National University of Computer and Emerging Sciences

School of Computing | Spring 2023 | Islamabad Campus

Serial No:

CS-1008 Numerical Computing

BS(CS)

Tuesday, February 27, 2023

Course Instructors

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Sessional I

Total Time: 1 Hour

Total Marks: 50

Signature of Invigilator

Student Name | Roll No | Section | Signature

DO NOT OPEN THE QUESTION BOOK OR START UNTIL INSTRUCTED.

Instructions:

1. Attempt on question paper. Attempt all of them. Read the question carefully, understand the question, and then attempt it.
2. No additional sheet will be provided for rough work. Use the back of the last page for rough work.
3. If you need more space write on the back side of the paper and clearly mark question and part number etc.
4. After asked to commence the exam, please verify that you have 5 pages different printed pages including this title page. There are a total of 4 questions.
5. Calculator sharing is strictly prohibited.
6. Use permanent ink pens only. Any part done using soft pencil will not be marked and cannot be claimed for rechecking.
7. Fit in all your answers in the provided space. You may use extra space on the last page if required. If you do so, clearly mark question/part number on that page to avoid confusion.

	Q-1	Q-2	Q-3	Q-4	Total
Marks Obtained					
Total Marks	15	15	10	10	50

Question # 1[15 marks]

(a) State few limitations/disadvantages of Taylor's polynomial. (2)

→ Taylor's polynomial give good approximation near centre.

→ Taking Large 'n' → degree, we are not sure whether it will give more accurate or even more distant approximations

(b) Density of air ρ varies with elevation h in the following manner: (10+3)

$h(km)$	0	3	6
$\rho(kg/m^3)$	1.225	0.905	0.652

(i) Express $\rho(h)$ as a quadratic function using Lagrange's method.

(ii) Use Lagrange polynomial to approximate $\rho(5)$

$$\rho(h) = \frac{(h-h_1)(h-h_2)}{(h_0-h_1)(h_0-h_2)}\rho(h_0) + \frac{(h-h_0)(h-h_2)}{(h_1-h_0)(h_1-h_2)}\rho(h_1)$$

$$+ \frac{(h-h_0)(h-h_1)}{(h_2-h_0)(h_2-h_1)}\rho(h_2)$$

$$= \frac{(h-3)(h-6)}{(0-3)(0-6)}1.225 + \frac{(h-0)(h-6)}{(3-0)(3-6)}0.905$$

$$+ \frac{(h-0)(h-3)}{(6-0)(6-3)}0.652$$

$$= 0.0037h^2 - 0.1178h + 1.225$$

$$\rho(5) = 0.7889 \ kg/m^3$$

Question # 2[15 marks]

A car traveling along a straight road is clocked at a number of points. The data from the observations are given in the following table, where the time is in seconds, the distance is in feet, and the speed is in feet per second.

$Time = t_i$	0	3	5	8
$Distance = f(t_i)$	0	225	383	623
$Speed = f'(t_i)$	75	77	80	74

Use a Hermite polynomial to predict the position of the car and its speed when $t = 7s$.

Construct Hermite by using divided difference

z_i	$f(z_i)$	F[,]	F[, ,]	F[, , ,]	F[, , , ,]			
0	0							
		75						
0	0		0					
		75		2/6				
3	225		2/3		-1/30			
		77		1/6		-1/100		
3	225		1		-1/12		9/3200	
		79		-1/4		1/80		-7/7680
5	383		1/2		1/60		43/9600	
		80		-1/6		-7/300		
5	383		0		-1/10			
		80		-2/3				
8	623		-2					
		74						
8	623							

$\boxed{H(7) = 544.0229}$

$$H_7(t) = f(z_0) + (t-z_0) f[z_0, z_1] + (t-z_0)^2 f[z_0, z_1, z_2] + (t-z_0)^2 (t-z_2) F[z_0, z_1, z_2, \ldots]$$

$$+ (t-z_0)^2 (t-z_2)^2 F[z_0, z_1, z_2, z_3] + (t-z_0)^2 (t-z_1)^2 (t-z_4) F[z_0, z_1, z_2, \ldots]$$

$$+ (t-z_0)^2 (t-z_2)^2 (t-z_4)^2 F[z_0, z_1, z_2, z_3, z_4, z_5]$$

$$+ (t-z_0)^2 (t-z_2)^2 (t-z_4)^2 (t-z_6) F(z_0 \ldots z_6)$$

$$= 0 + 75t + 1.4965t^2 - 2.3446t^3 + 1.1585t^4 - 2.2454 \ldots$$

$$+ 0.0244t^6 - 0.000911t^7$$

Question # 3[10 marks]

(a) Determine the missing entries in the following table (5)

x	$f[]$	$f[.]$	$f[..]$
$x_0 = 0.0$	$f[x_0] = -1.70464$		
		$f[x_0, x_1] = 9.2616$	
$x_1 = 0.4$	$f[x_1] = 2$		$f[x_0, x_1, x_2] = 0.923$
		$f[x_1, x_2] = 10$	
$x_2 = 0.8$	$f[x_2] = 6$		

$$F[x_0, x_1, x_2] = \frac{F[x_1, x_2] - F[x_0, x_1]}{x_2 - x_0}$$

$$0.923 = \frac{10 - ?}{0.8 - 0} \Rightarrow ? = 9.2616$$

$$F[x_1, x_2] = \frac{F[x_2] - F[x_1]}{x_2 - x_1} \Rightarrow 10 = \frac{6 - ?}{0.4} \Rightarrow ? = 2$$

$$F[x_0, x_1] = \frac{F[x_1] - F[x_0]}{x_1 - x_0} = \frac{2 - ?}{0.4}$$

(b) Use the above table to approximate $f(0.7)$. (5)

$$x_s = x_n + sh.$$

$$\Rightarrow 0.7 = 0.8 + sh.$$

$$\Rightarrow s = \frac{0.7 - 0.8}{h} = \frac{-0.1}{0.4} = -0.25$$

$$f(x_s) = f(x_n) + s\nabla f(x_n) + \frac{s(s+1)}{2!}\nabla^2 f(x_n)$$

$$= 6 + (-0.25)10 + \frac{(-0.25)(-0.25+1)}{2!}0.923$$

$$= 6 - 2.5 - 0.0865$$

$$= 3.4135.$$

Question # 4[10 marks]

Obtain the quadratic spline for the function $f(x)$ defined by the following data

x_i	2	5	7
$f(x_i)$	1	8	3

$2 \le x \le 5$

$S_0 = a_0 + b_0(x - x_0) + C_0(x - x_0)^2 = -3.67 + 2.33x$

$= 1 + \frac{7}{3}(x-2) + 0(x-2)^2$

$5 \le x \le 7$

$S_1 = a_1 + b_1(x - x_1) + C_1(x - x_1)^2 = -2.42x^2 + 26.5x - 64.08$

$= 8 + \frac{7}{3}(x-5) + -(2.42)(x-5)^2$

Step I Left end cdns

$S_0(x_0) = f_0 \Rightarrow a_0 = f_0 \Rightarrow \boxed{a_0 = 1}$

$S_1(x_1) = f_1 \Rightarrow a_1 = f_1 \Rightarrow \boxed{a_1 = 8}$

Step II Right end cdns

$S_0(x_1) = f_1 \Rightarrow a_0 + b_0(x_1 - x_0) + C_0(x_1 - x_0)^2 = 8$

$\Rightarrow 1 + b_0(3) + C_0(3)^2 = 8$

$\Rightarrow 3b_0 + 9C_0 = 7 \Rightarrow \boxed{b_0 = \frac{7}{3}}$

$S_1(x_2) = f_2 \Rightarrow a_1 + b_1(x_2 - x_1) + C_1(x_2 - x_1)^2 = 3$

$2b_1 + 4C_1 = -5 \Rightarrow 4C_1 = -5 - \frac{14}{3}$

$C_1 = \frac{-29}{12}$

$\boxed{C_1 = -2.42}$

Step III Continuity of 1st derivative

$S_0'(x_1) = S_1'(x_1)$

$b_0 + 2C_0(x_1 - x_0) = b_1 + 2C_1(x_1 - x_1)$

$\Rightarrow b_0 + 6C_0 = b_1$

Step IV

$S_0''(x_0) = 0 \Rightarrow \boxed{C_0 = 0}$

$\boxed{b_0 = b_1} \Rightarrow \boxed{b_1 = \frac{7}{3}}$