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# Model Based Trajectory Planning for Highly Automated Road Vehicles

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**Abstract:** The aim of this paper is to present a local trajectory planning method based on nonlinear optimization that is able to generate a dynamically feasible, comfortable and customizable trajectory for highly automated road vehicles. The presented algorithm is able to consider the nonholonomic dynamics of wheeled vehicles and ensures the dynamical feasibility of the planned trajectory by the model-based prediction of the vehicle's motion. The behavior of the vehicle is simulated with closed loop trajectory tracking control which allows to generate not only the trajectory of the vehicle but also the reference signal inputs for the controllers. The direct planning of the reference signals enables the vehicle to run exactly on the generated trajectory and eliminates the delays related to the inertia of the system.



*Keywords:* autonomous vehicles, trajectory planning, vehicle dynamics

## 1. INTRODUCTION AND MOTIVATION

Highly automated driving is one of most important and challenging research fields in today's vehicle industry as automated and autonomous driving is expected to contribute to the quality road transportation in various ways. Although the evolution of active and passive safety equipment made it possible to reduce the number of fatal road accidents significantly in the last decades, still many car crashes happen every day mainly due to human failure (WHO (2009)). Consequently, automation of vehicle driving could further increase the safety of road transportation. Another important social expectation is the simultaneous increase of energy efficiency and the decrease of emission which may also be enabled by the rise of degree of automation. A key aspect to the autonomous control of road vehicles is trajectory planning; the design of the vehicle's motion. The term trajectory is used in this paper for the collective of temporal functions (position, orientation, velocity, acceleration) which describe the state of motion of the vehicle.

In recent years, many different trajectory planning approaches have been developed due to the increasing interest in automated driving. Existing works can be broadly classified into three categories: simple geometric-based methods, heuristic-based methods and methods based on optimal control techniques.

Geometric based methods such as the works of You et al. (2015) and Ren et al. (2011) generate trajectories based on some parametric geometrical curves such as circular arcs, clothoids or splines. These algorithms calculate the parameters of the curves with the consideration of geometrical constraints (start and end points, initial and final values of the derivatives of the curve), the limited steering angle of the vehicle and the maximal allowed lateral acceleration of a mass point moving along the curve (Minh and Pumwa (2014)). Geometric based methods can be suitable mainly for low speed applications such as automated parking but become unfeasible as the velocity of the vehicle increases.

Heuristic-based approaches usually apply artificial intelligence techniques such as search-based methods, random sampling methods and machine learning methods. Search based methods such as A* create a discrete spatiotemporal lattice of the vehicle's surrounding to search for a collision-free path along the points of the lattice (Montemerlo et al. (2008); Ferguson et al. (2008c)). Random sampling methods such as RRT (Rapidly-exploring Random Tree) which is used in the works of Kuwata et al. (2009) and Pepy et al. (2006) firstly define some metrics for the proximity of two spatial points and sample random points in the space around the vehicle. Then, starting from the initial or the required end position of the vehicle, the algorithm builds up a tree structure from the sampled points. If the a random sample is found to be collision-free and close to a previous element of the tree by the predefined metrics is added to the tree. The process is continued until a branch of the tree approaches the required end (or initial) point of the vehicle, and a path is then evaluated along the tree. Machine learning methods such as SVM (Support Vector Machine) are supervised learning models with associated learning algorithms that analyze data and recognize patterns (Li et al. (2014)).

Most of the geometric-based and heuristic-based methods generate rather paths than trajectories. Because of this, additional effort is required to convert the computed path into a trajectory which happens usually with the assignment of some speed profile (Gu et al. (2013)).

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Optimal control based methods, like the one in Anderson et al. (2010) use optimal control techniques such as NLP (Nonlinear Programming) and MPC (Model Predictive Control) in order to evaluate the trajectory. Some fundamental approaches this work is based on can be found in Howard and Kelly (2007) as well as in Ferguson et al. (2008a,b) where the authors use optimization techniques in order to find the appropriate control input functions which drive a model of the vehicle to the desired end point. The response of the system to the given controls is calculated numerically by a model based prediction. Optimal control based methods enable the direct planning of trajectories instead of paths.

This paper is organized as follows. Section 2. describes the trajectory related requirements and the mathematical formulation of the planning. In Section 3. the presented nonlinear programming problem is described in details, while Section 4. explains the applied solution methods. Section 5. shows simulation results and Section 6. contains some conclusion remarks.

## 2. THE TRAJECTORY PLANNING PROBLEM

### *2.1 Trajectory Related Requirements*

Trajectories planned for vehicles with human passengers must be safe, dynamically feasible, comfortable and customizable according to individual needs. The most important requirement related to a trajectory is perhaps that it cannot lead to collision with static or dynamic obstacles. As the planning problem is very complex, it is difficult to handle all of the requirements simultaneously. Because of this, the obstacle avoidance is not in the scope of this paper. The presented method can be applied for trajectory candidate generation in a hierarchical framework where the candidates are later checked against collision by a higher level motion planer, like in Ferguson et al. (2008b).

Another key requirement is dynamical feasibility, which means that the planned trajectory must be traceable by the vehicle. In order to ensure the dynamic feasibility of the trajectory the nonholonomic dynamics of wheeled vehicles must be considered. Most of the works in literature however are dealing with highly simplified kinematic models of vehicle motion because the structure and formulation of geometric-based and heuristic-based approaches do not allow the inclusion of dynamic equations of motion (Sun et al. (2014); Li et al. (2015)). Dynamical feasibility is also related to safety, because the possible instability of the vehicle can easily lead to hazardous situations. Accordingly, the most important purpose of this paper is to present a method which can ensure the dynamical feasibility of the planned trajectory.

The comfort of a trajectory can be described by several dynamical quantities. As shown in Suzuki (1998) the comfort of the passengers decreases with the increasing magnitude of acceleration because this results in increasing force acting on them. The passenger discomfort also increases with the increasing magnitude of jerk, as high jerk means a rapid change in the magnitude and/or direction of acceleration (Förstberg (2000)). The tolerable magnitude of the components of acceleration and jerk parallel and perpendicular to the direction of motion is different. In order to make the planned trajectory comfortable, the magnitude of acceleration and jerk must be bounded and kept as low as possible. This requirement is not only important from comfort perspective but also safety relevant, as the vehicle can become instable in case of high side acceleration because of the decreasing adhesion on the tires.

The experienced level of comfort also depends on the individual demands of the users. This means it can be necessary to be able to customize the planned trajectories according to passenger preferences within the limits of safety and dynamical feasibility.

After the planning, the vehicle is driven on the trajectory by some tracking control. Although most of the works in literature does not concern with this (an exception is Kuwata et al. (2009)), it is also necessary to generate not only the trajectory but the corresponding reference signals depending on the realization of the trajectory tracking controller. Even with the application of the most state of the art control techniques, there will be delays in the tracking of reference signals caused by the inertia of the system. This means that the vehicle will not exactly follow the planned trajectory. It is also the aim of this paper to present a method which can minimize this kind of deviation with the direct planning of the reference signals of the controller.

### *2.2 Problem Description*

The aim of the local trajectory planning method is to move the vehicle from the given initial point A to a required nearby end point B on a dynamically feasible trajectory. The term local is used here to indicate that this trajectory is short-term, and the planning takes place in the local coordinate system of the vehicle.

Mathematically, one can describe the trajectory planning as a nonlinear programming problem. The aim here is to find the appropriate parameters $p$ for a set of parametrized control inputs $u(p)$ which will then drive the dynamic system which has the state $x$ and is defined by a set of differential equations $f(x, u(p))$ to satisfy a set of state constraints $C(x)$:

$$\dot{x} = f(x, u(p)), \tag{1}$$

$$C(x) = 0. \tag{2}$$

The trajectory can be expected to be optimal with regard to some performance qualifiers $Z_i$ beyond the satisfaction of the state constraints:

$$J(x) = \sum_i w_i Z_i(x) \rightarrow min \tag{3}$$

where $w_i$ are weighting factors.

The presented trajectory planning concept makes it possible to handle the requirements described in Section 2.1. While the dynamics of the system can be considered according to (1) and (2), the formulation in (3) also makes it possible to include the comfort related or even further requirements in the planning problem.

## 3. DYNAMICALLY FEASIBLE MOTION PLANNING

### *3.1 Model of Vehicle Dynamics*

The differential equations in (1) which govern the system are actually the equations of motion of the vehicle. In order to ensure the dynamical feasibility of the planned trajectory, a precise nonlinear single-track model of vehicle dynamics (Fig. 1.) is used.

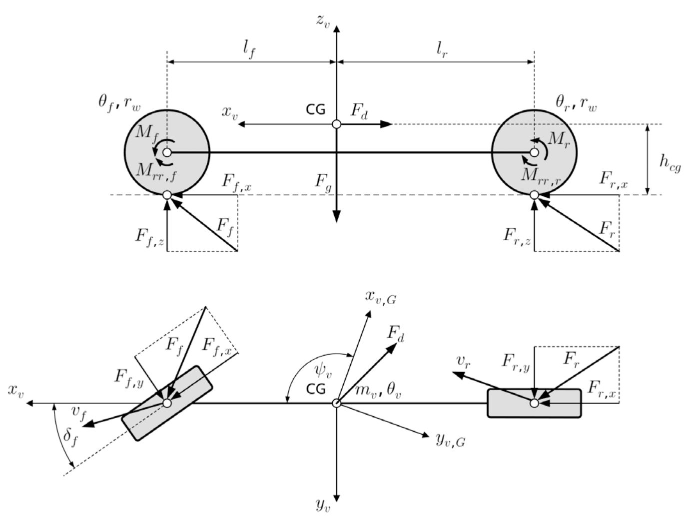


Fig. 1. Nonlinear single track vehicle model

The vehicle chassis is considered as a planar rigid body with three degree of freedom; translation in longitudinal $x_v$ and lateral $y_v$ directions, and rotation about the vertical axis $\psi_v$. The two front and rear wheels are represented with a single virtual wheel per axle which have a rotational degree of freedom about their axis denoted with $\rho_f$ and $\rho_r$ respectively. The parameters of the model are the mass $m_v$, the moment of inertia about the vertical axis $\theta_v$, and the center of gravity height $h_{cg}$ of the vehicle chassis, the moment of inertia of the front $\theta_f$ and rear $\theta_r$ wheels, the radius of the wheels $r_w$, and the horizontal distance from the center of gravity to the front $l_f$ and the rear $l_r$ axles.

The input vector of the system is:

$$u_v = [\delta_f \;\; M_v]^T, \tag{4}$$

where $\delta_f$ is the steering angle of the front wheel and $M_v$ is the total driving ($M_v > 0$) or braking ($M_v < 0$) torque.

The forces acting on the vehicle are the following. The front and rear tire forces are:

$$F_f = \begin{bmatrix} F_{f,x} \\ F_{f,y} \\ F_{f,z} \end{bmatrix} = m_v g \frac{l_r}{l_w + \beta_{lt}} \begin{bmatrix} c_{f,x} \\ c_{f,y} \\ 1 \end{bmatrix}, \tag{5}$$

$$F_r = \begin{bmatrix} F_{r,x} \\ F_{r,y} \\ F_{r,z} \end{bmatrix} = m_v g \frac{l_f}{l_w - \beta_{lt}} \begin{bmatrix} c_{r,x} \\ c_{r,y} \\ 1 \end{bmatrix}, \tag{6}$$

where $c_{f,x}$ and $c_{r,x}$ are the longitudinal, as well as $c_{f,y}$ and $c_{r,y}$ are the lateral adhesion coefficients of the front and rear wheels respectively, and $\beta_{lt}$ expresses the effect of load transfer between front and rear axles and can be calculated as:

$$\beta_{lt} = h_{cg}(c_{f,x} \cos \delta_f - c_{f,y} \sin \delta_f + c_{r,x}). \tag{7}$$

The adhesion coefficients express the portion of the vertical tire force which is transferred in the lateral or longitudinal direction. The adhesion coefficients of the front tire can be evaluated as:

$$c_{f,x} = \frac{s_{f,x}}{s_f} \sqrt{\frac{s_{f,x}^2 c_{f,x,x}^2 + s_{f,y}^2 c_{f,y,y}^2}{s_f^2}}, \tag{8}$$

$$c_{f,y} = \frac{s_{f,y}}{s_f} \sqrt{\frac{s_{f,x}^2 c_{f,x,x}^2 + s_{f,y}^2 c_{f,y,y}^2}{s_f^2}}, \tag{9}$$

where $s_{f,x}$ is the longitudinal and $s_{f,y}$ is the lateral slip of the front tire, $c_{f,x,x}$ and $c_{f,y,y}$ are the adhesion coefficients in the longitudinal and lateral directions with the consideration of only pure longitudinal or lateral wheel slips, and $s_f$ is the combined total slip defined as:

$$s_f = \sqrt{s_{f,x}^2 + s_{f,y}^2}. \tag{10}$$

The values of signed longitudinal and lateral tire slips of the front wheel are:

$$s_{f,x} = \frac{r_w \dot{\rho}_f - \dot{x}_{f,f}}{\max(r_w |\dot{\rho}_f|, |\dot{x}_{f,f}|)}, \tag{11}$$

$$s_{f,y} = -\frac{\dot{y}_{f,f}}{r_w |\dot{\rho}_f|}, \tag{12}$$

where $\dot{x}_{f,f}$ is the longitudinal and $\dot{y}_{f,f}$ is the lateral velocity of the front wheel center point in the own coordinate system of the wheel. These velocities can be evaluated according to Section 10.2.2 in Schramm et al. (2014). The adhesion coefficients considering unidirectional wheel slip $c_{f,x,x}$, $c_{f,y,y}$ are calculated based on the *Magic Formula* tire model in Section 4.3 of Pacejka (2006).

The vehicle model also considers the aerodynamic drag force which can be evaluated as:

$$F_d = \begin{bmatrix} F_{d,x} \\ F_{d,y} \\ 0 \end{bmatrix} = -\frac{1}{2} c_d \rho_a A_v \sqrt{\dot{x}_v^2 + \dot{y}_v^2} \begin{bmatrix} \dot{x}_v \\ \dot{y}_v \\ 0 \end{bmatrix}, \tag{13}$$

where $c_d$ is the aerodynamic drag coefficient and $A_v$ is the frontal area of the vehicle and $\rho_a$ is the volumetric mass density of air. The weight force of the vehicle which also effects the system can be expressed as:

$$F_g = [0 \;\; 0 \;\; -m_v g]^T. \tag{14}$$

The total driving or braking torque is distributed between the front and the rear axles with an arbitrary factor $\xi_M$ which can be time-variable to consider different braking and driving torque distribution. Accordingly, the driving or braking torques applied to the front and rear wheels are:

$$M_f = \xi_M M_v, \tag{15}$$

$$M_r = (1 - \xi_M) M_v. \tag{16}$$

Besides the driving or braking torques, the wheels are subject to rolling resistance torques $M_{rr,f}$ and $M_{rr,r}$. Rolling resistance torque of the first wheel can be expressed as:

$$M_{rr,f} = F_{f,z} f_f r_w \tanh \frac{4 \dot{x}_{f,f}}{v_{th}}, \tag{17}$$

where $f_f$ is the asymptotic rolling resistance coefficient and $v_{th}$ is the threshold velocity. The formulation in (17) makes it possible to eliminate the discontinuity of the rolling resistance torque at $\dot{x}_{f,f} = 0$. Equations (8) - (12) and (17) are also valid for the rear wheel with the changing of indexes $f$ to $r$.

The equations of motion of the vehicle chassis are derived in a vehicle fixed rotating local coordinate system, because the dynamical quantities are relevant from the passengers point of view. The state vector of the vehicle is defined as:

$$x = \left[x_{v,G}\ y_{v,G}\ \psi_v\ \dot{x}_v\ \dot{y}_v\ \dot{\psi}_v\ \dot{\rho}_f\ \dot{\rho}_r\right]^T, \tag{18}$$

where $x_{v,G}$ and $y_{v,G}$ are the coordinates of the vehicle chassis in the ground fixed global coordinate system, which is a stationary inertial frame of reference and coincides with the local coordinate system initially. The equations of state of the vehicle according to the formulation of (1) and the state vector in (18) are:

$$\begin{bmatrix} \dot{x}_{v,G} \\ \dot{y}_{v,G} \\ \dot{\psi}_v \\ \ddot{x}_v \\ \ddot{y}_v \\ \ddot{\psi}_v \\ \ddot{\rho}_f \\ \ddot{\rho}_r \end{bmatrix} = \begin{bmatrix} \dot{x}_v \cos\psi_v - \dot{y}_v \sin\psi_v \\ \dot{x}_v \sin\psi_v + \dot{y}_v \cos\psi_v \\ \dot{\psi}_v \\ \frac{1}{m_v}(F_{f,x} + F_{r,x} + F_{d,x} + m_v \dot{y}_v \dot{\psi}_v) \\ \frac{1}{m_v}(F_{f,y} + F_{r,y} + F_{d,y} - m_v \dot{x}_v \dot{\psi}_v) \\ \frac{1}{\theta_v}(l_f F_{f,y} - l_r F_{r.y}) \\ \frac{1}{\theta_f}(M_f - M_{rr,f} - r_f F_{f,x}) \\ \frac{1}{\theta_r}(M_r - M_{rr,r} - r_r F_{r,x}) \end{bmatrix}. \tag{19}$$

The physical meaning of equations in (19) is the following. The first two equations are the transformation of velocities from the local coordinate system to the global coordinate system. The forth, fifth, and sixth equations are the equations of motion of the vehicle chassis, and the two last equations are the equations of motion of the wheels. The equations of the chassis and the wheels are coupled by the adhesion coefficients in (5) and (6).

### *3.2 Parametrization of Input Functions*

The independent definition of the longitudinal and lateral characteristics of the trajectory is a common approach (Sun et al. (2014); Resende and Nashashibi (2010); Althoff et al. (2012)). Accordingly, the longitudinal velocity $v$ and the yaw rate $\omega$ are natural candidates for control inputs as the trajectory of the vehicle is effectively described by these two functions (Howard and Kelly (2007)).

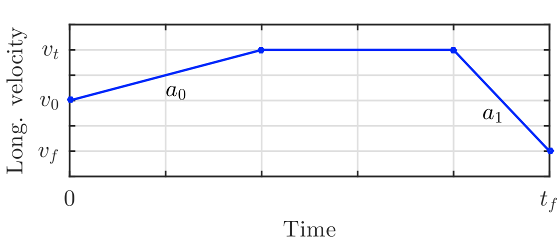


Fig. 2. Longitudinal velocity input

For the longitudinal velocity input function, a trapezoidal profile is applied (Fig. 2.). The usage of such piece-wise linear velocity profile is reasonable as it offers a sufficient velocity customization with a relatively small number of parameters. The parameters of the profile are the initial velocity $v_0$ and acceleration $a_0$, the travel velocity $v_t$, the final deceleration $a_f$ and velocity $v_f$ and the travel time along the trajectory $t_f$. Consequently, the parameter vector of the longitudinal velocity profile is:

$$p_v = \left[v_0\ a_0\ v_t\ a_f\ v_f\ t_f\right]^T. \tag{20}$$

With the appropriate selection of parameters, constant, linear, or linear ramp profiles can also be created freely.

The contour of a trajectory is primarily determined by the yaw rate profile. The yaw rate profile must be a smooth curve to avoid high transients and ensure the safety and comfort of the vehicle motion. An efficient parametrization of the yaw rate profile is a polynomial function of time. Fig. 3. shows a quintic polynomial profile. For the parametrization of the profile spline knot point

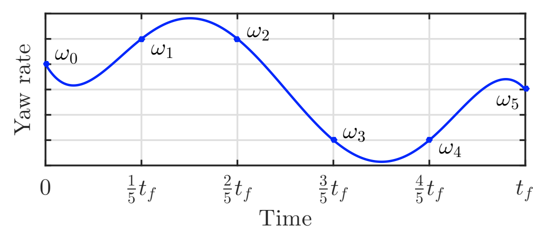


Fig. 3. Yaw rate input

parameters are used. The parameter vector of the yaw rate profile is:

$$p_\omega = \left[\omega_0\ \omega_1\ \dots\ \omega_n\ t_f\right]^T, \tag{21}$$

where $\omega_i, i = 0 \dots n$ are the knot points and $n$ is the degree of the applied polynomial function. The input vector of the system according to (2) is:

$$u(p) = \left[v(p_v)\quad \omega(p_\omega)\right]^T. \tag{22}$$

### *3.3 Motion Prediction with Closed Loop Control*

Examining the state vector of the vehicle in (18) it is conspicuous that the longitudinal velocity $\dot{x}_v$ and the yaw rate $\dot{\psi}_v$ are the states and not the inputs of the vehicle model. Yet, it would be advantageous to use these quantities as input functions according to (22) to effectively describe the trajectory. To be able to do so, a longitudinal velocity and yaw rate tracking controllers are implemented in order to provide the torque and steering angle inputs for the vehicle. The controllers are designed independently based on the linear models of longitudinal (Section 5.3) and lateral (Section 2.3) vehicle motion in Rajamani (2012). PI control is responsible for the longitudinal behavior while an LQ-servo controller is implemented for lateral control.

Assuming that the same controllers can be used to drive the real vehicle, this approach makes it possible to plan not only the trajectory, but also directly the reference signals which are needed to track it.

### *3.4 State Constraint Definition*

The purpose of the trajectory planning is that the vehicle reaches a prescribed final state $x_c$ at the end of its motion. Though it is not necessary to define a final value for each state variable, it is obvious that the final longitudinal

$x_{v,G,f}$ and lateral $y_{v,G,f}$ positions and the orientation $\psi_{v,f}$ of the vehicle have to be specified. Furthermore, it is also required to determine the final yaw rate $\dot{\psi}_{v,f}$ of the vehicle according to the final longitudinal velocity and road curvature. In order to ensure that two trajectories can connect to each other continuously, it is also reasonable to keep the final value of yaw acceleration $\ddot{\psi}_{v,f}$ at zero as well. The vector of the required final states of the vehicle can accordingly be defined as:

$$x_c = \left[x_{v,G,f}\ y_{v,G,f}\ \psi_{v,f}\ \dot{\psi}_{v,f}\ \ddot{\psi}_{v,f}\right]^T. \tag{23}$$

There are no longitudinal velocity or acceleration constraint defined because the longitudinal motion is not part of the optimization problem as described in Section 4.1. Assuming that the initial state of the vehicle $x_i$ is known at the time of planning and with the consideration of the closed loop system dynamics described by $f_{cl}(x, u(p))$, the final state of the vehicle can be calculated as:

$$x_f = x_i + \int_0^{t_f} f_{cl}(x, u(p))dt. \tag{24}$$

The formulation of state constraints according to (2) can now be written as:

$$C(x) = x_c - \widetilde{x}_f = 0, \tag{25}$$

where $\widetilde{x}_f$ contains the part of $x_f$ for which the required final values are prescribed.

### 3.5 Definition of the Cost Functional

Besides that the satisfaction of state constraints is the primary goal of the planning, the performance of the trajectory can be investigated as well according to (3). As described in Section 2.1 the high level of acceleration and jerk causes passenger discomfort and worsens the stability of the vehicle, which is undesirable. It is expected however to reach the required end state as soon as possible. Accordingly, the cost functional which expresses the quality of the trajectory can be chosen as:

$$J(x) = w_a \int_0^{t_f} \ddot{y}_v^2 + w_j \int_0^{t_f} \dddot{y}_v^2 + w_t t_f, \tag{26}$$

where $w_a$, $w_j$, and $w_t$ are weighting factors. It is important that the cost functional can contain arbitrary trajectory performance indicators. Accordingly, (26) is only one possible formulation which considers the comfort and the travel time of the trajectory. Minimal acceleration and minimal jerk cost functionals have previously been used in literature, but only based on particle kinematics (Sun et al. (2014); Althoff et al. (2012)). In (26) only the lateral acceleration and jerk are considered, because the longitudinal motion is not part of the optimization as presented in Section 4.1. With the proper selection of the weighting factors it is possible to customize the planned trajectory within the limits of dynamical feasibility. However, it may be difficult to find the correct values of the weighting factors that match the passenger's preference.

## 4. SOLUTION OF THE TRAJECTORY PLANNING PROBLEM

### 4.1 Reduction of Parameter Space

In order to reduce the size of the problem and keep the computational effort on an acceptable level, the parameter vector of the longitudinal velocity profile is considered to be known and is not part of the solution. According to this, the vector of free input function parameters is:

$$p = p_\omega. \tag{27}$$

This assumption is feasible as the parameters of the velocity profile can often be determined straight by the type of the motion (lane keeping or slowing down before an intersection) or legal regulations (maximal allowed velocity).

### 4.2 Constrained Planning

Considering only the basic formulation of the trajectory planning problem in (1) and (2) the correct input function parameters can be evaluated by the numerical solution of the state constraint equation specified in (25). For this, the determination of the final state that the vehicle actually reaches by the current value of the parameters is also necessary. Accordingly, the differential equations of the closed loop system are also solved numerically. An implicit method is required for the numerical integration because the wheel dynamics is known to be stiff and easily becomes unstable in case of explicit solvers (Rill (2007)). The solution of the equations of state of the closed loop system means that the generated trajectory is actually the model based prediction of the vehicle's motion. The state constraint equation can be solved with eg. the LMA (Levenberg-Marquardt) algorithm or with TR (Trust Region) techniques if the number of free parameters is equal to the number of state constraints (Marquardt (1963); Powell (1968)).

### 4.3 Constrained-optimal Planning

If the optimality of the trajectory is also required according to (3), the trajectory planning becomes a nonlinear constrained optimization problem, where the objective is to minimize the cost functional defined in (26) subject to the state constraints according to (25). The problem can be solved with state of the art nonlinear optimization techniques such as IPM (Interior Point Methods) or SQP (Sequential Quadratic Programming) (Byrd et al. (2000); Powell (1978)).

## 5. SIMULATION RESULTS

### 5.1 Generated Trajectories

To demonstrate the operation of the planner, simulations were carried out with the parameters of a mid-size car. The yaw rate reference signal was chosen to be cubic in case of the constrained planner, and quintic in case of the constrained-optimal planner, which means four and six free parameters respectively. The longitudinal velocity reference was a constant 20 m/s profile.

Fig. 4. shows a set of lane change like trajectories. The blue trajectories were generated with the constrained method described in Section 4.2, while the red trajectories were evaluated by the constrained-optimal method presented in Section 4.3. Regardless of the solution method all trajectories reach the prescribed final states which are denoted with black dots. The difference between the results of the

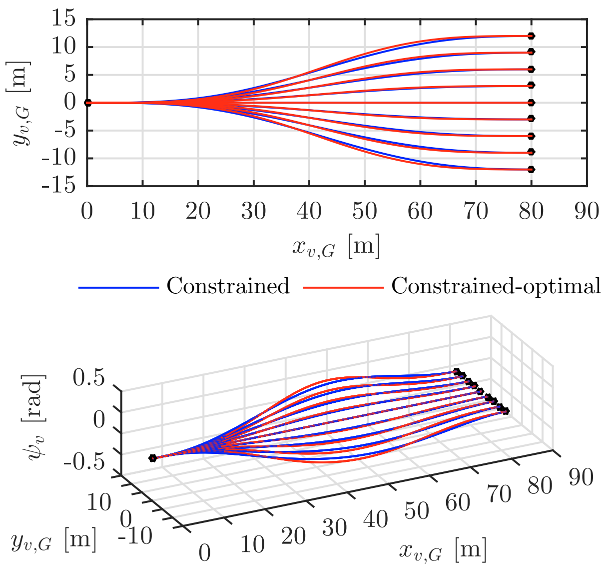


Fig. 4. Lane changing trajectories at $v = 20$ m/s

two methods seems not to be significant. Still, the trajectories planned by the constrained-optimization algorithm are reaching the final state ∼5% slower, but the highest values of lateral acceleration and jerk can even be ∼16% and ∼87% smaller respectively than in the pure constrained case.

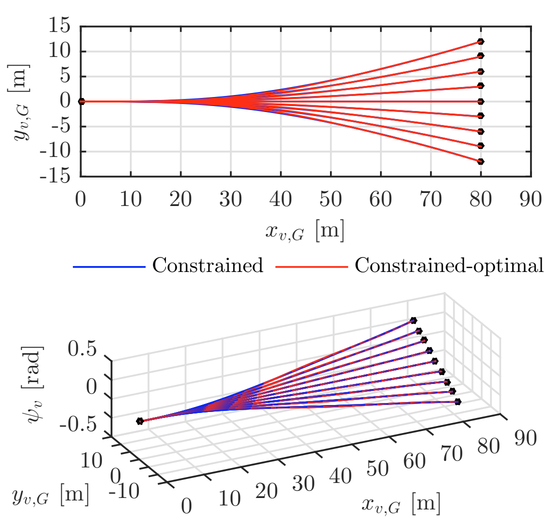


Fig. 5. Curved lane keeping trajectories at $v = 20$ m/s

Similarly, Fig. 5. shows a set of trajectories which are keeping a curved lane. The difference between the trajectories generated by the two algorithms is seemingly even less conspicuous here, but the application of the constrained-optimization method resulted also ∼5% slower trajectories while enabled that the highest values of lateral acceleration and jerk can even be ∼32% and ∼89% smaller respectively.

### 5.2 Direct Controller Reference Planning

In Fig. 6, the advantage of the direct reference signal planning is presented in case of lane keeping trajectories. The vehicle runs on the blue trajectories driven by the reference signals which are directly planned by the constrained-optimization method and obviously reaches the required end states as this was the target of the planning. Conversely, most of the planning methods in literature are not able to generate the reference signals directly. To simulate this case, reference signals are also calculated indirectly based on the previously planned trajectories. When driven by these subsequently planned reference signals, the vehicle runs on the red trajectories. It can be seen that the vehicle deviates from the required trajectories and misses the prescribed end states because of the settling time of the closed loop system in this case. The highest value of deviation is 1.72 m in position and $0.86^o$ in orientation. In contrast to almost all trajectory planning algorithms in literature, the method presented in this paper makes it possible to run exactly on the planned trajectories if not considering the inaccuracies and uncertainties of the model.

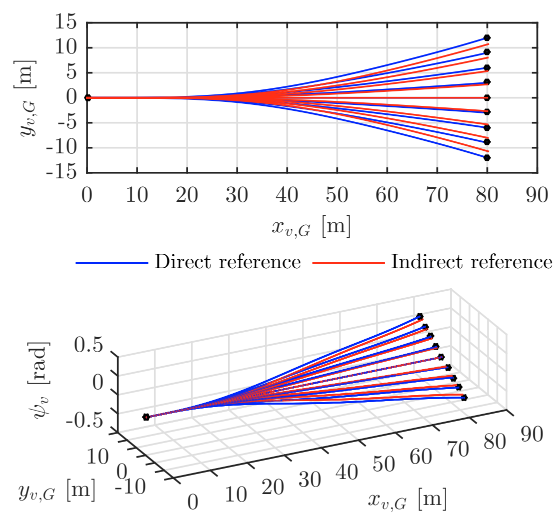


Fig. 6. Curved lane keeping trajectories at $v = 20$ m/s with directly and indirectly planned reference signals

## 6. CONCLUSION AND FUTURE WORK

In this paper a new nonlinear optimization based trajectory planner was presented for highly automated road vehicles. The method is able to plan dynamically feasible and comfortable trajectories keeping an eye also on customizability. The generated trajectories are the numerical calculation of the response of an appropriate model of vehicle dynamics in a closed loop system with the trajectory tracking controllers.

The results show that this approach makes it possible to generate the reference signals for the controllers directly and simultaneously thus enables the vehicle to run exactly on the planned trajectory apart from the inaccuracies of the model. It is also shown that the constrained optimization formulation of the planning problem contributes excessively to the comfort of the trajectory by the suitable cost functional.

Obviously, the described method requires considerable computational effort. The current implementation is not suitable for real-time application, it is the mission of a further research to allow the usage of the method on real-time embedded systems. A possible approach can be the appropriate selection of the initial guesses for the optimization methods based on off-line calculations. It is also important to investigate the sensitivity and robustness of the developed methods as the real plant always deviates from the applied system models. The performance of the presented technique needs to be examined also in case of rapid changes in road and environmental conditions. The proposed method may serve as a basis for complex trajectory planning and executing frameworks of the future.

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