CS2008: Numerical Computing All sections

Homework

Polynomial Fitting using Least Squares Approximation

Objective:

Fit a line (degree 1 polynomial), a degree 2 polynomial, and a degree 3 polynomial to three distinct sets of data points using the least squares method.

Data Sets:

1. Linear Data Points (Type 1):
   - (1, 2)
   - (2, 3)
   - (3, 4)
   - (4, 5)
   - (5, 6)

2. Quadratic Data Points (Type 2):
   - (1, 1)
   - (2, 4)
   - (3, 9)
   - (4, 16)
   - (5, 25)

3. Cubic Data Points (Type 3):
   - (1, 1)
   - (2, 8)
   - (3, 27)
   - (4, 64)
   - (5, 125)

Instructions and Mathematical Task:

1. Consider the Type 1 data and fit a Line (Degree 1):

   - Model: $P(x) = a_0 + a_1x$
   - Least Squares: Minimize $E = \sum (y_i - P(x_i))^2$.
   - System of Equations: This results in a 2 × 2 system:

   $$\begin{bmatrix} n+1 & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \end{bmatrix}$$

   - Output: Present the fitted line equation.

2. Consider the Type 2 data and fit a Polynomial of Degree 2:

   - Model: $P(x) = a_0 + a_1x + a_2x^2$
   - Least Squares: Minimize $E = \sum (y_i - P(x_i))^2$.
   - System of Equations: This results in a 3 × 3 system:

   $$\begin{bmatrix} n+1 & \sum x_i & \sum x_i^2 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \end{bmatrix}$$

   - Output: Present the fitted polynomial equation.

3. Consider the Type 3 data and fit a Polynomial of Degree 3:

   - Model: $P(x) = a_0 + a_1x + a_2x^2 + a_3x^3$
   - Least Squares: Minimize $E = \sum (y_i - P(x_i))^2$.

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- System of Equations: This results in a 4 × 4 system:

$$\begin{bmatrix} n+1 & \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 & \sum x_i^5 \\ \sum x_i^3 & \sum x_i^4 & \sum x_i^5 & \sum x_i^6 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \\ \sum x_i^3 y_i \end{bmatrix}$$

- Output: Present the fitted polynomial equation.

Discussion:

Briefly discuss how the least squares method works and the importance of choosing an appropriate degree for polynomial fitting.

Submission Requirements:

- Submit your calculations (classwork) and the fitted polynomial equations for all three data types.
- Provide a short explanation (100-150 words) on the least squares approximation method.