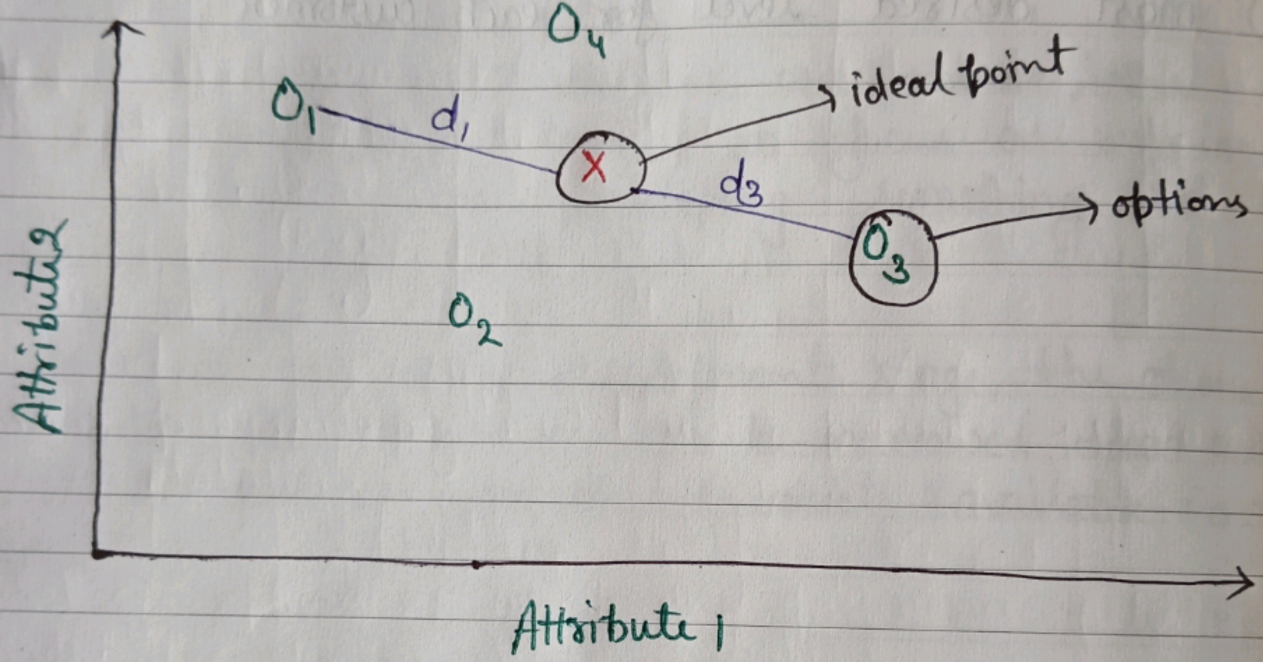
~~Week 10~~

~~Consumer Choice models~~

Conjoint Problem

Geometric explanation



(X_1, X_2) = Coordinates of the Ideal point

(Y_{11}, Y_{12}) = Coordinates of O_1

d_1 = Distance b/w O_1 and ideal point

If $d_1 < d_2$
then O_1 have more preference than O_2.

Optimization Formulation of Conjoint Problem

The logic

- A methodology for analyzing individual differences in preference judgements with regard to a set of options.
- The product options are represented as points in a multi-attribute space.
- Different consumers (Subject) correspond to different "ideal points" that denote their "most preferred" location.
- Given two options, the customer is supposed to prefer that option which is "closer" to his/her ideal point.
- As a measure of distance, normally either the Euclidean metric or the weighted Euclidean metric is used.

Illustrative example

	Attribute 1	Attribute 2
Product 1	1.5	12
Product 2	10	8
Product 3	2.3	4
Product 4	1	7

y_{31} y_{32}

Pairs	Pairwise preference data	
	Prefers	Over
(1,2)	1	2
(1,3)	1	3
(1,4)	4	1
(2,3)	2	3
(2,4)	2	4
(3,4)	4	3

Linear programming model using pairwise judgements:

Notations

- Set of options on which the preference judgement is made: $j = \{1, 2, ----, n\}$.
- The n options are described in terms of t dimension $P = \{1, 2, ---, t\}$
- The pre-specified location of the j^{th} option in the t-dimensional space is denoted by Y_j. That is $Y_j = \{y_{j,p}\}\ p \in P$.

- The ideal point of the subject is $X = \{x_p\}\ p \in P$, that is, the product location most preferred by the individual. (x_p can be positive, negative, or zero).

Problem Specific notations

8 • We have four variants to be compared: $n=4$.
$j = 1, 2, 3, 4$.

9 • The $4 (n=4)$ options are described on 2 dimensions.
$t=2$. $P = \{1, 2\}$.

10 • Specified location of the j^{th} option in the 2-dimensional space is denoted by Y_j. This has two coordinates:

11 $Y_j = \{Y_{j1}, Y_{j2}\}$. We ~~ha~~ have Y_1, Y_2, Y_3, Y_4.

12
$$Y_1 = (Y_{11}, Y_{12})$$
$$Y_2 = (Y_{21}, Y_{22})$$

1

• Ideal point for this consumer is $X = \{X_1, X_2\}$

2

3

4

5

6

LP model using pairwise judgements : Notations

- Then the unweighted and weighted distance of the j^{th} option from ~~of~~ the ideal point, respectively, is given by :

$$d_j^u = \left[\sum_{p \in P} (y_{j,p} - x_p)^2\right]^{1/2}, \forall j \in J$$

$$d_j^w = \left[\sum_{p \in P} \boxed{w_p}(y_{j,p} - x_p)^2\right]^{1/2}, \forall j \in J$$

→ Weighted distance

- Weights are non-negative for all the attributes ($w_p \geq 0$)
- Moreover, the squared distance is $S_j = (d_j^w)^2$.
- Let $\Omega = \{j, k\}$ denote the set of ordered pairs (j, k) where j designates the preferred product on a forced choice basis resulting from a paired comparison involving j and k.
- In this set-up, the only variables are weights (w_p) and the ideal point (x_p). All the others are parameters and are known apriori.
- Any optimization problem for which (W, x) is the

solution, will satisfy the (distance) inequality:

$$S_K \geq S_j$$

- Given option locations $Y_j = \{y_{j,p}\}$ and the set of ordered pairs Ω, determine the solution (W, X) such tha
(a) weights are non-negative and
(b) distance inequality is violated as less as possible.
- The objective function can be formulated as a minimization. And the objective function can be defined as the "poorness of fit".

$$\boxed{B = \text{poorness of fit} = \sum_{(j,k)\in\Omega} (S_j - S_k)^+}$$

$$(x)^+ = \max(0, x)$$
$$x = -2, \Rightarrow x^+ = 0$$
$$x = 2 \Rightarrow x^+ = 2$$
$$(S_j - S_K)^+ = \max(0, S_j - S_K)$$

Goodness of fit − Poorness of fit = Constant (h)

16 MONDAY
2022 MAY

WEEK 21
136-229

2022 APRIL

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

Final formulation

~~first~~ attribute

Let,

$$a_{jkp} = y_{kp}^2 - y_{jp}^2, \ \forall (j,k) \in \Omega \text{ and } p \in P$$

$$b_{jkp} = -2(y_{kp} - y_{jp}), \ \forall (j,k) \in \Omega \text{ and } p \in P$$

$$V = \{v_p\} = \{w_p x_p\}, \ p \in P \quad \Rightarrow x_p = \frac{V}{w_p}$$

$$\boxed{Z_{jk} = \max\left[0, -\left[\sum_{p\in P} w_p a_{jkp} + \sum_{p \in P} v_p b_{jkp}\right]\right]}$$

e.g. if $j=1, k=3, p=1$

$$a_{131} = (y_{31})^2 - (y_{11})^2$$

$$\boxed{\begin{aligned} A_p &= \sum_{(j,k)\in\Omega} a_{jkp} \text{ for } p \in P \\ D_p &= \sum_{(j,k)\in\Omega} b_{jkp} \text{ for } p \in P \end{aligned}}$$

Final formulation : LP

$$\text{Min} \sum_{(j,k)\in\Omega} z_{jk}$$

$$\left\{ x_p = \frac{\cancel{w_p}}{\cancel{v_p}} \quad \frac{v_p}{w_p} \right.$$

Subject to :

$$\sum_{p\in P} w_p a_{jkp} + \sum_{p\in P} v_p b_{jkp} + z_{jk} \geq 0 \text{ for } (j,k)\in\Omega$$

$$\sum_{p\in P} w_p A_p + \sum_{p\in P} v_p D_p = 1$$

* Pairwise choice = $\frac{n(n-1)}{2}$ → no. of columns

Example

Product (K)	Attribute 1 (P)	Attribute 2 (J)
1	1.5	12
2	10	8
3	2.3	4
4	1	7
5	9	1

n

$n = 5$

Pairwise choice $= \frac{n(n-1)}{2} = \frac{5 \times 4}{2} = 10$

Consumer Choices	j		K
Prefers	1	over	2
Prefers	3	over	1
Prefers	4	over	1
Prefers	5	over	1
Prefers	2	over	3
Prefers	2	over	4
Prefers	2	over	5
Prefers	4	over	3
Prefers	3	over	5
Prefers	4	over	5

$a_{jkp} = y_{kp}^2 - y_{jp}^2 , \forall (j,k) \in \Omega$ and $p \in P$

$b_{jkp} = -2(y_{kp} - y_{jp}) , \forall (j,k) \in \Omega$ and $p \in P$

$p \rightarrow$ Attribute 1 / Attribute 2

attribute	$y_{kp} \to k$	$y_{jp} \to j$
a_{121}	$y_{21} \to p$	$y_{11} \to p$
a_{311}	y_{11}	y_{31}
a_{411}	y_{11}	y_{41}
a_{511}	y_{11}	y_{51}
a_{231}	y_{31}	y_{21}
a_{241}	y_{41}	y_{21}
a_{251}	y_{51}	y_{21}
a_{431}	y_{31}	y_{41}
a_{351}	y_{51}	y_{31}
a_{451}	y_{51}	y_{41}

$\Rightarrow$

y_{kp}	y_{jp}	a_{jk1} $\cancel{(y_{kp}^2 - y_{jp}^2)}$	a_{jk2} $\cancel{-2(y_{kp} - y_{jp})}$
10	1.5	97.75	-80
1.5	2.3	-3.04	128
1.5	1	1.25	95
1.5	9	-78.75	143
2.3	10	-94.71	-48
1	10	-99	-15
9	10	-19	-63
2.3	1	4.29	-33
9	2.3	75.71	-15
9	1	80	-48
Total ~~35.5~~	~~64~~	-35.5	64

b_{jk1}	b_{jk2}
-17	8
1.6	-16
-1	-10
15	-22
15.4	8
18	2
2	14
-2.6	6
-13.4	6
-16	12
2	8

$A_p = \sum_{(j,k)\in\Omega} a_{jkp}$ for $p \in P$

$D_p = \sum_{(j,k)\in\Omega} b_{jkp}$ for $p \in P$

$A_1 = \sum a_{jk1} = -35.5$
$A_2 = \sum a_{jk2} = 64$

$D_1 = \sum b_{jk1} = 2$
$D_2 = \sum b_{jk2} = 8$

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

Final formulation: LP

$$\text{Min} \sum_{(j,k)\in\Omega} z_{jk}$$

Subject to:

$$\sum_{p\in P} w_p a_{jkp} + \sum_{p\in P} v_p b_{jkp} + z_{jk} \geq 0 \text{ for } (j,k)\in\Omega$$

$$\sum_{p\in P} w_p A_p + \sum_{p\in P} v_p D_p = 1$$

decision variables

$w_p \geq 0$ and v_p unrestricted for $p \in P$

$z_{jk} \geq 0$ for $(j,k)\in\Omega$

W_1	W_2	V_1	V_2
0.00676769	0.00991238	0.03813	0.0662

z_{12}	z_{31}	z_{41}	z_{51}	z_{23}	z_{24}	z_{25}	z_{43}	z_{35}	z_{45}
1	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	1

JUNE 2022

M	T	W	T	F	S	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

SATURDAY
MAY 2022
21

WEEK 21
141-224

$\sum \cancel{w_p} A_p$

$\sum w_p a_{jkp} + \sum v_p b_{jkp} + z_{jk}$		
$-1.05471E-15$	$\geq$	0
0.25	$\geq$	0
0.25	$\geq$	0
$6.66134E-16$	$\geq$	0
$1.11022E-15$	$\geq$	0
$6.66134E-16$	$\geq$	0
0.25	$\geq$	0
$4.44089E-16$	$\geq$	0
0.25	$\geq$	0
0.25	$\geq$	0
1	$=$	1

$$\boxed{\sum A_p w_p + \sum v_p D_p} = 1$$

$$\text{Min} \sum_{(j,k)\in\Omega} Z_{jk} \rightarrow \text{objective function} = 0.25$$

SUNDAY 22

Conjoint Analysis: Statistical method – Linear regression

Introduction :-

8 • A traditional conjoint analysis is really just a multiple regression problem.

9 • The respondant's rating for the product concepts form the dependent variable.

10 • The characterstic of the product (the attribute levels) are the independent (predictor) variables.

11 • The estimated betas associated with the independent variables are the ~~utt~~ utilities (preference score)
12 for the levels.

• The R-Square for the regression characterizes the
1 internal consistency of the respondants.

2 **Example**

3 Consumer choice process for a cell phone purchase
Three attributes – Brand, Battery Size and the
4 Camera resolution.

5 • We have three brands to choose from; two options for battery, and three choices for the
6 camera resolution.

• We assume that this is a full-factorial design. Therefore, the respondent (customer, buyer) is shown all the 18 combinations available. And they provide preference ratings for each of these combination

$X_1 = \begin{cases} 1 & \text{if brand is Samsung} \\ 0 & \text{otherwise} \end{cases}$

$X_2 = \begin{cases} 1 & \text{if brand = Vivo} \\ 0 & \text{otherwise} \end{cases}$

$X_3 = \begin{cases} 1 & \text{if brand = Xiomi} \\ 0 & \text{otherwise} \end{cases}$

$X_4 = \begin{cases} 1 & \text{if battery = 4500 mAh} \\ 0 & \text{otherwise} \end{cases}$

$X_5 = \begin{cases} 1 & \text{if battery = 6000 mAh} \\ 0 & \text{otherwise} \end{cases}$

$X_6 = \begin{cases} 1 & \text{if Camera = 20MP} \\ 0 & \text{otherwise} \end{cases}$

$X_7 = \begin{cases} 1 & \text{if Camera = 13MP} \\ 0 & \text{otherwise} \end{cases}$

$X_8 = \begin{cases} 1 & \text{if Camera = 8MP} \\ 0 & \text{otherwise} \end{cases}$

Regression Method for Conjoint Analysis

Example: Regression Statistics

- The regression equation is:

~~Y = 1.334~~

Intercept (coefficients) VIVO (coefficient) XIOMI (coefficient)

$$Y = 1.334 + 1.167 \times VIVO + 1.83 \times XIOMI +$$
$$\beta_0 + \beta_1 X_1 + \beta_2 X_2 +$$
$$2.2334 \times (6000\,mAh) + 4.83 \times (20MP) + 2.16 \times (13MB)$$
$$\beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5$$

- The regression coefficients are called "part-worth".
- The regression is significant
- Each of the explanatory variables also significant.

Partworths

- These are the level utilities for the attributes.
- The total worth of the product (option) is calculated from multiple attributes and multiple levels of attributes together.
- Utilities values for the separate parts of the product (assigned to the attributes) are the part worths
- Partworth for each level of each attribute:

Base level

Brand/Manufacturer	
Samsung	0
VIVO	1.167
XIOMI	1.834

Base level

Battery	
4500 mAh	0
6000 mAh	2.334

Base level

Camera resolution	
8MP	0
13MP	2.167
20MP	4.834

Range = Max value − Min value

~~Attribute~~ ~~Partworth range~~

Importance of each attribute

Attribute	Partworth range (Max − Min)
Brand	1.834 − 0 = 1.834
Battery	2.334 − 0 = 2.334
Camera	4.834 − 0 = 4.834

Total of ranges = 1.834 + 2.334 + 4.834 = 9.002

Attribute	Importance
Brand	1.834/9 = 20.37%
Battery	2.334/9 = 25.92%
Camera	4.834/9 = 53.7%