Lie Methods for Nonlinear Dynamics with Applications to Accelerator Physics

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Preface

John Wallis (1616-1703), Savilian Professor of Geometry at Oxford, was a mathematician and predecessor of Isaac Newton. His most important book, published in 1656, was *Arithmetica Infinitorum*. It introduced, among others, the concepts of negative and fractional exponents, and considered the problem of finding the areas under curves described by functions involving such exponents. He also introduced the symbol ∞. In 1685 he published *Algebra*.

His contemporary Thomas Hobbes (1588-1679), a philosopher and political theorist, read (or perhaps only paged through) this 1656 book, and described it as a "a scab of symbols as if a hen had been scraping there." Apparently taken by this simile, on another occasion he wrote of Wallis: "And for (your book) on *Conic Sections*, it is covered over with a scab of symbols that I had not the patience to examine whether it be well or ill demonstrated." He goes on to say: "Symbols, though they shorten the writing, yet do not make the reader understand it sooner than if it were written in words. $\cdots$ (with the use of symbols) there is a double labour of the mind, one to reduce your symbols to words, which are also symbols, another to attend to the ideas which they signify."

But, according to Leibniz (1646-1716), "In symbols one observes an advantage in discovery which is greatest when they express the exact nature of a thing briefly and, as it were, picture it; then indeed the labor of thought is wonderfully diminished."[1] Laplace (1749-1827) was even more enthusiastic when he wrote "Such is the advantage of a well-constructed language that its simplified notation often becomes the source of profound theories." And, according to Whitehead (1861-1947), "Civilization advances by extending the number of

[1]Leibniz invented much of modern calculus notation. He also introduced the term *dynamick* for what Newton (1642-1727) had previously called *rational mechanics*. But Newton objected to this name, not because of its "inadequacy to describe the subject matter", but rather because Leibniz had "set his mark upon the whole science of forces calling it Dynamick, as if he had invented it himself & is frequently setting his mark upon things by new names & new Notations". Leibniz was kinder to Newton when he wrote "Taking mathematics from the beginning of the world to the time of Newton, what he has done is much the better half." For a history of how Leibnizian notation came to be used in Great Britain, see the Web site `https://en.wikipedia.org/wiki/Analytical_Society`.

To Descartes (1596-1650) we owe the use of the symbols $a, b, c \cdots$ as constants, the symbols $x, y, z \cdots$ as variables, writing xx as x^2 etc., and, of course, honor for forging the connection between algebra and geometry (to create analytic geometry) by the use of Cartesian coordinates including making graphs of functions. To add to the list: Robert Recorde in 1540 introduced the $+$ and $-$ symbols for addition and subtraction and in 1557 introduced the equal sign $=$, William Oughtred in 1631 introduced the multiplication sign $\times$ and the trigonometric function symbols sin and cos, Johann Rahn in 1659 introduced the division sign $\div$ and the therefore sign $\therefore$, and William Jones in 1706 introduced use of the Greek letter π to denote the value that is the ratio of the circumference to the diameter for any circle and use of a dot above a letter to denote differentiation with respect to time.

important operations which we can perform without thinking of them."

The purpose of this book is to explore and illustrate how Lie-algebraic/map methods and Lie-algebraic concepts/symbols are broadly applicable to many areas of Nonlinear Dynamics including Accelerator Physics.

Reference

J. Mazur, *Enlightening symbols: a short history of mathematical notation and its hidden power*, Princeton University Press (2014).

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Minds: New ideas often emerge through close interaction between minds. Accordingly, these ideas do not solely belong to any of these individual minds, but rather are the fruit of their mutual interaction - an emergent property of exchange and interaction. Many of these minds are from the past and are known only through their writings or influences passed down through our ancestors. They are too numerous to mention here, but some will be acknowledged in the text, and more will be referenced in the Bibliography. Others are those of our contemporaries or near contemporaries. They include John Horvath, Richard K. Cooper, Robert Gluckstern, David Sutter, Eyvind Wichmann, David Judd, Robert Karplus, Alex Dessler, J. David Jackson, V. Bargmann, Louis Michel, Maury Tigner, Karl L. Brown, Alex Chao, John Irwin, Miguel Furman, Leo Michelotti, Martin Berz, Klaus Halbach, K. B. Wolf, Desmond Barber, Dobrin Kaltchev, Sateesh Mane, Peter Walstrom, Paul Channell, C. Thomas Mottershead, Filippo Neri, John Finn, David Douglas, Étienne Forest, Liam Healy, Robert Ryne, Govindan Rangarajan, Dan Abell, Marco Venturini, Chad Mitchell.

I was like a boy playing on the sea-shore, and diverting myself now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.

Isaac Newton

For in Him we live and move and have our being.

Acts 17:28

Assertion made by Saint Paul about the "unknown god" and attributed by Paul to an unnamed Greek poet, now thought to be Epimenides of Knossos because this line appears in his poem *Cretica*.

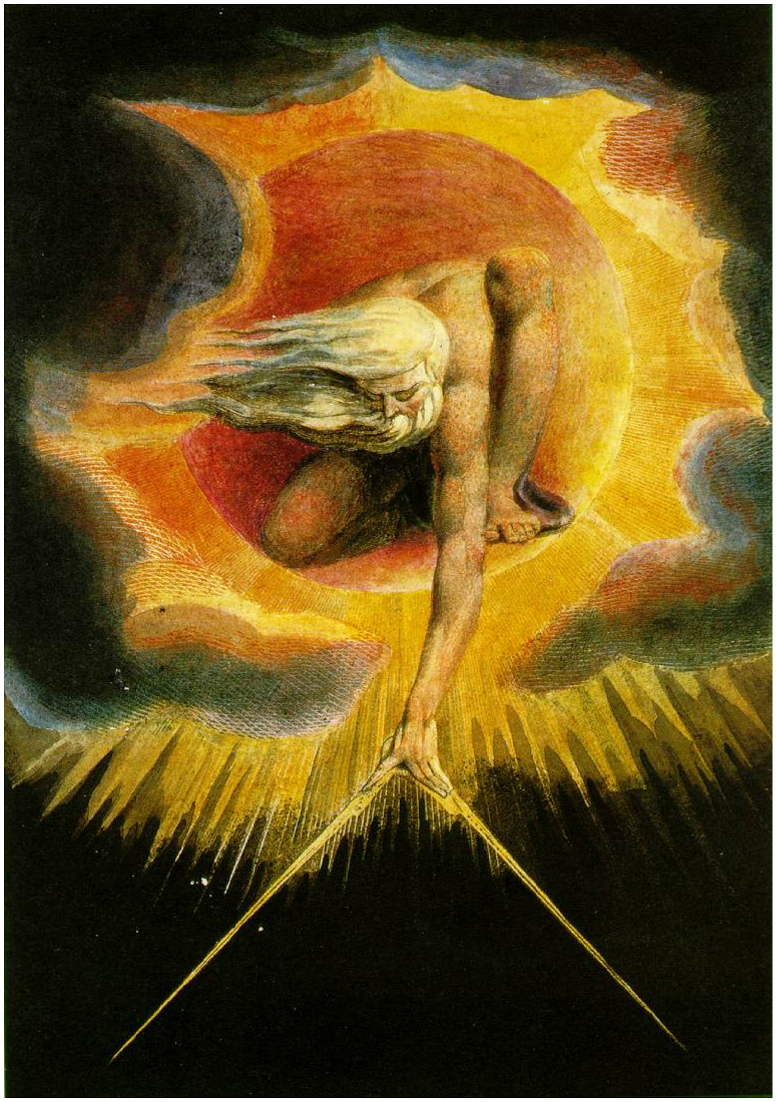

Figure 0.0.1: The Ancient of Days. “If the doors of perception were cleansed, everything would appear to man as it is: Infinite.” William Blake (1757-1827)