

# Least Square Regressor Selection Based Detection for Uplink 5G Massive MIMO Systems

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**Abstract**— Massive multiple-input multiple-output (MIMO) is a core component of next-generation 5G networks which groups together antennas at both transmitter and the receiver to provide high spectral and energy efficiency. However, uplink signal detection in massive MIMO system becomes inefficient and computationally complex with a larger number of antennas. In this paper, we propose an algorithm for the uplink detection based upon least square regressor selection problem. The results through simulations show that the proposed algorithm is computationally efficient and achieves near-optimal bit error rate (BER) performance in comparison to the conventional uplink detection algorithms. The proposed algorithm can provide a good tradeoff between BER and computational complexity and is suitable for uplink detection in massive MIMO systems.

**Keywords**— *Massive MIMO, 5G, spectral efficiency, regressor selection, BER*

## I. INTRODUCTION

With globalization, there has been a huge surge in mobile data traffic, and a drastic increase in demand for higher data rate, lower latency, better energy and spectral efficiency. Also, with the introduction of the Internet of Things (IoT), this growth in data traffic is expected in the next few decades as well. The fifth-generation network (5G) is expected to accommodate this huge increase in data traffic along with higher reliability and data rate, improved energy and spectral efficiency and with lower latency. MIMO is considered as a key technology for current wireless communication networks, and it has been widely adopted in 3GPP (third generation partnership project), LTE (long term evolution) and IEEE 802.11n standard [1]. Massive MIMO, which is an extension of MIMO, has recently been considered a promising technology to fulfill the requirement of 5G networks. Massive MIMO receiver is equipped with hundreds or even thousands of antennas, and it can serve tens of user simultaneously to provide better spectral efficiency and throughput as illustrated in Fig. 1 [2-4]. One of the

challenging issues in massive MIMO systems is uplink signal detection which increases exponentially with a large number of antenna terminals [2][5].

There have been numerous studies to find an optimal uplink detector for massive MIMO. The non-linear MIMO detector such as Maximum Likelihood (ML) and Sphere Decoder (SD) are practically infeasible as complexity is increased exponentially with the higher number of antennas [6]. The performance of several linear receivers has been studied [7-9]. Linear detectors such as Zero-Forcing (ZF), Minimum Mean Square Error (MMSE) are also inefficient for a large number of antennas as they involve matrix inversion. To avoid matrix inversion, Richardson method and Neumann series expansion (NSE) method have been considered, but complexity was minimally reduced and remained unaffordable when there were more than two terms in NSE [10– 14]. Other linear methods based on the Conjugate Gradient (CG) and Gauss-Seidel (GS) were also considered in [15][16]. The proposed algorithm is based on least square regressor selection problem [17] which finds the best fit from available regressors. This method is widely used

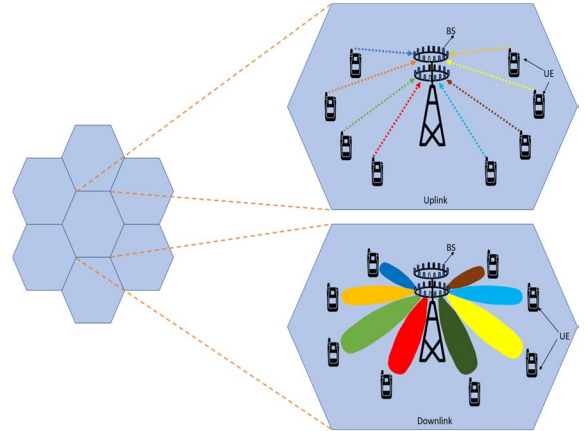


Figure 1: Massive MIMO Systems

because of its application in various fields such as finance, medicine, economics, machine learning, wireless communication, and statistics.

The rest of the paper is organized as follows: Section II presents the system model and section III describes the proposed algorithm. Simulation results including bit error rate (BER) and symbol error rate (SER) performance and computational complexity are discussed in section IV. Finally, section V concludes the paper and suggests some future work.

## II. SYSTEM MODEL

An uplink massive MIMO system with  $M$  antennas at the base station (BS) is considered and  $N$  ( $M \gg N$ ) number of active users having single antenna are simultaneously communicating with the base station as in Fig. 2. Perfect channel state information (CSI) is assumed between the base station and user, and the most commonly used Rayleigh fading channel is considered for this simulation. Each  $N$  users encodes their bitstream, and then each encoded bitstream is mapped into constellation point within the finite alphabet set, i.e., BPSK (binary phase shift keying), QPSK (quadrature phase shift keying) and QAM (quadrature amplitude modulation).

The received vector is denoted by  $y = [y_1, y_2, y_3 \dots y_N]^T$  where,  $y \in \mathbb{C}^M$  and the transmitted vector is denoted by  $s = [x_1, x_2, x_3 \dots x_N]$  where  $x \in \mathbb{C}^N$ . The received signal vector is given as:

$$y = Hx + w \quad (1)$$

In (1),  $H$  is uplink channel matrix of order  $M \times N$  whose elements are independent and identically distributed (i.i.d) with zero mean and unit variance, i.e.,  $H \sim \mathcal{CN}(0,1)$  and  $H \in \mathbb{C}^{M \times N}$ . And,  $w$  is Additive White Gaussian Noise (AWGN) and each element of  $w$  is i.i.d with zero mean and finite variance, i.e.,  $w \sim \mathcal{CN}(1, \sigma^2 I)$  and  $w \in \mathbb{C}^N$ . The ML of the (1) is equivalent to the Euclidean distance minimization  $\|y - Hx\|_2^2$  between  $y$  and  $Hx$  [1]:

$$\hat{x} = \arg \min_{x \in \mathbb{C}^N} \|y - Hx\|_2^2 \quad (2)$$

Equation (2) can be solved using various detection methods such as ZF, ML, and MMSE. As the number of user increases in massive MIMO, ML becomes computationally inefficient as it involves computation of the objective function for all  $x^N$ . Linear receivers such as ZF and MMSE involves inverting the matrix and factorizing the matrix of order  $M \times N$ . ZF performs a linear transformation of the received signal by applying the pseudoinverse  $S_{ZF}$  of  $H$ . ZF eliminates interference but suffers noise enhancement [18]. Solution for ZF is given as:

$$\hat{x} = S_{ZF} * y \quad (3)$$

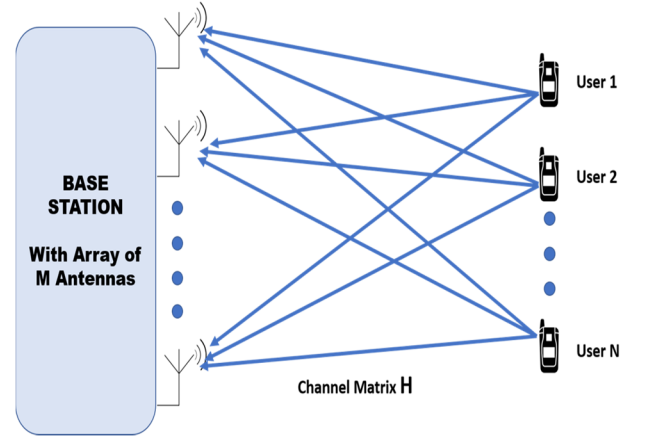


Figure 2: System Model for Uplink Massive MIMO

$$S_{ZF} = (H^H H)^{-1} H^H \quad (4)$$

The MMSE algorithm reduces noise and interference by reducing the error between the transmitted and the received vector [1]. The signal matrix for MMSE is given as:

$$S_{MMSE} = (H^H H + \frac{W_0}{E_s} I)^{-1} H^H \quad (5)$$

The linear detectors, ZF and MMSE, have lower computational complexity when compared to ML, but the error performance of ML is better than these two linear detectors.

## III. PROPOSED ALGORITHM FOR UPLINK DETECTION

Almost all the linear detection method requires inversion of the  $M \times N$  matrix which incurs at very high computational complexity for a higher number of antennas and users. We are proposing a low complexity and efficient uplink detection method for massive MIMO systems which avoids complex computations and also gives optimal BER performance. The proposed method is summarized in Algorithm 1.

In this method, inputs to the algorithm are received signal,  $y$  and channel matrix,  $H$ . At the beginning of the algorithm, we initialized dual variable  $\lambda$ , augmented Lagrangian parameter  $\rho > 1$ , step size  $0 < \gamma < 1$  and vector  $z$ . The  $x$ -update during the iteration requires inverse of gram matrix,  $A$ . The calculation of  $A$  involves matrix multiplication, where,  $\beta$  is the regularization parameter and  $I$  is an identity matrix [19]. To reduce the complexity of the algorithm, we did the inversion using Cholesky decomposition during the preprocessing step [20]. The Cholesky factorization, expresses matrix  $A$  as the product of the triangular matrix and its transpose given as:

$$A = \hat{R} * \hat{R}' \quad (6)$$

where  $\hat{R}$  is a lower triangular matrix and it is called

**Algorithm 1** Proposed Algorithm

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1: Inputs:  $y, H$ 
2: Initialization:
3:    $\hat{\lambda}_0 = \hat{z}_0 = 0$ 
4:    $\gamma = 0.25, \rho = 5$ 
5: Preprocessing:
6:    $A = H' * H + \beta * I$ 
7:    $\hat{R} = chol(A, lower)$ 
8:    $R = squeeze(\hat{R})$ 
9:    $Q = squeeze(\hat{R}')'$ 
10:   $\hat{H} = H' * y$ 
11: Iteration
12:  for  $k = 1$  to  $k_{max}$  do
13:     $P = \hat{H} + \rho * (z_k - u_k)$ 
14:     $x_k = (Q / (R/P))$ 
15:     $C = \{x_k | cardinality(x_k) \leq N\}$ 
16:     $z_k = \Pi_C(x_k + z_k)$ 
17:     $\lambda_k = \hat{u}_k + \gamma * (x_k - z_k)$ 
18:  End for
19: Output:  $x_k$ 

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the Cholesky factor of  $A$ . The matrix-vector multiplication of step ten in the proposed algorithm is also done during preprocessing to reduce the computation during the iteration. During the first step of the iteration, we minimized  $x$  and kept  $z$  and  $\lambda$  constant. This minimization was done using forward-backward solves [17]. The  $z$ -update of the algorithm is the projection onto a non-convex set  $C$ .

$$z_k = \Pi_C(x_k + z_k) \quad (7)$$

Here, the projection keeps  $N$  largest elements from the  $(x_k + z_k)$  and rest of the element are made zero. The cardinality function in this step gives the number of the non-zero element. In short,  $z$ -update is like intermediate sorting. The  $\lambda$  update is given by:

$$\lambda_k = \hat{u}_k + \gamma * (x_k - z_k) \quad (8)$$

where  $0 < \lambda < 1$  is the step size which ensures the convergence of the algorithm [19].

#### IV. SIMULATION RESULTS

In this section, we provide the simulation results to evaluate the performance of the proposed uplink detection algorithm. We compared the proposed algorithm with conventional massive MIMO detector such as MMSE and ZF detection algorithm. Table I shows the simulation parameters used for the simulation. Number of active users in the system are 8 or 16 and each user has single antennas and the number of antennas at the base station can be 16, 32, 64, 128 or 256. The noise variance is controlled by SNR and signal variance of 2 is used. The bandwidth of the system is 20 MHz and SNR values are varied from 0 dB to 30dB. A massive MIMO

TABLE I. SIMULATION PARAMETERS

Parameter	Value
System Bandwidth	20 MHz
Number of Users	8 or 16
Receive Antennas	16, 32 64, 128 or 256
Signal to Noise Ratio	0 to 30 dB
Noise Variance	Controlled by SNR
Signal Variance	2
Channel Model	Uncorrelated Rayleigh Fading
Modulation Scheme	BPSK, QPSK, 16QAM, 64QAM

system is considered where the base station has a large number of antennas, and users having single antenna send data simultaneously to the base station. In general, users may have more than one antenna, but for the simplicity of the simulation, it was assumed that each user is equipped with a single antenna. The symbols generated are transmitted via uncorrelated Rayleigh fading channel, AWGN noise is considered, and different modulation schemes such as BPSK, QPSK, 16QAM, and 64QAM are used to compare the simulation results. All the simulations are done in Matlab under Mac OS, with 3.4 GHz Intel Core i7 processor and 10GB of RAM.

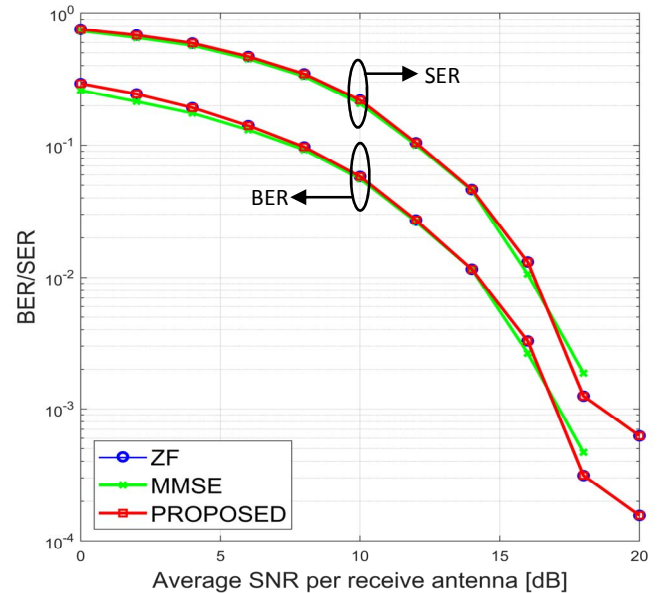


Figure 3: Comparison of bit error rate (BER) and symbol error rate (SER) performance of proposed algorithm with ZF and MMSE, with 32 base station antenna (M), 16 users (N) and 16QAM modulation.

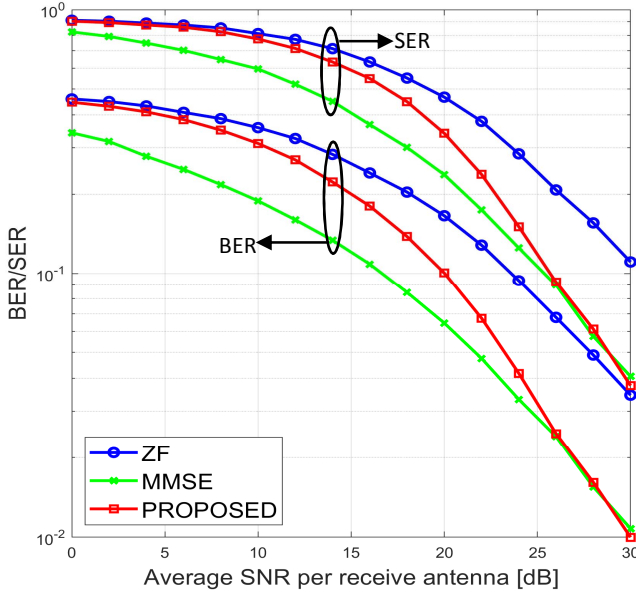


Figure 4: Comparison of bit error rate (BER) and symbol error rate (SER) performance of proposed algorithm with ZF and MMSE, with 16 base station antenna (M), 16 users (N) and 16QAM modulation.

#### A. BER/SER Performance

We access the BER and SER performance of proposed algorithm and compare it with ZF and MMSE algorithm. Fig.3, Fig.4, Fig.5 shows the BER/SER vs. SNR performance

for a varying number of base station antenna and users using 16QAM modulation. Fig.3 shows BER and SER performance for  $M=32$  and  $N=16$  using 16QAM modulation. As observed in Fig.3, the proposed algorithm has similar performance than ZF and MMSE for lower SNR values, and for higher SNR, the proposed algorithm outperforms the conventional MIMO detection algorithm. Fig.4 shows BER and SER performance for  $M=16$  and  $N=16$  using 16 QAM modulation. At lower SNR, the proposed algorithm has better error performance than ZF, but MMSE had better error performance. At very higher SNR, the proposed algorithm has better error rate than both MMSE and ZF.

Fig.5 shows error performance for  $M=8$  and  $N=8$  with 16QAM modulation. The proposed algorithm outperforms the ZF algorithm, but MMSE performs better than the proposed algorithm. Fig.6 shows the error performance of the proposed algorithm for the varying number of base station antennas with  $N=16$  users and 16QAM modulation. At  $\text{BER} = 10^{-4}$ , changing base station antenna from 64 to 128 gives 3.9dB gain. An additional gain of 3.5dB is achieved if we increase the base station antenna to 256. Thus, an increasing number of base station antenna improves the error performance. Fig.7 shows the performance of the proposed algorithm with various modulation scheme simulated with  $M=16$  and  $N=16$ . At  $\text{BER} = 10^{-1}$ , changing modulation scheme from BPSK to QPSK has an almost 3.3dB loss, and further 5.3dB loss occurs if the modulation scheme is changed to 16QAM. Thus, increasing the modulation order decreases the BER performance of the proposed algorithm.

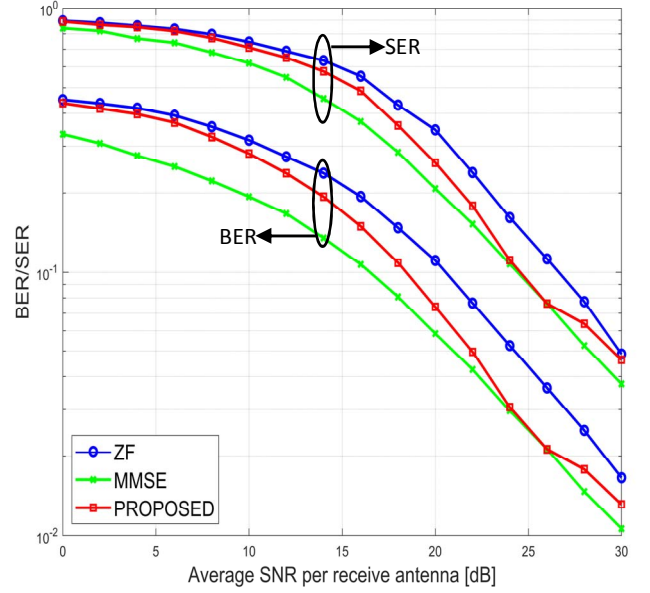


Figure 5: Comparison of bit error rate (BER) and symbol error rate (SER) performance of proposed algorithm with ZF and MMSE, with 8 base station antenna (M), 8 users (N) and 16QAM modulation.

#### B. Complexity Analysis

The computational complexity of the proposed algorithm mainly depends upon the number of base station antennas and complexity increases with a higher number of antennas. The complexity of the proposed algorithm is compared with conventional MIMO detection algorithms like ZF and MMSE. The preprocessing step in the proposed algorithm includes a matrix multiplication, a matrix inversion with Cholesky factorization and a matrix-vector multiplication. The matrix Multiplication has the cost of  $\mathcal{O}(M^2N)$ , the computational complexity of Cholesky function is  $\mathcal{O}(N^3)$ , and the matrix-vector computation has the complexity of  $\mathcal{O}(MN)$ . Since these three steps are computed just once outside of iteration, this does not have a significant impact on the overall complexity of the algorithm. The steps in the iteration include subsequent backlash operation which has the complexity of  $\mathcal{O}(N^2)$ , matrix-vector multiplication  $\mathcal{O}(MN)$ , vector sums having complexity  $\mathcal{O}(N)$  and some other small operations. To calculate total complexity, we only involve dominant terms, thus, the total complexity of the proposed algorithm is  $\mathcal{O}(kN^2)$ , where,  $k$  is a total number of iterations. The complexity of ZF and MMSE is in order of  $\mathcal{O}(MN^2)$ [21].

As the number of base station antennas is much higher than the number of iteration ( $M \gg k$ ), we can conclude that the proposed algorithm has lower computational complexity when compared to conventional uplink detection algorithms like ZF and MMSE.

#### V. CONCLUSION

In this paper, we proposed a novel uplink detection method for Massive MIMO system based on least square regressor selection problem. The BER and SER performance



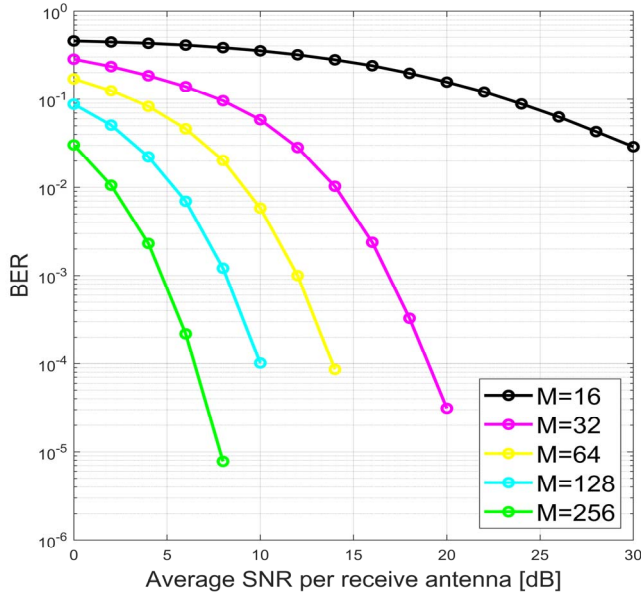


Figure 6: Comparison of bit error rate (BER) performance of proposed algorithm with different number of base station antenna ( $M$ ) with 16 users ( $N$ ) and 16QAM modulation.

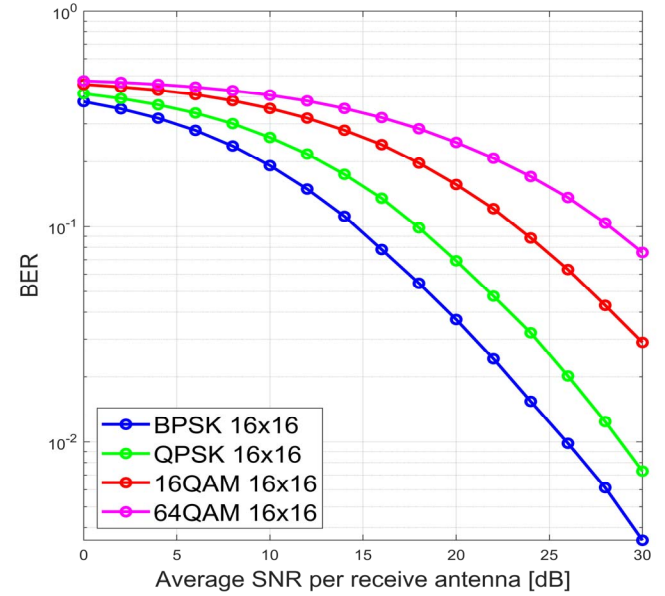


Figure 7: Comparison of bit error rate (BER) performance of proposed algorithm with different modulation schemes (BPSK, QPSK, 16QAM and 64QAM), with 16 base station antenna ( $M$ ) and 16 users ( $N$ ).

of the proposed algorithm was compared with conventional MIMO uplink detection algorithms, and simulation results show that the proposed algorithm always has better error performance than ZF and has comparable performance with MMSE at higher SNR. The computational efficiency of the proposed algorithm was calculated, and it was found that the proposed algorithm has lower computational complexity when compared to both ZF and MMSE. The simulation results also show that the error performance of the proposed algorithm becomes better with the higher number of base station antennas and the error performance reduces with higher modulation order. Thus, the proposed algorithm provides a good trade-off between BER and complexity and is suitable for uplink detection of 5G massive MIMO systems. In the future, we plan to test the proposed algorithm by including several realistic network parameters, and it would be interesting to test this algorithm with multi-antenna users as well.

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