

## Greibach Normal Form (GNF)

Convert the following grammar G into GNF

$$S \rightarrow XA \mid BB$$

$$B \rightarrow b \mid SB$$

$$X \rightarrow b$$

$$A \rightarrow a$$

1. Rewrite G in Chomsky Normal Form (CNF)

3 It is already in CNF

2. Re-label the variables

*S with  $A_1$*

*X with  $A_2$*

*A with  $A_3$*

*B with  $A_4$*

$$A_1 \rightarrow A_2 A_3 \mid A_4 A_4$$

$$A_4 \rightarrow b \mid A_1 A_4$$

$$A_2 \rightarrow b$$

$$A_3 \rightarrow a$$

3. Identify all productions which do not conform to any of the types listed below

$$A_i \rightarrow A_j x_k \quad j > i$$

$$Z_i \rightarrow A_v x_k \quad j \leq n$$

$$A_i \rightarrow a x_k$$

$x_k \in V^*$  (i.e.  $x_k$  is some string of zero or more variables)

$a \in T$  (i.e. 'a' is a single terminal)

4.  $A_4 \rightarrow A_1 A_4 \dots \dots \dots$  *identified*

5.  $A_4 \rightarrow A_1 A_4 \mid b$

To eliminate  $A_1$  we'll use the substitution rule

Substituting for  $A_1 \rightarrow A_2 A_3 \mid A_4 A_4$

$$A_4 \rightarrow A_2 A_3 A_4 \mid A_4 A_4 A_4 \mid b$$

The above two productions still do not conform to any of the types in step 3.

Substituting for  $A_2 \rightarrow b$

$$A_4 \rightarrow b A_3 A_4 \mid A_4 A_4 A_4 \mid b$$

Now we have to remove left recursive production  $A_4 \rightarrow A_4 A_4 A_4$

$$A_4 \rightarrow b A_3 A_4 \mid b \mid b A_3 A_4 Z \mid b Z$$

$$Z \rightarrow A_4 A_4 \mid A_4 A_4 Z$$

6. At this stage our grammar now looks like

$$A_1 \rightarrow A_2 A_3 \mid A_4 A_4$$

$$A_4 \rightarrow b A_3 A_4 \mid b \mid b A_3 A_4 Z \mid b Z$$

$$Z \rightarrow A_4 A_4 \mid A_4 A_4 Z$$

$$A_2 \rightarrow b$$

$$A_3 \rightarrow a$$

All rules now conform to one of the types in step 3.

***But the grammar is still not in Greibach Normal Form!***

7. All productions for  $A_2$ ,  $A_3$  and  $A_4$  are in GNF

$$\text{for } A_1 \rightarrow A_2 A_3 \mid A_4 A_4$$

Substitute for  $A_2$  &  $A_4$  to convert it to GNF

$$A_1 \rightarrow b A_3 \mid b A_3 A_4 A_4 \mid b A_4 \mid b A_3 A_4 Z A_4 \mid b Z A_4$$

$$\text{for } Z \rightarrow A_4 A_4 \mid A_4 A_4 Z$$

Substitute for  $A_4$  to convert it to GNF

$$Z \rightarrow b A_3 A_4 A_4 \mid b A_4 \mid b A_3 A_4 Z A_4 \mid b Z A_4 \mid b A_3 A_4 A_4 Z \mid b A_4 Z \mid b A_3 A_4 Z A_4 Z \mid b Z A_4 Z$$

8. Finally the Grammar in GNF is

$$A_1 \rightarrow b A_3 \mid b A_3 A_4 A_4 \mid b A_4 \mid b A_3 A_4 Z A_4 \mid b Z A_4$$

$$A_4 \rightarrow b A_3 A_4 \mid b \mid b A_3 A_4 Z \mid b Z$$

$$Z \rightarrow b A_3 A_4 A_4 \mid b A_4 \mid b A_3 A_4 Z A_4 \mid b Z A_4 \mid b A_3 A_4 A_4 Z \mid b A_4 Z \mid b A_3 A_4 Z A_4 Z \mid b Z A_4 Z$$

$$A_2 \rightarrow b$$

$$A_3 \rightarrow a$$