

Q1. [10 pts] Convert the following grammar to Chomsky Normal Form Grammar. Show all the intermediary steps in the correct order clearly to score full marks.

$S \rightarrow aAbB \mid ABC \mid a$

$A \rightarrow aA \mid a \mid CD$

$B \rightarrow CbC \mid b$

$C \rightarrow CC \mid \Delta$

$D \rightarrow CC \mid Db$

$E \rightarrow EE \mid \Delta$

Q2. [10 pts] Convert the following grammar to its equivalent Greibach Normal Form.

$S \rightarrow ab|a$

$A \rightarrow SS|b$

Q3. [10 pts] Use Pumping Lemma to prove that the following language is not Context-Free

$$A = \{ 0^n 1^m 0^n 1^{2n} \mid n, m, k > 0 \}$$

Q4. [10 pts] Design a PDA for **ODD PALINDROME** = { a, b, aaa, aba, bab, bbb, aaaaa, ... }

Q5. [10 pts] Design a Post Machine for the language $a^{2^n}b^na^n$ for $n \geq 0$.

Q6. [10 pts] For the language $a^{2n}b^nc^{2n}d^n$, where $n \geq 0$, design a 2-PDA.

Q7. [10 pts] Let L be some regular language in which all the words happen to have an even length. Let us define the new language $\text{Twist}(L)$ to be the set of all the words of L twisted, where by twisted we mean the first and second letters have been interchanged, and so on. For example, if

$L = \{ ba \ abba \ babb \ \dots \}$

$\text{Twist}(L) = \{ ab \ baab \ abbb \ \dots \}$

Build a Turing Machine that accepts $\text{Twist}(L)$. You are also **allowed** to use the sub programs of **INSERT** and **DELETE**. You may assume after **INSERT** operation, tape head points at the newly added cell while after **DELETE** operation, tape head points at the same location. You may leave the tape head at any location on the output string when the computation is done.

Status of tape on input is:

#	a	b	b	b	a	b	a	a	Δ	Δ	Δ	.	.
---	---	---	---	---	---	---	---	---	----------	----------	----------	---	---

↑

Status of tape at the output is:

#	b	a	b	b	b	a	a	a	Δ	Δ	Δ	.	.
---	---	---	---	---	---	---	---	---	----------	----------	----------	---	---

↑

Q8. [10 pts] Design a Turing machine that takes input a non-negative number x and performs the computable function $f(x) = 4x$. Assume the input is in unary notation, the tape head points in the start of the input. However, you may leave the tape head at any location on the output string when the computation is done. You are also **allowed** to use the sub programs of **INSERT** and **DELETE**. You may assume after **INSERT** operation, tape head points at the newly added cell while after **DELETE** operation, tape head points at the same location. An example is given below for your understanding:

Status of tape on input is:

#	1	1	Δ	Δ	Δ	.	.	.
---	---	---	---	---	---	---	---	---



Status of tape at the output is:

#	1	1	1	1	1	1	1	1	Δ	.
---	---	---	---	---	---	---	---	---	---	---



