

the input to h_1 is z_1

$$z_1 = w_1x_1 + w_2x_2 + b_1$$

$$z_1 = 0.15 * 0.05 + 0.20 * 0.10 + 0.35 = 0.3775$$

the output of $h_1 \Rightarrow$ we make sigmoid on $z_1 \Rightarrow g(z_1) = a_1$

$$a_1 = \frac{1}{1 + e^{-z_1}} = \frac{1}{1 + e^{-(0.3775)}} = 0.5932699921$$

the input to h_2 is z_2

$$z_2 = w_3x_1 + w_4x_2 + b_1$$

$$z_2 = 0.25 * 0.05 + 0.30 * 0.10 + 0.35 = 0.3925$$

the output of $h_2 \Rightarrow$ we make sigmoid on $z_2 \Rightarrow g(z_2) = a_2$

$$a_2 = \frac{1}{1 + e^{-z_2}} = \frac{1}{1 + e^{-(0.3925)}} = 0.5968843783$$

the input to o_1 is a_1 & a_2

$$z_3 = w_5a_1 + w_6a_2 + b_2 = 0.40 * 0.5932699921 + 0.45 * 0.5968843783 + 0.60 = 1.105905967$$

$$o_1 = \frac{1}{1 + e^{-z_3}} = 0.7513650695$$

the input to o_2 is a_1 & a_2

$$z_4 = w_7a_1 + w_8a_2 + b_2 = 0.50 * 0.5932699921 + 0.55 * 0.5968843783 + 0.60 = 1.224921404$$

$$o_2 = \frac{1}{1 + e^{-z_4}} = 0.7729284653$$

$$Error = \frac{1}{n} \sum_{i=1}^n (y(a_i) - a_i)^2 = \frac{1}{2} [0.01 + 0.01]$$

$$\text{Square error } [E] = \frac{1}{2n} \sum_{i=1}^n [y^i(\text{actual}) - a^i(\text{predict})]^2 \quad \& \quad n=1$$

$$E_{O1} = \frac{1}{2} [0.01 - 0.7513650695]^2 = 0.2748110831$$

$$E_{O2} = \frac{1}{2} [0.99 - 0.7729284653]^2 = 0.02356002559$$

$$\begin{aligned} \text{Total error } (E_{\text{total}}) &= E_{O1} + E_{O2} \\ &= 0.2748110831 + 0.02356002559 \\ &= 0.2983711087 \end{aligned}$$

$$\frac{\partial E_{\text{total}}}{\partial W5} = dW5 = dz_3 * a1$$

$$\begin{aligned} dz_3 &= a1 * g'(z_3) \\ &= 0.5932699921 * \left[\frac{1}{1+e^{z_3}} \left(1 - \frac{1}{1+e^{z_3}} \right) \right] \\ &= 0.1108320906 \end{aligned}$$

$$dW5 = 0.1108320906 * 0.5932699921 = 0.06575335351$$

$$\begin{aligned} W5 &= W5 - \alpha dW5 = 0.40 - (0.5)(0.06575335351) \\ &= 0.3671233232 \approx 0.3 \end{aligned}$$

$$\frac{\partial E_{\text{total}}}{\partial W6} = dW6 = dW5$$

$$\begin{aligned} W6 &= W6 - \alpha dW6 = 0.45 - 0.5 * 0.06575335351 \\ &= 0.4171233232 \approx 0.4 \end{aligned}$$

$$\frac{\partial E_{\text{total}}}{\partial W_7} = dW_7 = dZ_4 * a_2$$

$$dZ_4 = a_1 * g'(Z_4)$$

$$= 0.5932699921 * \left[\frac{1}{1 + e^{-Z_4}} \left(1 - \frac{1}{1 + e^{-Z_4}} \right) \right]$$

$$= 0.1041248477$$

$$dW_7 = 0.1041248477 * 0.5968843783$$

$$= 0.06215049496$$

$$W_7 = W_7 - \alpha dW_7 = 0.50 - 0.5 * 0.06215049496$$

$$= 0.4689247525 \approx 0.5$$

$$dW_8 = dW_7$$

$$W_8 = W_8 - \alpha dW_8 = 0.55 - 0.5 * 0.06215049496$$

$$= 0.5189247525 \approx 0.5$$

$$\frac{\partial E_{\text{total}}}{\partial W_1} = dW_1 = dZ_1 * i_1$$

$$dZ_1 = i_1 * g'(Z_1)$$

$$= 0.05 * \left[\frac{1}{1 + e^{-Z_1}} \left(1 - \frac{1}{1 + e^{-Z_1}} \right) \right]$$

$$= 0.01206503543$$

$$dW_1 = 0.01206503543 * 0.05 = 6.032517714 * 10^{-4}$$

$$W_1 = W_1 - \alpha dW_1 = 0.15 - 0.5 * dW_1 = 0.1496983741 \approx 0.149$$

$$\frac{\partial E_{total}}{\partial W_2} = dW_2 = dW_1$$

$$W_2 = W_2 - \alpha dW_2 = 0.20 - 0.5 * 6.032517714 * 10^{-4} \\ = 0.1996983741 \approx 0.199$$

$$\frac{\partial E_{total}}{\partial W_3} = dW_3 = dZ_2 * X_2$$

$$dZ_2 = X_2 g'(Z_2) \\ = 0.1 * \left[\frac{1}{1 + e^{Z_2}} \left(1 - \frac{1}{1 + e^{Z_2}} \right) \right]$$

$$= 0$$

$$\therefore dW_3 = 0$$

$$W_3 = W_3 - 0.5 * 0 = 0.25$$

$$dW_3 = dW_4 = 0$$

$$\therefore W_4 = W_4 - \alpha dW_4 = 0.30$$

⇒ After this round of back Propagation the total error is down
so is now = 0.291027924