would increase with the volatility. This situation accords with our cognition and pricing model.

Also, the premium grows with the increase of volatility, which implies volatility has a greater

impact on price of lookback option and these two options' growth rate with volatility is not same.