ANSWERS TO THE EXERCISES

1.1 Flow parameter:

$$Q / a_{t0} \propto \dot{m} / (\rho_{t0}\sqrt{kRT_{t0}}) \propto \dot{m}RT_{t0} / (P_{t0}\sqrt{kRT_{t0}}) \propto \dot{m}\sqrt{RT_{t0} / k} / P_{t0}$$

Speed parameter: $N / a_{t0} = N / \sqrt{kRT_{t0}}$

1.2 If $\beta_1 = 0$, then $C_{\theta 1} = 0$ and

$$\psi = C_{\theta 2} / U$$
$$R = 1 - C_{\theta 2} / (2U) = 1 - \psi / 2$$

So if $R = 0.5$, then $\psi = 1$ is the only acceptable value. From Eq. (1-24), the rotor discharge relative flow angle must be zero. Since the absolute flow angles into and out of the stage are zero, no inlet or exit guide vanes will be required.

1.3 $D_B = 1.2D_A$ and $A_B = 1.44A_A$. Therefore,

$$N_B = N_A / 1.2$$
$$Q_B = 1.44Q_A$$

For equivalence with the original compressor at 3,600 rpm, the scaled compressor must operate at 3,000 rpm and will supply 44% more flow capacity than the original.

1.4 For a 20% increase in flow capacity, a 20% increase in A_0 is needed, which requires a scale factor of $\sqrt{1.2}$. Hence the speed must be reduced by a factor of $\sqrt{1.2}$.

2.1 Equations (2-19), (2-27), (2-28) and (2-53) combine to yield

$$a_1^2 = kRT_1 = c_p(k-1)T_1$$

$$2c_pT_1 = 2a_1^2/(k-1)$$

$$T_{t1}/T_1 = 1+(k-1)C_{z1}^2/(2a_1^2) = 1+\tfrac{k-1}{2}M_1^2$$

Hence, from Eqs. (2-28) and (2-55),

$$(a_1/a_{t1})^2 = 1/(1+\frac{k-1}{2}M_1^2)$$

$$(\rho_1/\rho_{t1})^{k-1} = 1/(1+\tfrac{k-1}{2}M_1^2)$$

2.2 From Eq. (1-17),

$$H_2 = H_1 + UC_{\theta 2}$$

For a thermally and calorically perfect gas, T_t is a function of only H. Hence Eq. (2-52) yields

$$T_{t2id} = T_{t2} = T_{t1} + (H_2 - H_1)/c_p$$

In the absence of losses, the flow is isentropic and Eq. (2-55) yields

$$P_{t2id} = P_{t1}(T_{t2id}/T_{t1})^{\frac{k}{k-1}}$$

Equation (2-68), using the inlet velocity pressure as relevant base kinetic energy term, requires

$$P_{t2} = P_{t2id} - \overline{\omega}(P_{t1} - P_1)$$

2.3 For complete Mach number equivalence, the discharge Mach number and flow angle must be equivalent, i.e., $C_{\theta 2}/a_2$ and C_{z2}/a_2 must be equivalent. As shown in Exercise 2.1, when Mach number equivalence exists, this requirement can be restated to require equivalence on $C_{\theta 2}/a_{t2}$ and C_{z2}/a_{t2}. Mach number equivalence at the inlet requires equivalence on U/a_{t1}. From Exercise 2.2,

$$\Delta H = H_2 - H_1 = UC_{\theta 2}$$

$$\Delta H/a_{t1}^2 = (U/a_{t1})(C_{\theta 2}/a_{t2})a_{t2}/a_{t1}$$

$$(a_{t2}/a_{t1})^2 = T_{t2}/T_{t1} = 1+(kR/c_p)(\Delta H/a_1^2)$$

Hence equivalence on $\Delta H/a_{t1}^2$ is required to produce equivalence on $C_{\theta 2}/a_{t2}$, T_{t2}/T_{t1} and a_{t2}/a_{t1}. Neglecting losses, Eq. (2-55) requires

$$P_{t2} / P_{t1} = (T_{t2} / T_{t1})^{\frac{k}{k-1}}$$

$$\rho_{t2} / \rho_{t1} = (T_{t2} / T_{t1})^{\frac{1}{k-1}}$$

so equivalence on P_{t2}/P_{t1} and ρ_{t2}/ρ_{t1} is also achieved. Hence

$$\dot{m} / (\rho_{t2} a_{t2} A_2) = [\dot{m} / (\rho_{t1} a_{t1} A_1)][\rho_{t1} a_{t1} A_1 / (\rho_{t2} a_{t2} A_2)]$$

$$N / a_{t2} = N / a_{t1} [a_{t1} / a_{t2}]$$

Since the last terms on the right-hand side of the previous two equations satisfy equivalence, Mach number equivalence at the inlet of the blade row will produce Mach number equivalence at the discharge.

2.4 Denote the base case with the original working fluid by subscript B, and the new case by subscript N. Let T_{t2B} and ρ_{t2B} be the rotor exit conditions for the base case. For mass flow and speed equivalence at the rotor inlet,

$$\dot{m} / (\rho_t \sqrt{kRT}) = \text{constant}$$

$$N / \sqrt{kRT} = \text{constant}$$

Since the inlet conditions and R are identical for the two fluids, the mass flow and speed for the new case must be

$$\dot{m}_N = \dot{m}_B \sqrt{1.38 / 1.4}$$
$$N_N = N_B \sqrt{1.38 / 1.4}$$

For speed equivalence at the rotor exit,

$$N_N = N_B \sqrt{(1.38 T_{t2N}) / (1.4 T_{t2B})}$$

Hence $T_{t2N} = T_{t2B}$ is required to satisfy the equivalent speed condition at both the rotor inlet and exit. Mass flow equivalence at the rotor exit with $T_{t2N} = T_{t2B}$ requires

$$\dot{m}_N = \dot{m}_B \sqrt{1.38 / 1.4} (\rho_{t2N} / \rho_{t2B})$$

Thus, to achieve mass flow equivalence at both locations, $\rho_{t2N} = \rho_{t2B}$ is required. But Eq. (2-55) with $T_{t2N} = T_{t2B}$ requires

$$(\rho_{t2B} / \rho_{t1})^{0.4} = T_{t2B} / T_{t1} = (\rho_{t2N} / \rho_{t1})^{0.38}$$
$$\rho_{t2N} / \rho_{t1} = (\rho_{t2B} / \rho_{t1})^{0.4/0.38}$$

Hence the two conditions for Mach number equivalence cannot be satisfied at both the rotor inlet and exit. Since Mach number equivalence at both stations cannot be achieved for the same speed and mass flow, complete Mach number equivalence through the compressor is not achieved in this case.

2.5 Using Eq. (2-42), the temperature at which liquid will begin to form can be estimated from

$$\frac{T_c}{T} = 1 - \frac{3}{7(1+\omega)} \log_{10}\left(\frac{P}{P_c}\right)$$

For P = 200 kPa, T > 247.6° K is required to avoid liquid phase formation.

2.6 Use Eq. (2-56) to compute the temperature ratio, T_R, from pressure ratio, P_R, and η_{ad}. Use Eq. (2-57) to compute the polytropic efficiency. Hence

$$P_R = 3.0\text{: } T_R = 1.4338,\ \eta_p = 87.11\%$$
$$P_R = 5.0\text{: } T_R = 1.6868,\ \eta_p = 87.95\%$$

2.7 Use Eq. (2-56) to compute the stage temperature ratio, T_R, from the stage pressure ratio and stage η_{ad}. This yields a stage temperature ratio of 1.032477. For the three-stage compressor, the pressure ratio is $(1.1)^3$ = 1.331 and the temperature ratio is $(1.032477)^3$ = 1.10063. Then Eq. (2-56) yields the overall adiabatic efficiency of 84.59%. If the efficiencies are all polytropic, the stage temperature ratio, T_R, can be computed using Eq. (2-57). Then Eq. (2-57) can be used to compute the overall compressor efficiency as

$$\eta_p = \tfrac{k-1}{k} \ln(1.1^3) / \ln(T_R^3) = \tfrac{k-1}{k} \ln(1.1) / \ln(T_R)$$

So the overall compressor efficiency is identical to the individual stage efficiency.

2.8 From Eqs. (2-63) and (2-68), the discharge static and total pressures are

$$P_d = 200 + 0.6(30) = 218$$

$$P_{td} = 230 - 0.1(30) = 227$$

For a thermally perfect gas with no work or heat transfer, $T_{td} = T_{ti}$ = 300. Equation (2-55) yields the inlet and discharge static temperatures.

$$T_i = 300(200/230)^{(0.4/1.4)} = 288.26$$

$$T_d = 300(218/227)^{(0.4/1.4)} = 296.55$$

For adiabatic reversible (isentropic) flow, the discharge temperature would be

$$T_{d,rev} = 300(218/230)^{(0.4/1.4)} = 295.44$$

Hence the diffuser efficiency is given by Eqs. (2-52) and (2-62)

$$\eta_{diff} = \frac{c_p(295.44 - 288.26)}{c_p(296.55 - 288.26)} = 86.6\%$$

3.1 From Eq. (3-29) for axisymmetric, time-steady flow,

$$\frac{\partial(rW_\theta + \omega r^2)}{\partial m} = \frac{\partial rC_\theta}{\partial m} = 0$$

Similarly, Eq. (3-25) requires

$$\frac{\partial I}{\partial m} = 0$$

Inserting these results into Eq. (3-28) yields

$$\frac{\partial s}{\partial m} = 0$$

3.2 Direct substitution of $W_\theta = W_m \tan\beta'$ into Eq. (3-30) yields

$$\frac{W_m}{\cos^2\beta'}\frac{\partial W_m}{\partial n} + \frac{W_m^2 \tan\beta'}{r}\frac{\partial r\tan\beta'}{\partial n}$$
$$+\kappa_m W_m^2 + 2\omega W_m \tan\beta' \cos\phi = \frac{\partial I}{\partial n} - T\frac{\partial s}{\partial n}$$

For the stationary coordinate system, $\omega = 0$, $W \to C$, $\beta' \to \beta$ and $I \to H$. Hence

$$\frac{C_m}{\cos^2\beta}\frac{\partial C_m}{\partial n} + \frac{C_m^2 \tan\beta}{r}\frac{\partial r\tan\beta}{\partial n} + \kappa_m C_m^2 = \frac{\partial H}{\partial n} - T\frac{\partial s}{\partial n}$$

3.3 From the definition of the dot product and Eq. (3-61),

$$V = \sqrt{\vec{V}\cdot\vec{V}} = \sqrt{V_m^2 + V_n^2 + V_\theta^2}$$

$$\tfrac{1}{2}\vec{\nabla}V^2 = V\frac{\partial V}{\partial m}\vec{e}_m + V\frac{\partial V}{\partial n}\vec{e}_n + \frac{V}{r}\frac{\partial V}{\partial \theta}\vec{e}_\theta$$

3.4 For the stated conditions, Eqs. (3-21) and (3-27) combine to yield

$$\frac{\partial br\rho W_m}{\partial m} = 0$$

Equation (3-22) simplifies to

$$W_m\frac{\partial W_m}{\partial m} - \frac{\sin\phi}{r}[W_\theta + \omega r]^2 = -\frac{1}{\rho}\frac{\partial P}{\partial m}$$

Equation (3-23) and the definition of the angle ϕ yield

$$\frac{\partial(rW_\theta + \omega r^2)}{\partial m} = 0$$

and Eq. (3-25) simplifies to

$$\frac{\partial I}{\partial m} = 0$$

3.5 From the uniform flow assumption, the total mass flow is

$$\int_0^b r\rho u dy = \bar{r}\rho_e u_e(b - 2\delta) + \int_0^\delta r\rho u dy + \int_{b-\delta}^b r\rho u dy$$

where $\bar{r}$ is the average radius. Using the standard boundary layer approximation that r is constant for the last two terms and Eq. (3-35), yields

$$\dot{m}/(2\pi) = \int_0^b r\rho u dy = \bar{r}\rho_e u_e(b - 2\delta^*)$$

Similarly, the momentum flux is

$$\int_0^b r\rho u^2 dy = \bar{r}\rho_e u_e^2(b - 2\delta) + \int_0^\delta r\rho u^2 dy + \int_{b-\delta}^b r\rho u^2 dy$$

and Eq. (3-38) yields

$$\int_0^b r\rho u^2 dy = \bar{r}\rho_e u_e^2 (b - 2\delta^* - 2\theta)$$

3.6 Conservation of mass for incompressible flow before and after mixing requires

$$\bar{r}b\rho_e u_{mix} = \bar{r}\rho_e u_e (b - 2\delta^*)$$
$$u_{mix} = u_e(1 - 2\delta^* / b)$$

Conservation of momentum with constant static pressure requires

$$\bar{r}b(P_e + \rho_e u_{mix}^2) = \bar{r}bP_e + \bar{r}\rho_e u_e^2 b[1 - 2(\delta^* + \theta)/b]$$

Introducing the incompressible relation for total pressure,

$$P_{t,mix} + \tfrac{1}{2}\rho_e u_{mix}^2 = P_{t,e} + \rho_e u_e^2[\tfrac{1}{2} - 2(\delta^* + \theta)/b]$$

Introducing u_{mix} from the mass balance equation

$$P_{t,mix} + \tfrac{1}{2}\rho_e u_e^2(1 - 2\delta^*/b)^2 = P_{t,e} + \rho_e u_e^2[\tfrac{1}{2} - 2(\delta^* + \theta)/b]$$
$$P_{t,e} - P_{t,mix} = \rho_e u_e^2[\tfrac{1}{2}(1 - 2\delta^*/b)^2 - \tfrac{1}{2} + 2(\delta^* + \theta)/b]$$
$$P_{t,e} - P_{t,mix} = \tfrac{1}{2}\rho_e u_e^2[(2\delta^*/b)^2 + 4\theta/b]$$

5.1 From Eqs. (5-1), (5-2), (5-3) and (5-9)

$$\frac{\partial}{\partial m} = \frac{\partial\xi}{\partial m}\frac{\partial}{\partial\xi} + \frac{\partial\eta}{\partial m}\frac{\partial}{\partial\eta} = \frac{1}{\cos\beta}\frac{\partial}{\partial\xi} - \frac{\tan\beta}{S}\frac{\partial S}{\partial m}\frac{\partial}{\partial\eta}$$
$$\frac{\partial}{\partial\theta} = \frac{\partial\xi}{\partial\theta}\frac{\partial}{\partial\xi} + \frac{\partial\eta}{\partial\theta}\frac{\partial}{\partial\eta} = \frac{r}{S}\frac{\partial}{\partial\eta}$$

Substitution of these derivatives into the steady form of Eq. (3-21) using Eq. (3-27) yields

$$\frac{1}{\cos\beta}\frac{\partial br\rho W_m}{\partial\xi} - \frac{\tan\beta}{S}\frac{\partial br\rho W_m}{\partial\eta} + \frac{1}{S}\frac{\partial br\rho W_\theta}{\partial\eta} = 0$$

Converting to finite-difference form, using the difference approximation form given in Eqs. (5-35) and (5-36) and multiplying through by $4\Delta\xi\Delta\eta$, yields

$$
\begin{aligned}
&\frac{2\Delta\eta}{\cos\bar{\beta}}[(br\rho W_m)_{\xi+\Delta\xi,\eta}-(br\rho W_m)_{\xi-\Delta\xi,\eta}] \\
&-\frac{2\Delta\xi\bar{r}\bar{b}\tan\bar{\beta}}{\bar{S}}[(\rho W_m)_{\xi,\eta+\Delta\eta}-(\rho W_m)_{\xi,\eta-\Delta\eta}] \\
&+\frac{2\Delta\xi\bar{r}\bar{b}}{\bar{S}}[(\rho W_\theta)_{\xi,\eta+\Delta\eta}-(\rho W_\theta)_{\xi,\eta-\Delta\eta}]=0
\end{aligned}
$$

where the overbar designates values at point (ξ, η). Noting that $2\Delta m = 2\Delta\xi\cos\bar{\beta}$,

$$
\begin{aligned}
&\frac{2\Delta\eta\bar{S}}{\bar{r}}[(br\rho W_m)_{\xi+\Delta\xi,\eta}-(br\rho W_m)_{\xi-\Delta\xi,\eta}] \\
&-2\bar{b}\Delta m\tan\bar{\beta}[(\rho W_m)_{\xi,\eta+\Delta\eta}-(\rho W_m)_{\xi,\eta-\Delta\eta}] \\
&+2\bar{b}\Delta m[(\rho W_\theta)_{\xi,\eta+\Delta\eta}-(\rho W_\theta)_{\xi,\eta-\Delta\eta}]=0
\end{aligned}
$$

Checking the result for the control volume in Fig. 5-4, it can be seen that the second term in this difference equation does not precisely balance mass. The term $\tan\bar{\beta}$ should be evaluated at each boundary, instead of using a mean value. Thus there will be an inherent error in the numerical approximation to the continuity equation.

5.2 The required Taylor series are

$$\psi(m+\Delta m)=\psi(m)+\psi'(m)\Delta m+\tfrac{1}{2}\psi''(m)(\Delta m)^2+\tfrac{1}{6}\psi'''(m)(\Delta m)^3+\cdots$$

$$\psi(m-\Delta m)=\psi(m)-\psi'(m)\Delta m+\tfrac{1}{2}\psi''(m)(\Delta m)^2-\tfrac{1}{6}\psi'''(m)(\Delta m)^3+\cdots$$

Subtract the second equation from the first and divide by $2\Delta m$ to obtain

$$\psi'(m)=\frac{\psi(m+\Delta m)-\psi(m-\Delta m)}{2\Delta m}+O[(\Delta m)^2]$$

Hence the difference approximation is of second-order accuracy in Δm. Add the two Taylor series and divide by $(\Delta m)^2$ to obtain the second derivative difference approximation, which is also of second-order accuracy in Δm.

5.3 The required Taylor series are

$$\psi(m+2\Delta m)=\psi(m)+2\psi'(m)\Delta m+2\psi''(m)(\Delta m)^2+\tfrac{4}{3}\psi'''(m)(\Delta m)^3+\cdots$$

$$\psi(m+\Delta m)=\psi(m)+\psi'(m)\Delta m+\tfrac{1}{2}\psi''(m)(\Delta m)^2+\tfrac{1}{6}\psi'''(m)(\Delta m)^3+\cdots$$

Multiply the second equation by 4, subtract the first equation from it and divide by $2\Delta m$ to obtain

$$\psi'(m) = \frac{4\psi(m+\Delta m) - 3\psi(m) - \psi(m+2\Delta m)}{2\Delta m} + O[(\Delta m)^2]$$

which is also of second order accuracy in Δm.

5.4 The required Taylor series is

$$u(m+\Delta m) = u(m) + u'(m)\Delta m + \tfrac{1}{2}u''(m)(\Delta m)^2 + \tfrac{1}{6}u'''(m)(\Delta m)^3 + \cdots$$

Solve for the first derivative to obtain

$$u'(m) = \frac{u(m+\Delta m) - u(m)}{\Delta m} + O(\Delta m)$$

Hence this difference approximation is of first-order accuracy in Δm.

5.5 Under the conditions stated, the flow upstream of the blade at near steady-state conditions will be approximately axisymmetric and time steady. From Eq. (5-72), the quantity $Sb\rho W_m$ will be approximately constant, so its second derivative in Eq. (5-102) will be approximately zero. Similarly, the quantities I and $rC_\theta = rW_\theta + \omega r^2$ will be approximately constant, as seen from Eqs. (3-25) and (3-29), so their second derivatives in Eqs. (5-103) and (5-104) will be approximately zero. Had the quantity $Sb\rho$ been moved outside of the second derivative in Eq. (5-102), the stabilizing term would no longer be approximately zero unless Sb is constant. In contrast, if $Sb\rho$ were moved inside of the second derivatives in Eqs. (5-103) and (5-104), those stabilizing would cease to be near zero unless Sb is constant. The derivative terms in the stabilizing terms are chosen as the quantities most likely to be nearly constant at near steady-state conditions. Although these terms should normally be small, any numerical stability problem will cause them to become large, basically introducing as much numerical damping as needed for a stable solution.

5.6 (a) In general, estimate θ from the upstream value using Eq. (5-106) with δ^* and τ_w set to zero for the purpose of this initial guess. If the previous station is the leading edge, where $\theta = 0$, that procedure won't work. In that case, a reasonably safe initial guess could be obtained by setting θ from an assumed value of Re_θ that is well below transition, e.g., 50.

(b) The steps in the iteration to solution at each station might be as follows:

- Compute b_0 from Eq. (5-121)
- Compute K from Eq. (5-122)
- Compute Λ from Eq. (5-124)
- Compute δ' from Eq. (5-115)
- Compute δ^* from Eqs. (5-116) through (5-120)
- Compute τ_w from Eqs. (5-109) and (5-123)
- Recompute θ from Eq. (5-106)
- Repeat above steps until converged

5.7 (a) Estimate θ from the upstream value using Eq. (5-106) with δ^* and τ_w set to zero for the purpose of this initial guess. Estimate $(\delta - \delta^*)$ from Eq. (5-127) with E set to zero for the purpose of this initial guess.

(b) The steps in the iteration to solution at each station might be as follows:

- Compute H_1 from Eq. (5-128)
- Compute H_k from Eq. (5-132)
- Compute E from Eq. (5-133)
- Compute H and δ^* from Eqs. (5-131) and (5-129)
- Compute c_f from Eqs. (5-134) and (5-135)
- Recompute θ and $(\delta - \delta^*)$ from Eqs. (5-106) and (5-127)
- Repeat above steps until converged

6.1 For Eq. (6-8), $i^* = \alpha^* + \gamma - \kappa_1 = \alpha^* - \theta / 2 = f(\sigma, \theta)$.
For Eq. (6-12), $i^* = f(\beta_1^*, \sigma, \theta)$. But since $\beta_1^* = \kappa_1 + i^*$, the true independent variables can be expressed as $i^* = f(\kappa_1, \sigma, \theta)$

6.2 For Eq. (6-8), $\beta_1^* = \alpha^* + \gamma = f(\sigma, \theta, \gamma)$.
For Eq. (6-12), $\beta_1^* = i^* + \kappa_1 = f(\sigma, \theta, \kappa_1) = f(\sigma, \theta, \gamma)$.

6.3 Both Fig. 6-2 and 6-14 are from the same reference, and use the same definition of α^*. Hence if the positive and negative stall incidence angles are to be properly computed, Eq. (6-8) must be solved for α^*.

7.1 Convert the data at base (constant radius) blade angle data to κ_1 and κ_2. Interpolate for c and t_b / c at the mean radius, κ_1 at the inlet radius and κ_2 at the discharge radius. Estimate the stream surface angle from

$$\tan\phi = (r_2 - r_1) / (z_2 - z_1)$$

The effective geometry data in the stream surface are

$$\begin{aligned}
&c \to c / \cos\phi \\
&s = \pi(r_2 + r_1) / Z \\
&t_b / c \to \cos\phi\, t_b / c \\
&\tan\kappa_1 \to \cos\phi \tan\kappa_1 \\
&\tan\kappa_2 \to \cos\phi \tan\kappa_2
\end{aligned}$$

7.2 Calculation of the meridional gradient of W_m with simple finite differences will usually be based on data at points on opposite sides of blade rows. This may cause the computed gradient to be meaningless, since W_m may be strongly influenced by discontinuous changes in the swirl velocity, etc., imposed by the blades.

7.3 A choke condition in an axial-flow compressor will almost always be caused by choke in a blade row, with an associated abrupt increase in loss. The approach to the Mach number limit in Eq. (7-29) is normally just an indication that blade row choke has occurred. In the unlikely event of a true annulus choke, this should still be reasonably accurate, since hub-to-shroud gradients in W_m are seldom extreme in an axial-flow compressor. If the through-flow analysis were to be applied within the blade passage, the more rigorous choke criterion might be necessary.

7.4 The approximation should be quite accurate on interior stream surfaces, assuming a reasonable number of stream surfaces are used. However, it is somewhat of an extrapolation for the end-wall surfaces. Even there, it will offer acceptable accuracy unless the meridional gradient of W_m becomes excessive near the end-walls. By the definition, a positive mass flow rate passes between adjacent stream surfaces, so the singularity will occur only on end-walls, unless it is caused by numerical errors in the early iterations of the solution process. If an end-wall boundary layer solution is included, a singularity in the inviscid through-flow analysis should be suppressed by the abrupt increase in end-wall blockage prediction when the end-wall velocity becomes small.

8.1 From Eq. (8-11), the mass flow rate in a boundary layer is given by

$$\int_0^\delta \rho V_m dy = \rho_e V_{me}(\delta - \delta_1^*)$$

Hence a mass balance combining the two incoming flows yields

$$2\pi r(\rho_e V_{me})_{in}(\delta - \delta_1^*)^+ = 2\pi(r\rho_e V_{me})_{in}(\delta - \delta_1^*)_{in} + \dot{m}_{leak}$$

which results in Eq. (8-58).

8.2 From Eqs. (8-11) and (8-12), the meridional momentum flux in a boundary layer is given by

$$\int_0^\delta \rho V_m^2 dy = \rho_e V_{me}^2(\delta - \delta_1^* - \theta_{11})$$

Hence the leakage flow contributes no meridional momentum, so

$$2\pi r(\rho_e V_{me}^2)_{in}(\delta - \delta_1^* - \theta_{11})^+ = 2\pi(r\rho_e V_{me}^2)_{in}(\delta - \delta_1^* - \theta_{11})_{in}$$

Combining with the result in the previous exercise yields Eq. (8-60).

8.3 From Eqs. (8-12) and (8-13), the tangential momentum flux in a boundary layer is given by

$$\int_0^\delta \rho V_m V_\theta dy = \rho_e V_{me} V_{\theta e}(\delta - \delta_1^* - \theta_{12})$$

Hence to balance tangential momentum of the two incoming flows,

$$\begin{aligned} 2\pi r(\rho_e V_{me} V_{\theta e})_{in}(\delta - \delta_1^* - \theta_{12})^+ \\ = 2\pi(r\rho_e V_{me} V_{\theta e})_{in}(\delta - \delta_1^* - \theta_{12})_{in} + (\dot{m}U)_{leak} \end{aligned}$$

Combining with the result of Exercise 8.1 yields Eq. (8-62) for the case of leakage flow entering the boundary layer.

8.4 The process is the same as for the previous three exercises, except for the tangential momentum and direction of the leakage flow.

8.5 Substituting the power-law profiles into the designated equations yields

$$\delta_1^* = \int_0^\delta \left[1 - \left(\frac{y}{\delta}\right)^n\right] dy = \delta - \frac{\delta}{n+1}$$

$$\theta_{11} = \int_0^\delta \left[\left(\frac{y}{\delta}\right)^n - \left(\frac{y}{\delta}\right)^{2n}\right] dy = \delta \frac{n}{(n+1)(2n+1)}$$

which combine to yield Eq. (8-20).

$$\theta_{12} = \int_0^\delta \left[\left(\frac{y}{\delta}\right)^n - \left(\frac{y}{\delta}\right)^{n+m}\right] dy = \delta \frac{m}{(n+1)(n+m+1)}$$

which yields Eq. (8-21).

9.1 Trapezoidal-rule integration of the uncorrected data between stream surfaces 1 and 3 yields

$$I = \tfrac{1}{2}[(\Delta p_t')_1 + 2(\Delta p_t')_2 + (\Delta p_t')_3]\Delta\dot{m}$$

Integration of the corrected data yields

$$I_c = \tfrac{1}{2}[(\Delta p_t')_{1,c} + 2(\Delta p_t')_{2,c} + (\Delta p_t')_{3,c}]\Delta \dot{m}$$

Introducing Eq. (9-10),

$$I_c = \tfrac{1}{2}[2(\Delta p_t')_{2,c} - (\Delta p_t')_{3,c} + 2(\Delta p_t')_{2,c} + (\Delta p_t')_{3,c}]\Delta \dot{m} = 2(\Delta p_t')_{2,c}\Delta \dot{m}$$

From Eq. (9-9),

$$I_c = \tfrac{1}{2}[(\Delta p_t')_1 + 2(\Delta p_t')_2 + (\Delta p_t')_3]\Delta \dot{m} = I$$

9.2 For a circular-arc camberline, Eq. (4-7) gives the arc radius of curvature, R_C, as

$$R_c = c/[2\ \sin(\theta/2)]$$

Hence the camberline length, L, is given by

$$L = R_C\theta = c\theta\,/\,[2\sin(\theta\,/\,2)]$$

Dividing by the staggered spacing, $s\ \cos\gamma$, yields Eq. (9-15).

10.1 For a constant-work, repeating stage, $C_3 = C_1$ and $U^2\psi$ = constant. Hence, Eqs. (10-6) and (10-7) yield

$$h_2 - h_1 = U_c^2\psi_c - \tfrac{1}{2}(C_2^2 - C_1^2)$$

$$h_3 - h_1 = U_c^2\psi_c$$

Hence Eqs. (10-3) and (10-5) yield

$$R = 1 - \frac{C_{z2}^2 + C_{\theta 2}^2 - C_{z1}^2 - C_{\theta 1}^2}{2U_c^2\psi_c}$$

Differentiating with respect to r,

$$\frac{\partial R}{\partial r} = \frac{1}{U_c^2\psi_c}\left[C_{z1}\frac{\partial C_{z1}}{\partial r} + C_{\theta 1}\frac{\partial C_{\theta 1}}{\partial r} - C_{z2}\frac{\partial C_{z2}}{\partial r} - C_{\theta 2}\frac{\partial C_{\theta 2}}{\partial r}\right]$$

From Eq. (10-24),

$$C_z \frac{\partial C_z}{\partial r} + C_\theta \frac{\partial C_\theta}{\partial r} = -\frac{C_\theta^2}{r}$$

Hence

$$r_c \frac{\partial R}{\partial r} = \frac{1}{\psi_c} \frac{r_c}{r} \left[\frac{C_{\theta 2}^2}{U_c^2} - \frac{C_{\theta 1}^2}{U_c^2} \right]$$

Substituting Eqs. (10-20) and (10-21) to eliminate the velocity terms yields Eq. (10-45).

10.2 For a constant-work stage, Eq. (10-24) can be written as

$$\frac{r_c}{2} \frac{\partial (C_{z2} / U_c)^2}{\partial r} = -\frac{r_c^2 C_{\theta 2}}{r U_c} \frac{\partial [r C_{\theta 2} / (r_c U_c)]}{\partial r}$$

Equations (10-20) and (10-21), with n = -1 and m = 1, yield

$$C_{\theta 2} / U_c = (1 - R_c)(r / r_c) + \tfrac{1}{2} \psi_c (r_c / r)$$

$$r_c \frac{\partial [r C_{\theta 2} / (r_c U_c)]}{\partial r} = 2(1 - R_c)(r / r_c)$$

$$r_c \frac{\partial (C_{z2} / U_c)^2}{\partial r} = 4(1 - R_c)[(1 - R_c)(r / r_c) + \tfrac{1}{2} \psi_c (r_c / r)]$$

Integration of the last equation yields Eq. (10-48).

10.3 In the answer to Exercise 10.1, it is shown that

$$r_c \frac{\partial R}{\partial r} = \frac{1}{\psi_c} \frac{r_c}{r} \left[\frac{C_{\theta 2}^2}{U_c^2} - \frac{C_{\theta 1}^2}{U_c^2} \right]$$

Hence constant reaction requires

$$\frac{1}{\psi_c} \frac{r_c}{r} \left[\frac{C_{\theta 2}^2}{U_c^2} - \frac{C_{\theta 1}^2}{U_c^2} \right] = 0$$

Hence

$$\frac{C_{\theta 2}^2 - C_{\theta 1}^2}{U_c^2} = \frac{(C_{\theta 2} + C_{\theta 1})(C_{\theta 2} - C_{\theta 1})}{U_c^2} = 0$$

For a constant-work stage, $H_2 = H_1$, so Eq. (10-2) can be used to yield

$$\frac{(C_{\theta 2} + C_{\theta 1})\psi_c}{U_c} = 0$$

Since reaction is undefined for the trivial case of $\psi_c = 0$, the required condition is

$$C_{\theta 2} = -C_{\theta 1}$$

This condition also must be satisfied at the reference radius. Equations (10-9) and (10-10) are valid at the reference radius, so

$$2(1 - Rc) = 0$$

The only case where this is true is that of $R = 1$.

13.1 From Eq. (13-42) it is clear that the loss coefficient cannot be less than one. About the only design option available is to minimize the tangential loss, and possibly the exit cone loss, through the choice of r_3 and A_3. If the diffusers achieve the same discharge flow conditions, the diffuser type used has no effect on the scroll/collector loss.

13.2 From the information given, no firm conclusions can be reached about overall exhaust loss. The reduced scroll loss coefficient is due at least in part to the increase in kinetic energy supplied by the centrifugal impeller. The absolute exhaust system loss could actually be higher for the centrifugal stage configuration. The substitution is almost certain to reduce the curvature loss of the original diffuser and the tangential loss in the scroll. To justify the substitution, a performance analysis of the centrifugal stage and its exhaust system must be compared to the exhaust system analysis without the substitution, including any benefits from the additional pressure rise supplied by the centrifugal stage.

13.3 The axial diffuser will have the lower loss since there will be no curvature loss contribution. The main reason a curved diffuser might be chosen is to reduce the overall axial length of the compressor. If the flow exiting the compressor has significant C_θ, the curved diffuser could be more effective. The higher discharge radius will yield greater diffusion of C_θ through conservation of angular momentum.

13.4 The results will basically be as dependable as the correction model itself. While not all losses in a compressor are skin-friction-related, that simply means not all losses are Reynolds-number-related. If the correction model used accounts for that, the imposed roughness correction should be valid. If anything, it might be slightly conservative if the Reynolds number lies in a zone of transition from smooth to rough skin friction. The greatest uncertainty lies in the ability to assign a surface roughness that is consistent with the characteristic Reynolds number used in the correction model.

13.5 Calculate the value of surface roughness, e, that results in $Re_e = 60$. It is unnecessary to polish the surfaces to achieve a surface roughness less than that value. The constant, 2,000, in the definition of Re_e is rather insignificant and can be omitted in Eq. (13-34) for more general applications. Hence, the Reynolds number based on e is the significant parameter.

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ABOUT THE AUTHOR

Mr. Aungier is the manager of Advanced Technology for Elliott Turbomachinery Company Inc., Ebara Group in Jeannette, Pennsylvania. He has been active in fluid mechanics research and development for more than 36 years, 32 of those in turbomachinery aerodynamics, specializing in centrifugal compressors, axial-flow compressors and radial-inflow turbines. He has numerous publications in this field, primarily through the American Society of Mechanical Engineers. He is a graduate of Cornell University, where he received a masters degree in Aerospace Engineering and a bachelors degree in Engineering Physics.

Mr. Aungier started his career in 1966 as an officer in the U.S. Air Force, conducting research in hypersonic re-entry vehicle aerodynamics at the Air Force Weapons Laboratory in Albuquerque, New Mexico. He is the author of numerous Air Force and NASA publications, some of which are the basis for one of the analysis techniques described in this book. In 1970, Mr. Aungier joined the Research Division of Carrier Corporation in Syracuse, New York, where he spent 11 years managing and conducting applied research on the fluid dynamics of turbomachinery and air handling equipment. Most of his individual research was focused on the interests of Elliott Company (then a division of Carrier), including development of aerodynamic performance analysis techniques for axial-flow compressors, centrifugal compressors and radial-inflow turbines. In 1981, Mr. Aungier transferred to Elliott Company as manager of Compressor Development, where his interests expanded to include the development of systematic and efficient techniques for aerodynamic design of turbomachinery. His responsibilities were extended to include turbine aerodynamic development in 1983 and mechanical design and analysis in 1987. He continues to be an active contributor to turbomachinery aerodynamic technology, specializing in comprehensive aerodynamic design and analysis systems. In 2000, ASME Press published his first book, describing his centrifugal compressor aerodynamic design and analysis system. The present book provides a similar treatment of his axial-flow compressor aerodynamic design and analysis system.

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