Week - 8

MAY 2022

M	T	W	T	F	S	S
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30	31					
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Measuring the Efficiency of a Buisness Unit

THURSDAY
APRIL 2022
7
WEEK 15
097-268

Efficiency

Quantifying and comparing the efficiencies of the decision making units.

$$\text{Efficiency } (E) = \frac{\text{Actual output}}{\text{Rated output}}$$

or

$$\text{Efficiency } (E) = \frac{\text{Output}}{\text{Input}}$$

Productive efficiency (Production efficiency)

- Economics teaches us effective utilization of resources for the maximization of benefits (output).
- Productive efficiency is an aspect of economic efficiency focusing on maximizing the output under given constraints (without worrying about optimal allocation, or choice of products, etc.).
- The productive efficiency "frontier" are all the combinations of outputs such that the production of one product cannot be increased without sacrificing the output of the other (without any change in the technology).
-

- If the organization (any economic unit) is not on the frontier, it is inefficient.

Efficiency Measurement

- In the simplest way, efficiency is defined as the ratio of the output to the input

- Multiple types of inputs: Labor (white collar, blue collar, etc.); Infrastructure (factory, building, land, machinery etc.); Money (financial assets, loan, etc.).

- Essentially, resources goes as inputs

- Output can also be in many shapes and forms: Customers served/acquired; profits; sales volume, etc.

- How has the organization (or the economic unit) performed is the output.

Immediate questions

- How does the ratio of input to output work in presence of several inputs and several outputs?
- How do we, then, calculate the productive efficiency of an economic unit?

- More importantly, how do we compare several economic units on their efficiency?
- If an economic unit turns out to be inefficient, how can they become efficient?

Common approaches

- Operating ratios: Labor cost per transaction; sales per square feet; runs per innings.
- Problem: Doesn't reflect varying mix of inputs and outputs found in more diverse operations

- Financial ratios: Price to earning ratio (PE); Debt to equity ratio; Earnings per share (EPS)
- Problems:
  - Some inputs/outputs cannot be valued in currency terms.
  - Profitability is not the same as operating efficiency.

Single input, single output

input (X-axis) output (Y-axis) output/input

Sales office	Budget (INR)	Sales (INR)	Sales per INR invested
1	3,00,000	11,10,000	3.7
2	2,56,000	17,50,000	6.8
3	5,00,000	34,50,000	6.9
4	3,90,000	12,24,000	3.1
5	1,85,000	24,00,000	13.0

More inputs/outputs

- For two inputs and an output too, things are not difficult.

- Assume that each of the sales office has the same sales target: INR 10,00,000 (output). They have their budgets approved and the respective team sizes (input).

Sales office	Budget (INR)	Team size
1	3,00,000	13
2	2,56,000	9
3	5,00,000	7
4	3,90,000	10
5	1,85,000	14

One input, two outputs

- Let every sales office be given the same budget (INR 2,00,000). The sales achieved (in INR) and the potential sales leads (potential customers) are the outputs we track.

Sales office	→ x-axis Sales (INR)	→ y-axis No of leads
1	11,10,000	15
2	17,50,000	10
3	34,50,000	12
4	12,24,000	23
5	24,00,000	20

MAY 2022

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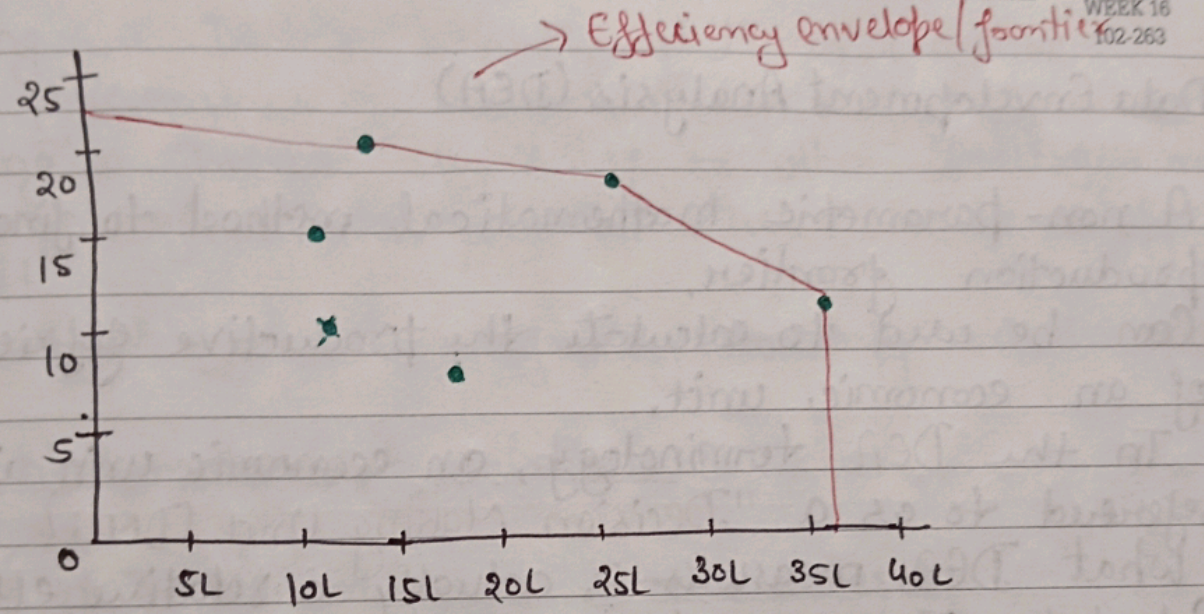
TUESDAY
APRIL 2022
12

WEEK 16
102-263



13 WEDNESDAY 2022 APRIL

2022 MARCH

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13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

WEEK 16
103-262

Data Envelopment Analysis (DEA)

- A non-parametric mathematical method to find the production frontier.
- Can be used to calculate the productive efficiency of an economic unit.
- In the DEA terminology, an economic unit is referred to as a "Decision Making Unit (DMU)
- What DEA measures is actually a relative efficiency - Calculate individual efficiencies for each DMU in a set of DMU's.
- Formulates an optimization problem for each DMU

DEA Logic

- For mutiple ~~units~~ inputs and multiple outputs define the weighted ratio.
- Solution: Let each DMU choose the input and the output weights to its advantage.
- Objective for each DMU: maximize its efficiency by choosing its $\boxed{\text{weight}}$ carefully.

$\downarrow$

Efficiency $(E) \leq 1$

- Constraints: Choose the weights such that using these weights, they shouldn't get an efficiency more than 1!

- Using a DMU's weights, if it can't achieve an efficiency of 1, then it is truly inefficient.
- Using a DMU's weight, if an other DMU gets an efficiency of 1, then that other DMU is really good!
- Note that we are referring to only relative efficiency.

DEA - Mathematical formulation

- K = # (no.) of DMU's considered in the dataset
- N = # of inputs considered for the DMU's
- M = # of outputs considered for the DMU's.
- I_{ik} = Recorded value of input i for the DMU K. ($i = 1, 2, \ldots, N$, $k = 1, 2, \ldots, K$)
- O_{jK} = Recorded value of output j for the DMU K. ($j = 1, 2, \ldots, M$, $K = 1, 2, \ldots, K$).
- X_{ik} = Weight assigned to input i by the DMU K. ($i = 1, 2, \ldots, N$, $K = 1, 2, \ldots, K$).
- Y_{jK} = Weight assigned to output j by the DMU K. ($j = 1, 2, \ldots, M$, $K = 1, 2, \ldots, K$).
- E_K = Efficiency of the DMU K, ($K = 1, 2, \ldots, K$)

- Efficiency is defined as the ratio of weighted outputs to the weighted input

$$\boxed{\text{Efficiency} = \frac{\text{Weighted Output}}{\text{Weighted Input}}}$$

- Each DMU defines its own efficiency using the weights they want to asign to their inputs and outputs.

- For a particular DMU k, the efficiency is:

$$\boxed{E_K = \frac{y_{1K}O_{1K} + y_{2K}O_{2K} + y_{3K}O_{3K} \cdots\cdots + y_{MK}O_{MK}}{x_{1K}I_{1K} + x_{2K}I_{2K} + x_{3K}I_{3K} + \cdots\cdots + x_{NK}I_{NK}}}$$

DEA - optimization problem

- Each DMU tries to maximize their own efficiency by adjusting the weights assigned to the inputs and the outputs.
- Only constraint on this: using these weights, none of the DMU's should get an efficiency more than 1!
- For each DMU K, the optimization problem is:

$$\text{Max}_{e}\ E_K$$

Subject to $E_K \leq 1, \quad K = 1, 2, \ldots K$

Decision variables: $x_{iK}, y_{jK} \geq 0, \forall i, \forall j$

Complexities

- Remember that efficiency is ratio of input to output.
- Weights are decision variables
- Hence the objective function and the constraints of this optimization problem are ratios of decision variables.
- That is, they are Non Linear.

- To linearize, we maximize the numerator of the efficiency equation for the DMU K.
- And normalize the denominator to 1
- For the constraints, we rearrange the efficiency terms to make it linear.

- For a DMU K,

weights

$$\text{Max } \underbrace{y_{1K}O_{1K} + y_{2K}O_{2K} + y_{3K}O_{3K} \dots + y_{MK}O_{MK}}_{\text{Numerator}}$$

output

Subject to

$$\underbrace{x_{1K}I_{1K} + x_{2K}I_{2K} + x_{3K}I_{3K} + \dots + x_{NK}I_{NK}}_{\text{Denominator}} = 1$$

$$\cancel{y_{1K}O_{11} + y_{2K}O_{21} + \dots + y_{MK}O_{M1}} \le$$

S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

Efficiency $(E_K) \leq 1$

8 $\dfrac{\text{Numerator}}{\text{Denominator}} \leq 1$

9

$\Rightarrow$ Numerator $\leq$ Denominator

10 $\downarrow$ Output $\quad\quad$ $\hookrightarrow$ Input

11 $y_{1K}O_{11} + y_{2K}O_{21} \cdots + y_{MK}O_{M1} \leq x_{1K}I_{11} + x_{2K}I_{21} \cdots + x_{NK}I_{N1}$

12 $y_{1K}O_{12} + y_{2K}O_{22} \cdots + y_{MK}O_{M2} \leq x_{1K}I_{12} + x_{2K}I_{22} \cdots + x_{NK}I_{N2}$

1 $\vdots$

2 $y_{1K}O_{1K} + y_{2K}O_{2K} \cdots + y_{MK}O_{MK} \leq x_{1K}I_{1K} + x_{2K}I_{2K} \cdots +$

3 $x_{NK}I_{NK}$

4 Decision variables: $x_{ik}, y_{jk} \geq 0, \forall i, \forall j.$

5

6

M	T	W	T	F	S	S
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9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

Data Envelopment Analysis – Example with one output and two inputs

DEA (Data Envelopment Analysis) – Linear Programming

- Each sales office has the same sales target: INR 10,00,000 (output). They have their budgets approved and the respective team sizes (inputs)

Sales office	Budget (INR)	Team size
1	3,00,000	13
2	2,56,000	9
3	5,00,000	7
4	3,90,000	10
5	1,85,000	14

Let us formulate linear programs to calculate the efficiency of each sales office.

Let us start with office #1. For this office,

- The only Output $O_{jk} = O_{11} = 10,00,000$
- The two inputs: (I_{ik}) Budget, $I_{11} = 3,00,000$; Team size, $I_{21} = 13$.
- We need one output weight (y_{11}), and two input weights (x_{11}, x_{21}).
- We expect the optimization problem to tell us the optimal values of the weights.

DEA-LP: Sales office 1

Max $y_{11} * 10{,}00{,}000$ } Numerator

Subject to $x_{11} * 3{,}00{,}000 + x_{21} * 13 = 1$ } Denominator

$y_{11} * 10{,}00{,}000 \leq x_{11} * 256000 + x_{21} * 9$

$y_{11} * 10{,}00{,}000 \leq x_{11} * 3{,}00{,}000 + x_{21} * 13$

$y_{11} * 10{,}00{,}000 \leq x_{11} * 5{,}00{,}000 + x_{21} * 7$

$y_{11} * 10{,}00{,}000 \leq x_{11} * 3{,}90{,}000 + x_{21} * 10$

$y_{11} * 10{,}00{,}000 \leq x_{11} * 1{,}85{,}000 + x_{21} * 14$

$E_k \leq 1$

Decision variables: $x_{11}, x_{21}, y_{11} \geq 0$

DEA-LP: Sales office 2

Same as above only change is that replace y_{11} with y_{12}, x_{11} with x_{12} and x_{21} with x_{22}.

Subject to $x_{12} * 256000 + x_{22} * 9 = 1$

Decision variables: $x_{12}, x_{22}, y_{12} \geq 0$

DEA-LP: Sales office 3

Same as above replace y_{12} with y_{13}, x_{12} with x_{13} and x_{22} with x_{23}

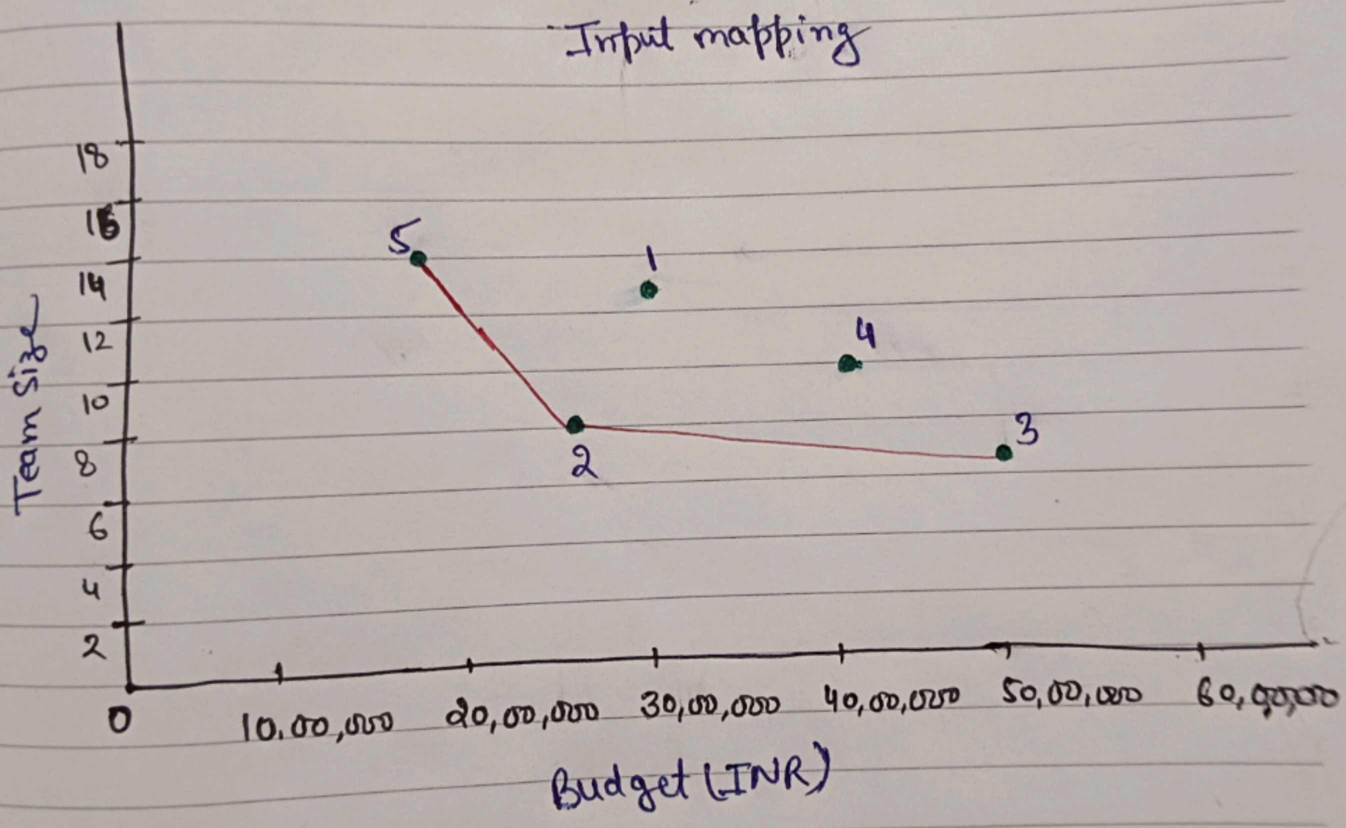
Max $y_{13} * 10{,}00{,}000$

Subject to $x_{13} * 5{,}00{,}000 + x_{23} * 7 = 1$

} copy from office 1 with replacement as mentioned above

Decision variables: $x_{13}, x_{23}, y_{13} \geq 0$

~~DEA LP.~~


Input mapping
Team size
18
16
14
12
10
8
6
4
2
0
5
1
4
2
3
10,00,000
20,00,000
30,00,000
40,00,000
50,00,000
60,00,000
Budget (INR)