MF 796 Assignment 5

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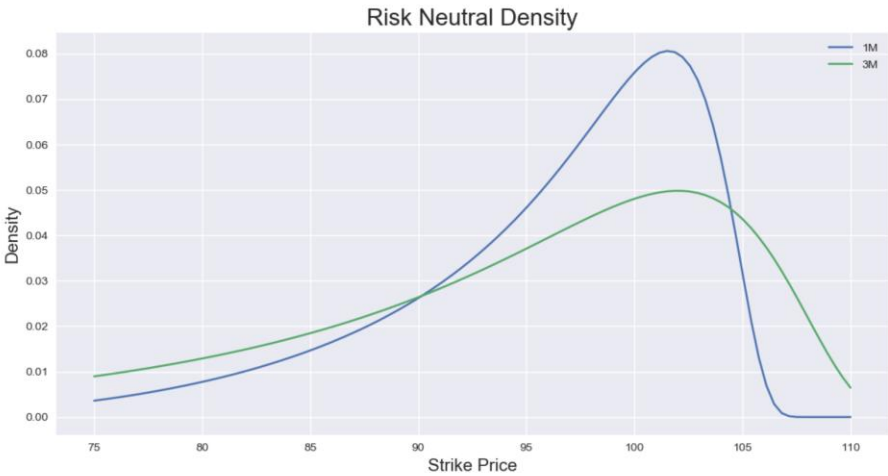
Problem 1

(a) Strike Price extracted is as follows:

	1M	3M
10P	89.138758	84.225674
25P	95.542103	93.470685
40P	98.700642	98.127960
50P	100.138720	100.338826
40C	101.262273	102.141761
25C	102.783751	104.532713
10C	104.395838	107.422225

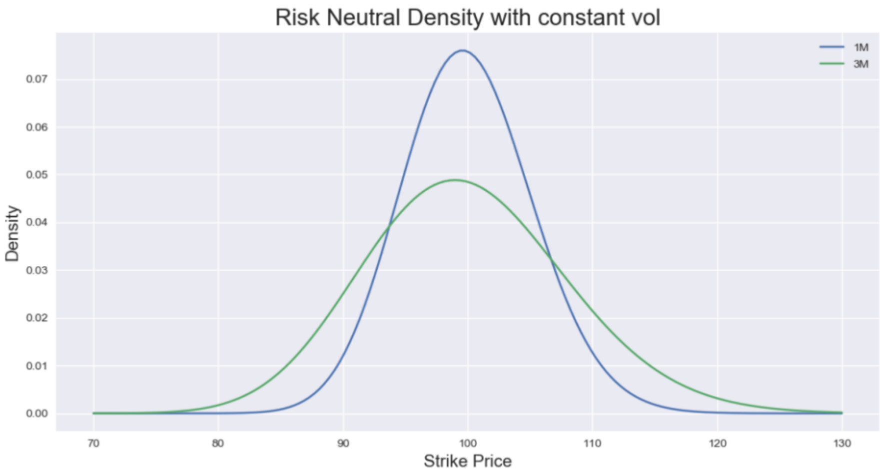
(b) The volatiliy function derived from interpolation is like the following figure:

(c) Risk Neutral Density



The results shown in the figure is consistent with our intuition. Both density present a skewed probability due to volatility skew. Also, the 3M option has a fatter tail than 1M option because the price is more likely to have great change when maturity increases.

(d) Risk Neutral Density with constant volatility



By using the constant volatility, both density has no skew.

(e) The price of the options:

Price of 1M European Digital Put Option with Strike 110: 0.98
Price of 3M European Digital Call Option with Strike 105: 0.32
Price of 2M European Call Option with Strike 100: 2.72

Problem 2

(a) In order to check for the arbitrage of option price, we need to check whether option prices are monotonically changing in strike; whether the rate of change relative to strike for call option is between -1 and 0, and the rate of change relative to strike for put option prices is between 0 and 1; whether option prices are convex with respect to changes in strike. As shown in the tables below, all the option data has no arbitrage opportunity. (False means no arbitrage)

Arbitrage check of Call:

	Monotonic	Delta	Convexity
49	False	False	False
140	False	False	False
203	False	False	False

Arbitrage check of Put:

	Monotonic	Delta	Convexity
49	False	False	False
140	False	False	False
203	False	False	False

(b) - (c) The optimized parameters with equal weight:

Start Params	**Lower Bounds**	**Upper Bounds**	**Params**	**Objective Func**
[0.25,0.12,0.4,0.25, 0.09]	[0.001,0,0,-1, 0]	[5.0, 2.0, 2.0, 1, 1.0]	[0.11,0.63,0.87,-0.81, 0.03]	35.72
[2, 0.2, 0.5, -1, 0.1]	[0.01,0,0,-1, 0]	[5.0, 2.0, 2.0, 1, 1.0]	[4.14,0.06,1.67,-0.81,0.04]	32.73
[0.2, 0.1, 0.5, 0.1, 0.5]	[0.001,0,0,-1, 0]	[2.5, 1, 1, 0.5, 0.5]	[3.05,0.06,1.43,-0.81, 0.04]	32.83
[0, 0.2, 0.2, 0, 0.2]	[0.01,0.01,0,-1,0]	[2.5, 1, 1, 0.5, 0.5]	[1.28,0.09,1,-0.82,0.03]	34.03

For the optimization, I use minimize function in Scipy package of Python. The optimization method I choose is 'SLSQP'. The parameters are kappa, theta, sigma, rho, v0. From the table above, we can see that the minimized objective function, which is the squared error, doesn't change significantly with different start parameters and bounds. However, with different bounds, we could see that the optimal value of kappa may vary while the other parameters would not change a lot. In summary

(d) The optimized parameters with varying weights:

Start Params	**Lower Bounds**	**Upper Bounds**	**Params**	**Objective Func**
[0.25,0.12,0.4,0.25, 0.09]	[0.001,0,0,-1, 0]	[5.0, 2.0, 2.0, 1, 1.0]	[3.27,0.05,1.15,-0.78,0.03]	256.36
[2, 0.2, 0.5, -1, 0.1]	[0.01,0,0,-1, 0]	[5.0, 2.0, 2.0, 1, 1.0]	[2.5,0.06,1.04,-0.78,0.03]	257.49
[0.2, 0.1, 0.5, 0.1, 0.5]	[0.001,0,0,-1, 0]	[2.5, 1, 1, 0.5, 0.5]	[2.45,0.06,1,-0.78,0.03]	258.03
[0, 0.2, 0.2, 0, 0.2]	[0.01,0.01,0,-1,0]	[2.5, 1, 1, 0.5, 0.5]	[1.28,0.09,1,-0.82,0.03]	256.24

Since the weight is inversely proportional to the bid-ask spread of the option price, the more liquid the option is, the greater weight the option would have in calculating the squared error. As we can see from the table above, the minimized objective function is also stable, and the parameters are roughly same as the equal weighted case.

Problem 3

(a) Delta comparison

Delta calculated by FFT is: 0.496
Delta calculated by BSM is: 0.365

The deltas calculated through different methods generate the different results. That is mainly because FFT algorithm includes the stochastic change of volatility while the volatility in BSM is constant. So I think the delta calculated by FFT is better.

According to the property of delta, if we hold a long position of call, we need to short delta position of underlying asset to build a delta neutral portfolio.

(b) Vega Comparison

Vega calculated by FFT is: 132.37
Vega calculated by BSM is: 50.04

From the results above, we can see that the Vega calculated by FFT is greater than the one through BSM. This is mainly because the rho in the Heston model is negative, we can expect the sensitivity of the option price calculated from Heston Model would be larger.