MAY 2022

M	T	W	T	F	S	S
30	31					1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

Week-9

Data Envelopment Analysis: Example with two outputs and one input

SATURDAY
APRIL 2022
23
WEEK 17
113-252

Every sales office be given the same budget (INR 2,00,000). The sales achieved (in INR) and the potential sales leads (potential customers) are the outputs.

Input

Output

Sales office		O_1 Sales (INR)		O_2 No of leads	
1	O_{11}	11,10,000	15	O_{21}	
2	O_{12}	17,50,000	10	O_{22}	
3	O_{13}	34,50,000	12	O_{23}	
4	O_{14}	12,24,000	23	O_{24}	
5	O_{15}	24,00,000	20	O_{25}	

$O_{11} = 11,10,000$, $O_{21} = 15$, $I_{11} = 2,00,000$

DEA - LP: Two output case

- like before we need to formulate an optimization problem for each of the sales office.
- Here, the only input is (I_{iK}) is the budget to run the office.
- The outputs are (O_{jK}): O_{1K} is the sales achieved; and O_{2K} potential leads.
- Corresponding to this, we would need one input weight (x_{1K}) and two output weights (y_{1K}, y_{2K}).

24
SUNDAY

MAY 2022

M	T	W	T	F	S	S
30	31					1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
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Week-9

Data Envelopment Analysis: Example with two outputs and one input

SATURDAY
APRIL 2022
23
WEEK 17
113-252

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4	O_{14}	12,24,000	23	O_{24}	
5	O_{15}	24,00,000	20	O_{25}	

$O_{11} = 11,10,000$, $O_{21} = 15$, $I_{11} = 2,00,000$

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24
SUNDAY

25 MONDAY
2022 APRIL

2022 MARCH

S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

WEEK 18
115-250

Efficiency (E) = $\frac{y_{11}O_{11} + y_{21}O_{21}}{x_{11}I_{11}}$ → Maximize numerator
→ Normalize denominator to 1

DEA-LP: Two output case - Sales office 1

Max y_{12} * 1110000 + y_{22} * 15

Subject to x_{11} * 2000000 = 1

y_{12} * 1110000 + y_{22} * 15 ≤ x_{11} * 2000000
y_{12} * 1750000 + y_{22} * 10 ≤ x_{11} * 2000000
y_{12} * 3450000 + y_{22} * 12 ≤ x_{11} * 2000000
y_{12} * 1224000 + y_{22} * 23 ≤ x_{11} * 2000000
y_{12} * 2400000 + y_{22} * 20 ≤ x_{11} * 2000000
Decision variables : $x_{11}, y_{11}, y_{21} \geq 0$

Example with multiple inputs and multiple outputs

MAY 2022

M	T	W	T	F	S	S
30	31					1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
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23	24	25	26	27	28	29

TUESDAY
APRIL 2022
26

WEEK 18
116-249

DEA:- LP - a Generic formulation

- For our comparison of the performance of the various sales offices, let us consider the full data.

	Inputs		Outputs	
Sales office	Budget (INR)	Team Size	Sales (INR)	No of leads
1	3,00,000	13	11,10,000	15
2	2,56,000	9	17,50,000	10
3	5,00,000	7	34,50,000	12
4	3,90,000	10	12,24,000	23
5	1,85,000	14	24,00,000	20

- Two inputs and two outputs - need to define the variables accordingly.
- Difficulty to image this on a 2D plot.

$I_{11} = 3,00,000$, $I_{21} = 11,10,000$ ~~13~~

$O_{11} = 11,10,000$, $O_{21} = 15$

Decision Variables { x_{11} x_{21} Input Weights
y_{11} y_{21} Output Weights

Efficiency $(E_1) = \dfrac{y_{11}O_{11} + y_{21}O_{21}}{x_{11}I_{11} + x_{21}I_{21}}$ → Maximizing numerator
→ Normalizing to 1

$E_1 \leq 1$, $E_2 \leq 1$, $E_3 \leq 1$ ....

$E_2 = \dfrac{y_{11}O_{12} + y_{21}O_{22}}{x_{11}I_{12} + x_{21}I_{22}} \leq 1$

27 WEDNESDAY
2022 APRIL

S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

2022 MARCH

WEEK 18
117-248

DEA : LP - Sales Office 1

Max $y_{11} * 11,10,000 + y_{21} * 15$

Subject to $x_{11} * 3,00,000 + x_{21} * 13 = 1$

$y_{11} * 11,10,000 + y_{21} * 15 \leq x_{11} * 3,00,000 + x_{21} * 13$

$y_{11} * 17,50,000 + y_{21} * 10 \leq x_{11} * 2,56,000 + x_{21} * 9$

$y_{11} * 34,50,000 + y_{21} * 12 \leq x_{11} * 5,00,000 + x_{21} * 7$

$y_{11} * 12,24,000 + y_{21} * 23 \leq x_{11} * 3,90,000 + x_{21} * 10$

$y_{11} * 24,00,000 + y_{21} * 20 \leq x_{11} * 1,85,000 + x_{21} * 14$

Decision variables: $x_{11}, x_{21}, y_{11}, y_{21} \geq 0$.

MAY 2022

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DEA - Prescription for inefficient units (One output and two inputs case)

THURSDAY
APRIL 2022
28

WEEK 18
118-247

Inefficient DMUs

- The graphical output and the optimization problem solution identifies the efficient and the inefficient DMUs.
- The efficient DMUs are NOT on the efficiency frontier.
- Of course, by moving towards the efficiency frontier (or the 'envelope')
- However, the move to the envelope is neither vertical nor horizontal.
- Important economic concepts: the types of efficiencies ("scale" vs "technical"), and disposability of inputs/outputs ("constant" vs "variable" return-to-scale).

Graphical calculation of HCU

- Coordinates of DMU1 are (3,00,000, 13). To generate sales of INR 10,00,000, it uses a budget of INR 3,00,000 and a team size of 13.
- The HCU is created as a combination of two efficient units as reference.
- For inefficient DMU1, DMUs 2 and 5 are the reference
- The HCU for 1 is found as a combination of 2 and 5.
- From simple geometry, we can find the HCU for 1 to be (2,37,540, 10.3).
- Conclusion: For the DMU1 called to be efficient, it needs to reduce its input: from the current budget of INR 3,00,000 to spend only INR 2,37,540, and

29 FRIDAY
2022 APRIL

2022 MARCH

S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

WEEK 18
119-246 reduce the team size from 13 to 10.3

8
9
10
11
12
1
2
3
4
5
6

Prescription for inefficient units (Two outputs and one input case)

MAY 2022

M	T	W	T	F	S	S
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SATURDAY
APRIL 2022
30

WEEK 18
120-245

Graphical calculation of HCU

- Coordinates of DMU_1 are (11,10,000, 15). With a budget of INR 2,00,000, the DMU_1 generates a sale of INR 11,10,000 and number of sales leads of 15.
- The HCU is a combination of two efficient units as reference.
- For inefficient DMU_1, DMU_3 & 4 and 5 are the reference.
- From simple geometry, we can find the HCU for 1 to be (16,26,000, 21.97).
- Conclusion: For the DMU_1 to be called efficient, its needs to increase its output: current sales of INR 11,10,000 needs to be increased to INR 16,26,000 and number of sales leads need to go up from 15 to 21.97.