An Efficient and Fast-convergent Detector for 5G and Beyond Massive MIMO Systems

Robin Chataut
Computer Science Department
Fitchburg State University
Fitchburg, MA, 01420, USA
rchataut@fitchburgstate.edu

Robert Akl
College of Engineering
University of North Texas
Denton, Texas, 76207, USA
Robert.Akl@unt.edu

Utpal Kumar Dey
College of Engineering
University of North Texas
Denton, Texas, 76207, USA
Utpal-KumarDey@my.unt.edu

***Abstract*—Massive MIMO (multiple-input multiple-output) is a sub-6GHz wireless access technology that can provide high spectral and energy efficiency and is considered as one of the key enabling technology for 5G, 6G, and beyond networks. The user signal detection during the uplink is one of the major challenges in massive MIMO systems due to the large number of antennas working together at both the user terminal and the base station. The current iterative methods do not offer great efficiency, and the conventional matrix inversion methods are computationally complex due to the large antennas involved in massive MIMO systems. In this paper, we propose a fast and efficient preconditioned iterative method by introducing a preconditioner based on ICF (Incomplete Cholesky Factorization). Additionally, we introduce a novel matrix initializer to further improve the convergence of the proposed algorithm. The numerical results, when compared to conventional methods, show that the proposed algorithm provides better error performance with optimal computational complexity.**

***Index Terms*—— Massive MIMO, 5G, 6G, beyond 5G, spectral efficiency signal detection, Gauss-Seidel, Cholesky factorization, hardware architecture**

I. INTRODUCTION

Over the past few years, wireless data traffic has increased tremendously with new technological innovations. Not just the cellular broadband, the innovation such as Internet of Things (IoT), Vehicle-to-vehicle (V2V) and Vehicle-to-everything (V2X) communication, virtual reality (VR), augmented reality (AR), mixed reality (MR), mobile cloud, and smart cities are also contributing to this increasing data traffic. A recent Ericsson mobility report has shown that the global mobile data traffic will be around 300 exabytes per month in 2026, which is almost four times the data that is generated today in 2021 [1]. The future generation networks (5G, beyond 5G, and 6G) are expected to accommodate this colossal growth in wireless data traffic. The 5G itself will accommodate more than half of the world's mobile data traffic in the next few years.

MIMO technology is widely adopted these days in many communication standards that include HSPA+ (3G), WiMAX, 4G Long Term Evolution (4G LTE), LTE-Advanced, IEEE 802.11n (Wi-Fi), and IEEE 802.11ac (Wi-Fi). The conventional MIMO technology has almost reached its throughput limits to accommodate this exponential growth in wireless traffic. Massive MIMO has become a promising technology to handle a large amount of data transmission over wireless networks [2], [3]. Massive MIMO is a new wireless access technology that uses a large number of antennas at the base station and provides high energy and spectral efficiency [4]–[7]. This technology can serve tens of users simultaneously with these hundreds and thousands of antenna terminals at the base station. A massive MIMO uplink and downlink system is shown in Fig. 1. However, the main challenge is the user signal detection at the base station, which becomes inefficient and computationally complex with large number of antenna terminals. The conventional signal detection algorithms that are generally used for MIMO systems are incapable of providing optimal performance with acceptable computational complexity when we have a system with these many antenna terminals.

A. Relevant Prior Art

Massive MIMO uplink signal detection is one of the major challenges in massive MIMO systems. With more antenna terminals, error performance degrades if the detection method is inefficient and computational complexity increases. In addition, unwanted interference is created due to the superimposition of the user signal at the base station, which further degrades the performance. Thus, an efficient and low complexity algorithm for signal detection is imminent for massive MIMO system implementation.

Several linear and non-linear methods have been proposed in recent years. Linear detectors such as Zero-Forcing (ZF), Minimum Mean Square Error (MMSE), and Maximum Likelihood (ML) provide optimal error performance. Although these linear detectors provide optimal error performance, these detectors, when used in massive MIMO systems, yield very high computational complexity due to the involvement of matrix inversion [8]–[10]. The non-linear detectors like Successive Interference Cancellation (SIC), ZF-SIC, and Sphere Decoder (SD) were also used; still, the computational complexity issue was not optimal [11], [12]. Several iterative methods are proposed recently [13]–[17]. Although these iterative methods have lowered the computational complexity, optimal error performance is still not achieved. This paper presents an efficient and fast convergent algorithm to detect uplink

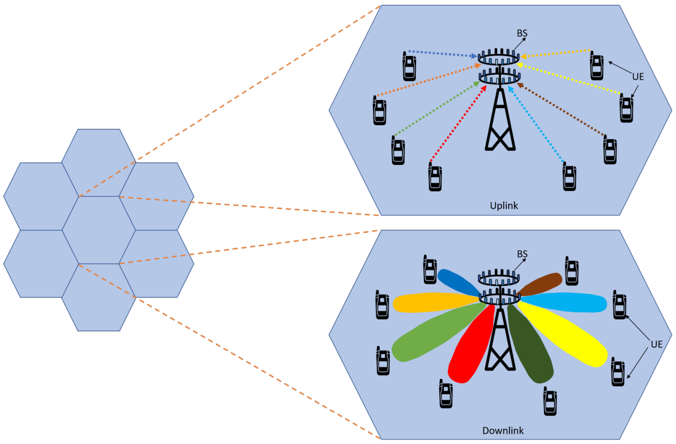


Fig. 1: A Massive MIMO uplink and downlink system.

signals in massive MIMO systems. Our proposed algorithm provides optimal error performance with acceptable computational complexity. The results from simulations verify the performance of the proposed algorithm.

B. *Contribution and Outline*

The main contributions of this paper :

1) We propose an efficient algorithm to address the uplink signal detection challenge in the massive MIMO systems.
2) We assess the proposed algorithm's error performance and computational complexity. We analyze the result by comparing it to the traditional algorithms.
3) The numerical results from the simulations show that the proposed algorithm has a better performance than the traditional massive MIMO uplink detection algorithms with optimal computational complexity.

The remainder of the paper is structured as follows: Section II describes the system model used for conducting the simulations. Section III presents the fundamentals of the proposed algorithm. The steps and parameters required for the simulation are presented in Section IV. Additionally, section IV also presents the numerical results obtained from the simulations and their analysis. Finally, Section V concludes the paper by summarizing the key ideas.

C. *Notations*

The matrices are represented with upper-case letters, and column vectors are represented with lower-case letters. The term independent and identically distributed (i.i.d.) Gaussian matrix refers to a matrix where each element is i.i.d. sampled from the normal distribution with zero mean and co-variance V. The Gaussian matrix is represented as $\mathcal{CN}(0, V)$ The matrix. The transpose, inverse, hermitian transpose of a matrix is denoted by $(.)'$, $(.)^{-1}$ and $(.)^H$ respectively. The identity matrix used is represented by I_M, which is a $M \times M$ identity matrix.

II. SYSTEM MODEL

Our system model consists of a massive MIMO base station having N antennas, serving K single antenna users simultaneously, where (N $\gg$ K). The system model is shown in Fig. 2. Each user will encode its bitstream using one of the BPSK (Binary Phase Shift Keying), 4-QAM (Quadrature Amplitude Modulation), 16-QAM, or 64-QAM modulation schemes. To remove the distortion of a signal when symbols interfere with each other, we have used a sufficiently long cyclic prefix. We assume that the magnitude of the user signal passing through the communication channel between the users and the base station will fade randomly according to a Rayleigh distribution.

The signal transmitted by the user is $x \in \mathbb{C}^K$, H channel vector between the user and the base station, where elements of $H \in \mathbb{C}^{N \times K}$ are independent and identically distributed with unit variance, and zero mean. w is the added circularly symmetric complex white Gaussian noise with zero mean and random variance. The base station receives the users signal simultaneously and can be represented as:

$$y = Hx + w \tag{1}$$

where, $y \in \mathbb{C}^N$ is the signal received at the base station.

ML detector is considered as optimal detector in terms of bit error rate performance. The estimate of the user signal is given as:

$$x = G^{-1}B \tag{2}$$

where, $G = (H^H H + \sigma^2 I)$ and $B = H^H y$ is regularized gramian matrix and σ is the ratio of the variance of noise per antenna and symbol energy. Unfortunately, this detector requires prohibitive complexity due to the large number of users requiring complex matrix inversion.

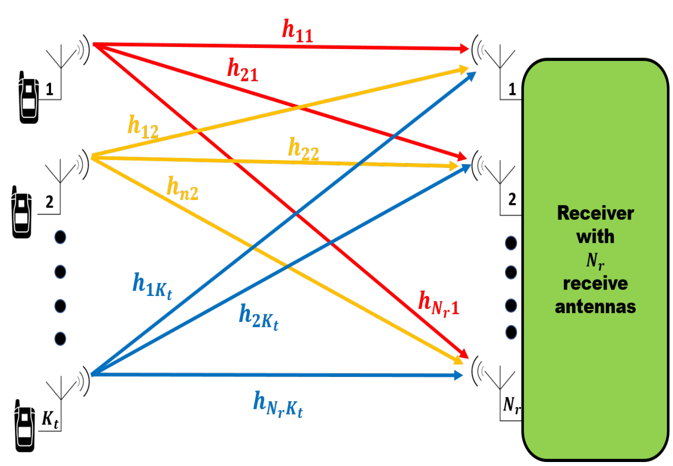


Fig. 2: System Model with N base station antenna serving K users.

III. PROPOSED ALGORITHM FOR UPLINK 5G MASSIVE MIMO SYSTEMS

This section presents the proposed algorithm. The proposed algorithm uses preconditioner and acceleration methods to improve the convergence of the algorithm. The number of iterations required is significantly reduced if we can make the algorithm converge faster, which ultimately will reduce the overall computational complexity of the algorithm. The inputs required are y, H, σ^2, K, N. We will compute most of the complex matrix-vector product outside of the loop iteration to avoid it computing multiple times during the iteration. Outside of the loop, it will be computed only once, which aids in reducing the computational complexity. To reduce the computational complexity of the overall system, (2) can be interpreted as a linear equation:

$$B = Gx \tag{3}$$

We can now split matrix G to derive the equation for the Gauss Siedel method. If D is a diagonal matrix, and $-U$ is an upper triangular matrix, and $-L$ is a lower triangular matrix, the matrix G can be split as:

$$G = D - U - L \tag{4}$$

We can now use GS method to solve (4):

$$x_{i+1} = \frac{(B + Ux_i)}{(D - L)} \tag{5}$$

where, x_i is estimated signal sent during the i_{th} iteration. Now, received signal Y can be represented using (3) and (4) as :

$$\begin{aligned}
B &= (D - U - L)x_{i+1} \\
B &= Dx_{i+1} - Ux_{i+1} - Lx_{i+1} \\
x_{i+1} &= \frac{B + Ux_i}{(D - L)} \\
x_{i+1} &= \frac{B + (D - L - G)x_i}{(D - L)} \\
x_{i+1} &= \frac{B + (D - L)x_i - Gx_i}{(D - L)} \\
x_{i+1} &= x_i + \frac{B - Gx_i}{(D - L)}
\end{aligned}$$

Refinement of the above equation in the matrix form is given as:

$$x_{i+1} = x_i + \frac{B - Gx_i}{(D - L)} \tag{6}$$

Then, the refinement of the GS method, derived in (7) is computed during the iteration to detect the user signal x_i. The refinement of the GS method and its convergence can be obtained from (6) and is defined in [18]:

$$x_{i+1} = \left[\frac{U}{(D - L)}\right]^2 x_i + \left[I + \frac{U}{(D - L)}\right]\left[\frac{B}{(D - L)}\right] \tag{7}$$

To further improve the acceleration of the algorithm, we introduce a preconditioner based on incomplete Cholesky factorization (ICF). Since we are dealing with a large sparse system, factorization is expensive and hard to store [19]. The incomplete factorization allows us to reduce the computation time and prescribe storage for each factor. These incomplete factors are used as preconditioners for GS iterative method. The ICF preconditioner used in this system is given as:

Algorithm 1 Proposed Algorithm for Massive MIMO Uplink Singal Detection

Inputs required: y, H, σ^2, K, N

Pre-processing Steps:

1: Calculate the grammian matrix: $G = \left(\sigma^2 * I\right) + (H^H * H)$
2: Change sparsity of the grammian matrix: $\hat{G} = sparse(G)$
3: Evaluate preconditioner using Cholesky Incomplete Factorization: $P_{ICF} = H' * H$
4: Compute diagonal, upper and lower triangular matrix:
$D = diagonal(\tilde{G})$
$U = upper(\tilde{G})$
$L = lower(\tilde{G})$
5: Apply preconditioner: $\tilde{B} = P_{ICF}^{-1} * \left(H^H * y\right)$
6: Estimate the initial solution: $x_0 = \frac{\tilde{B}}{(D-L-U)}$
Algorithm iteration:
7: **for** $i = 1\ to\ i_{max}$ **do**
8: $x_{i+1} = \left[\frac{U}{(D-L)}\right]^2 x_i + \left[I + \frac{U}{(D-L)}\right]\left[\frac{\tilde{B}}{(D-L)}\right]$
9: **End for**

Estimated output: x_i

$$P_{ICF} = H' * H; \tag{8}$$

where H is ”close” to L. The preconditioning operation with diagonal calculations are computationally efficient and will reduce the overall complexity of the proposed algorithm

Now, we precondition the initial system given in (3) with a symmetric, and positive definite matrix P_{ICF} and this preconditioner approximates the G^{-1} such that $||I - (P_{ICF})^{-1}A|| < 1$ [20].

$$(P_{ICF})^{-1}B = (P_{ICF})^{-1}Gx \tag{9}$$

To further improve the converge, we use an efficient initialization matrix to initialize user signal x. In general, the initial solution for GS iterative method is a set of zero vectors. But, the conventional initial solution is very far from the final solution, which requires more iterations to reach the final solution. More iteration also increases the computational complexity of the algorithm. The initial solution that we have used depends upon the previously calculated values of the lower triangular matrix, diagonal matrix, and upper triangular matrix. Thus computational complexity is not significantly increased.

$$x_0 = \frac{B}{(D - L - U)} \tag{10}$$

The step-wise summary of the proposed algorithm is presented in Algorithm 1.

IV. SIMULATION RESULTS and ANALYSIS

This section presents the simulation results to measure the performance of the proposed algorithm. The simulations were conducted under Windows OS, with a 3.4 GHz Intel Core i7 processor and 16GB of RAM. We have used various antenna configurations and modulation schemes to compare the performance of our algorithm with conventional massive MIMO detection algorithms. The ML algorithm provides the theoretical maximum performance under the optimal condition. Thus, we have used this as the benchmark to assess the performance of the proposed algorithm. To conduct the simulations, we have considered a system with 16 users having a single antenna, communicating simultaneously with the base station having a large number of antenna terminals (16 to 512). The Rayleigh fading channel is considered between the users and the base station with a system bandwidth of 20MHz and

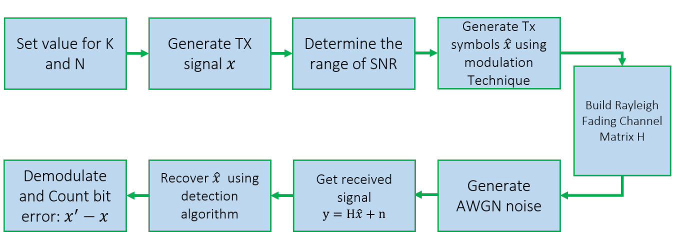


Fig. 3: General simulation steps.

TABLE I: Simulation Parameters

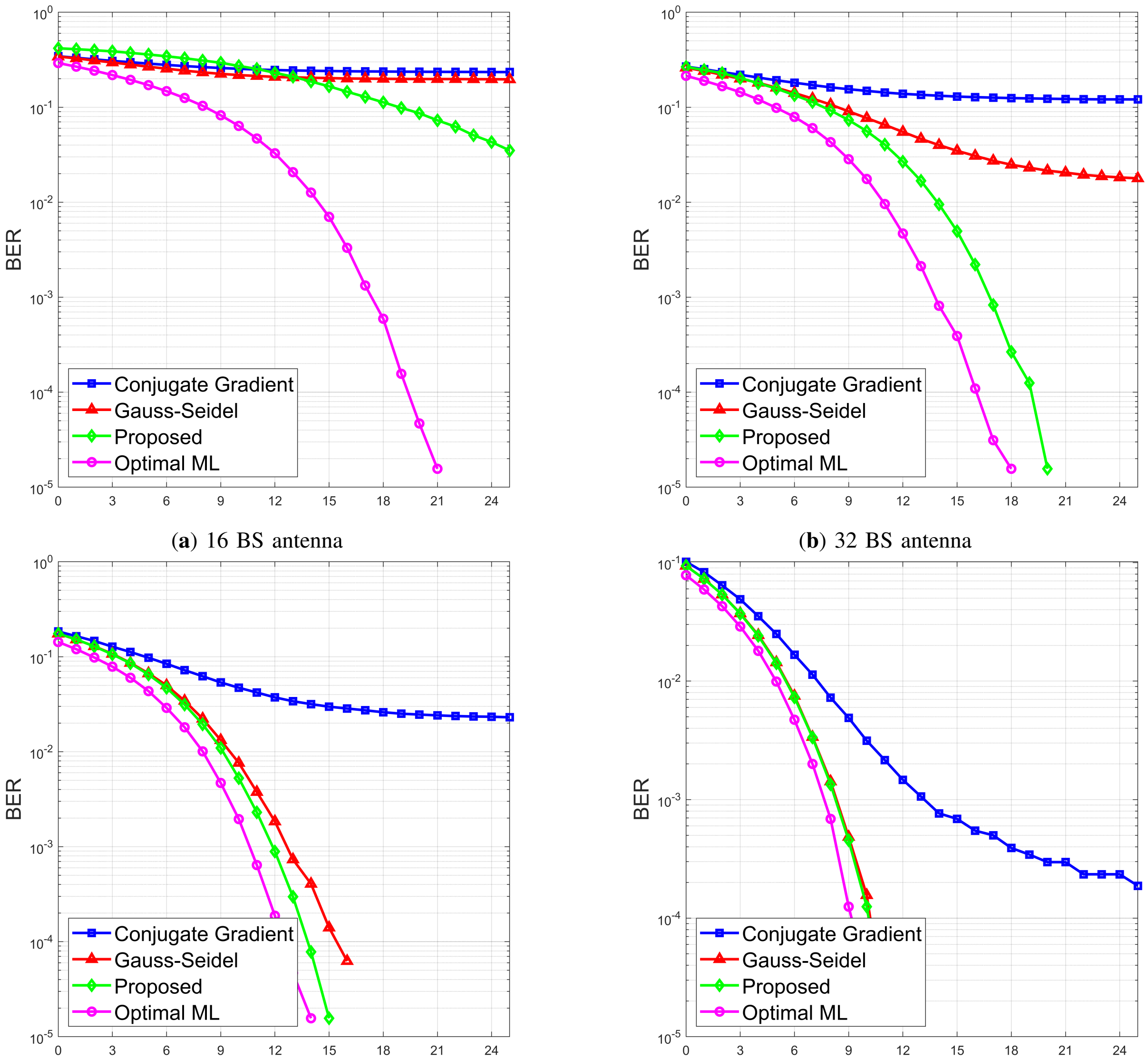
Parameter	Value
Number of Users	16
Number of Antenna at Base Station	16 to 512
Carrier Frequency	2.5 GHz
Bandwidth	20 MHz
Noise Variance	Controlled by SNR
Signal Variance	2
Signal to Noise Ratio	0 dB - 25dB
Channel Model	Uncorrelated Rayleigh Fading
Frame duration	10 ms
Slot duration	0.5 ms
Modulation Scheme	BPSK, QPSK, 16QAM, 64QAM

carrier frequency of 2.5 GHz is used. The signal-to-noise ratio range is from 0 dB to 25 dB, and a signal variance of 2 is used for the simulations. We did simulations with four modulation schemes, 64-QAM (Quadrature Amplitude Modulation), 16-QAM, QPSK (Quadrature Phase Shift Keying), and Binary Phase Shift Keying (BPSK). Fig. 3 summarizes the basic simulation steps and the Table. I summarizes the simulation parameters used for conducting the simulations.

A. BER Analysis

BER performance measure is the ‘ratio of the number of error bits transmitted to the total number of bits transmitted during a specific time’ [21]. We analyze the BER performance of the proposed algorithm and other traditional algorithms. Fig. 4 shows the performance with 16 user terminals, 16 to 128 base station antennas, and 16-QAM modulation. Fig. 4a shows the performance with 16 base station antennas. At lower SNR values, CG and GS have better error performance. However, at higher SNR values, the performance of the proposed algorithm improves notably. With 32 base station antennas as shown in Fig. 4b, the performance of the proposed algorithm exceeds the traditional algorithms over the range of SNR. At $BER = 10^{-3}$, the proposed algorithm achieves a 2.9 dB gain over the traditional GS algorithm. With 64 antennas at the base station, as shown in Fig. 4c, the performance of the proposed algorithm improves substantially and is near to optimal (ML). At $BER = 10^{-3}$, the proposed algorithm is leading the GS by 1.5 dB and has far better performance than the CG. Fig. 4d shows the performance with 128 antennas. With 128 antennas at the base station, performance is much closer to the optimal, and the performance of all the algorithms has improved.

We then assessed the performance of the proposed algorithm with various base station antenna configurations as shown in Fig. 5. This experiment was simulated with 16 users (N) and



(**a**) 16 BS antenna

(**b**) 32 BS antenna

(c) 64 BS antenna

(**d**) 128 BS antenna

Fig. 4: BER performance with 16 user terminals, 16 to 128 base station antennas, and 16-QAM modulation.

using 16-QAM modulation. The results from the simulation show that the increase in the number of antennas improves the BER performance of the system. At BER $=10^-3$, we get 5 dB gain when the number of base station antenna is changed from 32 to 64. Further, increasing the number of the antenna to 128 gives 4 dB additional gain. Almost 15 dB gain is achieved by 512 antennas system compared to the 32 antennas system. We also assessed the performance by changing the modulation order. Fig. 6 shows the BER performance of the proposed algorithm with different modulation schemes (BPSK, QPSK, 16QAM, and 64QAM). This experiment was conducted with 16 users (N) and using 16-QAM modulation. The results from the simulation show that the error performance of the proposed algorithm decreases with increasing modulation order. However, with higher modulation order, we can get a higher data rate. Thus, the selection of a modulation scheme is a very crucial design factor in massive MIMO systems, and it depends on the type of application we are using it for. There should always be a good balance between data rate and error performance.

B. Complexity Analysis

For any algorithm, the evaluation of computational complexity is essential. We have used Big $\mathcal{O}$ notation to compare the complexity of the proposed algorithm. Big $\mathcal{O}$ complexity is widely used in calculating the complexity of iterating algorithms [22]–[24], and it represents the worst-case scenario. The complexity of the proposed algorithm was found to be in the order of $\mathcal{O}(K^2 j)$, where K is the number of single antennas users, and j is the number of iterations. The proposed algorithm converges in very few iterations (~ 3). Compared to this, traditional linear algorithms such as ZF and MMSE

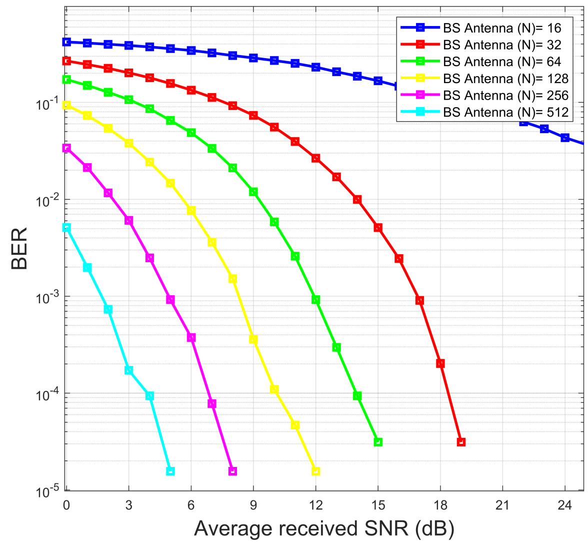


Fig. 5: BER performance of proposed algorithm with various BS antenna configuration. This experiment was conducted with with 16 users (N) and using 16-QAM modulation.

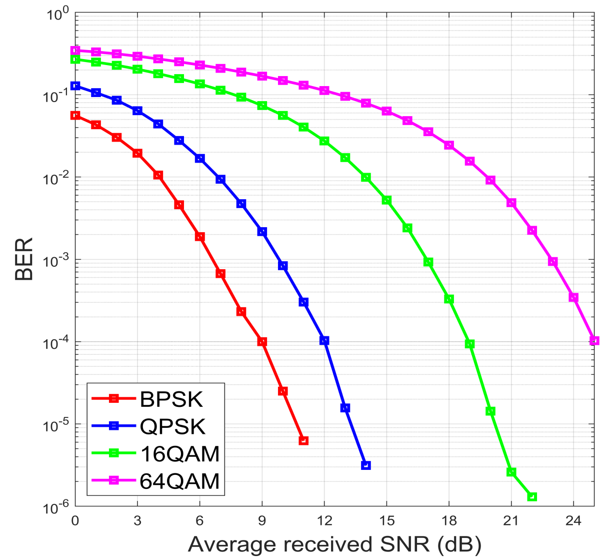


Fig. 6: BER performance of proposed algorithm with different modulation schemes (BPSK, QPSK, 16QAM and 64QAM). This experiment was conducted with with 16 users (N) and using 16-QAM modulation.

have higher complexity, which is in the order of $\mathcal{O}(NK^2)$ and traditional iterative algorithms such as CG and GS have the same complexity as the proposed algorithm in the order of $\mathcal{O}(K^2 j)$.

V. CONCLUSION

Massive MIMO is one of the emerging and key enabling technology for 5G and Beyond networks. The user signal detection during the uplink is one of the critical issues in this technology, and the need for an efficient algorithm is imminent to realize the promises of 5G and beyond networks. In this paper, we proposed an uplink signal detection algorithm for massive MIMO systems by introducing a preconditioner based on Incomplete Cholesky Factorization and a novel matrix initializer to improve the convergence and efficiency of the algorithm. The results achieved from simulations show that the proposed algorithm has better error performance when compared to the traditional iterative signal detection algorithm with optimal computational complexity. Furthermore, the performance of the proposed algorithm significantly improved when the number of base station antennas was increased, and lower modulation order was used. Thus for the realization of massive MIMO networks, the proposed algorithm provides a good trade-off between the error performance and computational complexity. In the future, we plan to the simulations with more users have multiple antennas. It would be interesting to incorporate some more realistic parameters in the channel model. The use of machine learning and deep learning in massive MIMO signal detection would also be a fascinating area to investigate. Overall, there are still many challenges and questions that need to be answered before we realize the promises made by future generation networks.

REFERENCES

[1] ”Mobile data traffic outlook,”*Ericsson.com*, June-2019.n[Online]. Available:https://www.ericsson.com/en/mobility-report/reports/june-2019/mobile-data-traffic-outlook. [Accessed: 24-July-2021].

[2] E. Telatar, ”Capacity of multi-antenna Gaussian channels,” *European Transactions on Telecommunications*, vol. 10, no. 6, pp. 585–596, Jun. 2001.

[3] D. Tse and P. Viswanath, *Fundamentals of wireless communication*, Cambridge Uni. Press, New York, NY, USA, 2005.

[4] Chataut, R.; Akl, R. Optimal pilot reuse factor based on user environments in 5G Massive MIMO. *Proceedings of the 2018 IEEE 8th Annual Computing and Communication Workshop and Conference (CCWC)*, Las Vegas, NV, USA, 8–10 January 2018; pp. 845–851.

[5] Alsharif, M.H.; Nordin, R.; Abdullah, N.F.; Kelechi, A.H. How to make key 5G wireless technologies environmental friendly: A review. *Trans. Emerg. Telecommun. Technol.* 2018, 29, e3254.

[6] Larsson, E.G., Tufvesson, F.; Edfors, O.; Marzetta, T.L. Massive MIMO for Next Generation Wireless Systems. *IEEE Commun. Mag*. 2014, 52, 186–195.

[7] 2. Akhlaghpasand, H.; Björnson, E.; Razavizadeh, S.M. Jamming Suppression in Massive MIMO Systems. *IEEE Trans. Circuits Syst. II Express Briefs*, 2020, 67, 182–186.

[8] Ghacham, S.; Benjillali, M.; Guennoun, Z. Low-complexity detection for massive MIMO systems over correlated Rician fading. *In Proceedings of the 2017 13th International Wireless Communications and Mobile Computing Conference (IWCMC)*, Valencia, Spain, 26–30 June 2017; pp. 1677–1682.

[9] Liang, Y.C.; Pan, G.M.; Bai, Z.D. Asymptotic performance of MMSE receivers for large systems using random matrix theory. *IEEE Trans. Inf. Theory*, 2007, 53, 4173–4190.

[10] A. Trimeche, N. Boukid, A. Sakly, and A. Mtibaa, ”Performance analysis of ZF and MMSE equalizers for MIMO systems,” *7th International Conference on Design and Technology of Integrated Systems in Nanoscale Era*, 2012, pp. 1-6.

[11] Elghariani, A.; Zoltowski, M. Successive interference cancellation for large-scale MIMO OFDM. *In Proceedings of the 2015 IEEE International Conference on Electro/Information Technology (EIT)*, Dekalb, IL, USA, 21–23 May 2015; pp. 657–661.

[12] Jeon, Y.; Lee, N.; Hong, S.; Heath, R.W. One-Bit Sphere Decoding for Uplink Massive MIMO Systems With One-Bit ADCs. *IEEE Trans. Wirel. Commun.* 2018, 4509–4521.

[13] Chataut, R.; Akl, R.; Dey, U.K.; Robaei, M. SSOR Preconditioned Gauss-Seidel Detection, and Its Hardware Architecture for 5G and beyond Massive MIMO Networks. *MDPI Electronics*, 2021, 10, 578.

[14] Wu, Z.; Zhang, C.; Xue, Y.; Xu, S.; You, X. Efficient architecture for soft-output massive MIMO detection with Gauss-Seidel method. *In Proceedings of the IEEE Circuits and Systems (ISCAS)*, Montreal, QC, Canada, 22–25 May 2016; pp. 1886–1889.

[15] Gao, X.; Dai, L.; Hu, Y.; Wang, Z.; Wang, Z. Matrix Inversion-Less Signal Detection Using SOR Method for Uplink Large-Scale MIMO Systems. *In Proceedings of the IEEE Global Communications Conference*, Austin, TX, USA, 8–12 December 2014; pp. 3291–3295.

[16] B. Yin et al., "VLSI Design of Large-Scale soft-output MIMO Detection Using Conjugate Gradients," *IEEE Int. Symp. Circuits Syst.*, Melbourne, Australia, May 24–27, 2014, pp. 1498–1501.

[17] Chataut, R.; Akl, R.; Dey, U.K. Least Square Regressor Selection Based Detection for Uplink 5G Massive MIMO Systems. *In Proceedings of the 2019 IEEE 20th Wireless and Microwave Technology Conference (WAMICON)*, Cocoa Beach, FL, USA, 8–9 April 2019; pp. 1–6.

[18] V.B. Kumar Bhatti and T.K Eneyeq, "A Refinement of Gauss-Seidel Method for Solving of Linear System of Equations," *Int. J. Contemp. Math. Sciences,* vol. 06, no. 3, pp. 117-121, 2011.

[19] Sirovljevic, R, "Incomplete Factorization Preconditioners for Least Squares and Linear and Quadratic Programming" *The University of British Columbia,* 2007.

[20] G. Fasshauer, Class Lecture, Topic: "Preconditioning" *477/577 NLA, Department of Applied Mathematics, Illinois Institute of Technology*, Chicago, IL, Aug. 2006.

[21] Valenta, V. Error Performance of BPSK and QPSK in Satellite Communications. *Department of Electrical and Electronic Technology, FEEC, BUT*, 2005.

[22] Chataut, R.; Akl, R.; Robaei, M. Accelerated and Preconditioned Refinement of Gauss-Seidel Method for Uplink Signal Detection in 5G Massive MIMO Systems. *In Proceedings of the 2020 10th Annual Computing and Communication Workshop and Conference (CCWC)*, Las Vegas, NV, USA, 6–8 January 2020; pp. 0083–0089.

[23] A Beginner's Guide to Big O Notation. 23 June 2009. [Online]. Available: https://rob-bell.net/2009/06/a-beginners-guide-to-big-o-notation [Accessed: 24-October-2021].

[24] Chataut, R.; Akl, R. Efficient and Low-Complexity Iterative Detectors for 5G Massive MIMO Systems. *In Proceedings of the 2020 16th International Conference on Distributed Computing in Sensor Systems (DCOSS)*, Marina del Rey, CA, USA, 25–27 May 2020; pp. 442–449.