

# Lecture 18:

## F2F RGB-D Registration (visual odometry)



RGB Image



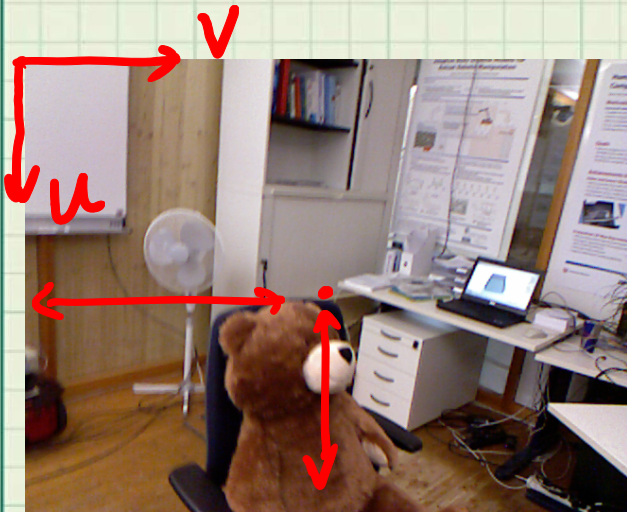
Depth Image



Microsoft Kinect



Goal: Track the camera pose temporally



$t_1$   
 $I_1$

$T \in SE(3)$

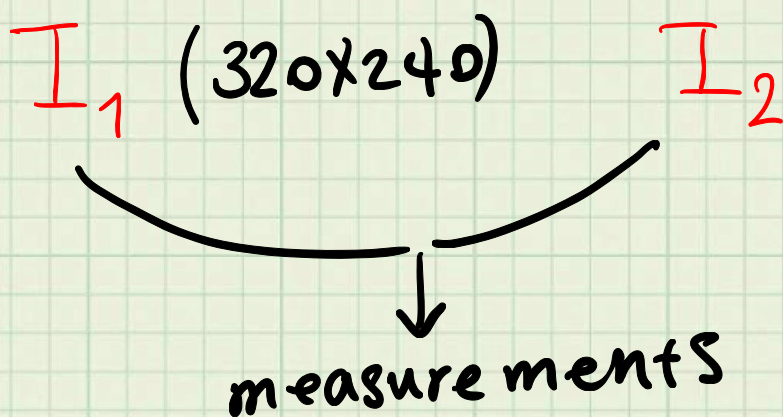
$t_2$   
 $I_2$

Camera projection model

$$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \quad \text{3D point}$$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \pi(P) = \begin{bmatrix} \frac{f_x x}{z} + C_x \\ \frac{f_y y}{z} + C_y \end{bmatrix} \quad \text{pixels on the image} \quad 2 \times 1$$





Assumption: Brightness constancy

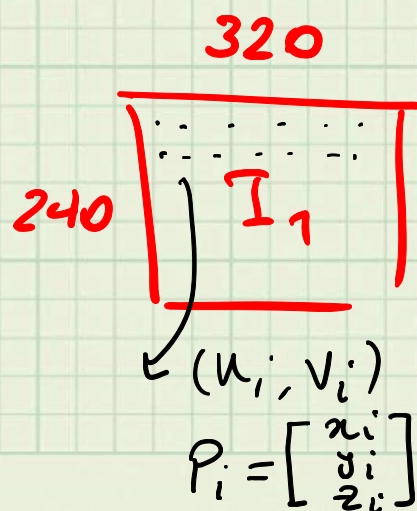
$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \pi(P_i), \quad I_i(u_i, v_i) \in \mathbb{R}^+$$

$$r_i(T) = I_2(\pi(\underbrace{T \cdot P_i}_{\text{wavy line}})) - I_1(\pi(P_i))$$

$$T \cdot P_i = R P_i + t$$

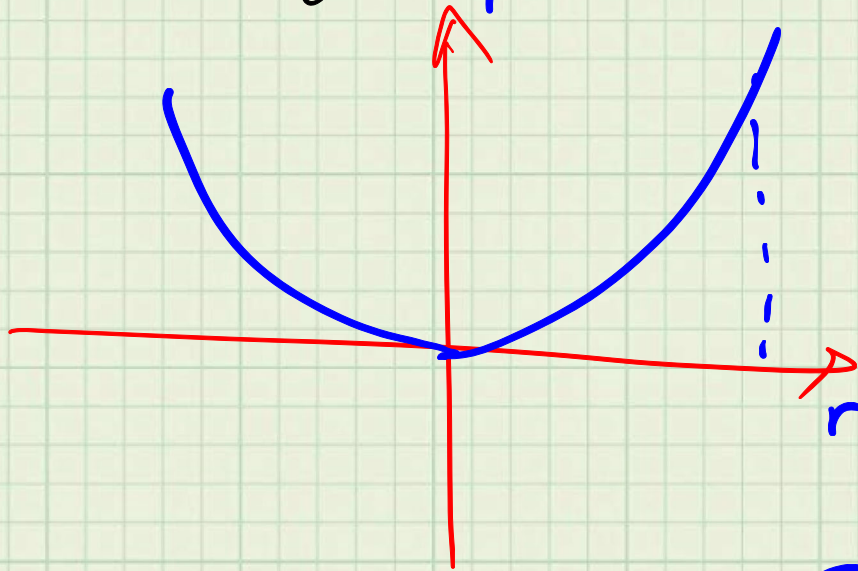
$$T = (R, t) \in SE(3)$$

$$i = 1, 2, \dots, 240 \times 320$$



# photometric Loss Function

$$T^* = \arg \min_{T \in SE(3)} \sum_i r_i^2(T)$$



M-estimator, a.k.a, Robust  
RGB-D VO

$$T^* = \arg \min_{T \in SE(3)} \sum \rho(r_i^2(T))$$

$\rho(\cdot)$  is the robust kernel or norm,  
e.g., Cauchy loss function.

We can solve this problem using IRLS.  
Ceres solver



$$\min_T r^T W r$$

$$r(\xi) \approx A \begin{matrix} \xi \\ 1 \times 6 \\ 6 \times 1 \end{matrix} + b \begin{matrix} 2 \times 1 \end{matrix}$$

$$A_i = \sqrt{w_i} \frac{\partial r_i}{\partial \xi} \quad (1 \times 6)$$

$$A = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_n \end{bmatrix}_{n \times 6}, \quad n = 240 \times 320$$

Normal Equation:

$$A^T A \xi = -\sqrt{w} A^T r$$

$$\left( \sum_i A_i^T A_i \right) \xi = - \sum_i \sqrt{w_i} A_i$$

$$H \xi = d \Rightarrow \xi = H \setminus d$$

Next Time:

Sensor fusion using factor graphs  
(graph SLAM)

