Compare three different ways to value an IRS

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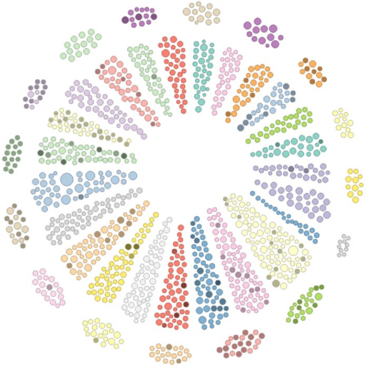


Motivation

Interest Rate Swaps (IRS) are the most actively non-exchange-traded (OTC) derivatives. According to the Bank of International Settlement they make up to 58.5% of all OTC derivative trades.

Below are some of the main reasons for companies to enter into an IRS:

+ Hedging risk of unfavorable interest rate fluctuations
+ Cost reduction of a loan
+ Reduction of uncertainty of future cash flows

Assumptions

The following assumptions are made in the course of the preceeding slides:

- For simplicity, the Principal amount is set to 1
- Simple compounding interest rates are used
- The valuations are based on positions in where we receive the fixed rate, so called Receiver Interest Rate Swaps

Interest Rate Swaps

Definition: A **Plain Vanilla Interest Rate Swap** (IRS) is an agreement between two parties to exchange payments of a fixed rate against a floating rate over a predetermined period of time at specific time points

⚠ Only exchange of streams of interest payments, no exchange of underlying principal amounts!

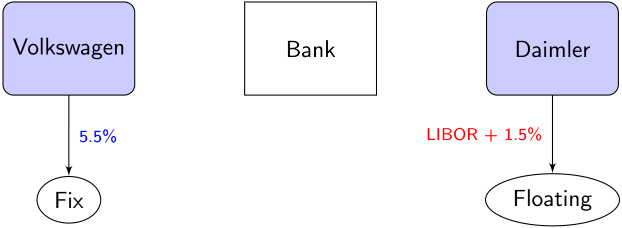
- receiver IRS (RIRS): fixed rate is received & floating rate is paid
- payer IRS (PIRS): fixed rate is paid & floating rate is received

Example:

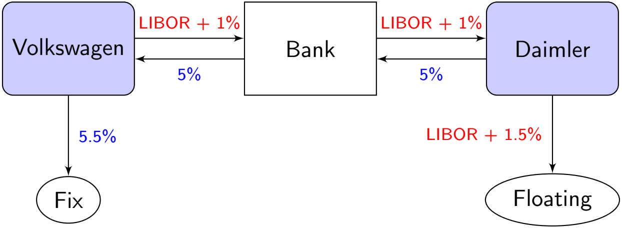
Initial situation before the IRS:

- VW is currently paying a fixed rate of 5.5% but wishes to pay a floating rate
- Daimler is currently paying a floating rate of LIBOR + 1.5% but wishes to pay a fixed rate

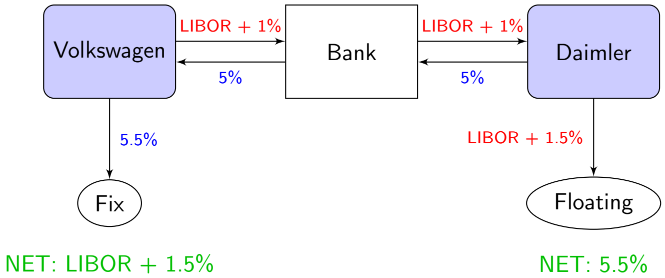
Initial situation visualized:



Both companies go to an intermediating bank in order to set up a swap:



Final payment structure for the companies loans:



Forward Rates

Definition: A **Forward Rate** is an interest rate applicable to a financial transaction that will take place in the future.

Due to arbitrage free investments the Forward Rate is implied in the Yield Curve and the following must hold (*where* $S \leq T$):

$$\overbrace{(1+r_S S)}^{\substack{\textit{Interest on}\\ \textit{an Investment}\\ \textit{from 0 to S}}} \cdot \underbrace{[1+F(0,S,T)(T-S)]}_{\substack{\textit{Interest on an Investment}\\ \textit{from S to T}}} = \overbrace{(1+r_T T)}^{\substack{\textit{Interest on}\\ \textit{an Investment}\\ \textit{from 0 to T}}}$$

$$\Rightarrow F(0,S,T) = \frac{1}{(T-S)} \cdot \left(\frac{1+r_T T}{1+r_S S} - 1 \right) = \frac{1}{(T-S)} \cdot \left(\frac{V(0,S)}{V(0,T)} - 1 \right)$$

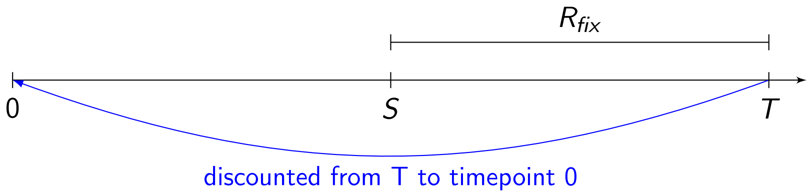
where $V(0,S) = \frac{1}{1+r_S S}$ and $V(0,T) = \frac{1}{1+r_T T}$.

Since the Principal is set to 1, $V(0,S)$ and $V(0,T)$ can be thought of as Discount Factors or in more general (when $P \neq 1$) as prizes of **Zero-Coupon-Bonds** with maturity in S respectively T years.

IRS Valuation

Forward Rate Agreements

Definition: A **Forward Rate Agreement** (FRA) is an agreement that a certain fixed interest rate will apply to a principal amount for a certain period of time, in exchange for an interest rate payment at the future interest rate.



Current **value of a FRA** paid-in-arrear $\widehat{=}$ discounted value of the payoff received at time T:

$$
\begin{aligned}
FRA_{R_{fix},S,T}\{R(t),t\} &= (1+R(0,T)\cdot T)^{-1}\cdot \overbrace{(T-S)\cdot(R_{fix}-R(S,T))}^{Net\ \ payoff} \\
&= V(0,T)\cdot(1+R_{fix}\cdot(T-S)) - (1+R(0,S)\cdot S)^{-1} \\
&= V(0,T)\cdot(T-S)\cdot R_{fix} + V(0,T) - V(0,S)
\end{aligned}
$$

IRS Valuation

- R_{fix} - fixed interest rate specified in the agreement
- $R(S,T)$ - future interest rate over the time period T-S
- $V(0,T) = \frac{1}{1+r_T\cdot T}$ - discount factor of a cashflow at time T where r_T ist the spot rate until time T

Valuation of Interest Rate Swaps

There are 3 different approaches for valuing an IRS, which are based on:

- Valuation and discounting of future cash flows
- Considering a Portfolio of Forward Rate Agreements (FRA)
- Valuing a Coupon and Floating Rate Bond

Valuation of Interest Rate Swaps - Cash Flow Approach

$$RIRS_{R_{fix},T}\{R(t),t\} = \sum_{i=0}^{n-1} \overbrace{V(0,t_{i+1})}^{\substack{\textit{Discount Factor of} \\ \textit{period } 0 \textit{ to } t_{i+1}}} \underbrace{(t_{i+1}-t_i)(R_{fix}-F(0,t_i,t_{i+1}))}_{\textit{Net Payment of period } t_i \textit{ to } t_{i+1}}$$

where $t_1,...,t_n$ are the dates when the payments are exchanged and $t_0 = 0$ & $t_n = T$.

Valuation of Interest Rate Swaps - FRA Approach

For simplicity, a plain vanilla IRS can be thought of as a portfolio of FRAs.

$$RIRS_{R_{fix},T}\{R(t),t\} = \sum_{i=0}^{n-1} FRA_{R_{fix},t_i,t_{i+1}}$$

where $t_1, \ldots, t_n$ are the dates when the payments are exchanged and $t_0 = 0$ & $t_n = T$.

IRS Valuation

Valuation of Interest Rate Swaps - Bond Approach

A further approach is to consider the fixed leg as a coupon bearing and the floating leg as a floating rate bond.

The coupon bearing bond can be valued in the following way:

$$FixedLeg_{R_K}\{R(t),t\} = \sum_{i=0}^{n-1} \underbrace{V(0,t_{i+1}) \cdot R_K \cdot (t_{i+1}-t_i)}_{\substack{Present\ Value\ of\ Coupon \\ Payment\ at\ time\ t_{i+1}}} + \overbrace{V(0,T)}^{\substack{Present\ Value \\ of\ the\ Principal}}$$

where $t_1, ..., t_n$ are the dates when the payments are exchanged and $t_0 = 0$ & $t_n = T$.

On the reset dates, the floating leg will always be traded at par (at its Principal), since the next coupon is equal to the rate used for discounting. If we consider today as the first reset date it holds that:

$$FloatingLeg = 1$$

And thus, the value of the RIRS can be calculated accordingly:

$$RIRS_{R_K,T}\{R(t),t\} = FixedLeg_{R_K}\{R(t),t\} - 1$$

IRS Valuation

R Simulation

⚠Code is also available in Python

Bibliography

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