Pattern Recognition
Assignment 1

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1 Generate the Data Matrix and the Label vector

Read data into torch.Tensor

[9]:
```
torch.set_default_dtype(torch.float64)

image_count_per_person = 10
person_count = 40
images = [Image.open(f"../data/s{j + 1}/{i + 1}.pgm") for j in␣
 ↪range(person_count) for i in range(image_count_per_person)]

all_data = np.array(images)
all_data.resize((image_count_per_person * person_count, images[0].width *␣
 ↪images[0].height))
all_data = Tensor(all_data)

labels = torch.tensor([i for i in range(person_count) for _ in␣
 ↪range(image_count_per_person)])
```

2 Split the Dataset into Training and Test sets

Split dataset into training and test data taking the even indexed rows for testing and the odd indexed rows for training

[10]:
```
odd_indices = [i for i in range(len(all_data)) if i % 2 == 1]
even_indices = [i for i in range(len(all_data)) if i % 2 == 0]

training_data = all_data[odd_indices]
test_data = all_data[even_indices]

training_labels = labels[odd_indices]
test_labels = labels[even_indices]
```

3 Classification using PCA

3.1 Original PCA Algorithm

Running Time: $O(d^3)$ to calculate the eigen values and eigen vectors of $\Sigma_{d\times d}$ matrix

ALGORITHM 7.1. Principal Component Analysis

PCA $(\mathbf{D}, \alpha)$:

$\boldsymbol{\mu} = \frac{1}{n}\sum_{i=1}^{n}\mathbf{x}_i$ `// compute mean`

$\mathbf{Z} = \mathbf{D} - \mathbf{1}\cdot\boldsymbol{\mu}^T$ `// center the data`

$\boldsymbol{\Sigma} = \frac{1}{n}\left(\mathbf{Z}^T\mathbf{Z}\right)$ `// compute covariance matrix`

$(\lambda_1, \lambda_2, \dots, \lambda_d) = \text{eigenvalues}(\boldsymbol{\Sigma})$ `// compute eigenvalues`

$\mathbf{U} = \begin{pmatrix}\mathbf{u}_1 & \mathbf{u}_2 & \cdots & \mathbf{u}_d\end{pmatrix} = \text{eigenvectors}(\boldsymbol{\Sigma})$ `// compute eigenvectors`

$f(r) = \frac{\sum_{i=1}^{r}\lambda_i}{\sum_{i=1}^{d}\lambda_i}$, for all $r = 1, 2, \dots, d$ `// fraction of total variance`

Choose smallest r so that $f(r) \geq \alpha$ `// choose dimensionality`

$\mathbf{U}_r = \begin{pmatrix}\mathbf{u}_1 & \mathbf{u}_2 & \cdots & \mathbf{u}_r\end{pmatrix}$ `// reduced basis`

$\mathbf{A} = \{\mathbf{a}_i \mid \mathbf{a}_i = \mathbf{U}_r^T\mathbf{x}_i, \text{for } i = 1, \dots, n\}$ `// reduced dimensionality data`

[4]:

```python
def pca1(data: Tensor, alpha: float) -> Tensor:
    centered_data: Tensor = data - data.mean(0)
    covariance: Tensor = 1 / len(data) * centered_data.T @ centered_data

    eigen_values, eigen_vectors = linalg.eigh(covariance)

    variance: float = covariance.trace().numpy()
    projected_variance = 0.0

    i = len(eigen_values) - 1
    while i >= 0 and projected_variance < alpha * variance:
        projected_variance += eigen_values[i]
        i -= 1

    eigen_vectors = torch.fliplr(eigen_vectors[:, i + 1:])

    return data @ eigen_vectors
```

[5]:

```python
# pca1(training_data, 0.8)
```

3.2 Enhanced PCA algorithm

Running Time: $O(n^3)$ to calculate the eigen values and eigen vectors of $\frac{1}{n}\mathbf{X}_{n\times d}\mathbf{X}_{d\times n}^T$ matrix
Reference: Section 12.1.4 from C. M. Bishop, Pattern Recognition and Machine Learning

[6]:
```python
def pca2(data: Tensor, alpha: float) -> Tensor:
    centered_data = data - data.mean(0)

    eigen_values, eigen_vectors = torch.linalg.eig(1 / len(data) * centered_data␣
 ↪@ centered_data.T)
    eigen_values = eigen_values.real
    eigen_vectors = eigen_vectors.real

    variance: float = (1 / len(data) * centered_data.T @ centered_data).trace().
 ↪numpy()
    projected_variance = 0.0

    idxs = torch.argsort(eigen_values, descending=True)
    eigen_values = eigen_values[idxs]
    eigen_vectors = eigen_vectors[:, idxs]

    i = 0
    while i < len(eigen_values) and projected_variance < alpha * variance:
        projected_variance += eigen_values[i]
        i += 1

    new_basis = centered_data.T @ eigen_vectors
    new_basis /= torch.sqrt(eigen_values * len(data))

    new_basis = new_basis[:, :i]

    return new_basis
```

[7]:
```python
alpha = 0.95

new_basis = pca2(training_data, alpha)

# project training data
projected_training_data = training_data @ new_basis
```

Classify test data using KNN

[4]:
```python
def knn(test_data: Tensor, training_data: Tensor, k: int):
    distance_matrix = torch.cdist(test_data, training_data)
    indices = torch.argsort(distance_matrix, dim=1)
    return indices[:, :k]
```

[25]:
```python
result_labels = training_labels[knn(test_data @ new_basis,␣
↪projected_training_data, 1)].mode(keepdim=True)[0]
```

tensor(6272.0381)

[10]:
```python
Latex(rf"for $\alpha$ = {alpha}, accuracy = {1 - torch.
↪count_nonzero(result_labels - test_labels[:, None]) / len(result_labels)}")
```

[10]: for $\alpha = 0.95$, accuracy $= 0.935$

[44]:
```python
def pca_classify(training_data: Tensor, test_data: Tensor, training_labels:␣
↪Tensor, test_labels: Tensor, alpha: float, k: int) -> float:
    new_basis = pca2(training_data, alpha)
    projected_training_data = training_data @ new_basis
    result_labels = training_labels[knn(test_data @ new_basis,␣
↪projected_training_data, k)].mode(keepdim=True)[0]
    return 1 - torch.count_nonzero(result_labels - test_labels[:, None]) /␣
↪len(result_labels)
```

[12]:
```python
alpha = 0.8
accuracy = pca_classify(training_data, test_data, training_labels, test_labels,␣
↪alpha, 1)
Latex(rf"for $\alpha$ = {alpha}, accuracy = {accuracy}")
```

[12]: for $\alpha = 0.8$, accuracy $= 0.9299999999999999$

[13]:
```python
alpha = 0.85
accuracy = pca_classify(training_data, test_data, training_labels, test_labels,␣
↪alpha, 1)
Latex(rf"for $\alpha$ = {alpha}, accuracy = {accuracy}")
```

[13]: for $\alpha = 0.85$, accuracy $= 0.94$

[14]:
```python
alpha = 0.9
accuracy = pca_classify(training_data, test_data, training_labels, test_labels,␣
↪alpha, 1)
Latex(rf"for $\alpha$ = {alpha}, accuracy = {accuracy}")
```

[14]: for $\alpha = 0.9$, accuracy $= 0.945$

[15]:
```python
alpha = 0.95
accuracy = pca_classify(training_data, test_data, training_labels, test_labels,␣
↪alpha, 1)
Latex(rf"for $\alpha$ = {alpha}, accuracy = {accuracy}")
```

[15]: for $\alpha = 0.95$, accuracy $= 0.935$

It is clear that the accuracy increases by increasing α ($\alpha \propto$ accuracy).
However, the accuracy seems to decreases as α goes from 0.9 to 0.95

4 Classification using LDA

4.1 Original LDA Algorithm

Running Time: $O(d^3)$ to calculate the eigen values and eigen vectors of $\Sigma_{d\times d}$ matrix

ALGORITHM 20.1. Linear Discriminant Analysis

LINEARDISCRIMINANT $(\mathbf{D} = \{(\mathbf{x}_i, y_i)\}_{i=1}^{n})$:

$\mathbf{D}_i \leftarrow \left\{\mathbf{x}_j \mid y_j = c_i, j = 1,\ldots,n\right\}, i = 1,2$ `// class-specific subsets`

$\boldsymbol{\mu}_i \leftarrow \text{mean}(\mathbf{D}_i), i = 1,2$ `// class means`

$\mathbf{B} \leftarrow (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2)(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2)^T$ `// between-class scatter matrix`

$\mathbf{Z}_i \leftarrow \mathbf{D}_i - \mathbf{1}_{n_i}\boldsymbol{\mu}_i^T, i = 1,2$ `// center class matrices`

$\mathbf{S}_i \leftarrow \mathbf{Z}_i^T\mathbf{Z}_i, i = 1,2$ `// class scatter matrices`

$\mathbf{S} \leftarrow \mathbf{S}_1 + \mathbf{S}_2$ `// within-class scatter matrix`

$\lambda_1, \mathbf{w} \leftarrow \text{eigen}(\mathbf{S}^{-1}\mathbf{B})$ `// compute dominant eigenvector`

[5]:
```
def LDA(D , y):
    n_features = len(D[0])
    classes = torch.unique(y)
    n_classes = len(classes)

    overall_mean = torch.mean(D ,dim=0)

    # between-class scatter
    Sb = torch.zeros((n_features , n_features))

    # within-class scatter
    S = torch.zeros((n_features , n_features))

    for i in classes:
        Kth_class = D[y == i]
        cur_mean = torch.mean(Kth_class , dim=0)

        # calculate between class scatter matrix
        centered_kth_mean = (cur_mean - overall_mean).unsqueeze(1)
        Sb += (Kth_class.shape[0] * Tensor.
↪matmul(centered_kth_mean,centered_kth_mean.T))

        # calculate within class scatter matrix
        centered_kth_class = Kth_class - cur_mean
        S += Tensor.matmul(centered_kth_class.T,centered_kth_class)


    #compute matrix (S^-1*B)
```

```python
    A = linalg.pinv(S) @ Sb

    #Compute the eignValues and eignVectors
    eigenvalues, eigenvectors = linalg.eig(A)
    eigenvalues, eigenvectors =eigenvalues.real, eigenvectors.real

    #Sort the eignValues and eignVectors
    idxs = torch.argsort(eigenvalues,descending=True)
    eigenvectors = eigenvectors[:,idxs]

    return eigenvectors[:,:n_classes-1]
```

[6]:
```python
def lda_classify(training_data, test_data, training_labels, test_labels, k):
    projection_matrix = LDA(training_data,training_labels)
    projected_training_matrix = training_data @ projection_matrix
    projected_test_matrix = test_data @ projection_matrix

    result_labels =␣
 ↪training_labels[knn(projected_test_matrix,projected_training_matrix, k)].
 ↪mode(keepdim=True)[0]
    return 1 - torch.count_nonzero(result_labels - test_labels[:, None]) /␣
 ↪len(result_labels)
```

[7]:
```python
k = 1

accuracy = lda_classify(training_data, test_data, training_labels, test_labels,␣
 ↪k)
print(f"accuracy = {accuracy}")
```

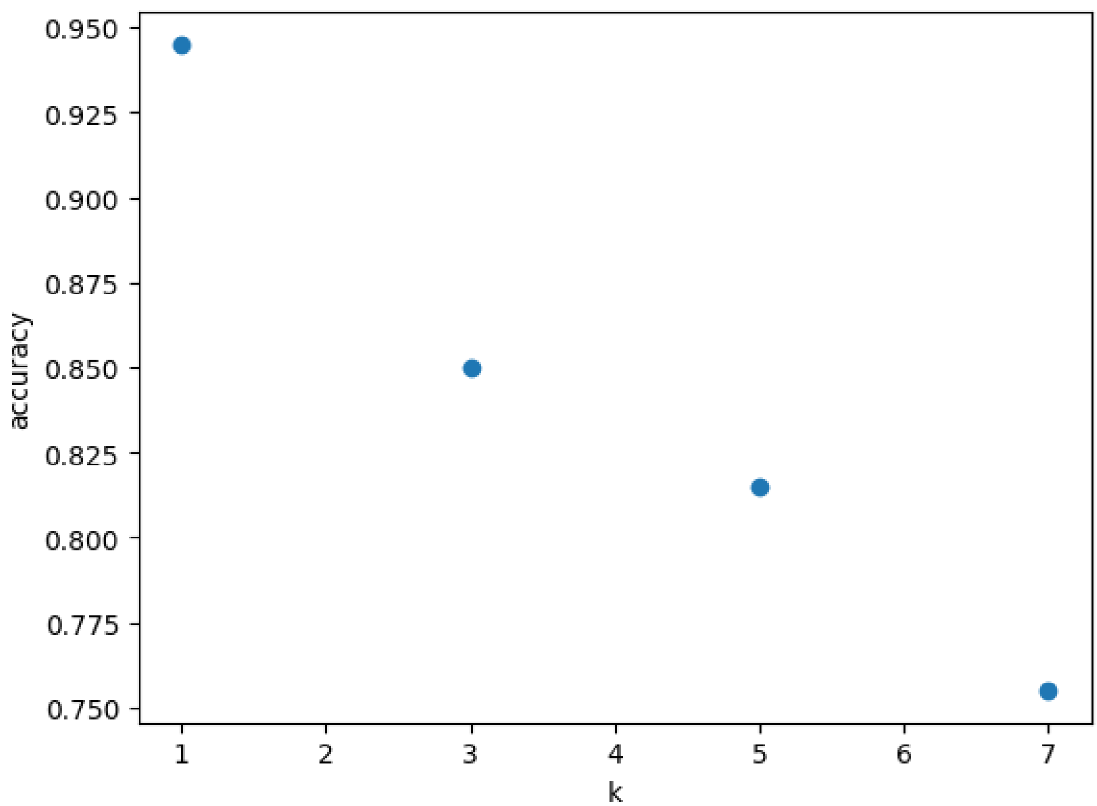
```
accuracy = 0.96
```

5 Classifier Tuning

5.1 PCA using $\alpha = 0.9$

[16]:
```
ks = [1, 3, 5, 7]
alpha = 0.9

accuracies = [pca_classify(training_data, test_data, training_labels,␣
 ↪test_labels, alpha, k) for k in ks]
plt.xlabel("k")
plt.ylabel("accuracy")
plt.scatter(ks, accuracies)
```

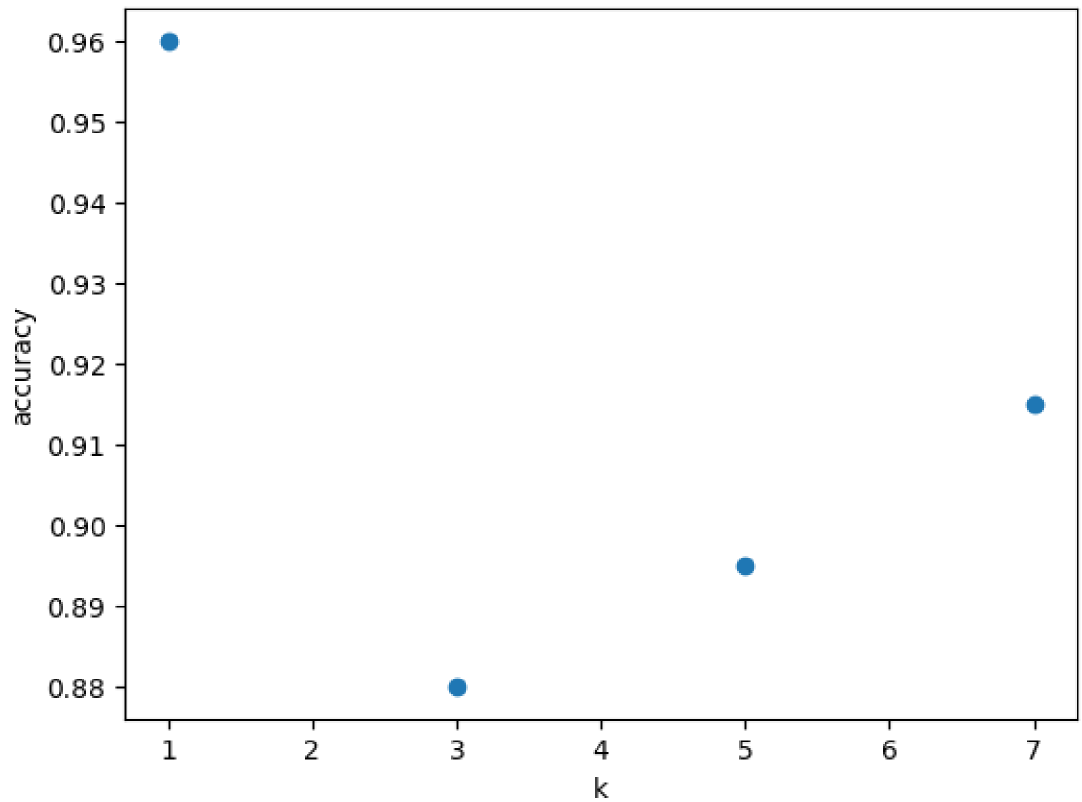
[16]: <matplotlib.collections.PathCollection at 0x1a51a12d110>



Using KNN classification with k = 1 gives the most accurate results

5.2 LDA

[ ]:
```
ks = [1, 3, 5, 7]

accuracies = [lda_classify(training_data, test_data, training_labels,␣
 ↪test_labels, k) for k in ks]
plt.xlabel("k")
plt.ylabel("accuracy")
plt.scatter(ks, accuracies)
```



Using KNN classification with k = 1 gives the most accurate results

6 Compare vs Non-Face Images

Read non-face data (iris dataset)

[11]:
```
iris_path_1 = "../data/iris/iris-setosa"
iris_path_2 = "../data/iris/iris-versicolour"
iris_path_3 = "../data/iris/iris-virginica"

iris_images = [Image.open(f"{iris_path_1}/{filename}") for filename in os.
 ↪listdir(iris_path_1)]
iris_images.extend([Image.open(f"{iris_path_2}/{filename}") for filename in os.
 ↪listdir(iris_path_2)])
iris_images.extend([Image.open(f"{iris_path_3}/{filename}") for filename in os.
 ↪listdir(iris_path_3)])
iris_images = [ImageOps.grayscale(iris_images[i].resize((92, 112))) for i in␣
 ↪range(len(iris_images))]

iris_data = np.array(iris_images)
iris_data.resize((iris_data.shape[0], iris_data.shape[1] * iris_data.shape[2]))
iris_data = Tensor(iris_data)
```

Split dataset and combine with face dataset

[12]:
```
iris_training_indices = [i for i in range(len(iris_data)) if i % 2 == 0]
iris_test_indices = [i for i in range(len(iris_data)) if i % 2 == 1]

iris_training_data = iris_data[iris_training_indices]
iris_test_data = iris_data[iris_test_indices]

iris_training_labels = Tensor([-1 for _ in range(len(iris_training_indices))])
iris_test_labels = Tensor([-1 for _ in range(len(iris_test_indices))])

face_non_face_training_data = torch.vstack((training_data, iris_training_data))
face_non_face_training_labels = torch.hstack((training_labels,␣
 ↪iris_training_labels))
```

6.1 Classify using PCA

[185]:
```python
alpha = 0.95
k = 1

non_face_accuracy = pca_classify(face_non_face_training_data, iris_test_data,␣
↪face_non_face_training_labels, iris_test_labels, alpha, k)
face_accuracy = pca_classify(face_non_face_training_data, test_data,␣
↪face_non_face_training_labels, test_labels, alpha, k)
print(f"accuracy at detecting non-faces = {non_face_accuracy}")
print(f"accuracy at detecting faces = {face_accuracy}")
```

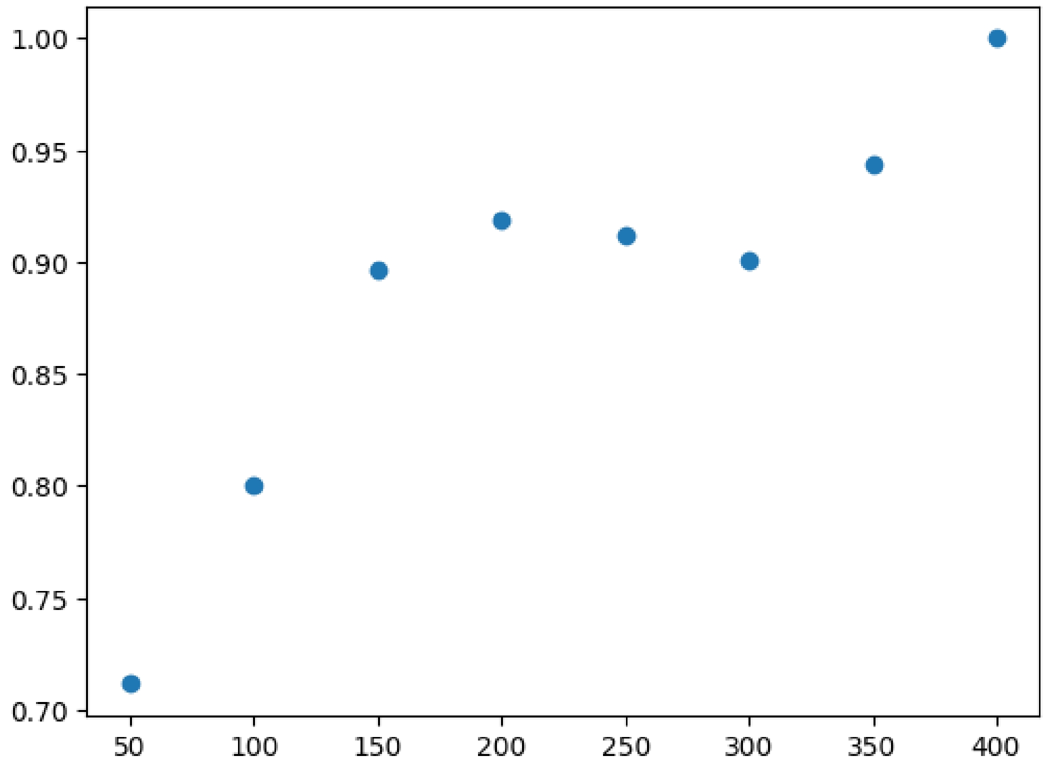
```
accuracy at detecting non-faces = 0.9380952380952381
accuracy at detecting faces = 0.935
```

6.2 Using different number of non-faces

[48]:
```python
alpha = 0.95
k = 1

number_of_non_faces = [50, 100, 150, 200, 250, 300, 350, 400]

non_face_accuracies = []
face_accuracies = []

for no in number_of_non_faces:
    iris_training_indices = [i for i in range(len(iris_data)) if i % 2 == 0]
    iris_test_indices = [i for i in range(len(iris_data)) if i % 2 == 1]

    iris_training_data = iris_data[:no]
    iris_test_data = iris_data[no:]

    iris_training_labels = Tensor([-1 for _ in range(len(iris_training_data))])
    iris_test_labels = Tensor([-1 for _ in range(len(iris_test_data))])

    face_non_face_training_data = torch.vstack((training_data,␣
↪iris_training_data))
    face_non_face_training_labels = torch.hstack((training_labels,␣
↪iris_training_labels))

    non_face_accuracies.append(pca_classify(face_non_face_training_data,␣
↪iris_test_data, face_non_face_training_labels, iris_test_labels, alpha, k))
    face_accuracies.append(pca_classify(face_non_face_training_data, test_data,␣
↪face_non_face_training_labels, test_labels, alpha, k))
```

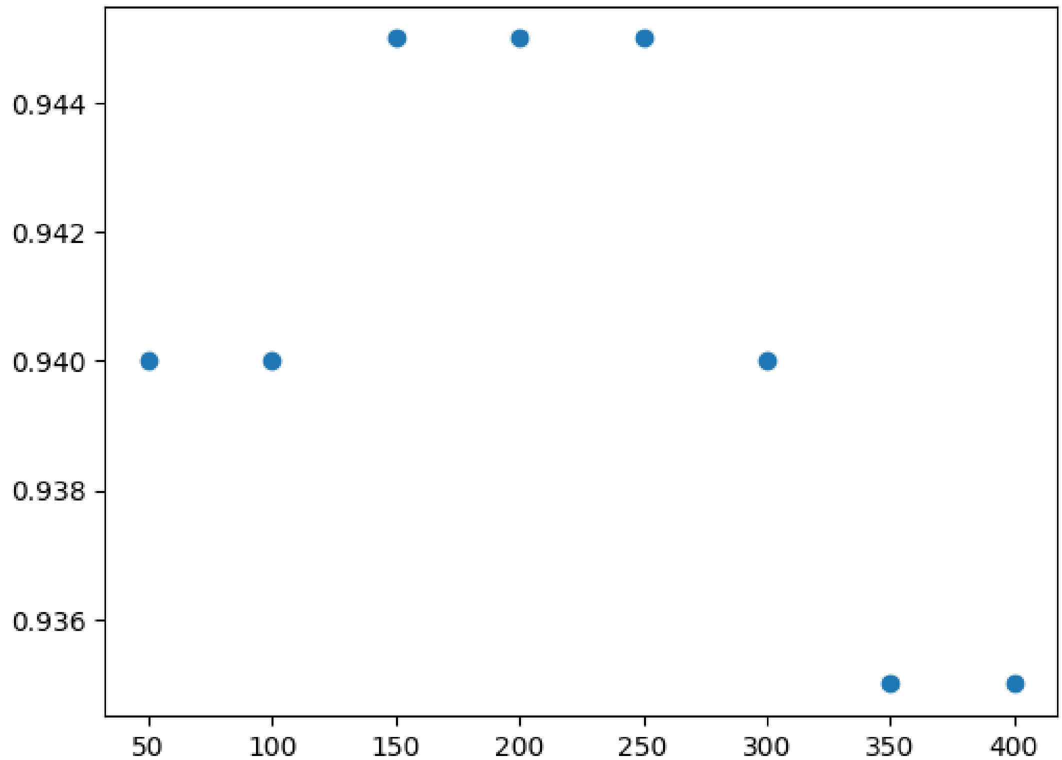
[49]:
```
plt.scatter(number_of_non_faces, non_face_accuracies)
```

[49]: <matplotlib.collections.PathCollection at 0x2d28f5f1e10>



[50]:
```
plt.scatter(number_of_non_faces, face_accuracies)
```

[50]: <matplotlib.collections.PathCollection at 0x2d28f5ea190>



6.3 Classify using LDA

Using 40 eigen vectors

[13]:
```
k = 1

non_face_accuracy = lda_classify(face_non_face_training_data, iris_test_data,␣
 ↪face_non_face_training_labels, iris_test_labels, k)
face_accuracy = lda_classify(face_non_face_training_data, test_data,␣
 ↪face_non_face_training_labels, test_labels, k)
print(f"accuracy at detecting non-faces = {non_face_accuracy}")
print(f"accuracy at detecting faces = {face_accuracy}")
```

```
accuracy at detecting non-faces = 0.9904761904761905
accuracy at detecting faces = 0.94
```

7 Bonus

7.1 PCA with 7 training images and 3 test images

Splitting dataset into training and test data

[15]:
```
training_indices = [i for i in range(len(all_data)) if i % 10 == 0 or i % 10 ==␣
↪1 or i % 10 == 3 or i % 10 == 4 or i % 10 == 6 or i % 10 == 7 or i % 10 == 9]
test_indices = [i for i in range(len(all_data)) if i % 10 == 2 or i % 10 == 5 or␣
↪i % 10 == 8]

training_data = all_data[training_indices]
test_data = all_data[test_indices]

training_labels = labels[training_indices]
test_labels = labels[test_indices]
```

[18]:
```
alpha = 0.8
accuracy = pca_classify(training_data, test_data, test_labels, alpha, 1)
Latex(rf"for $\alpha$ = {alpha}, accuracy = {accuracy}")
```

[18]: for $\alpha = 0.8$, accuracy $= 0.9833333333333333$

[19]:
```
alpha = 0.85
accuracy = pca_classify(training_data, test_data, test_labels, alpha, 1)
Latex(rf"for $\alpha$ = {alpha}, accuracy = {accuracy}")
```

[19]: for $\alpha = 0.85$, accuracy $= 0.9833333333333333$

[20]:
```
alpha = 0.9
accuracy = pca_classify(training_data, test_data, test_labels, alpha, 1)
Latex(rf"for $\alpha$ = {alpha}, accuracy = {accuracy}")
```

[20]: for $\alpha = 0.9$, accuracy $= 0.9833333333333333$

[21]:
```
alpha = 0.95
accuracy = pca_classify(training_data, test_data, test_labels, alpha, 1)
Latex(rf"for $\alpha$ = {alpha}, accuracy = {accuracy}")
```

[21]: for $\alpha = 0.95$, accuracy $= 0.9833333333333333$

7.2 LDA with 7 training images and 3 test images

[16]:
```
accuracy = lda_classify(training_data, test_data, training_labels, test_labels,␣
↪1)
print("accuracy = ",accuracy)
```

```
accuracy =  tensor(0.9833)
```

8 Bonus

8.1 Linear Regularized Discriminant Analysis(RDA)

[17]:
```python
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis
from sklearn.metrics import accuracy_score

# Create and fit the regularized discriminant analysis model
rda = LinearDiscriminantAnalysis(solver='eigen',shrinkage=0.5)
rda.fit(training_data, training_labels)

# Make predictions on the test set
y_pred = rda.predict(test_data)

# Calculate accuracy
accuracy = accuracy_score(test_labels, y_pred)
print("Accuracy:", accuracy)
```

```
Accuracy: 0.9833333333333333
```