

# Greibach Normal Form

≡ Tag	CFL
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A context-free grammar (CFG) is in Greibach Normal Form (GNF) if all production rules satisfy one of the following conditions:

- A non-terminal generating a terminal (e.g.;  $X \rightarrow x$ )
- A non-terminal generates a terminal followed by any number of non-terminals (e.g.;  $X \rightarrow xX_1X_2...X_3$ )

## Steps:

**Step 1.** If the given grammar is not in CNF, convert it to CNF.

**Step 2.** Change the names of non terminal symbols to  $A_1$  till  $A_N$  in same sequence.

**Step 3.** Check for every production rule if RHS has first symbol as non terminal say  $A_j$  for the production of  $A_i$ , it is mandatory that  $i$  should be less than  $j$ . Not great and not even equal.

If  $i > j$  then replace the production rule of  $A_j$  at its place in  $A_i$ .

If  $i = j$ , it is the left recursion. Create a new state  $Z$  which has the symbols of the left recursive production, once followed by  $Z$  and once without  $Z$ , and change that production rule by removing that particular production and adding all other production once followed by  $Z$ .

**Step 4.** Replace very first non terminal symbol in any production rule with its production until production rule satisfies the above conditions.

For converting a CNF to GNF always move left to right for renaming the variables.

$$S \rightarrow CA \mid BB$$

$$B \rightarrow b \mid SB$$

$$C \rightarrow b$$

$$A \rightarrow a$$

Change the names of the non-terminal symbols into some  $A_i$  in ascending order of  $i$ .

$S$  with  $A_1$

$C$  with  $A_2$

$A$  with  $A_3$

$B$  with  $A_4$

$$A_1 \rightarrow A_2 A_3 \mid A_4 A_4$$

$$A_4 \rightarrow b \mid A_1 A_4$$

$$A_2 \rightarrow b$$

$$A_3 \rightarrow a$$

Alter the rules so that the non-terminals are in ascending order, such that, if the production is of the form  $A_i \rightarrow A_j x$ , then,  $i < j$  and should never be  $\geq j$ .

$j$  is the subscript of the first non-terminal on the right side e.g. in  $A_1 \rightarrow A_2 A_3 \mid A_4 A_4$ ,  $j$  is  $(A_2)A_3$  and  $(A_4)A_4$

$$A_1 < A_2 \checkmark$$

$$A_1 < A_4 \checkmark$$

For 2nd production,

$$A_4 \rightarrow b A_1 A_4$$

$$A_4 > A_1 \times \text{ not in GNF}$$

Replace  $A_1$  with some other non-terminal.  
value. ( $A_1$ )

$$A_4 \rightarrow b A_2 A_3 A_4 \mid A_4 A_4 A_4$$

$$\text{Still } A_4 > A_2$$

$$A_4 \rightarrow b A_3 A_4 \mid A_4 A_4 A_4$$

Still  $A_4 > A_3$  but there is a non-terminal followed by non-terminals so it's also in GNF.

$$A_4 = A_4 \quad (i=j)$$

When  $i=j$ , remove left recursion.  
introduce a new variable.

$$A_4 \rightarrow b \mid b A_3 A_4 \mid A_4 A_4 A_4$$

$$Z \rightarrow A_4 A_4 Z \mid A_4 A_4$$

$$\rightarrow A_4 \rightarrow b Z \mid b A_3 A_4 Z \mid b A_3 A_4 \mid b A_3 A_4$$

updated grammar:-

$$A_1 \rightarrow A_2 A_3 \mid A_4 A_4 \quad \times$$

$$A_4 \rightarrow b \mid b A_3 A_4 \mid b Z \mid b A_3 A_4 Z \checkmark$$

$$Z \rightarrow A_4 A_4 \mid A_4 A_4 Z \quad \times$$

$$A_2 \rightarrow b \quad \checkmark$$

$$A_3 \rightarrow a \quad \checkmark$$

→ right side must not start with a non-terminal.

$$A_1 \rightarrow b A_3 \mid b A_4 \mid b A_3 A_4 A_4 \mid b Z A_4 \mid b A_3 A_4 Z A_4$$

$$\rightarrow Z \rightarrow b A_4 \mid b A_3 A_4 A_4 \mid b Z A_4 \mid b A_3 A_4 Z A_4 \mid$$

$$b A_4 Z \mid b A_3 A_4 A_4 Z \mid b Z A_4 Z \mid b A_3 A_4 Z A_4 Z$$

Final Grammar:-

$$A_1 \rightarrow bA_3 \mid bA_4 \mid bA_3A_4A_4 \mid bZA_4 \mid bA_3A_4ZA_4$$

$$A_4 \rightarrow b \mid bA_3A_4 \mid bZ \mid bA_3A_4Z$$

$$Z \rightarrow bA_4 \mid bA_3A_4A_4 \mid bZA_4 \mid bA_3A_4ZA_4 \mid bA_4Z \mid bA_3A_4A_4Z \mid bZA_4Z \mid bA_3A_4ZA_4Z$$

$$A_2 \rightarrow b$$

$$A_3 \rightarrow a$$

## ▼ Past Paper Qs

Convert the following grammar to its equivalent Greibach Normal Form.

$$S \rightarrow aAb \mid a$$

$$A \rightarrow SS \mid b$$

Convert the following grammar to its equivalent Greibach Normal Form.

$$S \rightarrow ab \mid a$$

$$A \rightarrow SS \mid b$$

**GNF:**

Convert to GNF. You must convert to intermediary grammar to get full marks.

$$S \rightarrow AB \mid \Delta$$

$$A \rightarrow AB \mid BB \mid a$$

$$B \rightarrow AB \mid a$$

Convert to GNF. You must convert to intermediary grammar to get full marks.

$$S \rightarrow AB$$
$$A \rightarrow AB \mid a$$
$$B \rightarrow AB \mid a$$

Convert the following grammar to GNF.

$$S \rightarrow Xa \mid Yb \mid YYYb$$
$$W \rightarrow ZZ \mid SZZ \mid W \mid \Delta$$
$$X \rightarrow Sb \mid b \mid SZZ$$
$$Y \rightarrow Sa \mid a \mid Z \mid \Delta$$
$$Z \rightarrow ZZ \mid SZZ$$