Week-5

APRIL 2022

M	T	W	T	F	S	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

Non-linear Demand Response Curve

FRIDAY
MARCH 2022
4

WEEK 10
063-302

Non-linear relationship

- Price - Demand relationship not linear.

Analysing constant elasticity model using Simple Linear Regression

Constant elasticity model

- The constant elasticity model is not linear.
- Transformations allow the use of regression analysis to describe a curved pattern.

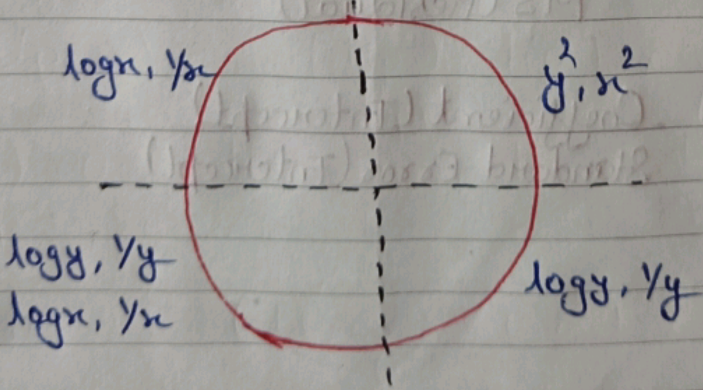
$$y = \beta_0 + \beta_1 x + \varepsilon$$

ε → error

y → response variable (dependent)

x → explanatory variable (independent)

- Select the correct transformation -



Week-5

APRIL 2022

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FRIDAY
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4
WEEK 10
063-302

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x → explanatory (independent) variable
ε → error

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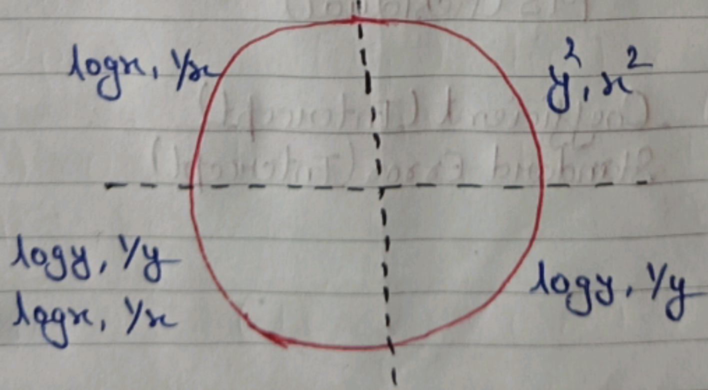


$$D = e\, C \cdot p^{-\varepsilon}$$

On taking natural log on both sides

$$\underbrace{LN(D)}_{\text{response }(y)} = \underbrace{LN(C)}_{\beta_0\,\text{(intercept)}} + \underbrace{\boxed{(-\varepsilon)}}_{\beta_1\,\text{(slope)}}\underbrace{\boxed{LN(P)}}_{\text{explanatory }(x)}$$

$$y = \beta_0 + \beta_1 x$$

$$MS(\text{Regression}) = \frac{SS(\text{Regression})}{df(\text{Regression})}$$

$$MS(\text{Residual}) = \frac{SS(\text{Residual})}{df(\text{Residual})}$$

Standard Error = Standard deviation of the error term

$$F\text{ Statistics} = \frac{MS(\text{Regression})}{MS(\text{Residual})}$$

$$\text{tstat} = \frac{\text{Coefficient (Intercept)}}{\text{Standard Error (Intercept)}}$$

Optimal Pricing – Revenue Maximization

Demand response curve

- Let us assume a linear relationship b/w Price and Demand.

$$D(p) = D_0 + m * p$$

Where $D(p)$ is the demand (as a function of price, p), D_0 is the market size (total demand when the price=0) and m is the slope.

From the last session, this relationship was:

$$D(p) = 5842.8 - 157.7 * p$$

Sales Revenue function

- The revenue obtained by selling product is called Revenue.

Revenue from sales is always calculated as

$$\text{Revenue} = \text{Demand} \times \text{Price}$$

$$R(p) = D(p) \times p$$

$$R(p) = (D_0 + m \times p) \times p = D_0 \times p + m \times p^2$$

$$\boxed{R_p = D_0 \times p + m \times p^2}$$

~~Demand response curve~~

Revenue maximization

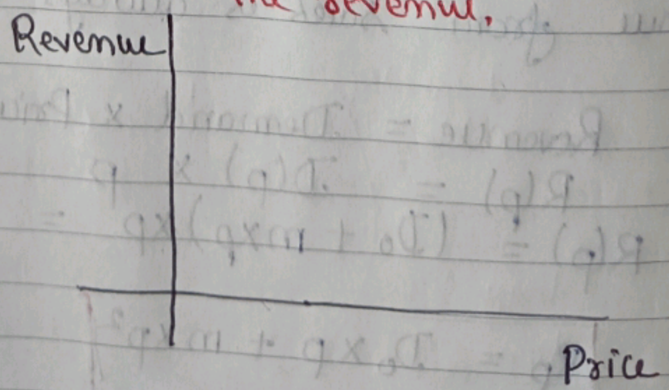
- We now find the optimal price that maximizes the revenue.
- From the first order Necessary Condition, we find the partial derivative of the revenue function w.r.t p, and set it to be zero.

e.g. $$R(p) = 5842.8 \times p - 157.7 \times p^2$$

$$\frac{\partial R(p)}{\partial(p)} = 5842.8 - 2 \times 157.7 \times p$$

$$\frac{\partial R(p)}{\partial(p)} = 0 \Rightarrow \boxed{p^* = \frac{5842.8}{2 \times 157.7} = 18.52}$$

optimal price that increases the revenue.



Profit function

- Typically, profit is the difference b/w the revenue and cost.
- Assume that the marginal cost of producing the good is c.
- The profit function

$$\boxed{\pi(p) = \text{Total Revenue} - \text{Total Cost} = D(p)\times p - D(p)\times c}$$

$$\pi(p) = D(p)\times(p-c) = (D_0 + m\times p)\times(p-c)$$

e.g. the profit function is,

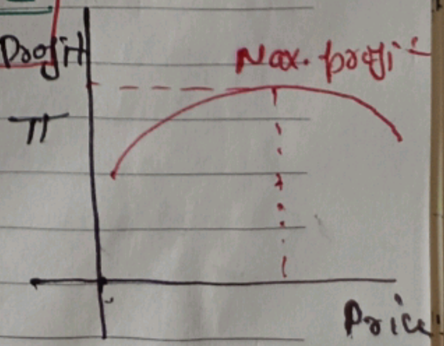
$$\pi(p) = (5842.8 - 157.7\times p)(p-c)$$

Profit maximization

- To find the optimal price that maximizes profit, we again use the First Order Necessary Condition.

$$\frac{\partial \pi(p)}{\partial p} = D_0 + 2\times m\times p - m\times c$$

$$\frac{\partial \pi(p)}{\partial p} = 0 \Rightarrow \boxed{p^* = \frac{D_0 - m\times c}{-2\times m}}$$



10 THURSDAY
2022 MARCH

WEEK 11
069-296

2022 FEBRUARY

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e.g. Let the marginal cost be 15
So the profit maximizing price is,

$$p^* = \frac{5842.8 + 157.7 \times 15}{2 \times 157.7} = 26.02$$

* The prices that maximises revenue need not be same the prices that maximize the profit.

Revenue Maximization versus Profit Maximization

Profit maximization

- Maximizing price

$$p^* = \frac{5842.8 + 157.7 \times 15}{2 \times 157.7} = 26.02$$

Revenue = Demand x Price
Profit = Demand x (p − c)
↳ c = marginal cost

Operations Research: Linear Programming and Duality in Spreadsheets and Python

Domain

8

Linear Programming

Integer Programming

9

Maximize $2X_1 + 3X_2$

Subject to

$X_1 + X_2 \leq 120$

11 $2X_1 + 3X_2 \leq 320$

$X_1, X_2 \geq 0$

Non-linear Programming

12

Linear Programming

1

2 Imagine you running an automobile firm which sells car in three segments - Hatchback, Sedan and SUV

3 at prices ₹5,00,000, ₹10,00,000 and ₹25,00,000 respectively.

Suppose that the manufacturing of cars primarily requires the following raw materials A and B. The firm has 1,20,000 units of resources A and 1,40,000 units of resource B available. The resource requirements for the manufacturing of each car variant is given below.

Requirements	Resource A	Resource B
Hatchback	15	20
Sedan	20	50
SUV	60	100

How many cars of each type should be produced to maximize revenue?

Solⁿ:- x_1 – No. of Hatchback cars produced
x_2 – No. of Sedan cars to be produced
x_3 – No. of SUV cars to be produced.

Objective function

Maximize $500000x_1 + 1000000x_2 + 2500000x_3$

- Resource A constraint
$$15x_1 + 20x_2 + 60x_3 \leq 120000$$
- Resource B constraint
$$20x_1 + 50x_2 + 100x_3 \leq 140000$$
- Non-negativity restrictions
$$x_1, x_2, x_3 \geq 0$$

Linear program

Primal

Maximize $500000x_1 + 1000000x_2 + 2500000x_3$
Subject to
$$15x_1 + 20x_2 + 60x_3 \leq 120000$$
$$20x_1 + 50x_2 + 100x_3 \leq 140000$$
$$x_1, x_2, x_3 \geq 0$$

15 TUESDAY
2022 MARCH

2022 FEBRUARY

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WEEK 12
074-291

Automobile firm
- Possesses resources A and B
- Manufactures and sells cars
- Aim: Maximize revenue

Buyer
- Purchase resources
- Aim: Minimize total cost.

Let Y_1, Y_2 be the costs of resource A and resource B

Dual

Minimize $120000 Y_1 + 140000 Y_2$

Subject to

$$15 Y_1 + 20 Y_2 \geq 500000$$
$$20 Y_1 + 50 Y_2 \geq 1000000$$
$$60 Y_1 + 100 Y_2 \geq 2500000$$
$$Y_1, Y_2 \geq 0$$

Dual variables : Shadow price or Marginal price of the resource at the optimum.

Primal-Dual relationship

Primal

Maximize $500000X_1 + 1000000X_2 + 2500000X_3$

Subject to

$$15X_1 + 20X_2 + 60X_3 \leq 120000$$
$$20X_1 + 50X_2 + 100X_3 \leq 140000$$
$$X_1, X_2, X_3 \geq 0$$

Dual

Minimize $120000Y_1 + 140000Y_2$

Subject to

$$15Y_1 + 20Y_2 \geq 500000$$
$$20Y_1 + 450Y_2 \geq 1000000$$
$$60Y_1 + 100Y_2 \geq 2500000$$
$$Y_1, Y_2 \geq 0$$

Primal	Dual
Maximization	Minimization
No. of Constraints	No. of variables
No. of variables	No. of constraints
Objective function coefficient	Right hand side in constraints
Right hand side in constraints	Objective function coefficient

Dual of the dual is the primal!

Dual

Minimize $120000Y_1 + 140000Y_2$
Subject to
$15Y_1 + 20Y_2 \geq 500000$
$20Y_1 + 50Y_2 \geq 1000000$
$60Y_1 + 100Y_2 \geq 2500000$
$Y_1, Y_2 \geq 0$

Converts to Standard form

Standard form

Maximize $-120000Y_1 - 140000Y_2$
Subject to
$-15Y_1 - 20Y_2 \leq -500000$
$-20Y_1 - 50Y_2 \leq -1000000$
$-60Y_1 - 100Y_2 \leq -2500000$
$Y_1, Y_2 \geq 0$

Finding the dual

Minimize $-500000X_1 - 1000000X_2 - 2500000X_3$
Subject to
$-15X_1 - 20X_2 - 60X_3 \geq -120000$
$-20X_1 - 50X_2 - 100X_3 \geq -140000$
$X_1, X_2, X_3 \geq 0$

Primal

Maximize $500000x_1 + 1000000x_2 + 2500000x_3$

Subject to

$$15x_1 + 20x_2 + 60x_3 \leq 120000$$
$$20x_1 + 50x_2 + 100x_3 \leq 140000$$
$$x_1, x_2, x_3 \geq 0$$

How to construct a dual?

Primal

Minimize $500x_1 + 100x_2 + 200x_3$

Subject to

$$15x_1 + 20x_2 + 60x_3 \geq 1200$$
$$20x_1 + 50x_2 + 100x_3 \leq 1400$$
$$x_1 \geq 0, \ x_2 \leq 0, \ x_3 \text{ unrestricted}$$

Convert to standard form

Define new variables $x_4, x_5, x_6 \geq 0$. Let $x_3 = x_4 - x_5$ and $x_6 = -x_2$.

Minimize $500x_1 - 100x_6 + 200(x_4 - x_5)$

Subject to

$$15x_1 - 20x_6 + 60(x_4 - x_5) \geq 1200$$
$$20x_1 - 50x_6 + 100(x_4 - x_5) \leq 1400$$
$$x_1, x_4, x_5, x_6 \geq 0$$

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Convert objective function to maximization
Convert $\geq$ constraint to $\leq$ constraint

Maximize $-500x_1 + 100x_6 - 200x_4 + 200x_5$

Subject to

$$-15x_1 + 20x_6 - 60x_4 + 60x_5 \leq -1200$$
$$20x_1 - 50x_6 + 100x_4 - 100x_5 \leq 1400$$
$$x_1, x_4, x_5, x_6 \geq 0$$

Dual

$-1200y_1 + 1400y_2$

Subject to

$$-15y_1 + 20y_2 \geq -500$$
$$20y_1 - 50y_2 \geq 100$$
$$-60y_1 + 100y_2 \geq -200$$
$$60y_1 - 100y_2 \geq 200$$
$$y_1, y_2 \geq 0$$