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Huber Fitting based ADMM Detection for Uplink 5G Massive MIMO Systems

Robin Chataut
Department of Computer Science and Engineering
University of North Texas
Denton, Texas, 76207, USA
RobinChataut@my.unt.edu

Robert Akl
Department of Computer Science and Engineering
University of North Texas
Denton, Texas, 76207, USA
Robert.Akl@unt.edu

Abstract— **Massive multiple-input multiple-output (Massive MIMO) has been considered as a promising technology to fulfill the requirement of fifth generation (5G) networks. Massive MIMO is designed to provide better throughput and spectral efficiency. However, the major challenge in this system is uplink signal detection which becomes more complicated with a higher number of base station antennas. In this paper, we propose an uplink detection algorithm which is based upon the alternating direction method of multipliers (ADMM) algorithm and Huber fitting. Results through Matlab simulation show that compared to conventional detection algorithms, our proposed algorithm can provide better error rate (BER) performance along with reduced computational complexity. We also improve the convergence rate of the algorithm by using the acceleration method.**

Keywords—Massive MIMO, 5G, uplink detection, ADMM, Huber fitting, BER

I. INTRODUCTION

MIMO is a widely adopted technology that uses multiple transmitters and receivers to transmit more data simultaneously. With the introduction of Internet-of-Things, machine-to-machine communication, high-quality video streaming, etc., there would be a massive increase in the data traffic, and this growth is expected in the next few decades as well [1]. Fifth generation networks(5G) are expected to accommodate this huge data traffic and address the current limitation on data rate, reliability, and energy efficiency [2]. Recently, Massive MIMO technology has been proposed, and it is believed that this technology has the potential to overcome the challenges associated with 5G systems such as data rate, bandwidth efficiency, and reliability. In Massive MIMO systems, the base station is equipped with hundreds and even thousands of antennas, and it can serve tens of users simultaneously [3]. There are many advantages of Massive MIMO, such as high spectral efficiency, antenna array gain, high reliability and robustness to interference and jamming. There are numerous challenges in Massive MIMO systems, but one of the critical challenge that we are focusing on this paper is uplink signal detection.

There has been an extensive study for achieving high performance and reduced complexity on Massive MIMO uplink detection methods. The conventional non-linear detection methods such as Sphere Decoder (SD) and Maximum Likelihood (ML) are not feasible as complexity increases exponentially with a number of antennas and modulation order [4]. Linear detection methods such as Zero-Forcing (ZF), Minimum Mean Square Error (MMSE) and V-BLAST detector have been considered, but these methods involve matrix inversion which makes them computationally inefficient with a large number of antennas [5-9]. Neumann series approximation method and the Richardson method avoids matrix inversion, but complexity is only slightly reduced [10-14]. Our proposed algorithm is based upon ADMM algorithm which was first introduced in [15] and is primarily designed for solving convex optimization problems by breaking bigger problems into smaller one. ADMM has come to global interest these days because of its application in various fields such as signal and image processing, machine learning, compressed sensing, wireless communication network, etc.

In this paper, we propose an efficient Massive MIMO uplink detection algorithm which is based on the ADMM algorithm and Huber fitting. ADMM makes a variable update in each iteration much easier, and during each iteration, variables are updated by solving an unconstrained convex optimization problem [16-17]. Huber fitting is a method mostly used for robust regression, and it makes function less sensitive to outliers in the data. We further improve the convergence of the proposed algorithm by using acceleration method described in [18].

The rest of the paper is organized as follows: Section II describes the system model. The proposed algorithm is presented in section III, and simulation results have been discussed in section IV. Finally, we conclude the paper in section V.

II. SYSTEM MODEL

We consider a Massive MIMO system in which base station is equipped with a considerable number of antennas M, and N (where M>>N) users equipment having single antenna are communicating with the base station as shown in fig. 1 [19]. We have assumed perfect Channel State Information (CSI) between all the users and the base station. A sufficiently long cyclic prefix is also considered such that there would be no intersymbol interference. The N users encode their own bitstream, and then the encoded bitstream is mapped into constellation points in the finite alphabet set (e.g., QPSK) $\mathcal{O}$.

The transmitted signal and the received signal vector are denoted by $x \in \mathbb{C}^N$ and $y \in \mathbb{C}^{\mathrm{M}}$ respectively. The received signal at the base station is given as:

$$y = H * x + w \tag{1}$$

where, $H \in \mathbb{C}^{M \times N}$ is the channel matrix whose elements are independent and identically distributed (i.i.d), i.e. $H \sim \mathcal{CN}\ (0,1)$ and $w \in \mathbb{C}^N$ is the Additive White Gaussian Noise (AWGN) and its each component w_i are i.i.d with zero mean and finite variance σ^2, i.e., $w \sim$ CN $(0, \sigma^2 I)$.

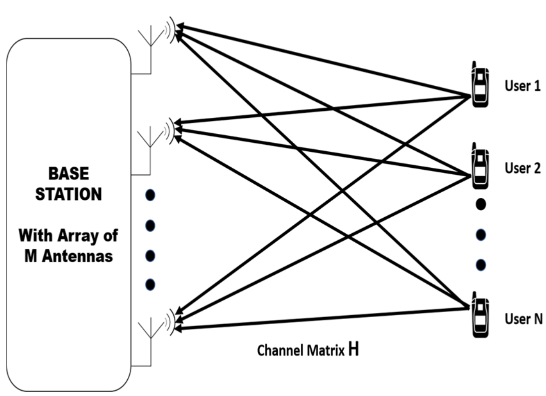

Figure 1: System Model for Massive MIMO uplink

The maximum likelihood of the problem one is equivalent to the Euclidean distance minimization $\|y - Hx\|_2^2$ between y and Hx:

$$\hat{x} = \underset{x \in x^N}{\arg\min}\ \|y - Hx\|_2^2 \tag{2}$$

This ML detector requires substantial computation for a large number of users. Hence, it is not a feasible detection method for massive MIMO uplink detection. To find a method which is efficient in both computation and performance, we can rewrite equation (2) into the following equivalent form:

$$\hat{x} = \underset{x,z \in \mathbb{C}^N}{\arg\min}\ \ \mathrm{g(z)} + \|y - Hx\|_2^2\ , \quad \text{where } \mathrm{z = x} \tag{3}$$

where, function g(z) is a convex regularizer function. For ZF detection, we have g(z) = 0 and for MMSE detection, g(z)= $N_0/(2E_z)\|z\|_2^2$. Where, E_z is the average transmit energy per user and N_0 is the noise variance. We can use convex polytype around the constellation $\mathcal{O}$ to find a better detection method than ZF and MMSE [20].

The augmented Lagrangian function $\mathcal{L}$ for equation (3) is given as follows:

$$\mathcal{L}(x, z, \lambda) = g(z) + \frac{1}{2}\|y - Hx\|_2^2 + \frac{\beta}{2}\|z - x - \lambda\|_2^2 \tag{4}$$

where, $\beta > 0$ is some fixed penalty parameter and λ is Lagrange parameter. Equation (4) can be solved by using the ADMM algorithm which minimizes the augmented Lagrangian. ADMM is primarily designed to solve convex optimization problems and one of the major advantages of ADMM is its flexibility towards the parallel computation. In each step of ADMM, we minimize the augmented Lagrangian over x and z to get the result.

Following are the steps in k ADMM iteration [21]:

$$x_{k+1} = argmin\ \{\tfrac{1}{2}\|y - Hx\|_2^2 + \tfrac{\beta}{2}\|z - x - \lambda\|_2^2\}$$

$$z_{k+1} = \Pi_c(x_k + z_k)$$

$$\lambda_{k+1} = \lambda_k + (x_{k+1} - z_{k+1})$$

where, Π_c is the Euclidean projection onto C and λ is the dual variable. First, we fix z and λ and minimize x; we then fix $x$and λ and minimize augmented Lagrangian over z and finally dual variable λ is updated.

We have used conventional MIMO detection algorithms such as ZF and MMSE to compare the performance of our proposed algorithm. ZF is a simple algorithm which reduces inter-antenna interference, and it performs a linear transformation of the received vector by applying the pseudo-inverse of the channel matrix. The equation for the ZF detector is given as:

$$S_{ZF}\ = (H^H H)^{-1} H^H$$

where, S_{ZF} is the pseudo inverse of the channel matrix and H^H is Hermitian transpose of channel matrix H.

MMSE is also a linear detector which reduces both interference and noise by minimizing the mean square error between the received and transmitted vector [1]. The signal matrix for MMSE is given as:

$$S_{MMSE} = (H^H H + \frac{W_0}{E_s} I)^{-1} H^H$$

where, W_0 is the noise variance and, E_s is the signal variance.

III. PROPOSED ALGORITHM FOR DETECTION

This section describes the proposed algorithm for uplink detection of Massive MIMO systems. The proposed algorithm is summarized in Algorithm 1. We first initialize $z, \lambda, \rho, \alpha, \gamma$. Where, $\rho > 1$ is an augmented Lagrangian parameter, $\alpha > 0$ is an over-relaxation parameter and $\gamma > 0$ is the step size.

During preprocessing, inversion of gram regularized matrix is required, which is required for the x-update, and x-update is done by taking the derivate of equation (4) as given in [15]. $\beta > 0$ is regularization parameter and I is an identity matrix [22]. We have used the Cholesky decomposition to calculate the inverse of the matrix, which is decomposition of the form:

$$A = R * R^T$$

$$A^{-1} = (R^{-1})^T * (R^{-1})$$

where, R is a lower triangular matrix with positive diagonal elements and R is called the Cholesky factor of A, and R^T is the conjugate transpose of R.

In the first step of the proposed algorithm iteration, keeping z and λ value fixed, we minimize x. For z-update, the proposed algorithm uses Huber fitting to update it during each iteration. Huber fitting function is defined as [23]:

$$minimize\ g^{hub}(y - Hx)$$

g^{hub} is the Huber penalty function, and for any scalar t, Huber penalty function is given by:

$$g^{hub}(t) = \begin{cases} t^2 / 2 & |t| \le 1 \\ |t| - 1/2 & |t| > 1 \end{cases}$$

This Huber penalty is used into ADMM algorithm during z-update. ADMM will remain the same except for the z-update which will now involve proximity operator of the Huber function. z-update is given as:

$$z_k = \frac{\rho}{1+\rho} * \left(x_k + \hat{\lambda}_k\right) + \frac{\mathbf{1}}{1+\rho} * S\left(x_k + \hat{\lambda}_k, \frac{\mathbf{1}}{1+\rho}\right)$$

The third step of ADMM iteration is λ-update. Then in next steps (steps 15-17 of the algorithm), we accelerate the performance of the algorithm by extrapolating the λ-update in each iteration and then we update z with the updated value of λ. In this method z- update is done twice, hence, it would increase the computational complexity, but this complexity is negligible when we compare it to the speed up that we can achieve with this modification [18].

Algorithm 1 Proposed Algorithm for Uplink Detection of Massive MIMO Systems

1: **Inputs**: y, H, N_0
2: **Initialization**:
3: $\lambda_0 = \hat{\lambda}_0 = z_o = \hat{z}_0 = 0$
4: $\gamma = 0.25, \alpha_0 = 1, \rho = 1.5$
5: **Preprocessing**:
6: $\beta = N_0 * E_S^{-1}$
7: $A = (H^H * H + \beta * I)$
8: $A = R * R^T$
9: $\tilde{R} = R^{-1}$
10: $\hat{y} = H^H * y$
11: **ADMM Iteration**
12: **for** $k = 1\ to\ k_{max}$ **do**
13: $x_k = (\tilde{R}^T * \tilde{R}) * (\hat{y} - \beta(\hat{\lambda}_k - \hat{z}_k))$
14: $z_k = \frac{\rho}{1+\rho} * \left(x_k + \hat{\lambda}_k\right) + \frac{1}{1+\rho} * S\left(x_k + \hat{\lambda}_k, \frac{1}{1+\rho}\right)$
15: $\lambda_k = \hat{\lambda}_k + \gamma(x_k - z_k)$
16: $\alpha_{k+1} = \frac{1+\sqrt{1+4\alpha_k^2}}{2}$
17: $\hat{\lambda}_{k+1} = \lambda_k + \frac{\alpha_k - 1}{\alpha_{k+1}} * (\lambda_k - \lambda_{k-1})$
18: $\hat{z}_{k+1} = z_k + \frac{\alpha_k - 1}{\alpha_{k+1}} * (z_k - z_{k-1})$
19: **End for**
20: **Output**: x_k

IV. SIMULATION RESULTS

In this section, we analyze the BER performance and computational complexity of the proposed algorithm. For the simulation, we consider a Massive MIMO system with 16 or 32 or 64 or 128 base station antennas and 16 users with a single antenna. These users may have more than one antenna, but for simplicity, we have assumed that these users just have a single antenna. The symbols are transmitted under the uncorrelated Rayleigh fading channel and QPSK, 16-QAM and 64-QAM constellation. The signal-to-noise ratio (SNR) range is between 1 to 25 dB, and the noise variance is controlled by SNR. All the simulations were done in Matlab under Mac OS, with 3.4 GHz Intel Core i7 processor and 10GB of RAM.

A. BER Performance

We evaluated the BER performance of the proposed algorithm and compared it with MMSE and ZF detection schemes. Fig. 2, Fig. 3, Fig. 4 shows the BER vs. SNR performance for QPSK, 16-QAM, and 64-QAM for the different number of base station antennas (M) respectively.

For M=16 antenna with QPSK modulation, we can see that the proposed algorithm has better BER performance than both MMSE and ZF algorithm. For M=32 or 64 with QPSK modulation, the proposed algorithm has better performance

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than ZF and almost similar performance as MMSE. For M=16 antenna with 16-QAM modulation proposed algorithm performs better than ZF and has the same performance as MMSE. For a higher number of antennas with 16-QAM modulation, the proposed algorithm has identical performance as ZF and MMSE detection methods.

For M=16 antenna with 64-QAM modulation proposed algorithm performs better than ZF and almost similar performance as MMSE at higher SNR. For a higher number of antennas (M=32 or 64) with 64-QAM modulation, the proposed algorithm has an identical performance as ZF and MMSE detection methods at only high SNR. At very low SNR, ZF and MMSE perform better than a proposed algorithm. For all the detection methods, we can see that BER performance is increased with the higher number of antennas.

From Fig. 2, Fig. 3, Fig. 4 we can also conclude that BER performance degrades with increasing modulation order. Fig.5 shows BER vs. SNR performance of the proposed algorithm with a various number of BS antennas with QPSK constellation. At BER=10^{-4} increasing BS antenna from 32 to 64 gives a gain of almost 6dB. An additional gain of 4 dB is obtained when the BS antenna is increased from 64 to 128. Thus, we can conclude that an increasing number of BS antennas gives significant improvement in BER performance.

B. Complexity Analysis

Since the proposed algorithm is an iterative algorithm, the complexity is dependent upon the number of iterations. The complexity of the proposed algorithm is calculated and compared with existing massive MIMO uplink detection

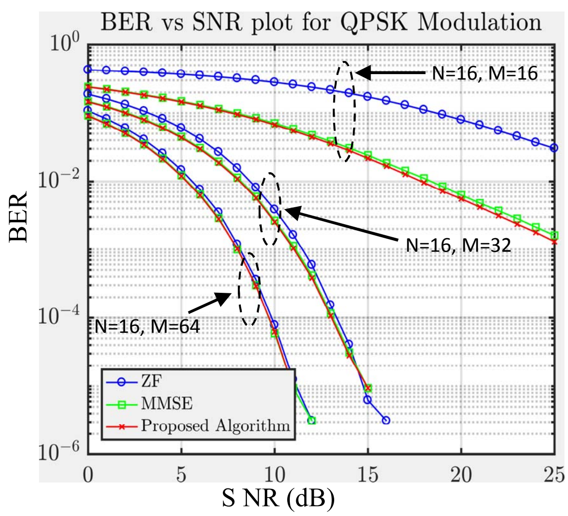

Figure 2: BER Vs. SNR for QPSK Modulation

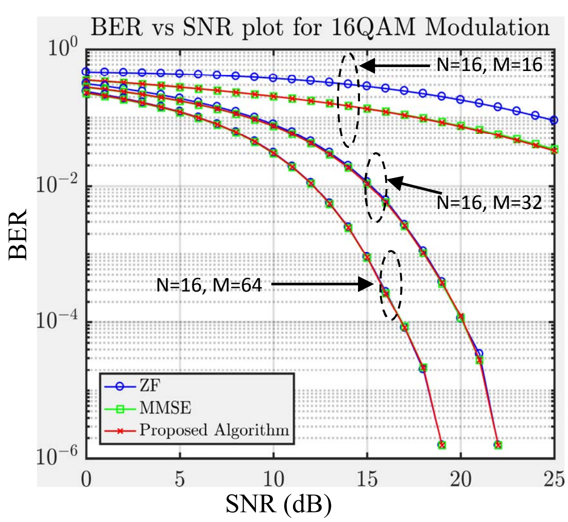

Figure 3: BER Vs. SNR for 16QAM Modulation

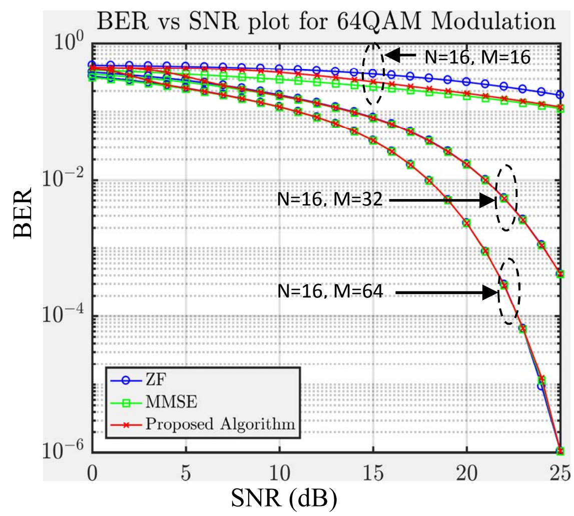

Figure 4: BER Vs. SNR for 64QAM Modulation

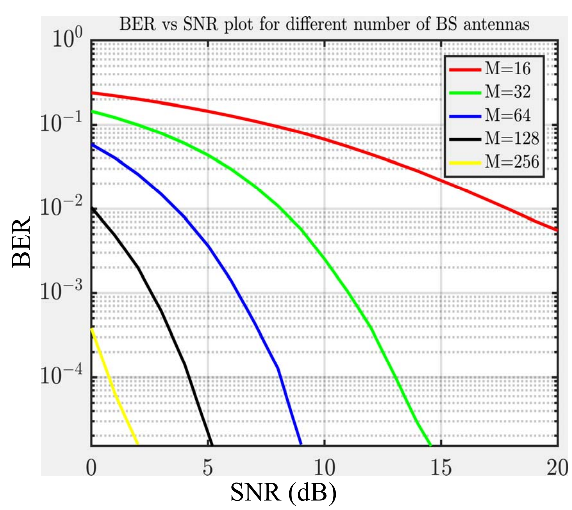

Figure 5: BER Vs. SNR for different number of BS antennas

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algorithms. Here, we analyze the complexity of the dominant terms and find the total complexity of the proposed algorithm. Preprocessing step involved a matrix inversion of order N x N with a cost of $O(N^3)$ and a matrix multiplication with a cost of O(MN). Since these two steps must be computed just once, it does not affect the overall complexity of the algorithm. The other steps of the algorithm involve just vector sums and matrix-vector multiplication with the cost of $O(N^2J)$, where J is the number of iterations. As we involve only the dominant term to calculate the overall complexity, the complexity of the proposed detector is in order of $O(N^2J)$. We compare this with the complexity of MMSE and ZF algorithm which as complexity in order of $O(MN^2)$ [24]. As M>>J, we can conclude that the proposed algorithm has reduced complexity when compared to conventional detection algorithm.

V. CONCLUSION

In this paper, we introduced a novel uplink detection algorithm suitable for massive MIMO systems. The proposed algorithm is based upon ADMM algorithm and Huber fitting method. We also described a method for faster convergence of the algorithm. The simulation result has shown that the proposed algorithm has better BER performance than conventional MIMO detection algorithms and it is computationally efficient as well. The proposed algorithm provides a good tradeoff between BER and complexity. Thus, the proposed algorithm can be effective for uplink detection in Massive MIMO systems.

In the future, it would be fascinating to test this algorithm by introducing several real-time network parameters in the system. Another future work could be testing this algorithm with hundreds and even thousands of base station antennas and with the added number of users.

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