# Applying GARCH-type models to capturing the stock market volatility characteristics with respect to the 2008 financial crisis

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**Abstract**

The severity of the global financial crisis in 2008 were unexpected, and it affected the volatility performance in financial markets. Due to the importance of this phenomenon, study of special volatility characteristics becomes more important. This project is mainly focusing on analyzing the changes in volatility characteristics of NASDAQ stock market affected by 2007-2008 financial crisis. The data set is divided into three time periods, i.e. pre-crisis period (2004-2006), during crisis (2007-2009) and post-crisis period (2010-2012). Both symmetric and asymmetric Generalized Autoregressive Conditional Heteroscedasticity models (GARCH) are used to model the volatility, and their fitting performances are compared through information criterion and forecasting evaluation. In addition, based on parameter comparations, the conclusion is shown that the impact of the crisis on the ARCH effect and leverage effect has been significant, but it just slightly affects the persistency of volatility. Moreover, the asymmetric GARCH models can capture these effects more accurate.

***Keywords***: GARCH models; stock market volatility; financial crisis;

## 1. Introduction

The study on volatility in the stock market is closely related to the measurement of risk. As volatility of stock market provides useful information in managing portfolio management, a variety of models are applied to forecast stock market movement and evaluate the performance of volatility. The global financial crisis happened in 2008 is considered to be the most devastating crisis since the 1929 Great Depression. It caused large swings in stock volatility and were followed by concerns about systemic risk of the financial system. In this project, the NASDAQ stock market is chosen since it is the second-largest stock exchange in the world by market capitalization, and it has been exposed to numerous periods of normal and abnormal volatility. Thus, questions regarding the influence by 2008 financial crisis to volatility in NASDAQ were valuable to discuss. Three different GARCH type models, symmetric GARCH(1,1) and asymmetric GJR-GARC(1,1) and EGARCH(1,1) will be used to measure market reaction to the deviations in volatility.

Empirical studies apply ARCH and GARCH type models to capturing and forecasting the stock market volatility. In a review of financial literature conducted on 93 published and working papers, Poon and Granger (2003) conducted an extensive analysis of forecasting performance of various volatility models and concluded that the GARCH type models deliver superior forecasting. According to their study, the GARCH family models factor in the more significant effect of a negative shock to volatility than a positive shock of the same magnitude. A measure of impact due to the 2008 financial crisis has been illustrated by Angabini and Wasiuzzaman (2011) using GARCH, GJR-GARC, and EGARCH. It showed that there was a significant increase in volatility and leverage effect in the Malaysian stock market, but a small decrease in the volatility persistence. Moreover, Lim and Sek (2013) compared the performances of GARCH, TGARCH and EGARCH models in capturing stock market volatility during pre-crisis, crisis and post-crisis periods. By evaluating out-of-sample error criteria, their result indicated that for the normal period (pre and post-crisis), symmetric GARCH model performs better than the asymmetric GARCH, but for fluctuated crisis period, asymmetric GARCH model is preferred.

The remainder of the project is organized as follows. In Section 2, the methodology of GARCH, EGARCH, and GJR-GARCH are introduced, and two kinds of loss function RMSE and MAE for model evaluation are also mentioned. Next, Section 3 presents the data analysis and model result in NASDAQ with different time-frames, and it also conducts ARCH-LM test and the out-of-sample forecast. Further discussions are also illustrated here. Finally, the conclusion is demonstrated in Section 4.

## 2. Methodology

### 2.1 GARCH type models

2.1.1 Generalized Autoregressive Conditional Heteroskedasticity (GARCH)

Bollerslev (1986) proposed the generalized-ARCH (GARCH) model by including lagged values of conditional variance, which is a better fit for modeling time series data when the data exhibits heteroskedasticity and volatility clustering. The main advantage of GARCH over ARCH is that, ARCH(p) modelling requires relatively high values of p for good fitting, while GARCH(1,1) is usually enough for fitting financial data. The general form of the GARCH(p,q) model is:

$$\begin{cases} x_t = \delta_t \varepsilon_t, \quad \varepsilon_t \sim NID(0,1) \\ \delta_t^{\ 2} = \mu + \sum_{i=1}^{p} \alpha_i x_{t-i}^2 + \sum_{j=1}^{q} \beta_i \delta_{t-j}^2 \end{cases}$$

where $\alpha_i x_{t-i}^2$ captures the ARCH effect and $\beta_j \sigma_{t-j}^2$ captures the GARCH effect.

However, there are some limitations to the GARCH model. The most important one is the GARCH model cannot capture the asymmetric performance that characterized in most financial time series, especially in stock markets, which means volatility response asymmetricity to good and bad news in the market. It is known as the “leverage effect”. Later, in order to improve this problem, Nelson (1991) proposed the asymmetric EGARCH model and Glosten, Jagannathan and Runkel (1993) proposed another asymmetric one, denoted as GJR-GARCH model.

2.1.2 Exponential GARCH (EGARCH)

The EGARCH model with the exponential nature of the conditional variance captures the effect of external unexpected shocks and reacts asymmetrically to the good and bad news on the predicted volatility. The general form of the EGARCH(p,q) model is:

$$\begin{cases} x_t = \delta_t \varepsilon_t, \quad \varepsilon_t \sim NID(0,1) \\ \log(\delta_t^{\ 2}) = \mu + \sum_{i=1}^{p} \alpha_i \left| \frac{x_{t-i}}{\delta_{t-i}} \right| + \sum_{j=1}^{q} \beta_i \log(\delta_{t-j}^2) + \sum_{k=1}^{p} \gamma_k \frac{x_{t-k}}{\delta_{t-k}} \end{cases}$$

The impact is asymmetric if $\gamma_k$ is not equal to zero whereas the impact of leverage effect can be included in the EGARCH model.

2.1.3 GJR-GARCH

Glosten, Jagannathan and Runkle (1993) proposed GJR-GARCH model, another asymmetric model.

$$\begin{cases} x_t = \delta_t \varepsilon_t, \quad \varepsilon_t \sim NID(0,1) \\ \delta_t^{\,2} = \mu + \sum_{i=1}^{p} \left[ \alpha_i + \gamma_i I_{(\varepsilon_{t-i}>0)} \right] \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_i \delta_{t-j}^2 \end{cases}$$

the sign of the indicator term captures the asymmetry where I is an indicator function and $\gamma_k$ represents the scale of this effect. When the residual $\varepsilon_{t-i}$ is smaller than zero, the indicator term I equals to one, otherwise, it equals to zero when the residual is not smaller than zero.

**2.2 Model evaluations**

This project uses two loss functions to determine which model has the best forecasting performance. Since it is not appropriate to contain only one loss function for the evaluation of volatility models, two error measured ways are taken into account: mean absolute error (MAE) and root mean square error (RMSE). Both ways' criterions are the smallest value error is, the better the predicting ability of the model.

2.2.1 Mean absolute error (MAE)

The MAE is a good indicator of average model forecasting performance. The MAE is given as,

$$MAE = n^{-1} \sum_{t=1}^{n} \left| r_t^2 - \hat{\delta}_t^2 \right|$$

2.2.2 Root mean square error (RMSE)

The RMSE is more appropriate to represent model forecasting performance than MAE when the error distribution is expected to be Gaussian. The RMSE is given as,

$$RMSE = n^{-1} \sqrt{\sum_{t=1}^{n} \left( r_t^2 - \hat{\delta}_t^2 \right)^2}$$

# 3. Empirical applications

## 3.1 Data analysis

The data used in this project is the daily rate of return in NASDAQ stock market from 2004 to 2012. The rate of returns should be required first by the following function:

$$R_t = 100 \cdot \ln\left(\frac{p_t}{p_{t-1}}\right)$$

where, $R_t$ is the rate of returns for time t and $p_t$ is the daily adj. close price for the stock index at time t. In order to explore the volatility persistence during the 2008 financial crisis, the whole rate of return data set is divided into three sub data sets. The first period covers the pre-crisis period from 2004.01.01 to 2006.12.31, since in 2004 the financial market was just recovering from the last Asian crisis. The second is the 2008 financial crisis period that from 2007.01.01 until 2009.12.31, and the last one is the period after the recovery of crisis which is from 2010.01.01 to 2012.12.31. Data was obtained from Yahoo finance.

Figure. 1 presents the rate of returns through different periods. It shows that volatility changes over time and it tends to be clustering. The volatility is relatively consistent over 2004 to 2006 and 2010 to 2012 compared to a huge fluctuation change from 2007 to 2009, which is the financial crisis period. Moreover, from figure 1, the large changes in volatility tend to be followed by the large changes, of either sign, and small changes tend to be followed by small changes, which means the volatility also clustering.

Figure. 1 Rate of returns over different periods.

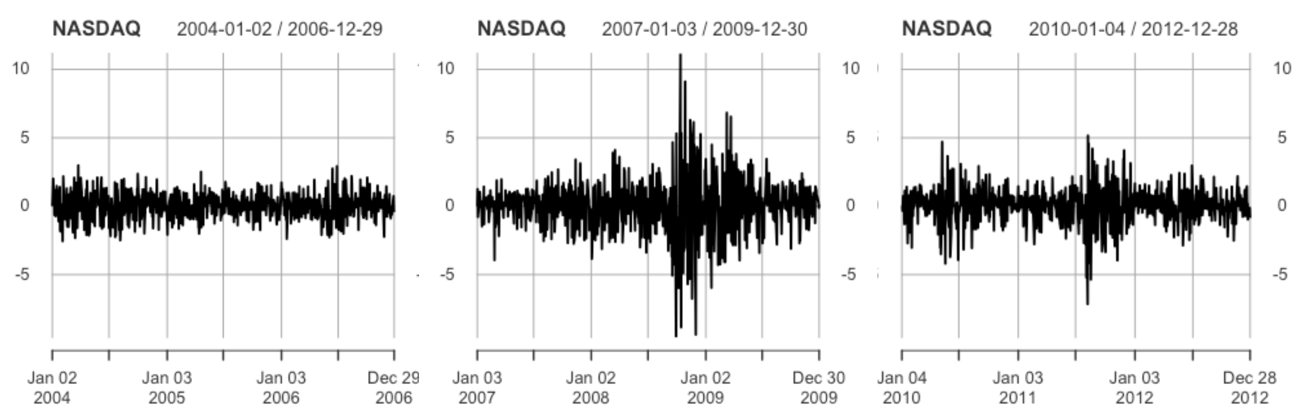

Table 1. Data analysis for NASDAQ

| | 2004-2006 | 2007-2009 | 2010-2012 |
|---|---|---|---|
| Observations | 755 | 755 | 755 |
| Mean | 0.02454804 | -0.00741216 | 0.03303197 |
| Median | 0.0615377 | 0.109067 | 0.07352311 |
| Maximum | 2.976504 | 11.15944 | 5.159188 |
| Minimum | -2.552368 | -9.587695 | -7.148908 |
| Std. Dev. | 0.9228503 | 1.927065 | 1.285192 |
| Skewness | -0.06064619 | -0.1147312 | -0.3598161 |
| Kurtosis | 3.174674 | 7.702133 | 5.959111 |
| Jarque-Bera | 151.747 | 703.263 | 293.9309 |
| (p-value) | $\leq 0.01$ | $\leq 0.01$ | $\leq 0.01$ |

The statistics analysis for the daily rate of returns are shown in Table 1. In general, there are large differences between the maximum returns and the minimum one, especially for the during crisis period, and the standard deviation is also the largest in this period among these three time-frames. Moreover, all mean are around zero and a slightly negative mean appears in the crisis period. The variables included are not normally distributed. All of the variables are slightly negatively skewed, and the kurtosis in each case is exceeded the benchmark for normal distribution, which is an indication of leptokurtosis. Specifically, by performing the Jarque-Bera test, its results also reject the normality for rate of return in each period with the p-value at 1% level.

Table 2. Unit root tests

| | 2004-2006 | 2007-2009 | 2010-2012 |
|---|---|---|---|
| Dickey-Fuller | -9.5614 | -8.2744 | -8.6496 |
| (p-value) | $\leq 0.01$ | $\leq 0.01$ | $\leq 0.01$ |

Table 2 above represents the result of Unit root tests. The Augmented Dickey-Fuller (ADF) test is applied to check whether the given time series is stationary or integrated. The test result shows that there is very strong evidence against the null hypothesis that the returns have a unit root. Thus, all of these returns are stationary with a constant mean across time no matter they are during financial crisis period or not.

### 3.2 Model evaluations

Firstly, ARMA models are used to estimate the conditional mean. Different ARMA models are examined based on information criteria methods (with the auto.arima function in R). Next, by performing the Portmanteau test and ARCH-LM test, the results in Table 3 illustrate the significant level of existing ARCH effect. Hence, since the returns exhibits ARCH effect, GARCH type models will be more appropriate to fit the data.

Table 3. ARCH heteroscedasticity test for ARMA models' residuals

| | 2004.01.01-2006.12.31 | | | 2007.01.01-2009.12.31 | | | 2010.01.01-2012.12.31 | | |
|---|---|---|---|---|---|---|---|---|---|
| Model | ARMA(0,0) | | | ARMA(2,1) | | | ARMA(1,1) | | |
| # of lags | 4 | 8 | 12 | 4 | 8 | 12 | 4 | 8 | 12 |
| PQ-test* | 16.3 | 41.0 | 68.0 | 199 | 452 | 727 | 15.3 | 39.8 | 66.6 |
| (p-value) | $\leq 0.01$ | $\leq 0.01$ | $\leq 0.01$ | 0.00 | 0.00 | 0.00 | $\leq 0.01$ | $\leq 0.01$ | $\leq 0.01$ |
| LM-test** | 205.4 | 94.1 | 58.1 | 462.8 | 122.6 | 67.1 | 205.7 | 93.5 | 58.1 |
| (p-value) | (0.00) | (0.00) | $\leq 0.01$ | (0.00) | (0.00) | $\leq 0.01$ | (0.00) | (0.00) | $\leq 0.01$ |

*: PQ-test stands for Portmanteau-Q test **: ARCH Lagrange-Multiplier test

The symmetric GARCH model and asymmetric EGARCH and GJR-GARCH models are examined in this project. Since GARCH type models can successfully capture the volatility present in the data with even a small lag (i.e. $p \leq 2, q \leq 2$), GARCH(1,1), EGARCH(1,1), and GJR- GARCH(1,1) are chosen to estimate the return data. The estimated coefficients of all models for three time-frame are shown in Table 4. In addition, in order to test the ARCH effect is whether been capture or not, ARCH-LM test is applied again, and the p-value results are shown in Table 5, which recommend that no presence of autocorrelation in the residuals of these GARCH type models.

Table 4. GARCH-type models coefficients

| | 2004.01.01-2006.12.31 | | | 2007.01.01-2009.12.31 | | | 2010.01.01-2012.12.31 | | |
|---|---|---|---|---|---|---|---|---|---|
| Coeff. | GARCH | EGARCH | GJR-GARCH | GARCH | EGARCH | GJR-GARCH | GARCH | EGARCH | GJR-GARCH |
| $\mu$ | 0.0123 | -0.0033 | 0.0136 | 0.0324 | 0.0148 | 0.0257 | 0.0399 | 0.0189 | 0.0485 |
| $\alpha_1$ | 0.0368 | -0.0632 | 0.0001 | 0.0840 | -0.1373 | 0.00 | 0.103 | -0.1939 | 0.00 |
| $\beta_1$ | 0.948 | 0.9885 | 0.9492 | 0.9055 | 0.9833 | 0.9558 | 0.8726 | 0.9406 | 0.8598 |
| $\gamma_1$ | | 0.0204 | 0.0648 | | 0.0984 | 0.0781 | | 0.1279 | 0.2114 |

Table 5. ARCH Lagrange-Multiplier test

| | 2004.01.01-2006.12.31 | | | 2007.01.01-2009.12.31 | | | 2010.01.01-2012.12.31 | | |
|---|---|---|---|---|---|---|---|---|---|
| Lags | GARCH | EGARCH | GJR_GARCH | GARCH | EGARCH | GJR_GARCH | GARCH | EGARCH | GJR_GARCH |
| 3 | 0.1868 | 0.06797 | 0.07345 | 0.5620 | 0.5766 | 0.9542 | 0.6539 | 0.8920 | 0.1135 |
| 5 | 0.4055 | 0.20710 | 0.25389 | 0.8302 | 0.8362 | 0.9885 | 0.8534 | 0.9602 | 0.2925 |
| 7 | 0.5205 | 0.33287 | 0.32388 | 0.9267 | 0.9107 | 0.9939 | 0.9524 | 0.9696 | 0.4515 |

*: All the values shown in the table are the p-value for ARCH Lagrange-Multiplier test

In order to find the best-fitted model, the results for information criteria method of GARCH(1,1), EGARCH(1,1), and GJR- GARCH(1,1) among three periods are compared in Table 6 below. In the pre-crisis period (2004-2006), although EGARCH is slightly better than the other two models based on the criteria values, we have to admit that the differences are small compared to the differences in the during-crisis. Both EGARCH(1,1) and GJR-GARCH(1,1) are appropriate to estimate the return during the financial crisis (2007-2009), it seems that they can capture the leverage effect compare to the symmetric GARCH(1,1). Moreover, in the post-crisis time (2010-2012), these three models have almost the same ability to fit the data.

Table 6. Information criteria method

| | 2004.01.01-2006.12.31 | | | 2007.01.01-2009.12.31 | | | 2010.01.01-2012.12.31 | | |
|---|---|---|---|---|---|---|---|---|---|
| I.C. | GARCH | EGARCH | GJR_GARCH | GARCH | EGARCH | GJR_GARCH | GARCH | EGARCH | GJR_GARCH |
| AIC | 2.6434 | 2.6181 | 2.6337 | 3.7159 | 3.6764 | 3.6727 | 3.0505 | 3.0195 | 3.0622 |
| BIC | 2.6679 | 2.6488 | 2.6888 | 3.7404 | 3.7070 | 3.7279 | 3.0568 | 3.0274 | 3.1174 |
| SIC | 2.6433 | 2.6181 | 2.6334 | 3.7158 | 3.6763 | 3.6725 | 3.0505 | 3.0195 | 3.0619 |
| HQIC | 2.6528 | 2.6299 | 2.6549 | 3.7253 | 3.6882 | 3.6940 | 3.0528 | 3.0233 | 3.0835 |

*: The best-fitted model is labeled in red.

To compare the changes in coefficients due to the financial crisis, the T-test for different periods' parameters were proposed. However, there are no significant changes respect to the whole coefficient tests since all p-values are larger than 1% level. In this case, Table 7 below presents the comparison of differences in each coefficient for the pre-crisis period and during-crisis period. The differences are calculated by the numerical difference, and the percentage is obtained by dividing the pre-crisis's coefficients.

In the symmetric GARCH model, the indicated parameter $\beta_1$ for GARCH effect which maintains the correlation between conditional variance $\sigma_t$ decreased by 4.5%, which implies the persistence of volatility decreased. While on the other hands, in these asymmetric GARCH models, the indicated parameter $\beta_1$ only performed a little percentage change, which implies the persistence is preserved in EGARCH and GJR-GARCH. In addition, all of these three models have experienced a great change in the ARCH effect with respect to the parameter $\alpha_1$. The asymmetric (leverage) effect within EGARCH and GJR-GARCH are shown by the coefficient $\gamma_1$. As shown in Table 7, by applying T-test for null hypothesis $\gamma_1 = 0$, the results illustrate that this parameter is significantly different from zero implying that the both periods have leverage effect and asymmetric.

Table 7. Model differences

| | 2004-2006 | 2007-2009 | Differences | Percentage |
|---|---|---|---|---|
| $\alpha_1$ | | | | |
| GARCH | 0.0368 | 0.0840 | 0.0472 | 128.3% |
| EGARCH | -0.0632 | -0.1371 | -0.0739 | 169.9% |
| GJR-GARCH | 0.0001 | 0.00 | -0.0001 | 100% |
| $\beta_1$ | | | | |
| GARCH | 0.9482 | 0.9055 | -0.0427 | 4.50% |
| EGARCH | 0.9885 | 0.9833 | 0.0052 | 0.5% |
| GJR-GARCH | 0.9492 | 0.9558 | 0.07 | 0.7% |
| $\gamma_1$ | | | | |
| GARCH | - | - | - | - |
| EGARCH | 0.0204 | 0.0948 | 0.0744 | 364.7% |
| GJR-GARCH | 0.0648 | 0.0781 | 0.0133 | 20.5% |

### 3.3 Forecasting Evaluation

These GARCH type models are evaluated by out-of-sample forecasting test using two loss functions RMSE and MAE. The out of sample period is 1 month for each period. Table 8 indicates the best forecasting model in EGARCH(1,1), and GJR-GARCH(1,1) is more exceptional when estimating the during-crisis period, which is corresponding to the in-sample test result for information criteria method above.

Table 8. Forecasting performance

| | 2002.01.01-2006.12.31 | | | 2002.01.01-2009.12.31 | | | 2002.01.01-2017.12.31 | | |
|---|---|---|---|---|---|---|---|---|---|
| Measure | GARCH | EGARCH | GJR_GARCH | GARCH | EGARCH | GJR_GARCH | GARCH | EGARCH | GJR_GARCH |
| RMSE | 0.0381 | 0.0155 | 0.0294 | 0.0585 | 0.0106 | 0.0689 | 0.0892 | 0.0279 | 0.0703 |
| MAE | 0.0373 | 0.0135 | 0.0238 | 0.0575 | 0.0097 | 0.0681 | 0.0889 | 0.0272 | 0.0699 |

## 4. Conclusion

This project evaluates different symmetric and asymmetric GARCH models to investigate the changes in the volatility of the NASDAQ stock market with respect to the 2008 financial crisis. The continuously compound return is used as the input, and the time is divided into three time periods, i.e. pre-crisis period (2004-2006), during crisis (2007-2009) and post-crisis period (2010-2012).

According to the statistical analysis, it shows that the returns are slightly negatively skewed and leptokurtosis in all periods. When the ARMA type models are used to fit the return data, the ARCH-LM test detected a strong ARCH effect and volatility clustering in the residuals. Thus, GARCH models are applied to estimate for these three time-frames. Based on the results in previous literature, symmetric GARCH(1,1) and asymmetric EGARCH(1,1), GJR-GARCH(1,1) are the most appropriate ones, and the rechecking results in ARCH-LM test also show the absence of ARCH effect after using the GARCH type models. By checking the information criteria and forecasting evaluation, EGARCH(1,1) seems to be the best model to fit the return series in either periods, and GJR-GARCH(1,1) is more exceptional when estimating the during-crisis period.

Both asymmetric EGARCH(1,1) and GJR-GARCH(1,1) illustrate the similar result for three periods. The coefficients $\gamma_1$ are significantly different from zero implying that all periods have leverage effect and asymmetric. A comparation for pre-crisis, during-crisis return series shows that the global financial crisis significantly increases the volatility and the leverage effect with a slightly increase in the volatility persistency. It indicates that the occurrence of financial crisis will not only cause the returns to be more fluctuating, but also increase the differences in changes of volatility between facing “bad news” and “good news”.

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