Chapter 2

THERMODYNAMICS

This chapter highlights some of the fundamental concepts from thermodynamics that are essential to the aerodynamic design and analysis of axial-flow compressors. These concepts provide the basis for defining and evaluating the energy transfer processes and for modeling the state properties of the working fluid. Unlike the centrifugal compressor, axial-flow compressors are most often applied to rather ideal working fluids, the most common being air. But the application of axial-flow compressors to non-ideal working fluids is becoming more common. Consequently, a basic description of non-ideal gas modeling is included. This description is sufficient for application of the techniques, but is less detailed than this author's previous description for centrifugal compressors (Aungier, 2000). Readers interested in more detail are referred to that earlier reference or to Aungier (1995, 1998).

NOMENCLATURE

A = Helmholtz energy, $dA = -PdV$
a = sound speed and gas constant defined in Eq. (2-32)
b = gas constant defined in Eq. (2-33)
C = velocity
c = gas constant defined in Eq. (2-34)
c_p = specific heat at constant pressure and pressure recovery coefficient
c_v = specific heat at constant volume
H = total enthalpy
h = enthalpy
k = ratio of specific heats = c_p / c_v
M = molecular weight
$\dot{m}$ = mass flow rate
n = exponent in Eq. (2-34), defined in Eq. (2-35)
P = pressure
q = specific heat transfer
$\dot{q}$ = heat transfer rate
R = gas constant
R_U = universal gas constant
s = specific entropy

T = temperature
u = specific internal energy
V = specific volume
w = specific work input
$\dot{w}$ = power input rate
z = compressibility factor
η = efficiency
μ = gas viscosity
ρ = gas density
ω = acentric factor
ξ = parameter defined in Eq. (2-69)
$\overline{\omega}$ = loss coefficient

Subscripts

ad = adiabatic-reversible (isentropic) process
c = critical point parameter
d = discharge condition
i = inlet condition
p = polytropic process
r = reduced parameter (normalized by its critical point value)
ref = reference condition
t = total condition
0 = condition ahead of an inlet guide vane
1 = condition ahead of a rotor blade row
2 = condition following a rotor blade row

Superscripts

0 = condition where the ideal gas model applies

2.1 FIRST AND SECOND LAWS OF THERMODYNAMICS

The first law of thermodynamics covers the basic principle of conservation of energy. The first law can be applied to compressors with one restriction. Since a compressor is an open system, steady flow is the only case to which the first law is applicable. If $\dot{w}$ is the power input to the compressor and $\dot{q}$ is the heat transfer between the compressor and its surroundings, the first law of thermodynamics can be written

$$\dot{q} + \dot{w} = \dot{m}\Delta[u + \tfrac{1}{2}C^2 + P/\rho] \tag{2-1}$$

where $\dot{m}$ is the mass flow rate, u is the specific internal energy, C is velocity, P is pressure, ρ is density and any change in potential energy due to gravitational

force is neglected. The term "specific" designates a parameter measured per unit mass of fluid. The terms in brackets specify the internal energy, kinetic energy and the flow work, P/ρ. The term flow work refers to the work necessary to move the fluid across the boundaries of the system. Only when steady flow crosses the system's boundaries does P/ρ provide a direct measure of the flow work, which restricts Eq. (2-1) to the steady flow case. The normal practice in fluid dynamics applications is to combine the internal energy and flow work terms into a single parameter called the enthalpy, h.

$$h = u + P / \rho \tag{2-2}$$

The above thermodynamic conditions are simple state variables or static conditions. It is useful to introduce a special class of thermodynamic conditions referred to as total or stagnation conditions. A total thermodynamic condition is defined as the value of a parameter that will exist if the fluid is brought to rest with no transfer of heat or external work, i.e., all kinetic energy is completely recovered. Total conditions will generally be designated with a subscript, t. Total enthalpy occurs so often that an exception will be made and H will be used instead of h_t. Total enthalpy is given by

$$H = h + \tfrac{1}{2}C^2 \tag{2-3}$$

Heat transfer can normally be neglected for flow through a compressor. There are obvious exceptions, such as when a heat exchanger is included in the system. If heat transfer is neglected, the flow is called adiabatic and Eqs. (2-1) through (2-3) combine to yield

$$\dot{w} = \dot{m}(H_d - H_i) \tag{2-4}$$

where the subscripts i and d refer to the compressor's inlet and discharge conditions, respectively. Equation (2-4) is the basic energy equation for steady, adiabatic flow through a compressor.

The second law of thermodynamics introduces the concept of a reversible process. A process is referred to as reversible if the system and its surroundings can be returned to their original states after the process has occurred. If that is not the case, the process is called irreversible. Processes influenced by heat transfer or friction effects are common examples of irreversible processes. The specific entropy, s, is defined as

$$ds = \frac{dq_{rev}}{T} \tag{2-5}$$

where T is the temperature, q is the specific heat transfer and the subscript, rev, designates a reversible process. The second law of thermodynamics can be expressed as

$$\Delta s \geq 0 \qquad (2\text{-}6)$$

Hence, an adiabatic, reversible process is also a constant entropy or isentropic process. Indeed, it is fairly common practice in turbomachinery to use the terms adiabatic and isentropic interchangeably, although the latter is clearly the more precise term. Entropy provides a fundamental measure of the irreversibility of a process, i.e., the inefficiency or losses associated with the process. If the first law of thermodynamics is applied to a fluid element in a closed system,

$$(dq)_{rev} = Tds = du + dw = du + PdV \qquad (2\text{-}7)$$

where w is the specific work and $V = 1/\rho$ is the specific volume. Equations (2-2) and (2-7) provide a fundamental thermodynamic equation for entropy that is valid for any process, i.e.,

$$Tds = dh - VdP \qquad (2\text{-}8)$$

2.2 EFFICIENCY

The aerodynamic quality of a compressor or a component of a compressor is commonly measured in terms of efficiency, which is a measure of actual performance relative to an ideal performance that would be achieved by some reversible process. Figure 2-1 illustrates a typical enthalpy-entropy diagram for a compressor or a portion of a compressor that includes at least one rotor row, such that work has been done on the fluid. The inlet conditions are designated as P_{ti}, T_{ti}, s_i and H_i. The compressor does work on the fluid to produce discharge conditions P_{td}, T_{td}, s_d and H_d. Note that static and total conditions are, by definition, related by a reversible process. Hence, there is no difference between total and static entropy and the subscript, t, can be omitted for s. One measure of efficiency is to compare the actual process to an ideal adiabatic (isentropic) process. As seen in Fig. 2-1, an isentropic process could produce the change in total pressure with an enthalpy rise ΔH_{ad}, commonly referred to as the adiabatic head. The actual process required an enthalpy rise of ΔH. Since lines of constant pressure always diverge on an h-s diagram, ΔH is always larger than ΔH_{ad} for a non-isentropic process. Hence, the adiabatic or isentropic efficiency, η_{ad}, is defined as

$$\eta_{ad} = \frac{\Delta H_{ad}}{\Delta H} \qquad (2\text{-}9)$$

where ΔH_{ad} is given by

$$\Delta H_{ad} = \int_{P_{ti}}^{P_{td}} \frac{dP}{\rho} \; ; (s = \text{const}) \qquad (2\text{-}10)$$

This development of efficiency has considered a compressor operating between inlet and discharge total conditions. Hence it is often called the total-to-total

adiabatic efficiency. It is often the case that the fluid kinetic energy at the compressor discharge serves no useful purpose to a specific application. In that case, P_{td} may be replaced by the static pressure, P_d, in Eq. (2-10) to yield the total-to-static adiabatic efficiency. The additional substitution of P_i for P_{ti} yields the static-to-static adiabatic efficiency. Some care is required to understand which basic definition is being used when interpreting efficiency data. It is not uncommon for the term adiabatic efficiency to be used for any of the above three types without qualification.

Adiabatic efficiency is the most common definition used for axial-flow compressors. But it has a definite weakness as a means of evaluating the aerodynamic quality of a design. As illustrated in Fig. 2-1, constant pressure lines diverge on an *h-s* diagram. This means that two compressors having the same basic aerodynamic design quality, but operating at different pressure ratios, will have different adiabatic efficiencies. Hence, adiabatic efficiency is not particularly useful to an aerodynamic designer seeking to evaluate the true aerodynamic quality of a compressor or a stage design. Another consequence of this thermodynamic effect is that the adiabatic head of a multistage compressor is not equal to the sum of the stage adiabatic heads. Polytropic efficiency is a more useful definition, which eliminates this undesirable thermodynamic effect. Polytropic efficiency is sometimes referred to as the "small-stage" or "true aerodynamic" efficiency. Instead of using a path of constant entropy as the reversible path, polytropic efficiency uses a path of constant efficiency defined by
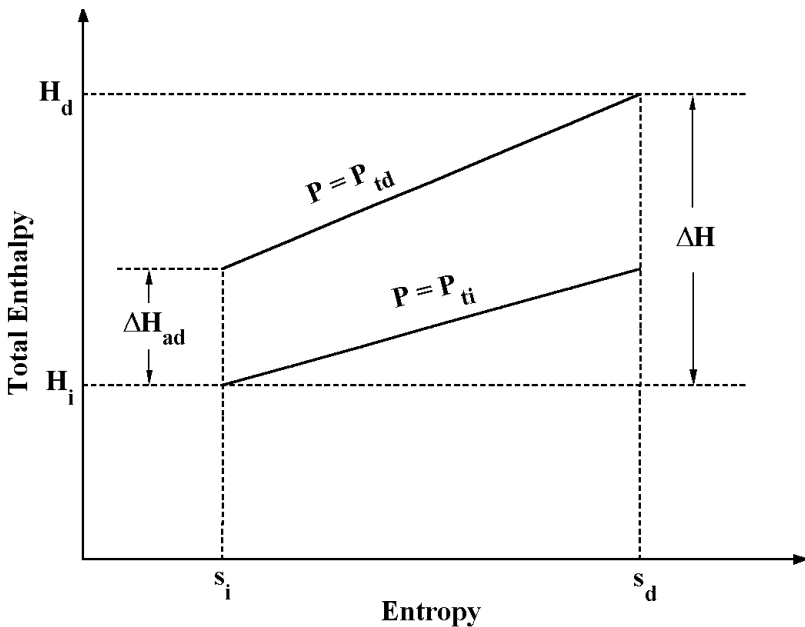

FIGURE 2-1   An Enthalpy-Entropy Diagram

$$\eta_p = \frac{1}{\rho} \frac{dP}{dh} \tag{2-11}$$

where η_p is defined such that the path passes through the two end points of the process, e.g., (P_{ti}, H_i) and (P_{td}, H_d). This is straightforward in principle, but less so in practice. For many years, common practice was to approximate a polytrope by a path defined by $P/\rho^{\,e} = $ constant. This approximation is appropriate for ideal gases, but can introduce large errors for non-ideal gases. Models that use polytropic efficiency with non-ideal gases have been reported by Shultz (1962) and Mallen and Saville (1977). Huntington (1985) studied the problem in detail and showed that the Mallen-Saville model yields excellent accuracy. Huntington proposed a slightly better model by including an intermediate point on the path, but, for convenient use, this had the disadvantage of requiring a numerical method. Hence, this writer employs the Mallen-Saville model in all cases. This model uses an empirical path defined by

$$T\frac{ds}{dT} = \text{constant} \tag{2-12}$$

Equations (2-8) and (2-12) can be combined to yield the polytropic head, ΔH_p, as

$$\Delta H_p = \Delta H - (s_d - s_i)(T_{td} - T_{ti}) / \ln(T_{td} / T_{ti}) \tag{2-13}$$

to provide the total-to-total polytropic efficiency, η_p, by

$$\eta_p = \frac{\Delta H_p}{\Delta H} \tag{2-14}$$

Extension of Eqs. (2-13) and (2-14) to total-to-static and static-to-static efficiency is analogous to adiabatic efficiency as previously discussed. Basically, it is now possible to employ polytropic efficiency with no more difficulty than adiabatic efficiency. It is only necessary to have an appropriate equation of state to use either model.

2.3   FLUID EQUATION-OF-STATE FUNDAMENTALS

Thermodynamics contributes one of the fundamental governing equations for compressor aerodynamic design and analysis, commonly called the equation of state. To be more precise, there are actually two equations of state required. The first is the thermal equation of state, which supplies a relationship among the fundamental state variables, typically in the functional form $P = P(\rho, T)$. The second is the caloric equation of state, which relates the energy content of the fluid to state variables, typically in the functional form $h = h(T, P)$ or $u = u(T, P)$. These equations of state may be derived from kinetic theory or statistical mechanics, or they may be developed empirically from experimental data.

Axial-flow compressors are usually applied to rather simple fluids, the most common being air. Consequently, books on axial-flow compressors usually provide a very limited discussion of the equation of state applicable only to very ideal working fluids. But the application of axial-flow compressors to very non-ideal fluids is becoming more common, so this chapter provides a broader discussion.

A fluid is considered to be an ideal or thermally perfect gas if P, T and ρ can be related by the simple linear thermal equation of state

$$P = \rho R T \qquad \qquad (2\text{-}15)$$

where R is a constant for the fluid. The gas constant, R, is related to the universal gas constant, R_U, and the fluid's molecular weight, M

$$R = R_U / M \qquad \qquad (2\text{-}16)$$

where R_U = 8314 Pa-m³/(kmole-°K) in metric units. All working fluids exhibit non-ideal behavior under appropriate conditions. Figure 2-2 is a schematic of a pressure-enthalpy diagram for any working fluid. It is seen that the fluid may be liquid, vapor or both at various state points. Clearly, it will not be possible for a thermally perfect gas equation of state to model all possible state points. This is true even when the equation is restricted to the vapor phase. Figure 2-2 illustrates the location of the fluid's critical point, which is defined as the highest
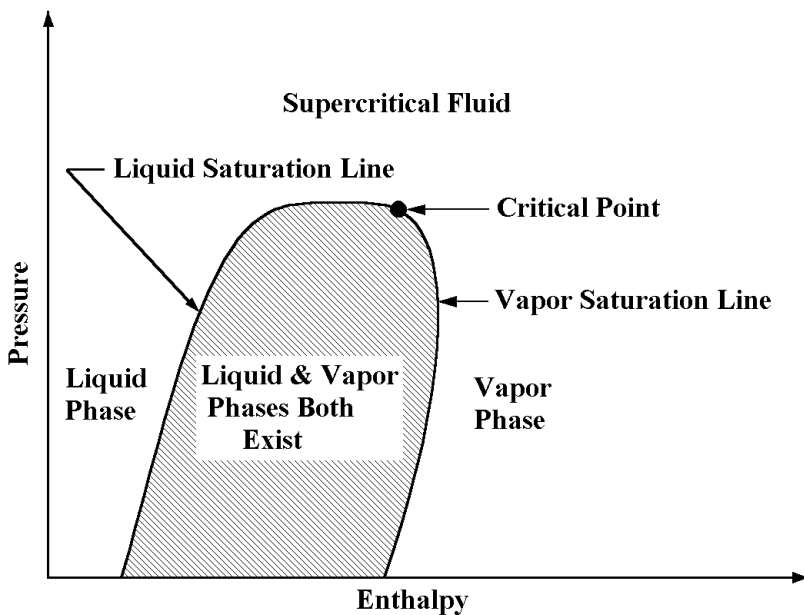

FIGURE 2-2   A Pressure-Enthalpy Diagram Schematic

temperature at which liquid and vapor can coexist. Experimental measurements of critical-point properties (T_c, P_c, ρ_c) are commonly made and almost never conform to Eq. (2-15). Normally, the thermally perfect gas equation of state can provide a reasonable approximation if T is much greater than T_c and P is much less than P_c. At pressures well above P_c, the fluid is said to be in the supercritical regime. Again, it is known from experiment that supercritical fluids almost always show non-ideal gas behavior.

Yet the role of the thermally perfect gas equation of state is far more significant than just providing a useful approximation. Indeed, at sufficiently low values of density, all fluids follow the perfect gas law. This provides a dramatic simplification to the process of modeling non-ideal gases. Under conditions where the gas is thermally perfect, it can be shown that the energy content of a fluid is independent of pressure, i.e., $h^0 = h^0(T)$ and $u^0 = u^0(T)$, where the superscript, 0, designates conditions where the fluid is thermally perfect. This means it is always possible to define the caloric equation of state as a function of only one state variable.

2.4 THE CALORIC EQUATION OF STATE

From the previous discussion, it is seen that the caloric equation of state can be specified in a general form for use in either an ideal or non-ideal gas model. Under low-density conditions where the gas is thermally perfect, the specific heats at constant pressure and at constant volume are defined as

$$c_p^0(T) = \left(\frac{\partial h^0}{\partial T} \right)_P \tag{2-17}$$

$$c_v^0(T) = \left(\frac{\partial u^0}{\partial T} \right)_V \tag{2-18}$$

For a thermally perfect gas, it can be shown that

$$c_p^0(T) - c_v^0(T) = R \tag{2-19}$$

The caloric equation of state can be specified by supplying either $c_p^0(T)$ or $c_v^0(T)$ and using Eqs. (2-17) through (2-19), i.e.,

$$h^0(T) = h^0(T_{ref}) = \int_{T_{ref}}^{T} c_p^0(T) dT \tag{2-20}$$

$$u^0(T) = u^0(T_{ref}) = \int_{T_{ref}}^{T} c_v^0(T) dT \tag{2-21}$$

where h^0 and u^0 can be assigned any desired values at a reference state point (T_{ref}, P_{ref}). Specific heat correlations as a function of temperature are readily available for most fluids of interest (e.g., Ried, et al., 1977, 1987; and Yaws, 1999). For the

special case where the specific heats are constant, the gas is referred to as a calorically perfect gas. In that case, Eqs. (2-20) and (2-21) can be directly integrated to yield analytical equations, i.e.,

$$h^0(T) = h^0(T_{ref}) + c_p^0(T - T_{ref}) \qquad (2\text{-}22)$$

$$u^0(T) = u^0(T_{ref}) + c_v^0(T - T_{ref}) \qquad (2\text{-}23)$$

Most books on axial-flow compressors limit attention to thermally and calorically perfect gases. This simplifies the writer's task by reducing some concepts to simple, analytical forms. This practice is avoided in this book because it results in some unfortunate consequences. Even experienced turbomachinery engineers have occasionally applied these simplified concepts beyond their limits of validity, resulting in some unfortunate conclusions. It has often encouraged developers of computerized aerodynamic analyses to incorporate special thermodynamic relations that are limited to this special class of fluids. A numerical analysis rarely benefits from these simplified methods, and it can be very difficult to generalize such an analysis later, should that become necessary. It is quite possible to have hundreds of such relations scattered through an analysis, presenting a formidable problem when attempting to generalize it.

2.5   ENTROPY AND THE SPEED OF SOUND

An entropy equation is essential to all aspects of compressor aerodynamic design and analysis. The efficiency definitions introduced previously in this chapter all require an entropy equation. Even the simplest aerodynamic analyses require an entropy equation to convert between total and static thermodynamic conditions. This conversion is an isentropic process, accomplished by imposing a change in enthalpy (the kinetic energy) at constant entropy. An entropy equation is also required to impose a non-isentropic loss mechanism such as a total pressure loss. It is useful to develop the entropy equation consistent with the caloric equation of state defined for a thermally perfect gas. This will be needed for the non-ideal gas model described later in this chapter. For any pressure where the gas is thermally perfect, the specific entropy is given by Eq. (2-8), i.e.,

$$s^0(T,P) = s^0(T_{ref},P_{ref}) + \int_{T_{ref}}^{T} \frac{c_p^0(T)}{T} dT - R\ln(P/P_{ref}) \qquad (2\text{-}24)$$

where s^0 can be assigned any desired value at any reference state point (T_{ref}, P_{ref}). For a calorically perfect gas, this simplifies to

$$s^0(T,P) = s^0(T_{ref},P_{ref}) + c_p^0\ln(T/T_{ref}) - R\ln(P/P_{ref}) \qquad (2\text{-}25)$$

The speed of sound is also an essential parameter governing the performance of a compressor. From fundamental thermodynamics, the speed of sound, a, for any gas is given by

$$a^2 = \left(\frac{\partial P}{\partial \rho}\right)_s = k\left(\frac{\partial P}{\partial \rho}\right)_T \qquad (2\text{-}26)$$

where k is the ratio of specific heats.

$$k = c_p / c_v \qquad (2\text{-}27)$$

For a thermally perfect gas, this yields

$$a^0 = \sqrt{kRT} \qquad (2\text{-}28)$$

2.6 THE THERMAL EQUATION OF STATE FOR REAL GASES

The general thermal equation of state for a real gas is

$$P / (\rho RT) = z(T,P) \qquad (2\text{-}29)$$

where z is the compressibility factor. This equation applies to any fluid, with $z = 1$ for the special case of a thermally perfect gas. The compressibility factor can be obtained from generalized tabular data (e.g., Nelson and Obert, 1954; and Pitzer et al., 1955). But that is rarely done today since many excellent real gas equations of state are available. The real gas equation of state directly provides the many other thermodynamic parameters required and yields much better computational speed when used in numerical methods. The simple two-parameter equations-of-state are a good choice for general aerodynamic design and analysis. They offer good accuracy, excellent computational speed and easy access to the required gas property data for almost any working fluid or fluid mixture. The Redlich-Kwong equation (Redlich and Kwong, 1949) and various modifications to it (Aungier, 1994, 1995; Barnes, 1973; Soave, 1972; and Wilson, 1966) are recognized as being among the most accurate of the two-parameter equations. Aungier (1994, 1995) evaluated these five equations of state in considerable detail for twelve different compounds over a wide range of temperatures and pressures. Specific emphasis was placed on covering a wide range of accentric factors, ω, since that parameter is used by the various modified Redlich-Kwong equations to improve the prediction accuracy. Accentric factor (Pitzer et al., 1955) is defined as

$$\omega = -\log_{10}(P_v / P_c) - 1 \, ; \, T / T_c = 0.7 \qquad (2\text{-}30)$$

where P_v is pressure on the vapor saturation line and the subscript c designates a critical point property (Fig. 2-2). Table 2-1 lists the compounds investigated, together with their accentric factors. Based on this evaluation, it was concluded that only the original Redlich-Kwong equation and Aungier's modified Redlich-Kwong are suitable for general turbomachinery aerodynamic design and analysis. The modified equations of Barnes, Soave and Wilson all showed improved

Table 2-1 List of Compounds

Compound	ω
Ammonia	0.2550
Carbon Dioxide	0.2250
Ethylene	0.0868
Helium	−0.464
Hydrogen	−0.220
I-Butane	0.1848
Methane	0.0080
N-Pentane	0.2539
Nitrogen	0.0400
Propane	0.1520
Refrigerant R134a	0.3254
Steam	0.3440

accuracy over the original equation for the types of compounds and thermodynamic property ranges for which they were developed. But they were found to have serious deficiencies for $\omega < 0$, and they exhibited reduced accuracy for supercritical fluids (i.e., P > P$_c$, Fig. 2-2). Consequently, only the original Redlich-Kwong equation and Aungier's modified form will be described here. The original Redlich-Kwong equation is

$$P = \frac{RT}{V-b} - \frac{a}{V(V+b)\sqrt{T_r}} \tag{2-31}$$

where $T_r = T / T_c$ is the reduced temperature, and

$$a = 0.42747 R^2 T_c^2 / P_c \tag{2-32}$$
$$b = 0.08664 RT_c / P_c \tag{2-33}$$

Equations (2-32) and (2-33) are derived from the thermodynamic stability condition, which requires that the first and second partial derivatives of P with respect to V must both equal zero at the critical point. For its application, this equation requires only the critical temperature and critical pressure for the fluid (and the caloric equation of state data). It offers very good accuracy over a wide range of thermodynamic conditions. It does have one well-known deficiency near the critical point for the fluid. For any fluid, this equation yields $z = 1/3$ at the critical point, which is not typical of most fluids. Basically, this equation should never be used for points close to the critical point.

Aungier's modified Redlich-Kwong equation of state is

$$P = \frac{RT}{V-b+c} - \frac{a}{V(V+b)T_r^n} \tag{2-34}$$

where a and b are given by Eqs. (2-32) and (2-33), respectively, n is given by

$$n = 0.4986 + 1.1735\omega + 0.4754\omega^2 \qquad (2\text{-}35)$$

and the constant, c, is calculated directly by applying Eq. (2-34) at the critical point with all critical point properties specified. It is included to remove the deficiency in the original equation for points near the critical point. Aungier's equation requires two more specified parameters than does the original equation, i.e., ω and either V_c or z_c. Note that if $c = 0$ and $n = 0.5$, Aungier's equation reduces to the original equation. If $a = b = c = 0$, the thermally perfect gas equation of state is obtained. This makes it very easy to develop a computerized equation-of-state package for use in any aerodynamic analysis, which can offer all three equations of state.

Figure 2-3 illustrates the benefits of Aungier's modified Redlich-Kwong equation over the original form. This figure shows a comparison of the prediction accuracy of the two equations for about 25% of the data points considered in the evaluation in Aungier (1994, 1995). Tabular (P, V, T) data from the literature were used for that purpose. The two equations of state were used to predict P from the tabular values of T and V, which were compared to the tabular value of P to compute the error. It is seen that Aungier's model provides a significant improvement in prediction accuracy for a vast majority of the points considered. Aungier (1994, 1995) notes that Aungier's model resulted in about a 50% reduction in the root-mean-square error for the complete set of data considered. This writer uses Aungier's model for all real gas problems, but maintains the original model as a
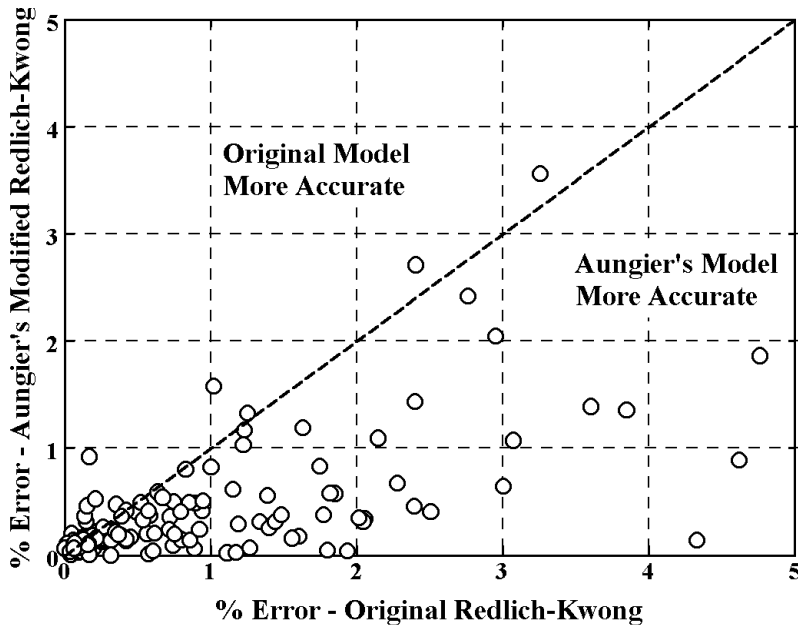

FIGURE 2-3   State-Point Prediction Accuracy

viable alternative. No equation-of-state is infallible, so it is a good idea to have an alternative available.

Either of the above equations of state can be used if the accentric factor and the critical point properties are known. This is a real advantage offered by these equations, since those data are readily available for virtually any compound likely to be encountered in turbomachinery applications. Ried and Sherwood (1966), and Reid et al. (1977, 1987) and Yaws (1999) are good sources for these gas property data.

Equations (2-31) and (2-34) can also be applied to gas mixtures. If the subscript, i, designates the ith compound in a mixture of N compounds and x_i is its mole fraction, the constants for the mixture are given by

$$\sqrt{a} = \sum_{i=1}^{N} x_i \sqrt{a_i} \qquad (2\text{-}36)$$

$$b = \sum_{i=1}^{N} x_i b_i \qquad (2\text{-}37)$$

$$z_c = \sum_{i=1}^{N} x_i z_{ci} \qquad (2\text{-}38)$$

$$\omega = \sum_{i=1}^{N} x_i \omega_i \qquad (2\text{-}39)$$

$$M = \sum_{i=1}^{N} x_i M_i \qquad (2\text{-}40)$$

$$c_p = \sum_{i=1}^{N} x_i c_{pi} \qquad (2\text{-}41)$$

The gas constant for the mixture is given by $R = R_U / M$. The effective values of T_c and P_c for the mixture can be computed from a, b and R, using the definitions in Eqs. (2-32) and (2-33). Then c can be computed from Eq. (2-34). Note that Eq. (2-41) requires that c_p be the specific heat per mole rather than per unit mass. For the original Redlich-Kwong equation, $c = 0$, and Eqs. (2-38) and (2-39) are not used.

A mixture equation of state formed in this manner is applicable only to the vapor phase of the fluid. Although the above real-gas models apply to the more general two-phase flow problem, the composition of the two phases may be different and vary with state-point conditions, requiring special treatment. Axial-flow compressors are not intentionally applied to two-phase flows, so the restriction to the vapor phase is not a concern in this application. Centrifugal compressors often encounter a special case of two-phase flow when intercoolers are located between stages. These intercoolers may cause some components to liquefy and drop out of the mixture. This is commonly referred to as liquid-knockout, and can be modeled by an equilibrium flash calculation. This is not commonly needed for axial-flow compressors and will not be discussed in this book. Should this capability be needed, Aungier (2000) describes the equilibrium-flash calculation using the equations of state presented here.

When the real-gas equation of state is restricted to the vapor phase, it is prudent to avoid accidental excursions into the "wet region" where liquid may form. In a numerical analysis, this can easily occur due to numerical errors in iterative solutions that are far from convergence, often terminating the analysis with a fatal error. The vapor saturation pressure, P_v, is known to vary proportional to $1/T$ (Ried and Sherwood, 1966). The gas property specifications required for Aungier's model include two points on the vapor saturation line shown in Fig. 2-2. One point is the critical point, while the other follows from the definition of the accentric factor in Eq. (2-30). Thus, the vapor saturation pressure can be approximated by

$$\log_{10}(P_v / P_c) = 7(1 + \omega)(1 - T_c / T) / 3 \tag{2-42}$$

This equation can easily be inverted to predict the vapor saturation temperature as a function of pressure. These relations can be used to limit the relevant independent thermodynamic conditions in all calculations intended only for the vapor phase to prevent fatal errors from these accidental excursions into the wet region. In some aerodynamic analyses, this simple precaution can yield a dramatic improvement in the reliability of the analysis.

2.7 THERMODYNAMIC PROPERTIES OF REAL GASES

Specification of the caloric equation of state, h^0 or u^0, has been limited to state points where the fluid is thermally perfect. For non-ideal fluids, h and u are functions of pressure as well as temperature. The calculation of the thermodynamic properties of a non-ideal fluid is best accomplished using departure functions (Ried et al., 1977). Departure functions are defined as the difference between the actual value of a parameter and its value under conditions where the fluid is thermally perfect, e.g., the quantity $h - h^0$ is the enthalpy departure function. To employ this process, the pressure at which the fluid can be considered to be thermally perfect, P^0, must be specified. The precise value is not important, but it will be a low (but non-zero) value of pressure where the thermally perfect gas approximation can be considered to be valid. Equation (2-15) supplies the corresponding specific volume, i.e., $V^0 = RT / P^0$. If A = the Helmholtz energy, Ried et al. (1977) shows that the relevant departure functions are

$$A - A^0 = -\int_{\infty}^{V}(P - RT / V)dV - RT \ln(V / V^0) \tag{2-43}$$

$$s - s^0 = -\left(\frac{\partial(A - A^0)}{\partial T}\right)_V \tag{2-44}$$

$$h - h^0 = (A - A^0) + T(s - s^0) + RT(z - 1) \tag{2-45}$$

$$u - u^0 = (A - A^0) + T(s - s^0) \tag{2-46}$$

For the Redlich-Kwong equation of state, the departure functions are

$$h - h^0 = PV - RT - \frac{a}{b}(n+1)T_r^{-n} \ln\left[\frac{V+b}{V}\right] \tag{2-47}$$

$$s - s^0 = R\ln\left[\frac{V}{V^0}\frac{V-b+c}{V}\right] - \frac{na}{bT}T_r^{-n}\ln\left[\frac{V+b}{V}\right] \tag{2-48}$$

where $c = 0$ and $n = 0.5$ for the original Redlich-Kwong equation of state. Basic thermodynamics provides the other parameters commonly required for aerodynamic analysis, i.e.,

$$c_p = \left(\frac{\partial h}{\partial T}\right)_P \tag{2-49}$$

$$c_v = \left(\frac{\partial u}{\partial T}\right)_V = c_p - R\left[z + T\left(\frac{\partial z}{\partial T}\right)_P\right]^2 \Big/ \left[z - P\left(\frac{\partial z}{\partial P}\right)_T\right] \tag{2-50}$$

$$a^2 = k\left(\frac{\partial P}{\partial \rho}\right)_T = \frac{kzRT}{1 - \frac{P}{z}\left(\frac{\partial z}{\partial P}\right)_T} \tag{2-51}$$

2.8 THERMALLY AND CALORICALLY PERFECT GASES

When the fluid can be considered thermally perfect ($z = 1$) and calorically perfect (c_p, c_v, and k are constants), equation-of-state calculations are greatly simplified. If the subscript, ref, designates conditions at an arbitrary reference state point (T_{ref}, P_{ref}) where h and s can be assigned arbitrary values, Eqs. (2-22) and (2-25) can be written

$$h = h_{ref} + c_p(T - T_{ref}) \tag{2-52}$$

$$s = s_{ref} + c_p \ln(T / T_{ref}) - R\ln(P / P_{ref}) \tag{2-53}$$

Combining Eqs. (2-3) and (2-52), the total and static temperatures are related by

$$c_p(T_t - T) = \tfrac{1}{2}C^2 \tag{2-54}$$

For an isentropic or adiabatic-reversible process Eq. (2-53) requires

$$T / T_{ref} = (P / P_{ref})^{\frac{k-1}{k}} = (\rho / \rho_{ref})^{k-1} \tag{2-55}$$

Equations (2-54) and (2-55) provide simple analytical expressions to relate total and static conditions. Similarly, the efficiency calculations in Eqs. (2-9) and (2-14) simplify to

$$\eta_{ad} = \frac{(P_{td} / P_{ti})^{\frac{k-1}{k}} - 1}{T_{td} / T_{ti} - 1} \qquad (2\text{-}56)$$

$$\eta_p = \frac{k-1}{k} \frac{\ln(P_{td} / P_{ti})}{\ln(T_{td} / T_{ti})} \qquad (2\text{-}57)$$

Reduction of these thermodynamic relations to simple analytical form can yield substantial reductions in computation time for computerized aerodynamic analyses. If the fluid is calorically imperfect, numerical integration of Eqs. (2-20) and (2-21) will normally be necessary. If the fluid is thermally imperfect, the departure functions of Eqs. (2-47) and (2-48) must also be applied. In either case, numerical iteration is required for isentropic processes to converge to the required constant entropy condition. This is the primary source of increased computation time. Typically, an analysis will perform a massive number of isentropic calculations to relate total and static conditions and to relate conditions in rotating and stationary coordinate systems. A calorically imperfect, thermally perfect fluid requires nearly as much computation time as a general non-ideal gas. Hence, the thermally and calorically perfect gas model offers significant advantages when it can be employed.

2.9   THE PSEUDO-PERFECT GAS MODEL

The pseudo-perfect gas model (Aungier, 1998, 2000) is a very useful concept that often provides all of the benefits of the thermally and calorically perfect gas model for cases where that model is not adequate. Its use requires some care in formulating an aerodynamic analysis. It is necessary to avoid use of any state relations that assume the relationships between c_p, c_v, R and k expressed in Eqs. (2-19) and (2-27). This really does not complicate the analysis, but the advantages of the pseudo-perfect gas model cannot be exploited unless this is done. The concept is to use fictitious values of c_p, c_v, R and k in an otherwise standard thermally and calorically perfect gas model. If the fictitious constants are represented with an overbar, they can be calculated from any two state points, designated by subscripts 1 and 2.

$$\overline{R} = R\sqrt{z_1 z_2} \qquad (2\text{-}58)$$

$$\overline{c}_p = (h_2 - h_1) / (T_2 - T_1) \qquad (2\text{-}59)$$

$$\overline{c}_v = (u_2 - u_1) / (T_2 - T_1) \qquad (2\text{-}60)$$

$$\overline{k} = \ln(P_2 / P_1) / \ln(\rho_2 / \rho_1) \qquad (2\text{-}61)$$

Equation (2-61) yields the isentropic exponent, which can be quite different from the ratio of specific heats for a real gas. The pseudo-perfect gas model is generally adequate as long as the values of z_1 and z_2 are not too different. This is usually true for compressor components operating on a non-ideal fluid. It can also be applied to applications such as multistage compressor performance analysis,

simply by recomputing the fictitious constants between stages or between blade rows. The modest stage pressure ratios in axial-flow compressors almost always ensure that the model is sufficient. Indeed, this writer has used it extensively for industrial centrifugal compressor performance analysis, where stage pressure ratios are much higher and non-ideal working fluids are common. No case has been encountered to date where the pseudo-perfect gas model yields results significantly different from a full non-ideal gas model.

This writer's normal practice is to select the inlet total conditions and the conditions corresponding to sonic flow as the two state points to define the fictitious constants—that includes most conditions likely to be encountered in a compressor blade row. The full real gas model is used to compute the constants. Then the pseudo-perfect gas model is used in the actual analysis. This writer uses a computerized equation-of-state module that contains the pseudo-perfect gas model as well as the other equations of state described in this chapter. This module is supported by a gas property database from which the equation-of-state for the fluid or fluid mixture is easily formulated. All aerodynamic design and analysis computer programs use this module to perform all equation-of-state calculations. Even for thermally perfect fluids, the pseudo-perfect gas model yields substantial reductions in computation time by using a calorically perfect gas approximation. Its adequacy for those cases is a virtual certainty.

2.10 COMPONENT PERFORMANCE PARAMETERS

The adiabatic and polytropic efficiencies introduced previously in this chapter can be used to evaluate the performance of a compressor, a compressor stage or even a single rotor blade row. But they have no meaning for a stationary compressor component, since total enthalpy is constant for those components. Yet, it is useful to be able to evaluate a stationary component's performance against some ideal, reversible process. Most components in a compressor serve to diffuse the flow to convert kinetic energy into static pressure. The most important of these is the exhaust diffuser, although stator and exit guide vane rows are also stationary, diffusing components. One method of evaluating a diffusing component is known as the diffuser efficiency. Its definition is really quite similar to the adiabatic efficiency introduced previously, except that static enthalpy is used as its basis. Figure 2-4 presents a schematic of an h-s diagram for a diffuser. Flow enters the diffuser with a velocity C_i and exits the diffuser with a lower velocity C_d. Since the total enthalpy, H, is constant, Eq. (2-3) can be used to compute the static enthalpy at the inlet, h_i, and at the discharge, h_d. It is seen from Fig. 2-4 that this process results in an increase in static enthalpy of Δh. Since irreversible losses such as friction occur, the entropy increases from s_i to s_d. Now, trace the line of constant pressure for the discharge pressure back to the inlet entropy. This shows that the static enthalpy increase required for an adiabatic, reversible process to produce the same pressure increase is Δh_{ad}. Therefore, a diffuser efficiency can be defined as

$$\eta_{diff} = \frac{\Delta h_{ad}}{\Delta h} \tag{2-62}$$
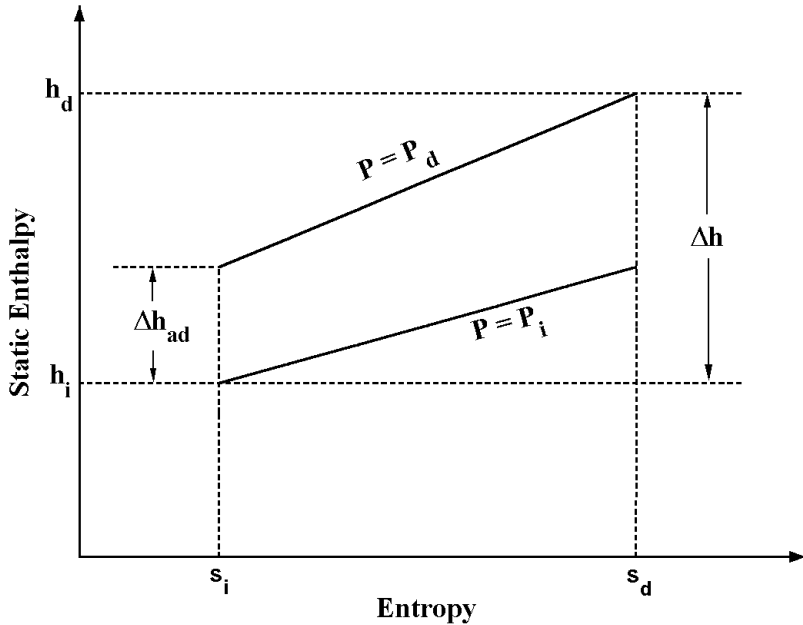
FIGURE 2-4 A Diffuser *h-s* Diagram

A more common parameter used to evaluate the performance of a diffuser is the static pressure recovery coefficient, c_p. This expresses the ratio of the static pressure rise to the available kinetic energy at the inlet. Usually, the available kinetic energy is expressed as a "velocity pressure," which is the difference between the total and static pressure.

$$c_p = \frac{P_d - P_i}{P_{ti} - P_i} \tag{2-63}$$

Hence, c_p is the fraction of the available kinetic energy that has been recovered as static pressure by the diffuser.

Although compressors are basically diffusing machines, there are some compressor components that accelerate the flow. Inlet passages and inlet guide vanes are typical examples. In those cases, the component can be evaluated using the nozzle efficiency. Figure 2-5 illustrates the parameters used to define it. The flow accelerates from the inlet velocity, C_i, to a higher discharge velocity, C_d. Due to irreversible processes such as wall friction, the entropy increases from s_i to s_d and the total pressure decreases from P_{ti} to P_{td}. If the flow is adiabatic (no heat transfer), the total enthalpy, H, is constant for this process. Equation (2-3) relates H, h and C at the inlet and the discharge. This is illustrated in Fig. 2-5, along with the ideal discharge velocity, C_{ad}, which would be produced by an isentropic process between
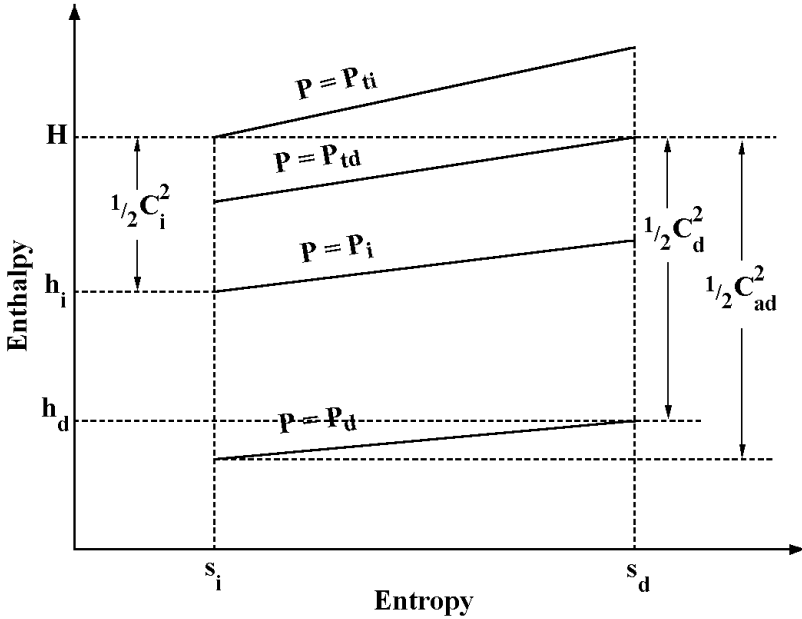
FIGURE 2-5   A Nozzle *h-s* Diagram

the actual inlet pressure, P_i, and discharge pressure, P_d. A nozzle efficiency can be defined as the ratio of the actual increase in kinetic energy to the increase in kinetic energy corresponding to the isentropic process, i.e.,

$$\eta_{noz} = \frac{C_d^2 - C_i^2}{C_{ad}^2 - C_i^2} \tag{2-64}$$

Occasionally, a simpler definition is used: It employs the velocity pressure, $P_t - P$, in place of C^2 as the measure of kinetic energy and uses $P_{ti} - P_d$ as the ideal (no loss in total pressure) discharge kinetic energy. Simple substitution of these alternate kinetic energy terms into Eq. (2-64) yields

$$\eta_{noz} = 1 - \frac{P_{ti} - P_{td}}{P_i - P_d} \tag{2-65}$$

The most fundamental measure of irreversibility is the increase in entropy. But an entropy increase is difficult to interpret, so total pressure loss is more commonly used. If Eq. (2-8) is applied to total conditions, with total enthalpy constant,

$$\frac{\Delta P_t}{\rho} = \frac{P_{ti} - P_{td}}{\rho} = T \Delta s \tag{2-66}$$

It is known from observation that total pressure loss is usually proportional to kinetic energy, so a logical definition of loss coefficient can be obtained by dividing by $^1/_2 C^2$, i.e., the loss coefficient can be defined as

$$\overline{\omega} = \frac{\Delta P_t}{\frac{1}{2}\rho C^2} \tag{2-67}$$

Experience has shown that use of velocity pressure as a measure of kinetic energy provides a better definition, i.e.,

$$\overline{\omega} = \frac{\Delta P_t}{P_t - P} \tag{2-68}$$

It is found that the second definition is much less sensitive to changes in the Mach number. This is very important in axial-flow compressor performance analysis. Typically, blade row performance is predicted with empirical loss coefficient models derived from low-speed cascade tests, but applied to blade rows with relatively high inlet Mach numbers. The flow is essentially incompressible (Mach number essentially zero) in the low-speed cascade tests, so the two definitions of loss coefficient are basically identical. But when empirical loss coefficient models are applied to predict losses in higher Mach number blade rows, Eq. (2-68) is found to yield substantially better results.

Equations (2-67) and (2-68) are ambiguous with regard to what kinetic energy is to be used in the denominator. The usual practice is use of the largest kinetic energy relevant to the component. For diffusing components, such as diffusers and compressor blade rows, the inlet kinetic energy is usually employed. For accelerating components, such as turbine blade rows, the discharge kinetic energy is usually employed. The formulation for loss coefficient assumed constant total enthalpy, which is far from true for rotor blade rows. But chapter 3 will show that total enthalpy viewed in a frame of reference rotating with the blades (the relative total enthalpy) is constant for axial flow. So, the above definitions can be used for rotors in that context. There is a subtle thermodynamic effect present when applying empirical loss coefficient models to rotating blade rows. In fact, the flow is not usually precisely axial, i.e., there is usually some change in radius as a streamline passes through a rotor. Consequently, even the relative total enthalpy is not usually constant through the blade row. For axial-flow compressors, this effect is so small that it can be neglected in virtually all cases. The situation is quite different in centrifugal and mixed-flow compressors, where consideration of this thermodynamic effect is essential. Aungier (2000) develops a correction procedure to apply empirical loss coefficients to rotating blade rows with a significant change in radius from inlet to discharge. Since it is not necessary for axial-flow compressors, it will not be included here. But investigators attempting to combine axial-flow compressor stages with mixed-flow or radial-flow stages should consult that reference.

2.11   GAS VISCOSITY

Gas viscosity is often required for aerodynamic analysis. A generalized estimation method compatible with the previous equation-of-state models is provided by Dean and Stiel (1965), and is reviewed by Ried et al. (1966). It is necessary to depart from this book's practice of maintaining independence from specific units in this case. The Dean and Stiel model predicts viscosity in centipoises and employs a dimensional parameter defined by

$$\xi = T_c^{1/6} / \left(\sqrt{M} P_c^{2/3} \right) \tag{2-69}$$

where T_c is in degrees Kelvin and P_c is in atmospheres. The low-pressure fluid viscosity is given by

$$\mu^0 \xi = (3.4 \cdot 10^{-4}) T_r^{8/9}; \quad T_r \leq 1.5 \tag{2-70}$$

$$\mu^0 \xi = 0.001668 \left(0.1338 T_r - 0.0932 \right)^{5/9}; \quad T_r > 1.5 \tag{2-71}$$

Then, the viscosity at any pressure is defined by a departure function

$$(\mu - \mu^0)\xi = (1.08 \cdot 10^{-4})[\exp(1.439 \rho_r) - \exp(-1.111 \rho_r^{1.858}) \tag{2-72}$$

Ried et al. (1966) indicate that accuracy within 5% can be expected for non-polar molecules, with somewhat larger errors possible for polar molecules. This viscosity estimation method is quite compatible with the equation-of-state models previously recommended in this chapter as long as the fluid is in the vapor phase. In principle, the real gas equations of state apply to the liquid or two-phase fluid also, but this viscosity model does not. The critical point data needed for Aungier's modified Redlich-Kwong equation of state are sufficient for this gas viscosity model also, but appropriate care is required to adjust the units of T_c and P_c in Eq. (2-69) and the μ predicted.

2.12   A COMPUTERIZED EQUATION-OF-STATE PACKAGE

One of the most useful tools for the developer of axial-flow compressor design and analysis software is a computerized equation-of-state package that can be used as a module in any other analysis to handle all equation-of-state input, output and thermodynamic calculations. Some useful features to include are the following.

- Provide the non-ideal gas, ideal gas (with variable specific heats) and pseudo-perfect gas as options always available to the user. Be sure to take advantage of opportunities for reduced computation time for the simpler models. For a basic ideal gas, the pseudo-perfect gas model

functions directly as a thermally and calorically perfect gas model. Include the capability to specify pseudo-perfect gas data directly, or to have these data computed from the equation of state and the specified gas mixture.

- Maintain a database of gas property data for compounds you need. Establish the capability to set up any of the equation-of-state models from this data simply by selecting compounds in the mixture and supplying their mole fractions. For the models described in this chapter, the data needed are M, P_c, T_c, V_c (or z_c), ω and $c_p(T)$. This writer uses a third-order polynomial fit of c_p as a function of T, consistent with Ried et al. (1977), which is a good source of all the data required for a very wide range of compounds. Note that setup of the pseudo-perfect gas model from the equation of state must be requested by the main analysis when appropriate thermodynamic conditions are known, as described previously in this chapter. Include a routine to accomplish this on demand.
- Include logic to save and recover gas property data so that input files of other programs maintain a common format for all aerodynamic software. It is a good idea to include the logic for basic checks on data validity to be sure the equation of state is complete. Reserve a special code for the equation-of-state model to signal to the program using this module that data is lacking or invalid.
- It is useful to include a standard output routine that can be used to insert a description of the mixture and the equation of state in use in output files for other programs.
- Include routines for the basic thermodynamic calculations needed in aerodynamic analysis. Standard functions likely to be needed include $V(T, P)$, $P(T, V)$, $T(V, P)$, $a(T, P)$, $h(T, P)$, $s(T, P)$, $\eta_{ad}(T_i, P_i, T_d, P_d)$ and $\eta_p(T_i, P_i, T_d, P_d)$. A routine to compute the isentropic change in P and T for a specified change in h is essential for conversion between static and total thermodynamic conditions and between rotating and stationary coordinate systems. A routine to compute the speed of sound at sonic flow conditions for known total conditions is often useful.
- Include the capability to edit the thermodynamic model to modify the compounds in the mixture, their mole fractions and the equation-of-state model to be used.
- The Dean and Stiel (1965) viscosity estimation model can be included to treat all gases and gas mixtures formed from the gas property database. If pseudo-perfect gas data are specified rather than calculated from the equation of state, viscosity data will also need to be specified.

Using this approach, development of aerodynamic design and analysis software will involve minimal logic related to the equation of state. In general, axial-flow compressor state calculations should be limited to the vapor phase using the constraint expressed in Eq. (2-42). If special consideration involving liquid phases is necessary, the approximate liquid phase models described in Aungier (1998, 2000) and the equilibrium flash calculation described in Aungier (2000) may be useful.

EXERCISES

2.1 Consider uniform axial flow at the inlet of a compressor with no inlet guide vanes (i.e., stations 0 and 1 are identical) with velocity C_{z1} and $C_{\theta 1} = 0$. For a thermally and calorically perfect gas, show that ratios ρ_1/ρ_{t1} and a_1/a_{t1} are functions of Mach number $M_1 = C_{z1} / a_1$ (these results were used in Chapter 1 without derivation to develop the equivalent flow and speed parameters).

2.2 The flow at station 1 of Exercise 2.1 passes through a rotor exiting at station 2 with $C_{z2} = C_{z1}$ and with a swirl velocity, $C_{\theta 2}$, at the mean radius where the blade speed is $\omega r = U$. The total pressure loss coefficient across the blade row, based on the inlet velocity pressure, is $\overline{\omega}$. For a thermally and calorically perfect gas, derive expressions for the ideal (no loss) discharge total temperature and pressure, T_{t2id} and $P_{t2id,}$ and the actual values, T_{t2} and P_{t2}, at the mean radius in terms of the inlet parameters P_{t1}, T_{t1}, H_1 and P_1. Use the Euler turbine equation, Eq. (1-17), to compute the change in total enthalpy across the rotor.

2.3 For the rotor row of Exercise 2.2, extend the Mach number equivalence parameters of Chapter 1 to include an expression for the equivalent total enthalpy rise across the rotor. Show that if the flow is isentropic (i.e., that $\overline{\omega} = 0$), Mach number equivalence at the inlet of the blade row will produce Mach number equivalence at the discharge.

2.4 The rotor row of Exercises 2.1 and 2.2 is to be operated with a new working fluid. The operation with the new working fluid will start with the same inlet conditions (P_{t1}, T_{t1}, ρ_{t1}) as for the original working fluid. Its operating speed and mass flow rate will be determined by requiring Mach number equivalence at station 1. The two fluids have the same gas constant, R, but the original working fluid has $k = 1.4$ and the new working fluid has $k = 1.38$. Assuming that the flow is isentropic, show that both the equivalent speed and flow parameters cannot be satisfied at station 2 (the rotor exit). Is complete Mach number equivalence achieved in this case?

2.5 An axial-flow compressor is to be operated with propane as the working fluid with an inlet pressure of 200 kPa. Fluid property data for propane are $M = 44.1$, $T_c = 369.83°$ K, $P_c = 4249.6$ kPa and $\omega = 0.152$. Estimate the lowest inlet temperature that can be used to avoid the risk of liquid erosion due to two-phase flow.

2.6 Two axial-flow compressors operate on a thermally and calorically perfect gas with $k = 1.4$. Both compressors have an adiabatic efficiency of 85%. The pressure ratios of the two compressors are 3.0 and 5.0. Calculate the polytropic efficiencies of the two compressors.

2.7 An axial-flow compressor consists of three repeating stages, each with a stage pressure ratio of 1.1 and a stage adiabatic efficiency of 85%. The working fluid is a thermally and calorically perfect gas with $k = 1.4$. Compute the overall adiabatic efficiency of the three-stage compressor. Repeat this problem assuming all efficiencies are polytropic.

2.8 Consider adiabatic flow of a calorically and thermally perfect gas with $k = 1.4$ through a diffuser. The inlet total temperature is 300° K, the inlet total pressure is 230 kPa, the inlet static pressure is 200 kPa, the static pressure recovery coefficient is 0.6 and the total pressure loss coefficient based on the inlet velocity pressure, $(P_{ti} – P_i)$, is 0.1. Compute the diffuser efficiency, η_{dif}.