National University
Of Computer and Emerging Sciences

Assignment-02

In partial
fulfillment of the requirements
for the course of

FA2024-CS3005

Theory of Automata

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BSCS-C

Question: 01

(a)

	0	1
+A	D	B
B	E	C
C	F	A
D	A	E
E	B	F
F	C	D

Label

$$P=\begin{bmatrix} B & C & D & E & F \\ EC & FA & AE & BF & CD \end{bmatrix} \quad Q=\begin{bmatrix} A \\ DB \end{bmatrix}$$

Group

$$\begin{bmatrix} B & C & D & E & F \\ PP & PQ & QP & PP & PP \end{bmatrix} \begin{bmatrix} A \\ PP \end{bmatrix}$$

Partition

$$\begin{bmatrix} B & E & F \\ PP & PP & PP \end{bmatrix} \begin{bmatrix} C \\ PQ \end{bmatrix} \begin{bmatrix} D \\ QP \end{bmatrix} \begin{bmatrix} A \\ PP \end{bmatrix}$$

Label

$$R=\begin{bmatrix} B & E & F \\ EC & BF & CD \end{bmatrix} \quad S=\begin{bmatrix} C \\ FA \end{bmatrix} \quad T=\begin{bmatrix} D \\ AE \end{bmatrix} \quad U=\begin{bmatrix} A \\ DB \end{bmatrix}$$

Group

$$\begin{bmatrix} B & E & F \\ RS & RR & ST \end{bmatrix} \begin{bmatrix} C \\ RU \end{bmatrix} \begin{bmatrix} D \\ UR \end{bmatrix} \begin{bmatrix} A \\ TR \end{bmatrix}$$

Partition

$$\begin{bmatrix} B \\ RS \end{bmatrix} \begin{bmatrix} E \\ RR \end{bmatrix} \begin{bmatrix} F \\ ST \end{bmatrix} \begin{bmatrix} C \\ RU \end{bmatrix} \begin{bmatrix} D \\ UR \end{bmatrix} \begin{bmatrix} A \\ TR \end{bmatrix}$$

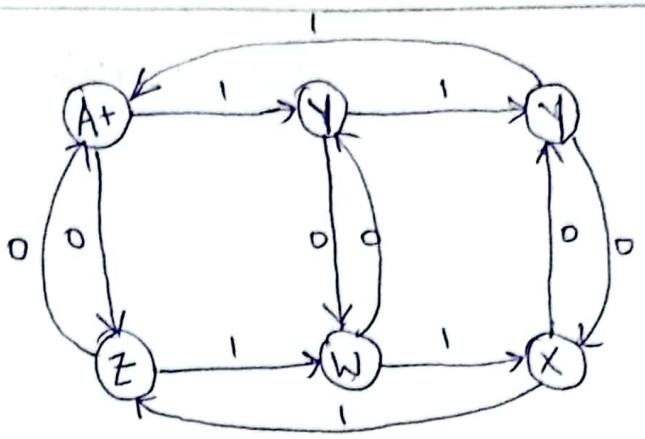
Label

$$V=\begin{bmatrix} B \\ EC \end{bmatrix} \quad W=\begin{bmatrix} E \\ BF \end{bmatrix} \quad X=\begin{bmatrix} F \\ CD \end{bmatrix} \quad Y=\begin{bmatrix} C \\ FA \end{bmatrix} \quad Z=\begin{bmatrix} D \\ AE \end{bmatrix} \quad A=\begin{bmatrix} A \\ DB \end{bmatrix}$$

Group

$$\begin{bmatrix} B \\ WY \end{bmatrix} \begin{bmatrix} E \\ VX \end{bmatrix} \begin{bmatrix} F \\ YZ \end{bmatrix} \begin{bmatrix} C \\ XA \end{bmatrix} \begin{bmatrix} D \\ AW \end{bmatrix} \begin{bmatrix} A \\ ZV \end{bmatrix}$$

	0	1
V	W	Y
W	V	X
X	Y	Z
Y	X	A
Z	A	W
+A	Z	Y



which is exactly the same as the original DFA. So it can't be reduced.

(b)

	0	1
q_0	q_1	q_3
q_1	q_2	q_4
q_2	q_1	q_4
q_3	q_2	q_4
+q_4	q_4	q_4

Label

$$P=\begin{bmatrix} q_0 & q_1 & q_2 & q_3 \\ q_1q_3 & q_2q_4 & q_1q_4 & q_2q_4 \end{bmatrix} \quad Q=\begin{bmatrix} q_4 \\ q_4q_4 \end{bmatrix}$$

Group

$$\begin{bmatrix} q_0 & q_1 & q_2 & q_3 \\ PP & PQ & PQ & PQ \end{bmatrix} \begin{bmatrix} q_4 \\ QQ \end{bmatrix}$$

Partition

$$\begin{bmatrix} q_0 \\ PP \end{bmatrix} \begin{bmatrix} q_1 & q_2 & q_3 \\ PQ & PQ & PQ \end{bmatrix} \begin{bmatrix} q_4 \\ QQ \end{bmatrix}$$

Label

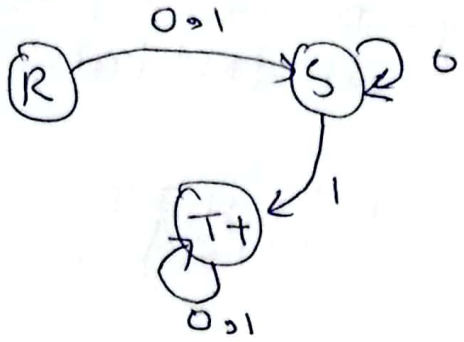
$$R=\begin{bmatrix} q_0 \\ q_1q_3 \end{bmatrix} \quad S=\begin{bmatrix} q_1 & q_2 & q_3 \\ q_2q_4 & q_1q_4 & q_2q_4 \end{bmatrix} \quad T=\begin{bmatrix} q_4 \\ q_4q_4 \end{bmatrix}$$

Group

$$\begin{bmatrix} q_0 \\ SS \end{bmatrix} \begin{bmatrix} q_1 & q_2 & q_3 \\ ST & ST & ST \end{bmatrix} \begin{bmatrix} q_4 \\ TT \end{bmatrix}$$

cannot be partitioned further

	0	1
R	S	S
S	S	T
+T	T	T



(c)

	a	b
A	B	F
B	A	F
C	G	A
D	H	B
+E	A	G
+F	H	C
+G	A	D
H	A	C

Label

$$J=\begin{bmatrix} A & B & C & D & H \\ BF & AF & GA & HB & AC \end{bmatrix} \quad K=\begin{bmatrix} E & F & G \\ AG & HC & AD \end{bmatrix}$$

E: unreachable

Group

$$\begin{bmatrix} A & B & C & D & H \\ JK & JK & KJ & JJ & JJ \end{bmatrix} \quad \begin{bmatrix} F & G \\ JJ & JJ \end{bmatrix}$$

Partition

$$\begin{bmatrix} A & B \\ JK & JK \end{bmatrix} \begin{bmatrix} C \\ KJ \end{bmatrix} \begin{bmatrix} D & H \\ JJ & JJ \end{bmatrix} \begin{bmatrix} F & G \\ JJ & JJ \end{bmatrix}$$

Label

$$L=\begin{bmatrix} A & B \\ BF & AF \end{bmatrix} \quad M=\begin{bmatrix} C \\ GA \end{bmatrix} \quad N=\begin{bmatrix} D & H \\ HB & AC \end{bmatrix} \quad P=\begin{bmatrix} F & G \\ HC & AD \end{bmatrix}$$

Group

$$\begin{bmatrix} A \\ LP \end{bmatrix}\begin{bmatrix} B \\ LP \end{bmatrix}\begin{bmatrix} C \\ PL \end{bmatrix}\begin{bmatrix} D \\ NL \end{bmatrix}\begin{bmatrix} H \\ LM \end{bmatrix}\begin{bmatrix} F \\ NM \end{bmatrix}\begin{bmatrix} G \\ LN \end{bmatrix}$$

Partition

$$\begin{bmatrix} A \\ LP \end{bmatrix}\begin{bmatrix} B \\ LP \end{bmatrix}\begin{bmatrix} C \\ PL \end{bmatrix}\begin{bmatrix} D \\ NL \end{bmatrix}\begin{bmatrix} H \\ LM \end{bmatrix}\begin{bmatrix} F \\ NM \end{bmatrix}\begin{bmatrix} G \\ LN \end{bmatrix}$$

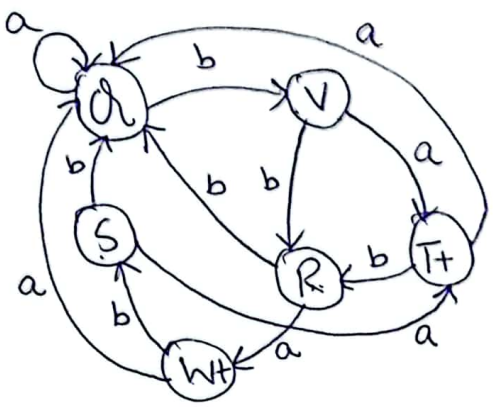
Label

$$Q=\begin{bmatrix} A \\ BF \end{bmatrix}\begin{bmatrix} B \\ AF \end{bmatrix}\ R=\begin{bmatrix} C \\ GA \end{bmatrix}\ S=\begin{bmatrix} D \\ HB \end{bmatrix}\ T=\begin{bmatrix} H \\ AC \end{bmatrix}\ V=\begin{bmatrix} F \\ HC \end{bmatrix}\ W=\begin{bmatrix} G \\ AD \end{bmatrix}$$

Group

$$\begin{bmatrix} A \\ QV \end{bmatrix}\begin{bmatrix} B \\ QV \end{bmatrix}\begin{bmatrix} C \\ WQ \end{bmatrix}\begin{bmatrix} D \\ TQ \end{bmatrix}\begin{bmatrix} H \\ QR \end{bmatrix}\begin{bmatrix} F \\ TR \end{bmatrix}\begin{bmatrix} G \\ QS \end{bmatrix}$$

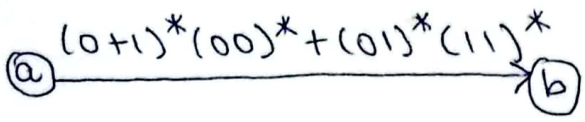
	a	b
Q	Q	V
R	W	Q
S	T	Q
T	Q	R
+V	T	R
+W	Q	S

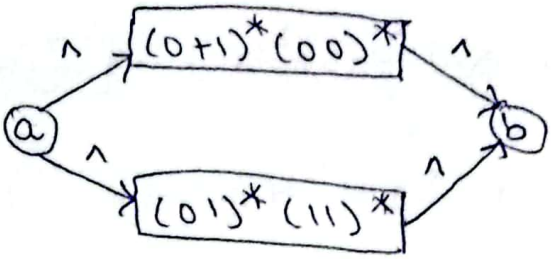


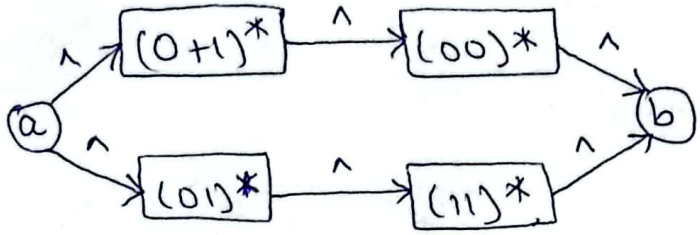
Question:02

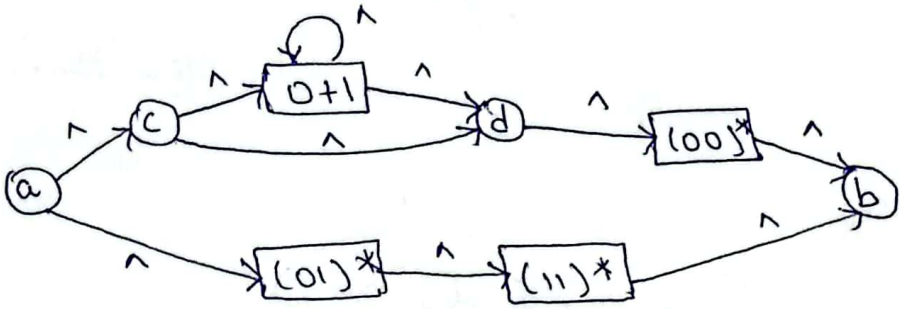
(a) Convert RE to Null NFA

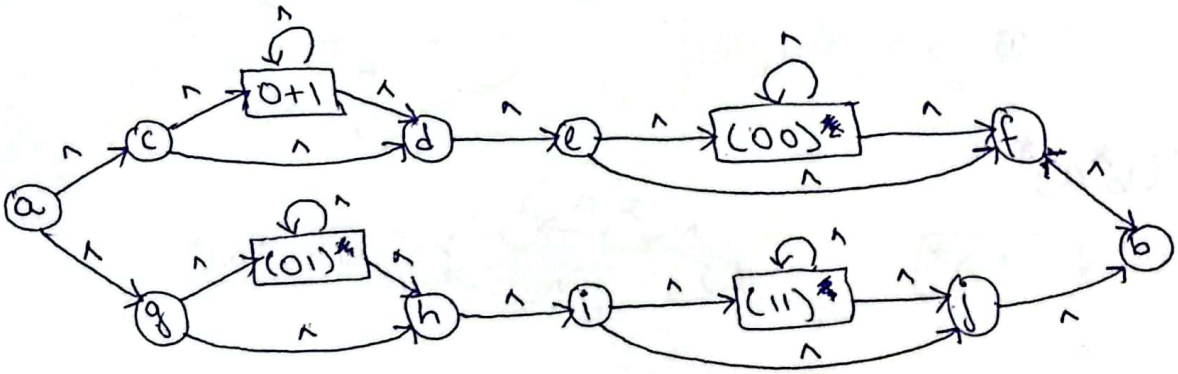
1. $(0+1)^*(00)^* + (01)^*(11)^*$

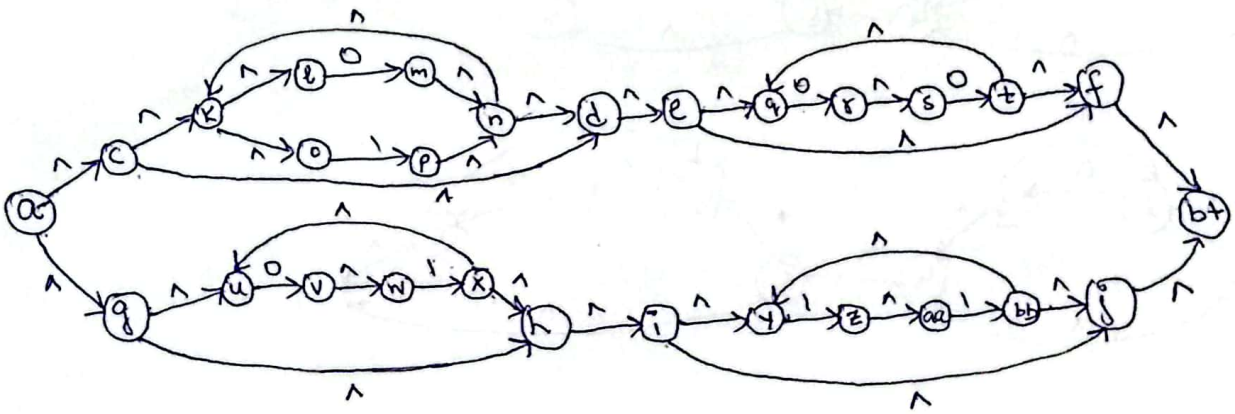




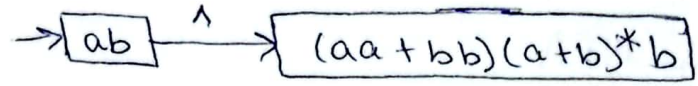


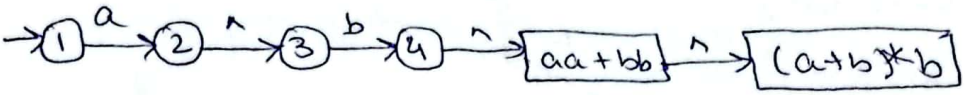


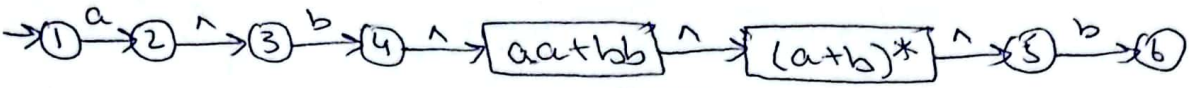


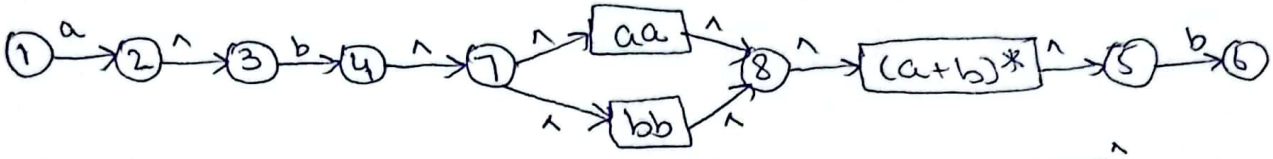


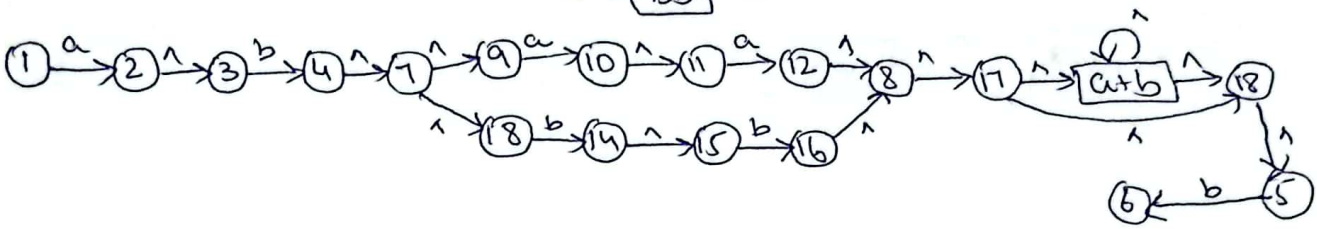
2. $ab(aa+bb)(a+b)^*b$

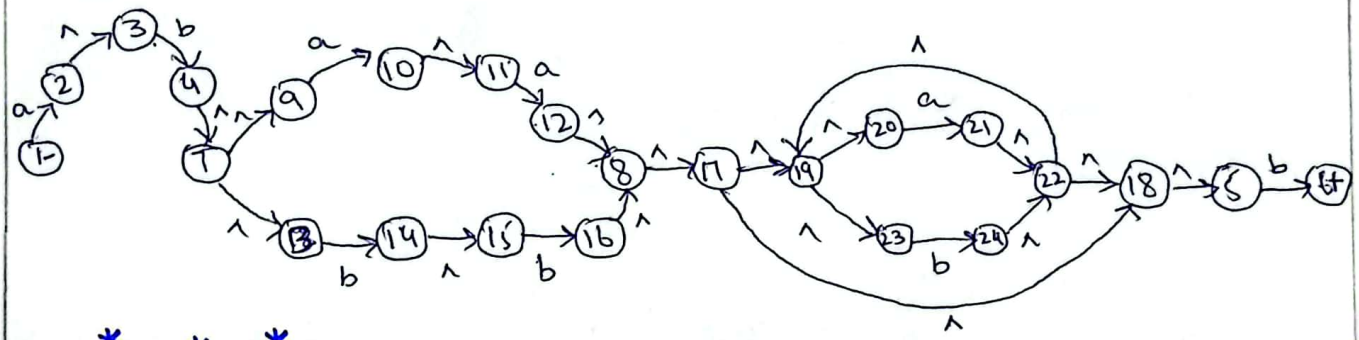




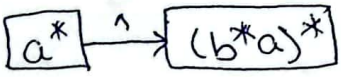


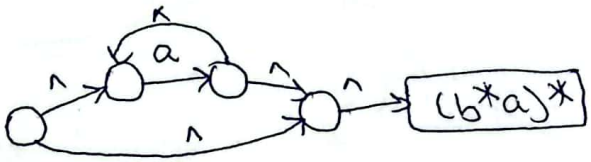


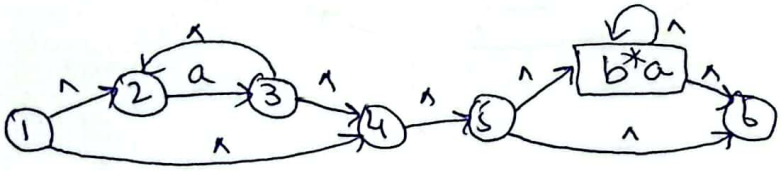


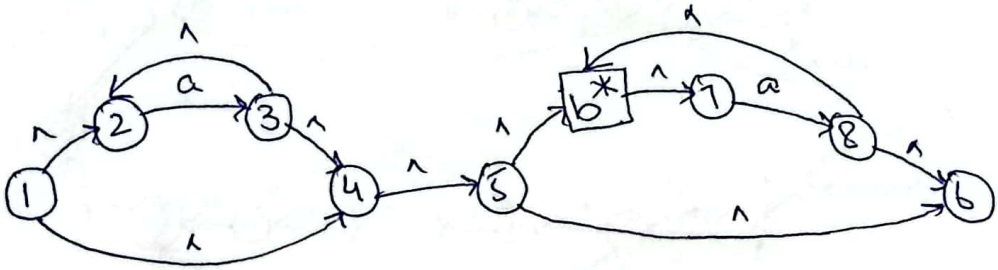


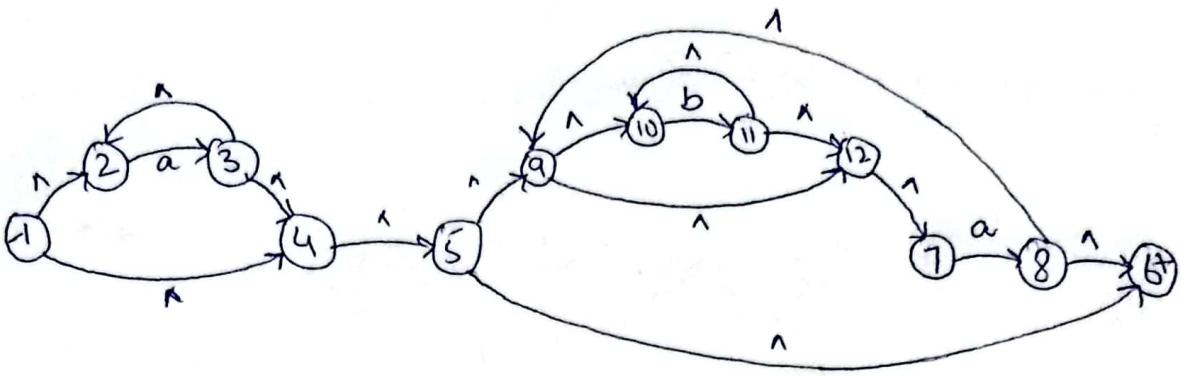
3. $a^*(b^*a)^*$



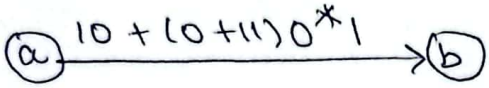


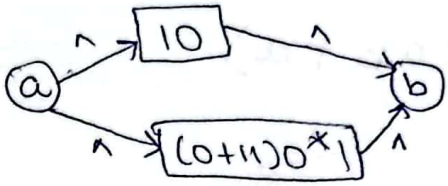


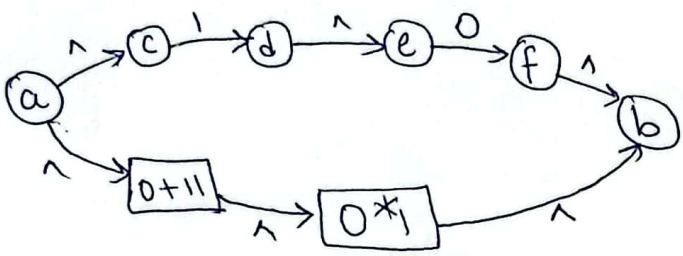


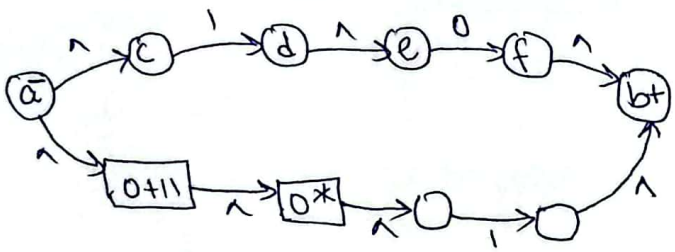

1
2
a
3
4
5
9
10
b
11
12
7
a
8
bt

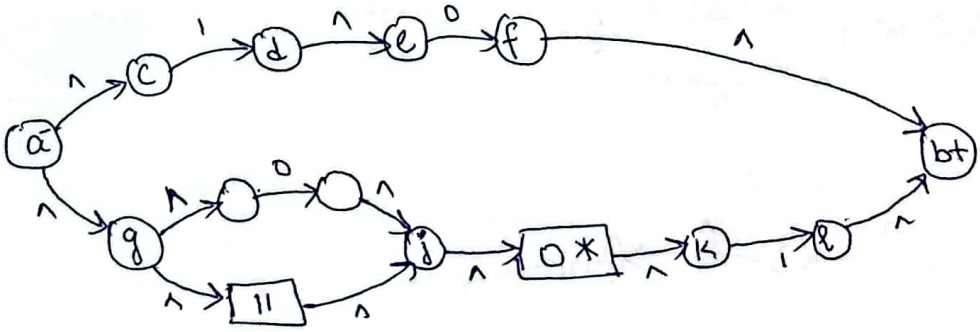
4. $10 + (0 + 11)0^*1$

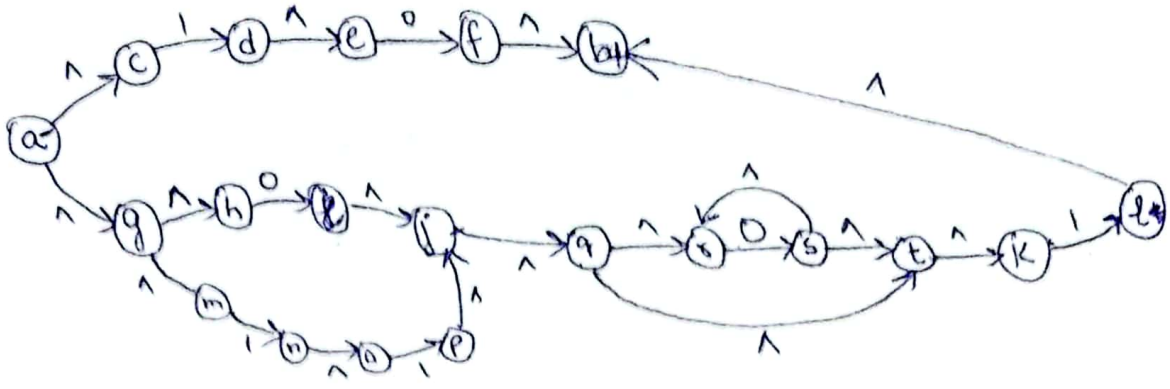

a
10 + (0+11)0*1
b


a
10
b
(0+11)0*1

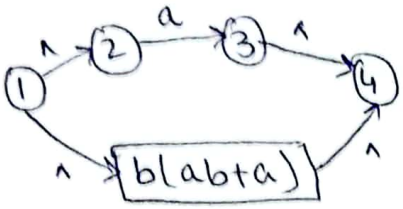

a
c
1
d
e
0
f
b
0+11
0*1

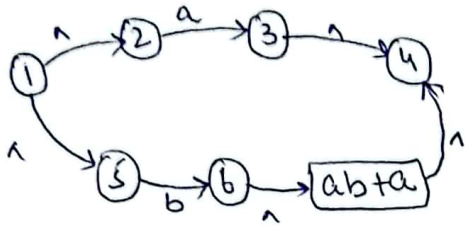

a
c
1
d
e
0
f
bt
0+11
0*

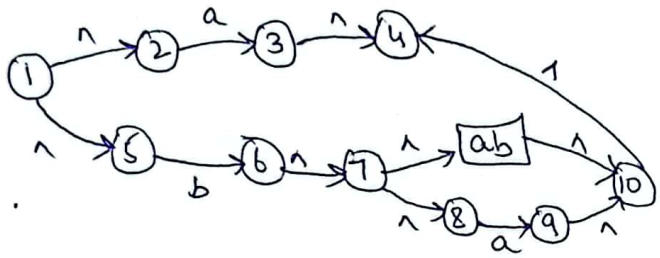

a
c
1
d
e
0
f
bt
g
0
11
i
0*
k
l

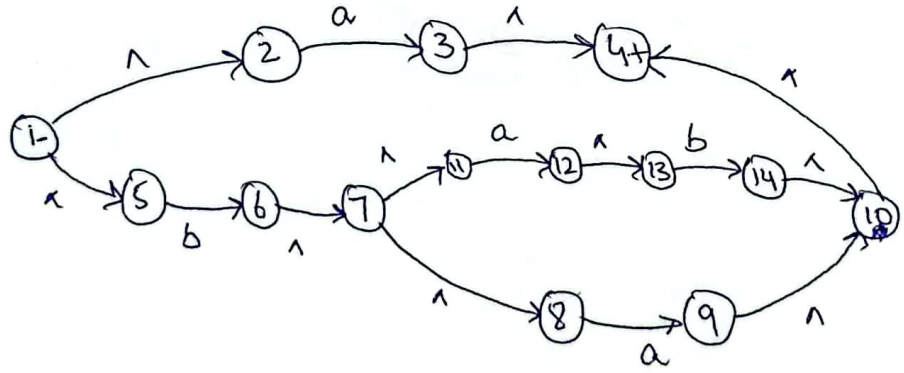


5. $a+b(ab+a)$









(b) Convert Null NFA to DFA

(1) λ-closure-a = {a, c, d, e, q, f, k, l, o, g, u, h, i, y, j, b}$_{A+}$

$A_0 = \{m, v\}_B$

$A_1 = \{p, z\}_C$

λ-closure-B = {m, v, r} = {m, v, r, n, k, o, l, d, e, q, f, b, w, s}$_{B+}$

$B_0 = \{m, r, t\}_D$

$B_1 = \{p, x\}_E$

λ-closure-C = {p, z} = {p, z, n, d, e, q, f, b, k, o, aa, l}$_{C+}$

$C_0 = \{r, m\}_F$

$C_1 = \{p, bb\}_G$

λ-closure-D = {m, r, t} = {m, r, t, n, d, e, q, f, b, k, l, o, s}$_{D+}$

$D_0 = \{r, m, t\}_D$

$D_1 = \{p\}_H$

λ-closure-E = {p, x} = {p, x, n, k, l, o, d, e, q, f, b, h, i, y, j, u}$_{E+}$

$E_0 = \{m, r, v\}_B$

$E_1 = \{p, z\}_C$

λ-closure-F = {r, m} = {r, m, s, n, k, l, o, d, e, q, f, b}$_{F+}$

$F_0 = \{t, m, r\}_D$

$F_1 = \{p\}_H$

λ-closure-G = {p, bb} = {p, bb, n, d, e, q, f, b, k, l, o, j, y}$_{G+}$

$G_0 = \{r, m\}_F$

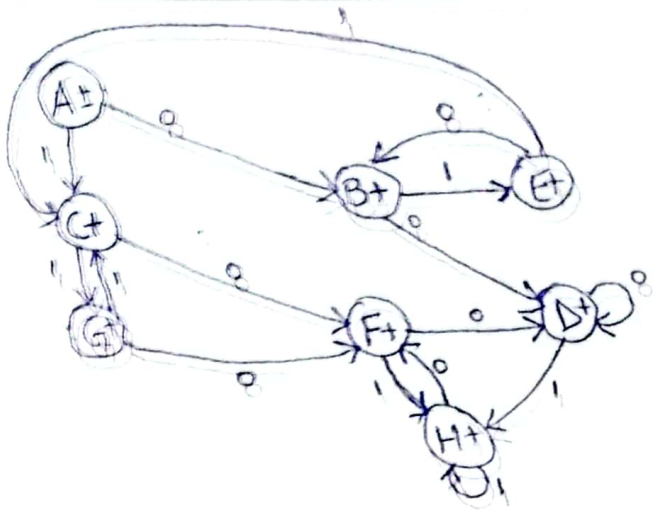
$G_1 = \{p, z\}_C$

λ-closure-H = {p} = {p, n, k, l, o, d, e, q, f, b}$_{H+}$

$H_0 = \{m, r\}_F$

$H_1 = \{p\}_H$

		0	1
→	A	B	C
	B	D	E
	C	F	G
	D	A	H
	E	B	C
	F	D	H
	G	F	C
*	H	F	H



2. ab(aa+bb)(a+b)*b

k-closure 1 : {1}A

A_a = {2}B A_b = {}

k-closure B : {2,3}B

B_a = {} B_b = {4}C

k-closure C : {4,7,13,9}C

C_a = {10}D C_b = {14}E

k-closure D : {10,11}D

D_a = {12}F D_b = {}

k-closure E : {14,15}E

E_a = {} E_b = {16}G

k-closure F : {12,8,17,19,20,23,18,5}F

F_a = {21}H F_b = {24,6}I

k-closure G : {16,8,17,19,20,23,18,5}G

G_a = {21}H G_b = {24,6}I

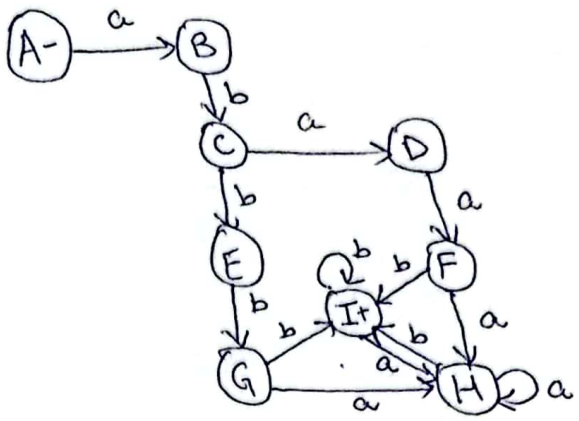
k-closure H : {21,22,18,5,19,20,23}H

H_a = {21}H H_b = {6,24}I

k-closure I : {24,6,22,18,5,19,20,23}I

I_a = {21}H I_b = {6,24}I

	a	b
-A	B	-
B	-	C
C	D	E
D	F	-
E	-	G
F	H	I
G	H	I
H	H	I
+I	H	I



(3)

λ-closure 1: {1, 2, 4, 5, 9, 10, 12, 7, 6}$_{A^-}$

A_a = {3, 8}$_B$ A_b = {11}$_C$

λ-closure B: {3, 8, 4, 5, 6, 9, 10, 12, 7, 2}$_{B+}$

B_a = {8, 3}$_B$ B_b = {11}$_C$

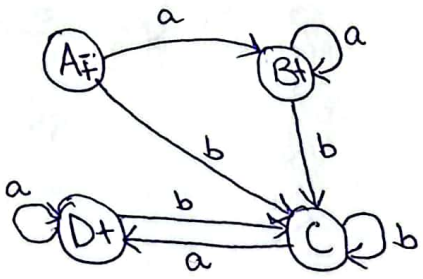
λ-closure C: {11, 10, 12, 7}$_C$

C_a = {8}$_D$ C_b = {11}$_C$

λ-closure D: {8, 6, 9, 10, 12, 7}$_{D+}$

D_a = {8}$_D$ D_b = {11}$_C$

	a	b
±A	B	C
+B	B	C
C	D	C
+D	D	C



(4)

λ-closure a : {a, c, q, h, m}A-

$A_0 = \{i\}_B$ $A_1 = \{d, n\}_C$

λ-closure B : {i, j, q, r, t, K}B

$B_0 = \{s\}_D$ $B_1 = \{l\}_E$

λ-closure C : {d, n, e, o}C

$C_0 = \{f\}_{F^+}$ $C_1 = \{p\}_G$

λ-closure D : {s, r, t, K}D

$D_0 = \{s\}_D$ $D_1 = \{l\}_{E^+}$

λ-closure E : {l, b}E+

$E_0 = \{\}$ $E_1 = \{\}$

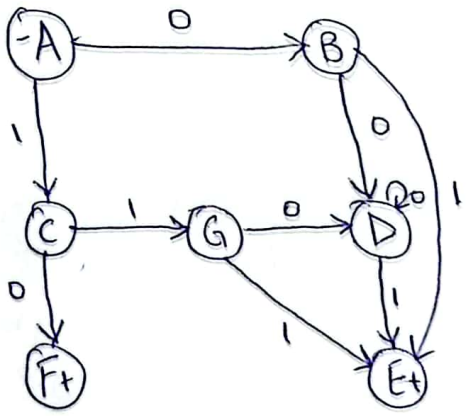
λ-closure F : {f, b}F+

$F_0 = \{\}$ $F_1 = \{\}$

λ-closure G : {p, j, q, r, t, K}G

$G_0 = \{s\}_D$ $G_1 = \{l\}_E$

	0	1
A	B	C
B	D	E
C	F	G
D	D	E
+E	-	-
+F	-	-
G	D	E



(5)

λ-closure 1 : {1, 2, 5} A-

A_a = {3} B A_b = {6} C

λ-closure B : {3, 4} B+

B_a = {} B_b = {}

λ-closure C : {6, 7, 11, 8} C

C_a = {12, 9} D C_b = {}

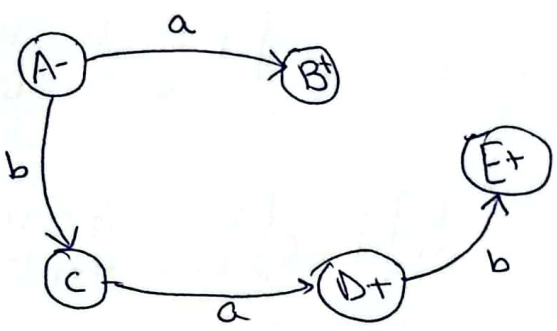
λ-closure D : {12, 9, 13, 10, 4} D+

D_a = {} D_b = {14} E

λ-closure E : {14, 10, 4} E+

E_a = {} E_b = {}

	a	b
- A	B	C
+ B	–	–
C	D	–
+ D	–	E
+ E	–	–



(c) DFA to Minimized DFA

(1)

Label

$$P = \begin{bmatrix} A & B & C & D & E & F & G & H \\ BC & DE & FG & DH & BC & DH & FC & FH \end{bmatrix}$$

Group

$$\begin{bmatrix} A & B & C & D & E & F & G & H \\ PP & PP & PP & PP & PP & PP & PP & PP \end{bmatrix}$$

can't be partitioned further!

	0	1
P	P	P

(2)

Label

$P=\begin{bmatrix} A & B & C & D & E & F & G & H \\ B- & -C & DE & F- & -G & HI & HI & HI \end{bmatrix}$ $Q=\begin{bmatrix} I \\ HI \end{bmatrix}$

Group

$\begin{bmatrix} A & B & C & D & E & F & G & H \\ P- & -P & PP & P- & -P & PQ & PQ & PQ \end{bmatrix}$ $\begin{bmatrix} I \\ PQ \end{bmatrix}$

Partition

$\begin{bmatrix} A & D \\ P- & P- \end{bmatrix}\begin{bmatrix} B & E \\ -P & -P \end{bmatrix}\begin{bmatrix} C \\ PP \end{bmatrix}\begin{bmatrix} F & G & H \\ PQ & PQ & PQ \end{bmatrix}\begin{bmatrix} I \\ PQ \end{bmatrix}$

Label

$R=\begin{bmatrix} A & D \\ B- & F- \end{bmatrix}$ $S=\begin{bmatrix} B & E \\ -C & -G \end{bmatrix}$ $T=\begin{bmatrix} C \\ DE \end{bmatrix}$ $U=\begin{bmatrix} F & G & H \\ HI & HI & HI \end{bmatrix}$ $V=\begin{bmatrix} I \\ HI \end{bmatrix}$

Group

$\begin{bmatrix} A & D \\ S- & U- \end{bmatrix}\begin{bmatrix} B & E \\ -T & -U \end{bmatrix}\begin{bmatrix} C \\ RS \end{bmatrix}\begin{bmatrix} F & G & H \\ UV & UV & UV \end{bmatrix}\begin{bmatrix} I \\ UV \end{bmatrix}$

Partition

$\begin{bmatrix} A \\ S- \end{bmatrix}\begin{bmatrix} D \\ U- \end{bmatrix}\begin{bmatrix} B \\ -T \end{bmatrix}\begin{bmatrix} E \\ -U \end{bmatrix}\begin{bmatrix} C \\ RS \end{bmatrix}\begin{bmatrix} F & G & H \\ UV & UV & UV \end{bmatrix}\begin{bmatrix} I \\ UV \end{bmatrix}$

Label

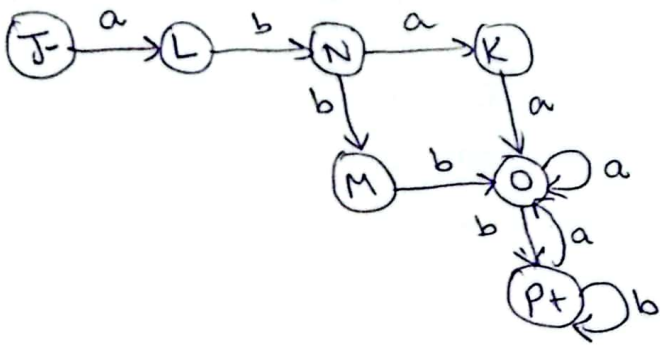
$J=\begin{bmatrix} A \\ B- \end{bmatrix}$ $K=\begin{bmatrix} D \\ F- \end{bmatrix}$ $L=\begin{bmatrix} B \\ -C \end{bmatrix}$ $M=\begin{bmatrix} E \\ -G \end{bmatrix}$ $N=\begin{bmatrix} C \\ DE \end{bmatrix}$ $O=\begin{bmatrix} F & G & H \\ HI & HI & HI \end{bmatrix}$ $P=\begin{bmatrix} I \\ HI \end{bmatrix}$

Group

$\begin{bmatrix} A \\ L- \end{bmatrix}\begin{bmatrix} D \\ O- \end{bmatrix}\begin{bmatrix} B \\ -N \end{bmatrix}\begin{bmatrix} E \\ -O \end{bmatrix}\begin{bmatrix} C \\ KM \end{bmatrix}\begin{bmatrix} F & G & H \\ OP & OP & OP \end{bmatrix}\begin{bmatrix} I \\ OP \end{bmatrix}$

Exam

	a	b
-J	L	–
K	O	–
L	–	N
M	–	O
N	K	M
O	O	P
+P	O	P



(3)

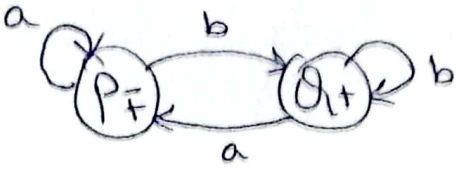
Label

$P=\begin{bmatrix} A & B & D \\ BC & BC & DC \end{bmatrix}$ $Q=\begin{bmatrix} C \\ DC \end{bmatrix}$

Group

$\begin{bmatrix} A & B & C \\ PQ & PQ & PQ \end{bmatrix}$ $\begin{bmatrix} D \\ PQ \end{bmatrix}$

	a	b
P	P	Q
Q	P	Q



(4)

Label

$P=\begin{bmatrix} A & B & C & D & G \\ BC & DE & FG & DE & DE \end{bmatrix}$ $Q=\begin{bmatrix} E & F \\ -- & -- \end{bmatrix}$

group

$\begin{bmatrix} A & B & C & D & G \\ PP & PQ & QP & PQ & PQ \end{bmatrix}$ $\begin{bmatrix} E & F \\ -- & -- \end{bmatrix}$

partition

$\begin{bmatrix} A \\ PP \end{bmatrix}\begin{bmatrix} B & D & G \\ PQ & PQ & PQ \end{bmatrix}\begin{bmatrix} C \\ QP \end{bmatrix}\begin{bmatrix} E & F \\ -- & -- \end{bmatrix}$

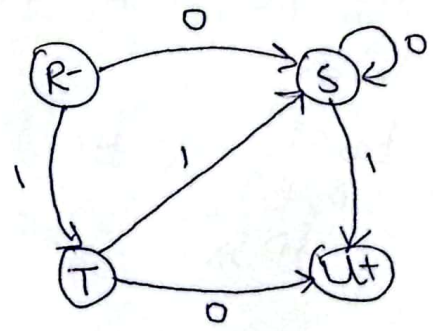
Label

$R=\begin{bmatrix} A \\ BC \end{bmatrix}$ $S=\begin{bmatrix} B & D & G \\ DE & DE & DE \end{bmatrix}$ $T=\begin{bmatrix} C \\ FG \end{bmatrix}$ $U=\begin{bmatrix} E & F \\ -- & -- \end{bmatrix}$

Group

$\begin{bmatrix} A \\ ST \end{bmatrix}$ $\begin{bmatrix} B & D & G \\ SU & SU & SU \end{bmatrix}$ $\begin{bmatrix} C \\ US \end{bmatrix}$ $\begin{bmatrix} E & F \\ -- & -- \end{bmatrix}$

	0	1
R	S	T
S	S	U
T	U	S
+ U	–	–



Label

$P = \begin{bmatrix} A & C \\ BC & D \end{bmatrix}$ $Q = \begin{bmatrix} B & D & E \\ -- & -E & -- \end{bmatrix}$

group

$\begin{bmatrix} A & C \\ QP & Q- \end{bmatrix} \begin{bmatrix} B & D & E \\ -- & -Q & -- \end{bmatrix}$

partition

$\begin{bmatrix} A \\ QP \end{bmatrix} \begin{bmatrix} C \\ Q- \end{bmatrix} \begin{bmatrix} B & E \\ -- & -- \end{bmatrix} \begin{bmatrix} D \\ -Q \end{bmatrix}$

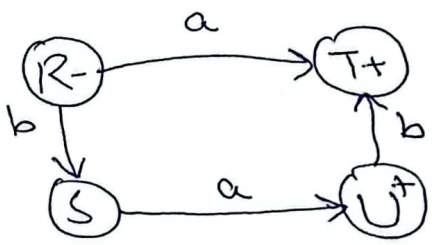
label

$R = \begin{bmatrix} A \\ BC \end{bmatrix}$ $S = \begin{bmatrix} C \\ D- \end{bmatrix}$ $T = \begin{bmatrix} B & E \\ -- & -- \end{bmatrix}$ $U = \begin{bmatrix} D \\ -E \end{bmatrix}$

group

$\begin{bmatrix} A \\ TS \end{bmatrix} \begin{bmatrix} C \\ U- \end{bmatrix} \begin{bmatrix} B & E \\ -- & -- \end{bmatrix} \begin{bmatrix} D \\ -T \end{bmatrix}$

		a	b
-	R	T	S
	S	U	—
+	T	—	—
+	U	—	T



Question:03

(a)

λ-closure A : $\{A, B, D\}_{A+}$

$A_a = \{D\}_B$ $A_b = \{C\}_C$ $A_c = \{C\}_{BC}$

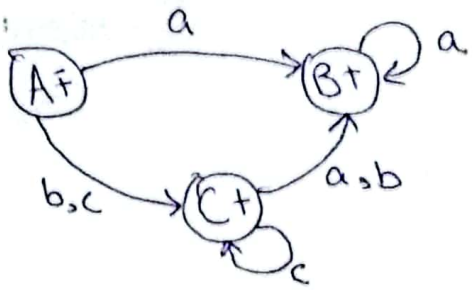
λ-closure B : $\{D\}_{B+}$

$B_a = \{D\}_B$ $B_b = \{\}$ $B_c = \{\}$

λ-closure C : $\{C, D, B\}_{C+}$

$C_a = \{D\}_B$ $C_b = \{D\}_B$ $C_c = \{C\}_C$

	a	b	c
∓ A	B	C	C
+ B	B	–	–
+ C	B	B	C



(b)

λ-closure q_0: $\{q_0, q_3, q_1\}_{A\mp}$

$A_a = \{q_1, q_2\}_B$ $\quad A_b = \{q_3\}_C$

λ-closure B: $\{q_1, q_2, q_5, q_3\}_{B+}$

$B_a = \{q_2\}_D$ $\quad B_b = \{q_3, q_5\}_E$

λ-closure C: $\{q_3, q_1\}_{C+}$

$C_a = \{q_2\}_D$ $\quad C_b = \{q_3\}_C$

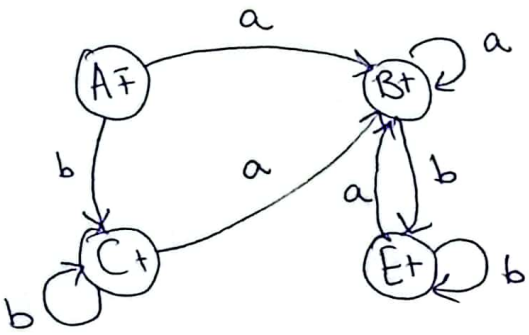
λ-closure D: $\{q_2, q_5, q_3, q_1\}_{D+}$

same as B so ignore.

λ-closure E: $\{q_3, q_5, q_1\}_{E+}$

$E_a = \{q_2\}_D$ $\quad E_b = \{q_5, q_3\}_E$

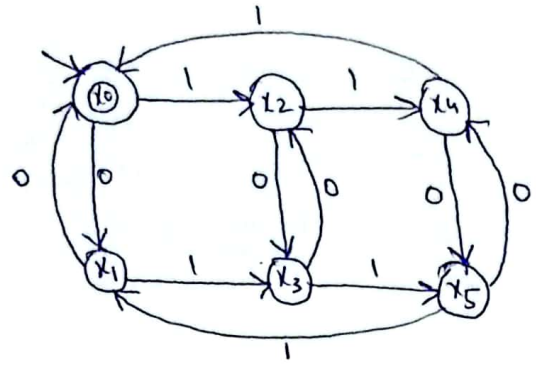
	a	b
± A	B	C
+ B	B	E
+ C	B	C
+ E	B	E



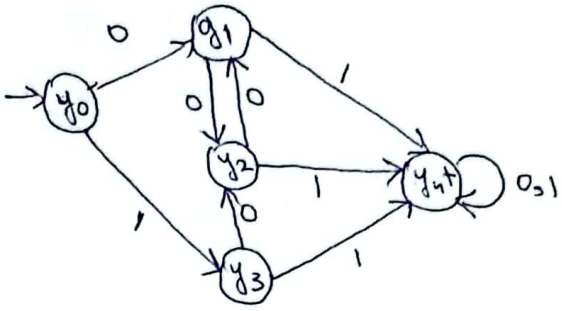
Question: 4

(a)

DFA-1 + DFA-2



DFA-1



DFA-2

z_1	0	1
x_0	x_1	x_2
y_0	y_1	y_3

$z_2 = x_1 + y_1$ $z_3 = x_2 + y_3$

z_2	0	1
x_1	x_0	x_3
y_1	y_2	y_4

$z_4 = x_0 + y_2$ $z_5 = x_3 + y_4$

z_3	0	1
x_2	x_3	x_4
y_3	y_2	y_4

$z_6 = x_3 + y_2$ $z_7 = x_4 + y_4$

z_4	0	1
x_0	x_1	x_2
y_2	y_1	y_4
	z_2	$z_8 = x_2 + y_4$

z_5	0	1
x_3	x_2	x_5
y_4	y_4	y_4
	z_8	$z_9 = x_5 + y_4$

z_6	0	1
x_3	x_2	x_5
y_2	y_1	y_4
	$z_{10} = x_2 + y_1$	z_9

z_7	0	1
x_4	x_5	x_0
y_4	y_4	y_4
	z_9	$z_{11} = x_0 + y_4$

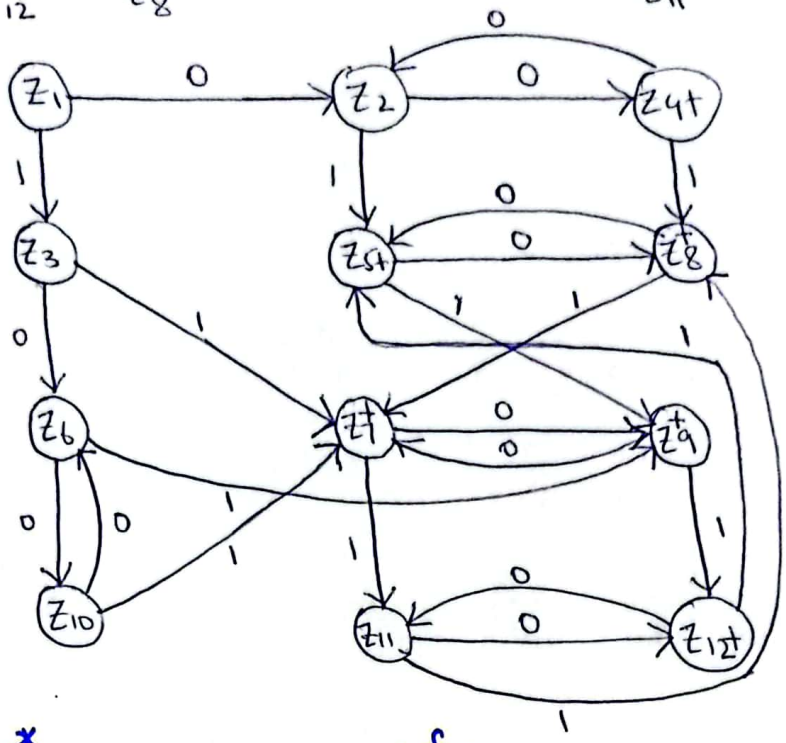
z_8	0	1
x_2	x_3	x_4
y_4	y_4	y_4
	z_5	z_7

z_9	0	1
x_5	x_4	x_1
y_4	y_4	y_4
	z_7	$z_{12} = x_1 + y_4$

z_{10}	0	1
x_2	x_3	x_4
y_1	y_2	y_4
	z_6	z_7

z_{11}	0	1
x_0	x_1	x_2
y_4	y_4	y_4
	z_{12}	z_8

z_{12}	0	1
x_1	x_0	x_3
y_4	y_4	y_4
	z_{11}	z_5

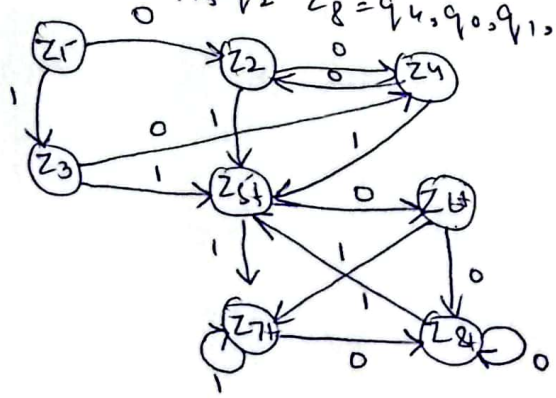


(DFA-2)*

(fig)

States

	0	1
$\rightarrow z_1 = q_0$	$z_2 = q_1$	$z_3 = q_3$
$z_2 = q_1$	$z_4 = q_2$	$z_5 = q_4, q_0$
$z_3 = q_3$	$z_4 = q_2$	$z_5 = q_4, q_0$
$z_4 = q_2$	$z_2 = q_1$	$z_5 = q_4, q_0$
$+z_5 = q_4, q_0$	$z_6 = q_4, q_0, q_1$	$z_7 = q_4, q_0, q_3$
$+z_6 = q_4, q_0, q_1$	$z_8 = q_4, q_0, q_1, q_2$	$z_7 = q_4, q_0, q_3$
$+z_7 = q_4, q_0, q_3$	$z_8 = q_4, q_0, q_1, q_2$	$z_7 = q_4, q_0, q_3$
$+z_8 = q_4, q_0, q_1, q_2$	$z_8 = q_4, q_0, q_1, q_2$	$z_5 = q_4, q_0$



(c)

DFA-1 Minimization:

	0	1
x_0	x_1	x_2
x_1	x_0	x_3
x_2	x_3	x_4
x_3	x_2	x_5
x_4	x_5	x_0
x_5	x_4	x_1

Label

$P=\begin{bmatrix} x_0 \\ x_1x_2 \end{bmatrix}$ $Q_1=\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \\ x_0x_3 & x_3x_4 & x_2x_5 & x_5x_0 & x_4x_1 \end{bmatrix}$

Group

$\begin{bmatrix} x_0 \\ QQ \end{bmatrix}\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \\ PQ & QQ & QQ & QP & QQ \end{bmatrix}$

Partition

$\begin{bmatrix} x_0 \\ QQ \end{bmatrix}\begin{bmatrix} x_1 \\ PQ \end{bmatrix}\begin{bmatrix} x_2 & x_3 & x_5 \\ QQ & QQ & QQ \end{bmatrix}\begin{bmatrix} x_4 \\ QP \end{bmatrix}$

Label

$R=\begin{bmatrix} x_0 \\ x_1x_2 \end{bmatrix}$ $S=\begin{bmatrix} x_1 \\ x_0x_3 \end{bmatrix}$ $T=\begin{bmatrix} x_2 & x_3 & x_5 \\ x_3x_4 & x_2x_5 & x_4x_1 \end{bmatrix}$ $U=\begin{bmatrix} x_4 \\ x_5x_0 \end{bmatrix}$

Group

$\begin{bmatrix} x_0 \\ ST \end{bmatrix}\begin{bmatrix} x_1 \\ RT \end{bmatrix}\begin{bmatrix} x_2 & x_3 & x_5 \\ TU & TT & US \end{bmatrix}\begin{bmatrix} x_4 \\ TR \end{bmatrix}$

Partition

$\begin{bmatrix} x_0 \\ ST \end{bmatrix}\begin{bmatrix} x_1 \\ RT \end{bmatrix}\begin{bmatrix} x_2 \\ TU \end{bmatrix}\begin{bmatrix} x_3 \\ TT \end{bmatrix}\begin{bmatrix} x_5 \\ US \end{bmatrix}\begin{bmatrix} x_4 \\ TR \end{bmatrix}$

Label

$A=\begin{bmatrix} x_0 \\ x_1x_2 \end{bmatrix}$ $B=\begin{bmatrix} x_1 \\ x_0x_3 \end{bmatrix}$ $C=\begin{bmatrix} x_2 \\ x_3x_4 \end{bmatrix}$ $D=\begin{bmatrix} x_3 \\ x_2x_5 \end{bmatrix}$ $E=\begin{bmatrix} x_5 \\ x_4x_1 \end{bmatrix}$ $F=\begin{bmatrix} x_4 \\ x_5x_0 \end{bmatrix}$

Group

$\begin{bmatrix} x_0 \\ BC \end{bmatrix}\begin{bmatrix} x_1 \\ AD \end{bmatrix}\begin{bmatrix} x_2 \\ DF \end{bmatrix}\begin{bmatrix} x_3 \\ CE \end{bmatrix}\begin{bmatrix} x_5 \\ FB \end{bmatrix}\begin{bmatrix} x_4 \\ EA \end{bmatrix}$

		0	1
±	A	B	C
	B	A	D
	C	D	F
	D	C	E
	E	F	B
	F	E	A

states are same as the original one.

Can't be minimized!

DFA-2 Minimization:

	0	1
q_0	q_1	q_3
q_1	q_2	q_4
q_2	q_1	q_4
q_3	q_2	q_4
q_4	q_4	q_4

Label

$$P=\begin{bmatrix} q_0 & q_1 & q_2 & q_3 \\ q_1q_3 & q_2q_4 & q_1q_4 & q_2q_4 \end{bmatrix} \quad Q=\begin{bmatrix} q_4 \\ q_4q_4 \end{bmatrix}$$

Group

$$\begin{bmatrix} q_0 & q_1 & q_2 & q_3 \\ PP & PQ & PQ & PQ \end{bmatrix} \begin{bmatrix} q_4 \\ QQ \end{bmatrix}$$

Partition

$$\begin{bmatrix} q_0 \\ PP \end{bmatrix} \begin{bmatrix} q_1 & q_2 & q_3 \\ PQ & PQ & PQ \end{bmatrix} \begin{bmatrix} q_4 \\ QQ \end{bmatrix}$$

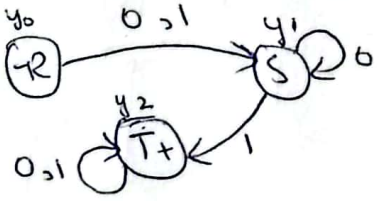
Label

$$R=\begin{bmatrix} q_0 \\ q_1q_3 \end{bmatrix} \quad S=\begin{bmatrix} q_1 & q_2 & q_3 \\ q_2q_4 & q_1q_4 & q_2q_4 \end{bmatrix} \quad T=\begin{bmatrix} q_4 \\ q_4q_4 \end{bmatrix}$$

Group

$$\begin{bmatrix} q_0 \\ SS \end{bmatrix} \begin{bmatrix} q_1 & q_2 & q_3 \\ ST & ST & ST \end{bmatrix} \begin{bmatrix} q_4 \\ TT \end{bmatrix}$$

		0	1
−	R	S	S
	S	S	T
+	T	T	T



Min DFA1 + Min DFA2

z_1	0	1
x_0	x_1	x_2
y_0	y_1	y_1

$z_2 = x_1 + y_1$ $\quad z_3 = x_2 + y_1$

z_2	0	1
x_1	x_0	x_3
y_1	y_1	y_2

$z_4 = x_0 + y_1$ $\quad z_5 = x_3 + y_2$

z_3	0	1
x_2	x_3	x_4
y_1	y_1	y_2

$z_6 = x_3 + y_1$ $\quad z_7 = x_4 + y_2$

z_4	0	1
x_0	x_1	x_2
y_1	y_1	y_2

$z_2 = x_1 + y_1$ $\quad z_8 = x_2 + y_2$

z_5	0	1
	x_3	x_2	x_5
	y_2	y_2	y_2

$z_8 = x_2 + y_2$ $\quad z_9 = x_5 + y_2$

z_6	0	1
	x_3	x_2	x_5
	y_1	y_1	y_2

$z_3 = x_2 + y_1$ $\quad z_9 = x_5 + y_2$

z_7	0	1
	x_4	x_5	x_0
	y_2	y_2	y_2

$z_9 = x_5 + y_2$ $\quad z_{10} = x_0 + y_2$

z_8	0	1
	x_2	x_3	x_4
	y_2	y_2	y_2

$z_5 = x_3 + y_2$ $\quad z_7 = x_4 + y_2$

z_9	0	1
	x_5	x_4	x_1
	y_2	y_2	y_2

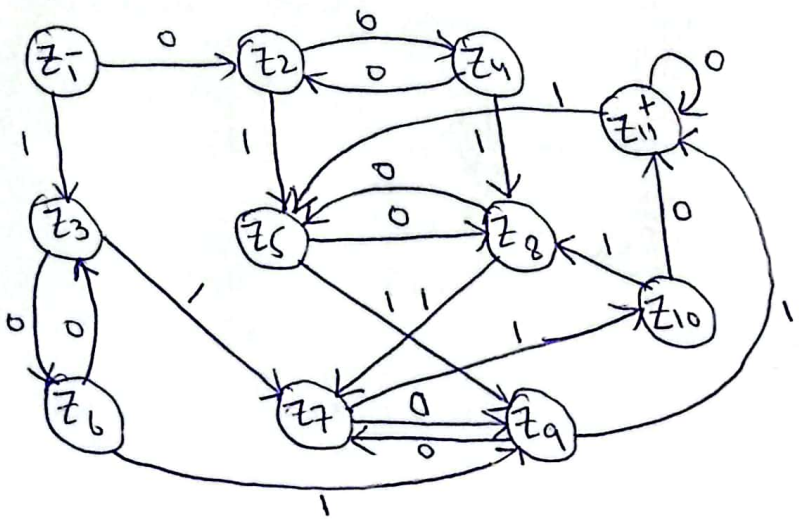
$z_7 = x_4 + y_2$ $\quad z_{11} = x_1 + y_2$

z_{10}	0	1
	x_0	x_1	x_2
	y_2	y_2	y_2

$z_{11} = x_1 + y_2$ $\quad z_8 = x_2 + y_2$

z_{11}	0	1
	x_1	x_0	x_3
	y_2	y_2	y_2

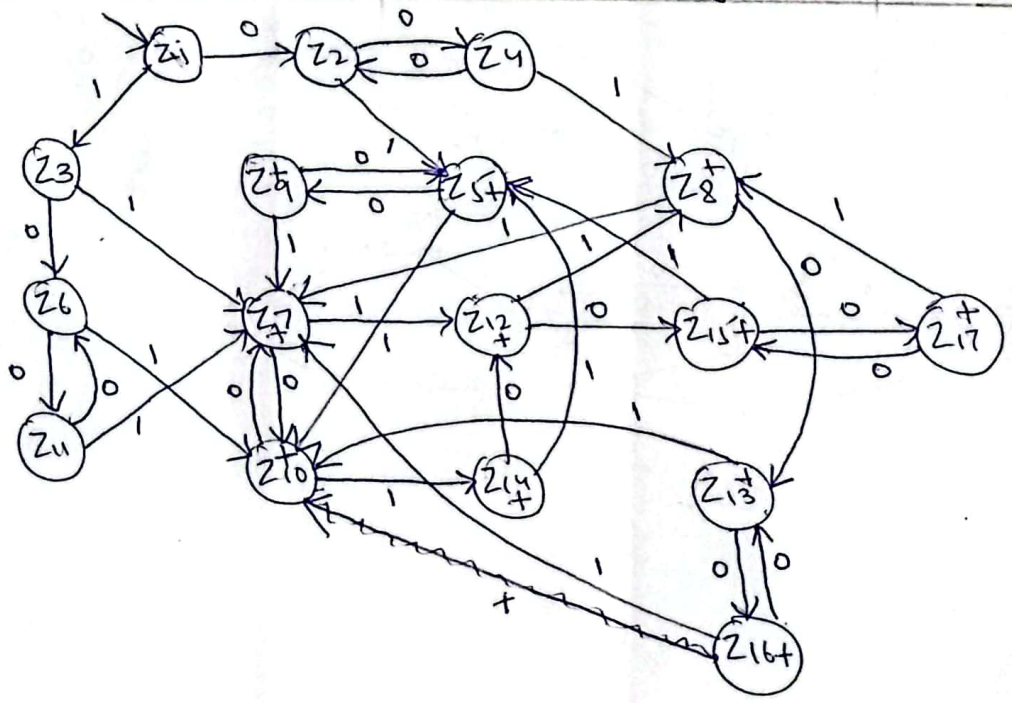
$z_{11} = x_0 + y_2$ $\quad z_5 = x_3 + y_2$



(a)
(b)

DFA1. DFA2

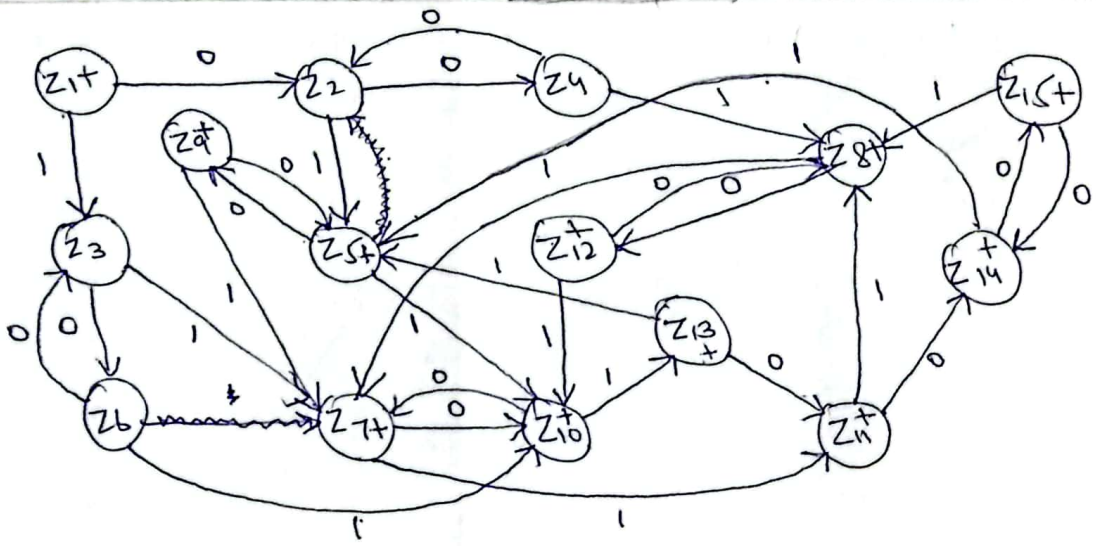
Old States	0	1
$z_1 = x_0, y_0$	$z_2 = x_0, y_1$	$z_3 = y_3, x_2$
$z_2 = x_1, y_1$	$z_4 = x_0, y_0, y_2$	$z_5 = x_3, y_4$
$z_3 = y_3, x_2$	$z_6 = x_3, y_2$	$z_7 = x_4, y_4$
$z_4 = x_0, y_0, y_2$	$z_2 = x_1,$	$z_8 = x_2, y_3, y_4$
$+z_5 = x_3, y_4$	$z_9 = x_2, y_4$	$z_{10} = x_5, y_4$
$z_6 = x_3, y_2$	$z_{11} = x_2, y_1$	$z_{10} = x_5, y_4$
$+z_7 = x_4, y_4$	$z_{10} = x_5, y_4$	$z_{12} = x_0, y_0, y_4$
$+z_8 = x_2, y_3, y_4$	$z_{13} = x_3, y_2, y_4$	$z_7 = x_4, y_4$
$+z_9 = x_2, y_4$	$z_5 = x_3, y_4$	$z_7 = x_4, y_4$
$+z_{10} = x_5, y_4$	$z_7 = x_4, y_4$	$z_{14} = x_1, y_4$
$z_{11} = x_2, y_1$	$z_6 = x_3, y_2$	$z_7 = x_4, y_4$
$+z_{12} = x_0, y_0, y_4$	$z_{15} = x_1, y_1, y_4$	$z_8 = x_2, y_3, y_4$
$+z_{13} = x_3, y_2, y_4$	$z_{16} = x_2, y_1, y_4$	$z_{10} = x_5, y_4$
$+z_{14} = x_1, y_4$	$z_{12} = x_0, y_0, y_4$	$z_5 = x_3, y_4$
$+z_{15} = x_1, y_1, y_4$	$z_{17} = x_0, y_2, y_0, y_4$	$z_5 = x_3, y_4$
$+z_{16} = x_2, y_1, y_4$	$z_{13} = x_3, y_2, y_4$	$z_7 = x_4, y_4$
$+z_{17} = x_0, y_2, y_0, y_4$	$z_{15} = x_1, y_1, y_4$	$z_8 = x_2, y_3, y_4$



(d)

Min DFA1 . Min DFA2

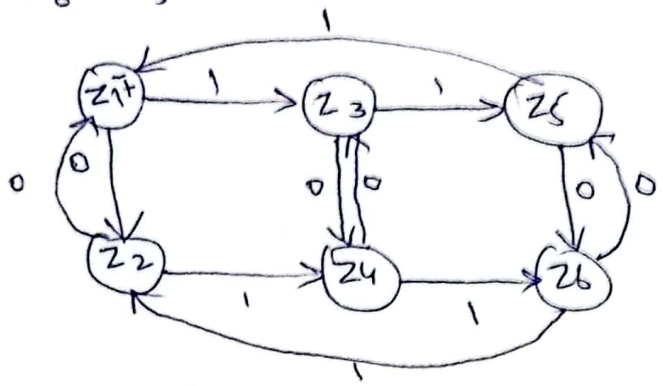
Old States	0	1
± $z_1 = x_0, y_0$	$z_2 = x_1, y_1$	$z_3 = x_2, y_1$
$z_2 = x_1, y_1$	$z_4 = x_0, y_0, y_1$	$z_5 = x_3, y_2$
$z_3 = x_2, y_1$	$z_6 = x_3, y_1$	$z_7 = x_4, y_2$
$z_4 = x_0, y_0, y_1$	$z_2 = x_1, y_1$	$z_8 = x_2, y_1, y_2$
+ $z_5 = x_3, y_2$	$z_9 = x_2, y_2$	$z_{10} = x_5, y_2$
$z_6 = x_3, y_1$	$z_3 = x_2, y_1$	$z_{10} = x_5, y_2$
+ $z_7 = x_4, y_2$	$z_{10} = x_5, y_2$	$z_{11} = x_0, y_0, y_2$
+ $z_8 = x_2, y_1, y_2$	$z_{12} = x_3, y_1, y_2$	$z_7 = x_4, y_2$
+ $z_9 = x_2, y_2$	$z_5 = x_3, y_2$	$z_7 = x_4, y_2$
+ $z_{10} = x_5, y_2$	$z_7 = x_4, y_2$	$z_{13} = x_1, y_2$
+ $z_{11} = x_0, y_0, y_2$	$z_{14} = x_1, y_1, y_2$	$z_8 = x_2, y_1, y_2$
+ $z_{12} = x_3, y_1, y_2$	$z_8 = x_2, y_1, y_2$	$z_{10} = x_5, y_2$
+ $z_{13} = x_1, y_2$	$z_{11} = x_0, y_0, y_2$	$z_5 = x_3, y_2$
+ $z_{14} = x_1, y_1, y_2$	$z_{15} = y_0, y_1, y_2, x_0$	$z_5 = x_3, y_2$
+ $z_{15} = y_0, y_1, y_2, x_0$	$z_{14} = x_1, y_1, y_2$	$z_8 = x_2, y_1, y_2$



(e)

(DFA-1)*

	0	1
$Z_1 = x_0$	$Z_2 = x_1$	$Z_3 = x_2$
$Z_2 = x_1$	$Z_1 = x_0$	$Z_4 = x_3$
$Z_3 = x_2$	$Z_4 = x_3$	$Z_5 = x_4$
$Z_4 = x_3$	$Z_3 = x_2$	$Z_6 = x_5$
$Z_5 = x_4$	$Z_6 = x_5$	$Z_1 = x_0$
$Z_6 = x_5$	$Z_5 = x_4$	$Z_2 = x_1$



Same