Midterm 1: Algebra and Limits Tips

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1 Midterm Information

Same place and same time as our usually Tuesday meetings. Don't accidentally go to the Thursday building! You are allowed to use a **hand written** sheet of note (A4 paper, 8x11 in), front and back.

2 Algebra

Here are the algebra tricks we talked about in class.

2.1 Power Rules

Rules concerning powers:

$$\begin{aligned} x^{-a} &= \frac{1}{x^a} \\ x^a \cdot x^b &= x^{a+b} \\ \frac{x^a}{x^b} &= x^a \cdot x^{-b} = x^{a-b} \\ \left(x^a\right)^b &= x^{a \cdot b} \\ x^{\frac{a}{b}} &= \sqrt[b]{x^a} \end{aligned}$$

2.2 Root Rules

Rules concerning roots:

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$$
$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$
$$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$$

2.3 Other Rules and Tips

Some other helpful definitions:

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

3 Limits

Steps to solve a limit $\lim_{x \to x_0} f(x)$.

1. First plug in the value x_0 into $f(x)$ and see what $f(x_0)$ looks like. If it is just a number (or $\infty + \infty = \infty$ or $-\infty - \infty = -\infty$), great we found our limit! If it is indeterminate we have more work to do. Remember that indeterminate comes in the form of

$$\frac{0}{0}$$
$$\frac{\pm\infty}{\pm\infty}$$
$$\infty - \infty$$

   There are also the four following cases we discussed in class

$$\frac{c}{0^+} = \infty$$
$$\frac{c}{0^-} = -\infty$$
$$\frac{-c}{0^+} = -\infty$$
$$\frac{-c}{0^-} = \infty$$

2. If we have an indeterminate case, then there are a few things we can do depending on the problem.

   If there is a root term in a fraction we can multiple the top and bottom by the conjugate. For example

$$\lim_{x\to 0} \frac{1-\sqrt{x+1}}{x}\left(\frac{1+\sqrt{x+1}}{1+\sqrt{x+1}}\right) = \cdots = -\frac{1}{2}$$

   If we have a nice polynomial we can try to factor it and cancel some terms. (Factor the top term and cancel with the bottom)

$$\lim_{x\to 5} \frac{x^2-2x-15}{x-5} = \cdots = 8$$

   Sometimes it is nice to divide top and bottom by the highest degree of x.

$$\lim_{x\to\infty} \frac{x+\frac{1}{x}}{x^2} = \cdots = 0$$