Math 569 Homework 1
Due April 12
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Problem 1 *Solve the PDE:*

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0, \quad -\infty < x < \infty, t > 0$$

subject to the initial conditions:

$$u(x,0) = u_0(x) = \begin{cases} -1, & -\infty < x \leq -a, a > 0, \\ \frac{x}{a}, & -a < x < a, \\ 1, & a \leq x < \infty. \end{cases}$$

Solution.
Consider the homogeneous PDE

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0, \quad -\infty < x < \infty, t > 0$$

subject to the initial conditions:

$$u(x,0) = u_0(x) = \begin{cases} -1, & -\infty < x \leq -a, \\ \frac{x}{a}, & -a < x < a, \\ 1, & a \leq x < \infty. \end{cases}$$

where $a > 0$.

□

Problem 2 *Consider the initial value problem in infinite domain:*

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0,$$

with

$$u(x,0) = u_0(x) = \begin{cases} 1, & x \le 0, \\ 1-x, & 0 < x < 1, \\ 0, & x \ge 1. \end{cases}$$

(a) Find where and when a shock first forms.

(b) Solve the problem and sketch or plot the solution before when a first shock forms.

(c) Find the shock speed using Rankine-Hugoniot condition.

(d) Solve the problem and sketch or plot the solution after the shock has formed.

Solution. This is a solution □

Problem 3 *Solve the PDE:*

$$\frac{\partial u}{\partial t} + y\frac{\partial u}{\partial x} = u, -\infty < x < \infty, t > 0,$$

subject to the initial condition:

$$u(x, 0) = u_0(x) = 2x, 0 \leq x \leq 2.$$

Where in the $x - t$ plane is the solution valid.

Solution.
□