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Accelerated and Preconditioned Refinement of Gauss-Seidel Method for Uplink Signal Detection in 5G Massive MIMO Systems

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***Abstract*—Massive MIMO (Multiple Input Multiple Output) is a sub-6 GHz wireless access technology, and it is one of the key enabling technologies for the next-generation wireless networks. Massive MIMO groups together antennas at both transmitter and the receiver and achieves high spectral and energy efficiency. Although massive MIMO offers immense benefits, it has to overcome some implementation challenges to make this system a reality. One of the fundamental issues in the massive MIMO system is the uplink signal detection, which becomes inefficient and computationally complex with a large number of antennas. In this paper, we propose an accelerated and preconditioned refinement of the Gauss-Seidel method for uplink signal detection in massive MIMO systems. We further increase the convergence rate of the proposed algorithm by applying Jacobi preconditioner and introducing a novel matrix initialization scheme. The simulations results show that the proposed algorithm achieves near-optimal Bit Error Rate (BER) performance, and it is computationally efficient, compared to conventional detection algorithms. Additionally, we also propose a novel hardware architecture for our proposed algorithm, which helps to identify the required physical components and their interrelationships.**

***Index Terms*—— 5G, bit error rate, complexity, Gauss-Seidel, hardware architecture, hardware efficiency, Massive MIMO, signal detection, spectral efficiency**

I. Introduction

There has been a tremendous surge in wireless data traffic over the past few years due to globalization. It has been predicted that wireless data traffic will be more than 131 exabytes per month by the end of 2024, and the majority of this data traffic will be carried by 5G networks [1]. Also, more than 90 percent of the wireless data traffic will come from cell phones [2]. With the rapid evolution of the Internet of Things (IoT) and Machine-to-Machine (M2M) communication, the growth in data traffic is expected to increase much faster. The fifth-generation 5G network has to address the increasing demands in wireless data traffic with improved speed, reliability, and accuracy. Massive MIMO is the most gripping sub-6 GHz wireless access technology to deliver the needs of 5G networks. Massive MIMO is considered one of the key enabling technology for 5G networks. It is an advancement of the contemporary MIMO systems which uses hundreds or even thousands of antennas at the base station to serve tens of users simultaneously [3]- [6]. Massive MIMO provides a vast improvement in spectral and energy efficiency. Some other advantages of massive MIMO are huge array gain, robustness to interference and jamming, low power consumption, low latency, and enhanced security. Various implementation issues have to be addressed before Massive MIMO can be implemented in 5G networks. One of the fundamental problems in massive MIMO deployment for 5G is uplink signal detection, which becomes computationally complex and inefficient with a large number of antennas. Fig. 1 shows a massive MIMO uplink and downlink system.

There has been numerous research to find an optimal detector for uplink signal detection in massive MIMO systems. The non-linear detectors like Successive Interference Cancellation (SIC) and Sphere Decoder (SD) provide optimal performance. However, the computational complexity increases with the large number of antennas, which makes them infeasible to use

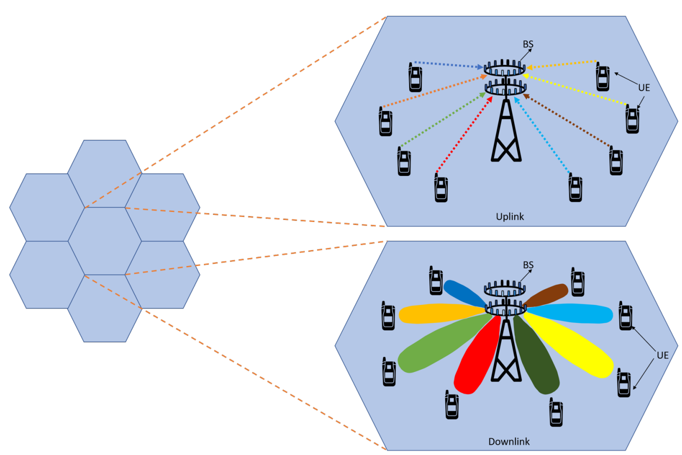

Fig. 1. Massive MIMO.

for Massive MIMO systems [7], [8]. Several linear detectors like ML, ZF, and MMSE have been considered for massive MIMO [9] - [11]. These linear detection methods involve complicated matrix inversion, which makes them computationally inefficient for a large number of antennas. To reduce the computational complexity, Neumann Series Approximation (NSA) [12], Jacobi Iterative [13], and Richardson method [14] were proposed but complexity was slightly reduced. Linear methods such as Gauss-Seidel (GS) [15] and Conjugate Gradient (CG) [16] were also considered for Massive MIMO, but the optimal performance was not achieved. In this paper, we propose a novel Accelerated and Preconditioned Refinement of Gauss-Seidel (APRGS) algorithm for uplink signal detection in massive MIMO systems. The proposed algorithm improves the Bit Error Rate (BER) performance and reduces computational complexity. The acceleration and preconditioning method used in the algorithm improves the convergence rate of the algorithm. A novel initialization scheme has been proposed for the acceleration of the algorithm, and the simplest Jacobi preconditioner has been used for preconditioning. Additionally, we have proposed a hardware architecture for the proposed APGRS algorithm to identify the required physical components and their interrelationships.

The rest of the paper is organized as follows: Section II presents the system model used for the simulation, Section III presents the proposed uplink signal detection algorithm, and Section IV presents the hardware architecture for the proposed algorithm. The simulation results are analyzed in section V, and finally, section VI concludes the paper summarizing the key ideas.

II. System Model

We have considered a Massive MIMO uplink system equipped with M antennas at the base station and simultaneously communicating with N (M $\gg$ N) single-antenna users, as shown in Fig.2. A user may have more than one antenna, but for simplicity of the simulation, we have assumed that the user has a single antenna. The Rayleigh fading channel with perfect Channel State Information (CSI) is considered between the user and the base station. Each of N users encodes their own bit-stream and thus encoded bit-stream is mapped into constellation point in the finite alphabet set $\mathcal{O}$ as BPSK (binary phase-shift keying), QPSK (quadrature phase-shift keying) or QAM (quadrature amplitude modulation) [17]. A sufficiently long cyclic prefix is assumed, which helps to mitigate the effect of Inter-Symbol Interference (ISI) [18]. The signal transmitted by the user is $x \in \mathbb{C}^N$, and the signal received at the base station during uplink is given as :

$$y = Hx + n \tag{1}$$

where, $y \in \mathbb{C}^M$ is the signal received at the base station, H is the channel vector between the user terminal and the base station,and elements of $H \in \mathbb{C}^{M\times N}$ are independent and identically distributed (i.i.d) with zero mean and unit variance, i.e., $H \sim \mathcal{CN}(0, 1)$. n is the Additive White Gaussian Noise (AWGN) and each element of $n \in \mathbb{C}^M$ is i.i.d with zero mean and finite variance, i.e., $n \sim \mathcal{CN}(0, \sigma^2 I)$. The Minimum Mean Square Error (MMSE) detector to estimate the transmitted signal x is given by:

$$x = (H^H H + \sigma^2 I)^{-1} H^H y \tag{2}$$

Equation (2) can be represented as:

$$x = A^{-1} Y \tag{3}$$

where, $A = (H^H H+\sigma^2 I)$ and $Y = H^H y$. In order to avoid the complicated matrix inversion,to reduce the computational complexity of the system, (3) can be interpreted as linear equation:

$$Y = Ax \tag{4}$$

Let us consider splitting of A as:

$$A = D - L - U \tag{5}$$

where $-L$ is a lower triangular matrix, D is a diagonal matrix, and $-U$ is an upper triangular matrix. The conventional GS method for solving (5) is given by:

$$x_{k+1} = (D - L)^{-1}(Y + Ux_k) \tag{6}$$

where, x_k is estimated transmitted signal at k_{th} iteration. Equation (6) converges for any initial value of x_o, if H is a Hermitian positive definite.Using (5) and (6), received signal Y can be written as:

$$\begin{aligned} Y &= (D - L - U)x \\ (D - L)x &= Y + Ux \\ (D - L)x &= Y + (D - L - A)x \\ (D - L)x &= Y + (D - L)X - Ax \\ x &= x + (D - L)^{-1}(Y - Ax) \end{aligned}$$

The iterative refinement of the above equation in the matrix form is given as:

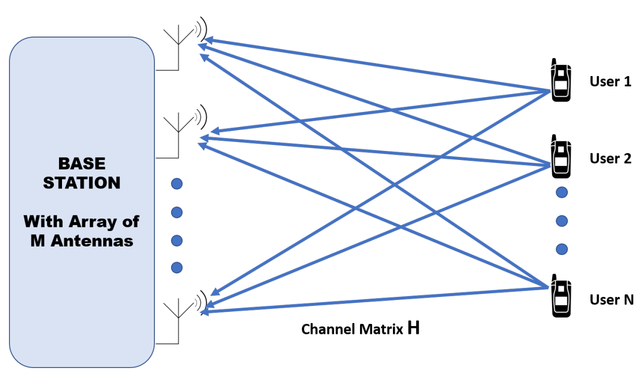

Fig. 2. System model for uplink massive MIMO systems.

$$x_{k+1} = x_{k+1} + (D - L)^{-1}(Y - Ax_{k+1}) \tag{7}$$

Now, we update (7) using the value of x_{k+1} obtained in (6) as:

$$x_{k+1} = (D-L)^{-1}(Y+Ux_k)+(D-L)^{-1}(Y-Ax_{k+1}) \tag{8}$$

The refinement of the GS method can be obtained from (8) as given in [19]:

$$x_{k+1} = [(D-L)^{-1}U]^2x_k+[I+(D-L)^{-1}U](D-L)^{-1}Y \tag{9}$$

The convergence of this refinement method is presented in [19]. To accelerate the proposed algorithm, we applied Jacobi preconditioning, and to perform the preconditioning, the initial system presented in (4) is transformed into a preconditioned system as :

$$P^{-1}Ax = P^{-1}Y \tag{10}$$

where, $P \approx A$ is a non-singular, symmetric, and positive definite matrix. The preconditioner should approximate A^{-1} such that $||I - P^{-1}A|| < 1$ [20]. In this paper, we have used Jacobi preconditioner,which is given as $P = diagonal(A)$. The preconditioning operation with diagonal calculations are computationally efficient and will reduce the overall complexity of the proposed algorithm. To further accelerate the proposed algorithm, we introduce a novel initialization matrix to initialize user signal x. In the conventional GS method, a set of zero vector is used as the initial solution x_0. However, this initial solution is not near the final solution, which reduces the convergence rate and increases computational complexity. Since we have a limited number of iterations, the faster convergence of the solution is highly significant. Our proposed initial solution is dependent on the received vector, lower triangular matrix, diagonal matrix, and the upper triangular matrix and is approximated as:

$$x_0 = (D - L - U)^{-1}Y \tag{11}$$

III. Proposed Algorithm for Uplink 5G Massive MIMO Systems

This section presents the proposed Accelerated and Preconditioned Refinement of the Gauss-Seidel algorithm used for uplink detection in 5G massive MIMO systems. The acceleration and precondition methods make the algorithm converge faster. The faster convergence of the algorithm means there will be less iteration required, and the overall computational complexity of the algorithm will be reduced. Additionally, a hardware architecture for the proposed algorithm is presented to identify the required physical components. The inputs to the algorithm are y, H, and N_0. All the matrix-vector multiplications are computed outside of the loop during preprocessing to reduce the overall computational complexity of the algorithm. The regularized gramian matrix, $A = \left(H^H * H + \ \gamma * I\right)$ is computed where the regularization parameter γ is the ratio

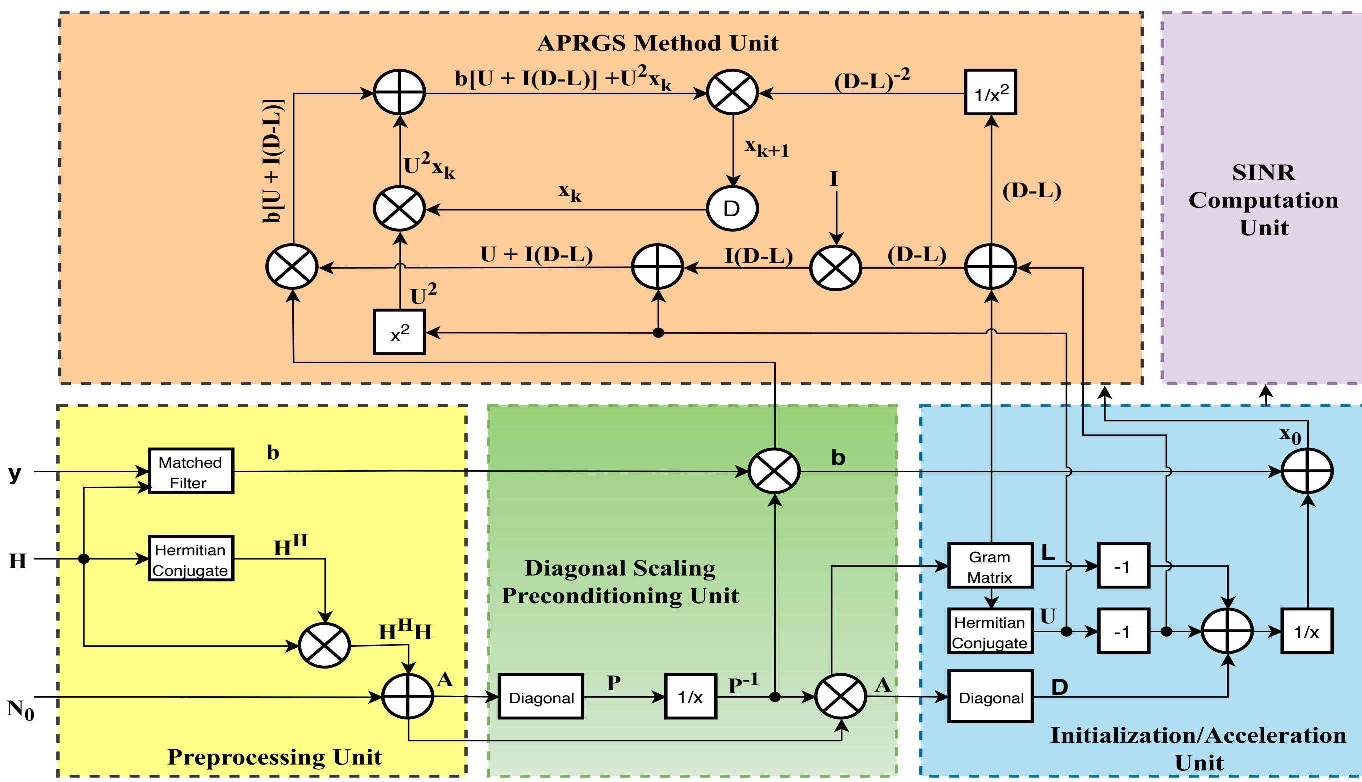

Fig. 3. Proposed hardware architecture for the APRGS algorithm for uplink signal detection in Massive MIMO systems.

Algorithm 1 Proposed APRGS Detection Method

Inputs: y, H, N_0

Preprocessing:

$A = \left(H^H * H + \ \gamma * I\right)$

$b = \left(H^H * Y\right)$

$P = diag(A)$

$\hat{A} = P^{-1}A$

$\hat{b} = P^{-1}b$

$D = diag(\hat{A})$

$U = Upper(\hat{A})$

$L = Lower(\hat{A})$

$x_0 = (D - U - L)^{-1}\hat{b}$

for $k = 1$ *to* k_{max} **do**

  $temp1 = [(D - L)^{-1}U]^2 x_k$

  $temp2 = [I + (D - L)^{-1}U](D - L)^{-1}\hat{b}$

  $x_{k+1} = temp1 + temp2$

End for

Output: x_k

of the variance of complex noise per receive antenna and average symbol energy. To accelerate the proposed algorithm, the Jacobi Preconditioning is applied to the gram matrix before computing the upper triangular, lower triangular, and diagonal matrix required for the x-update of the proposed APRGS method. The user signal, x_k, is then initialized with novel proposed initialization matrix x_0 derived in (11), which further increases the convergence of the proposed algorithm. Then, the refinement of the GS method, derived in (9) is computed during iteration to detect the user signal x_k. Since the proposed method does not require the complicated matrix inversion, it is computationally efficient than the conventional MIMO detection algorithms like MMSE and ZF.

IV. Proposed Hardware Architecture

The proposed hardware architecture for the APRGS algorithm for uplink detection in 5G massive MIMO systems is shown in Fig. 3. The proposed hardware architecture consists of five units; a preprocessing unit, diagonal scaling preconditioning unit (Jacobi preconditioning unit), initialization or acceleration unit, APRGS method unit, and SINR computation unit. The inputs to the architecture are y, H, and N_0. The preprocessing unit computes the gram matrix and the matrix-vector multiplication to reduce the computational complexity during the iteration. The diagonal scaling preconditioning unit and acceleration unit increases the convergence rate of the proposed algorithm, which further reduces the computational complexity. The APRGS method unit computes the update of user signal x and detects the signal transmitted by the user. Finally, the SINR computation unit computes the SINR for the detected user signal.

V. Simulation Results

In this section, we present the simulations results and evaluate the performance of the proposed APRGS method by comparing it with traditional massive MIMO uplink detection algorithms like Conjugate Gradient (CG), Gauss-Seidel (GS), and optimal Maximum Likelihood Detector (ML).

A massive MIMO system with a large number of the base station antenna is considered, and these antennas are assumed to be simultaneously communicating with 16 users having a single antenna. Table I shows all the simulation parameters used for conducting this simulation, and Fig. 4 presents the required simulation steps. The system bandwidth of 5 MHz, a signal variance of 2, and noise variance depending upon the current SNR value are considered. The most widely used Rayleigh fading channel is assumed between the user and the base station, where the channel gain from any base station antenna to a user terminal is described by a zero-mean circularly symmetric Gaussian random variable. The modulation schemes, BPSK, QPSK, 16-QAM, and 64-QAM, are used to compare the performance of different algorithms. All the simulations are conducted in Matlab under Mac OS, with 3.4 GHz Intel Core i7 processor and 10GB of RAM.

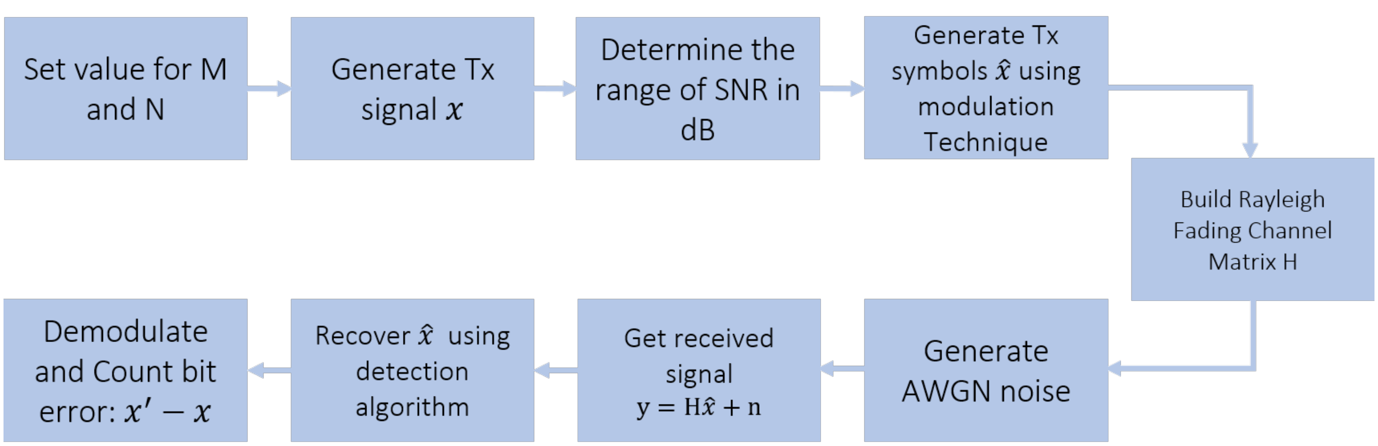

Fig. 4. Some general steps required for conducting the simulation.

A. *BER Performance*

We assess the BER performance of the proposed APRGS algorithm compared with various conventional detection algorithms over a range of SNR with 32 and 64 antennas at the base station, respectively. Fig. 5 shows that with 32 antennas at the base station, the performance of the proposed APRGS algorithm outperforms the traditional GS and CG algorithm over the range of SNR. At BER = $10^{-}3$, the proposed method achieved 3dB gain when compared to the traditional GS method. Fig. 6 shows the BER performance of the proposed algorithm with 64 antennas at the base station. The BER performance for all the algorithms improved with the higher number of antennas, and the performance of the proposed algorithm was found to be near-optimal. At BER = $10^{-}3$, the proposed algorithm achieved 1dB gain when compared to the ML method and 2.5 dB gain when compared to the traditional GS method. The proposed algorithm outperforms all the traditional algorithms and achieved near-optimal BER performance.

Fig.7 shows the performance of the proposed APRGS algorithm with an increasing number of antennas at the base station with 16 users and the 16QAM modulation. The simulation results show that the performance of the proposed algorithm increases with the number of base station antenna, and at BER $=10^{-}3$, increasing base station antenna from 32 to 256 gives almost 13dB gain. Thus, increasing the number of the antenna at the base station boosts the performance of the proposed algorithm. Fig. 8 shows the performance of the proposed APRGS algorithm with different modulation schemes. The results through the simulation show that the performance of the proposed algorithm decreases with an increasing modulation order. However, higher modulation schemes can be used in situations where a high data rate is considered more crucial than error performance.

TABLE I
SIMULATION PARAMETERS

Parameter	**Value**
System Bandwidth	20 MHz
Number of Users	16
Number of Antenna at Base Station	64
Noise Variance	Controlled by SNR
Signal Variance	2
Signal to Noise Ratio	5dB - 25dB
Channel Model	Uncorrelated Rayleigh Fading
Modulation Scheme	BPSK, QPSK, 16QAM, 64QAM

B. *Complexity Analysis*

The measurement of computational complexity is one integral part of evaluating the performance of the massive MIMO systems. Since the proposed APRGS algorithm is an iterative algorithm, the computational complexity of the

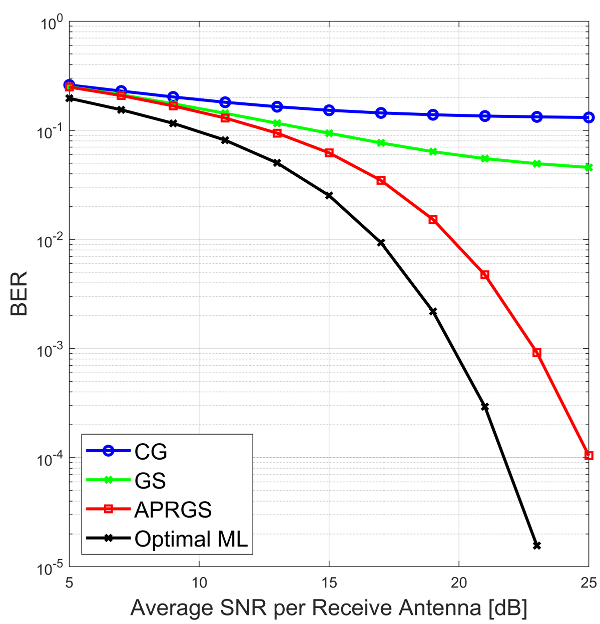

Fig. 5. Comparison of bit error rate (BER) performance of proposed APRGS algorithm with 32 base station antenna (M), 16 users (N) and using 64QAM modulation.

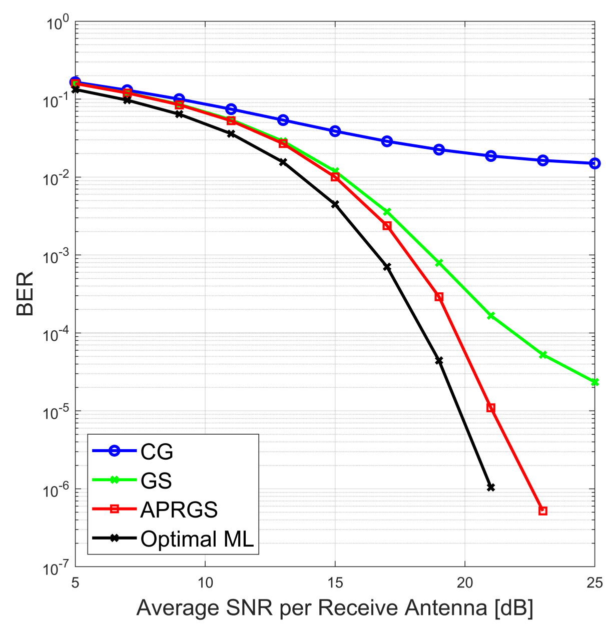

Fig. 6. Comparison of bit error rate (BER) performance of proposed APRGS algorithm with 64 base station antenna (M),16 users (N) and using 64QAM modulation.

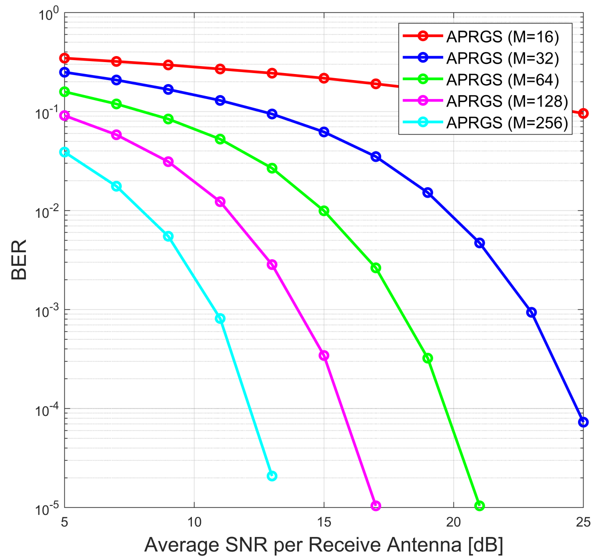

Fig. 7. Comparison of bit error rate (BER) performance of proposed APRGS algorithm with different number of base station antenna (M) with 16 users (N) and using 16QAM modulation.

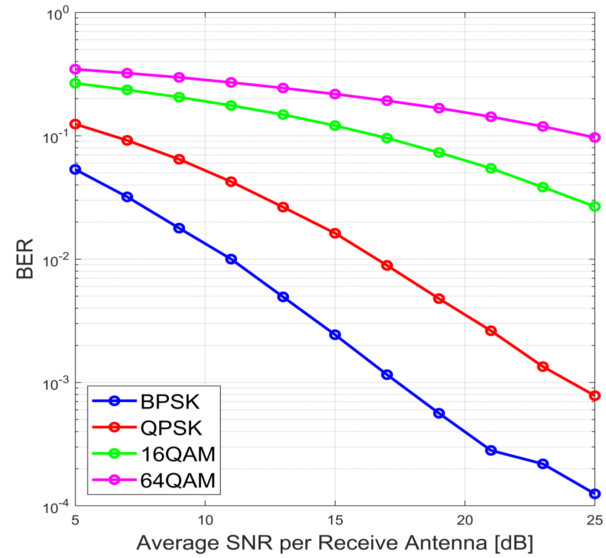

Fig. 8. Comparison of bit error rate (BER) performance of proposed APRGS algorithm with different modulation schemes (BPSK, QPSK, 16QAM and 64QAM), with 16 base station antenna (M) and 16 users (N).

algorithms largely depends upon the number of iterations. We computed the big O complexity of the proposed APRGS algorithm and compared it with the computational complexity of conventional detection algorithms. The big O notation describes the performance or complexity of the algorithm, and it depicts the worst-case scenario explicitly [21]. We computed the complexity of the proposed algorithm, and it was found to be in the order of O(kN^2), where k is the number of iterations. Since relatively fewer iterations were required for the algorithm to converge, the computational complexity of the proposed algorithm and the conventional algorithms like GS and CG were similar and in the order of O(kN^2). The computational complexity of traditional massive MIMO detectors like MMSE and ZF are in order of O(MN^2) [22]. As the number of antennas at bases station is much larger than the number of iterations ($M \gg k$), we can conclude that the proposed algorithm has better performance than traditional MMSE and ZF algorithms.

VI. Conclusion

In this paper, we proposed a novel user signal detection algorithm for uplink massive MIMO systems. We applied the Jacobi preconditioner and introduced a novel initialization scheme to improve the convergence rate of the proposed algorithm. Additionally, we proposed a hardware architecture for the proposed APRGS algorithm to identify the required physical components and their interrelationship. The simulations results show that the proposed APRGS method achieves near-optimal BER performance and outperforms the conventional massive MIMO detection algorithms. Also, in terms of computational complexity, the proposed algorithm performed better than conventional detection algorithms. The simulation results also show that the performance of the proposed algorithm improved with the higher number of base station antenna, and the performance gets reduced with higher modulation order.

Thus, the proposed algorithm provides a good trade-off between BER performance and the computational complexity and is suitable for uplink signal detection in massive MIMO systems. In the future, it would be interesting to test the experiment by adding several realistic network parameters and with more number of antennas at the base station and the user terminal.

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