1. mean

2. variance

Variance

The variance of returns is the average squared deviation of returns from the mean return calculated as:

$$\text{Variance } \sigma^2 = \frac{\sum_{i=1}^{i=n} (r_i - \bar{r})^2}{n} \tag{4.2}$$

Deviations from the mean $(r_i - \bar{r})$ are squared; this avoids the problem of negative deviations cancelling with positive deviations.

3. Annualized Returns

ANNUALIZED RETURNS

When comparing returns over long periods it is easier to think in terms of standardized periods – annual returns being the most convenient. The average annual return over a number of years can be calculated arithmetically or geometrically as follows:

$$\text{Arithmetic average or average return} \quad r_A = \frac{f}{n} \times \sum_{i=1}^{i=n} r_i \tag{2.24}$$

$$\text{Geometric average or annualized return} \quad r_G = \left(\prod_{i=1}^{n} (1 + r_i) \right)^{f/n} - 1 \tag{2.25}$$

where: n = the number of periods under analysis

f = the number of periods within the year (monthly $f = 12$, quarterly $f = 4$).

Average and annualized returns are calculated in Exhibit 2.21:

Exhibit 2.21 Average and annualized returns

Annual returns:

2003	10.5%
2002	−5.6%
2001	23.4%
2000	−15.7%
1999	8.6%

Arithmetic average:

$$\frac{10.5\% - 5.6\% + 23.4\% - 15.7\% + 8.6\%}{5} = 4.24\%$$

Geometric average or annualized return:

$$(1.105 \times 0.944 \times 1.234 \times 0.843 \times 1.086)^{1/5} - 1 = 3.3\%$$

It is poor performance measurement practice to annualize returns for periods less than 1 year. It is inappropriate to assume the rate of return achieved in the year to date will continue for the remainder of the year.

4. Annualized Std Dev

Standard deviation

For analysis it is more convenient to use our original non-squared units of return; therefore, we take the square root of the variance to obtain the standard deviation:

$$\text{Standard deviation } \sigma = \sqrt{\frac{\sum_{i=1}^{i=n}(r_i - \bar{r})^2}{n}} \tag{4.3}$$

A higher standard deviation would indicate greater uncertainty, variability or risk. In this version of standard deviation n not $n-1$ is used in the denominator. The use of $n-1$ would calculate the sample standard deviation. For large n it will make little difference whether n or $n-1$ is used. Since the majority of performance analysts tend to use n, for the sake of consistency and comparability I prefer to use n.

Equation (4.3) calculates standard deviation based on the periodicity of the data used, daily, monthly, quarterly, etc. For comparison, standard deviation is normally annualized for presentation purposes.

To annualize standard deviation we need to multiply by the square root of the number of observations in the year:

$$\text{Annualized standard deviation } \sigma^A = \sqrt{t} \times \sigma \tag{4.4}$$

where: t = number of observations in year (quarterly = 4, monthly = 12, etc.).

For example, to annualize a monthly standard deviation multiply by $\sqrt{12}$ and for a quarterly standard deviation multiply by $\sqrt{4}$ or 2.

5. BernardoLedoitRatio 所有盈利的和/所有亏损的和

The gain-los- ratio of any zeroprice portfolio x: is defined as:

E[x ˜ +]/E[x ˜ -],

where x ˜ +_ =max(0, x ˜ +) and x ˜ - = max(0, x- ˜) represent the positive and negative parts of the payoff, respectively.

- The gain-loss ratio summarizes the attractiveness of any zero-price portfolio.
  - A gain-loss ratio of one implies that the investment is fairly priced,
  - a gain-loss ratio above one implies the existence of an attractive investment opportunity

  https://www.econ.uzh.ch/dam/jcr:ffffffff-961c-1dd9-ffff-fffffe522c8e/Bernardo_Ledoit_2000.pdf

GAIN-LOSS RATIO (GLR)

The **Gain-Loss Ratio** or **Bernardo and Ledoit ratio** was introduced by Bernardo and Ledoit (2000). The gain-loss ratio is an alternative to the Sharpe ratio. The Gain-Loss Ratio is a downside risk measure similar to the Omega ratio, Sortino ratio, and the Kappa ratio The Gain-Loss ratio compares the expected value of positive returns to the expected value of negative returns. Positive returns are returns that exceed a certain threshold. Similarly, negative returns are the returns that do not exceed the threshold. Thus the Gain-Loss formula is the following:

$$GLR_i(\tau) = \frac{\text{Expected Value}(R^+)}{\text{Expected Value}(R^-)}$$

```
BernardoLedoitRatio <- function (R, ...)
{
    R <- checkData(R)
    if (ncol(R)==1 || is.null(R) || is.vector(R)) {
        R = na.omit(R)
        r1  = R[which(R > 0)]
        r2 = R[which(R < 0)]
        result = sum(r1)/-sum(r2)
        return(result)
    }
    else {
        result = apply(R, MARGIN = 2, BernardoLedoitRatio, ...)
        result<-t(result)
        colnames(result) = colnames(R)
        rownames(result) = paste("Bernardo and Ledoit ratio", sep="")
        return(result)
    }
}
```

6. Burke ratio

The Burke ratio is a drwdown-based measure of risk-adjusted performance.

BURKE RATIO FORMULA

The Burke ratio formula is the following

$$BR = \frac{r_P - r_F}{\sqrt{\sum_{t=1}^{d} D_t^2}}$$

where d is number of drawdowns, rP is the portfolio return, rF is the risk free rate and Dt the t-th drawdown. Clearly, the ratio looks a bit like the Sharpe ratio, but with a different denominator.

MODIFIED BURKE RATIO

In addition to the standard Burke ratio, some investors also use a modified version of the Burke ratio. The modified Burke is defined as follows

$$MBR = \frac{r_P - r_F}{\sqrt{\sum_{t=1}^{d} \frac{D_t^2}{n}}}$$

where n is the number of data points. Thus, the modified Burke ratio adjusts the original ratio by taking into consideration the number of drawdowns that has occured over the period we are analyzing.

- *Drawdown*

DRAWDOWN FORMULA

Having discussed the concept, we now discuss how to calculate it. Let's denote the drawdown at time t as DD_t. In that case, the asset's *current* DD_t equals

$$DD_t = min(0, \frac{p_t - p_{max}}{p_{max}})$$

where p_{max} is the historical peak and p_t is the current value of the investment or portfolio. In addition to the current DD, the average DD of an investment is perhaps more informative about the average level of DD the investor can expect from a portfolio or investment

$$\frac{1}{T}\sum_{t=1}^{T} DD_t$$

7. Calmar ratio

The Calmar ratio is a performance measure that consider the drawdown of a fund to assess the fund's performance.

- calmar ratio vs. sterling ratio:
  - sterling ratio: average annual rate of return for the last 36 month/ maximum drawdown for the last 36 month
  - calmar ratiio: on the monthly basis

CALMAR RATIO FORMULA

The Calmar ratio (CR) formula is the following

$$CR = \frac{\text{annualized return}}{|MDD|}$$

Both the annualized return and the maximum drawdown (MDD) are based on the last 36 months of monthly returns. To calculate the annualized return and the drawdown, see the page on how to annualize returns and how to calculate the drawdown. The maximum drawdown is the highest drawdown over the period considered.

8. Conditional Drawdown 5%

CONDITIONAL DRAWDOWN DEFINITION

Another related measure used in risk management is the so-called conditional drawdown. This is the average portfolio drawdown experienced in excess of a certain cut-off threshold α

$$(1-\alpha)\cdot 100\% \text{ of drawdowns}$$

9. Downside Deviation

Downside deviation only considers the kind of volatility that investors dislike (only associated with negative returns)

- MAR (minimum acceptable return)

MAR ratio is a measurement of returns adjusted for risk that can be used to compare the performance of commodity trading advisors, hedge funds and trading strategies. The MAR ratio is calculated by dividing the compound annual growth rate (CAGR) of a fund or strategy since inception by its biggest drawdown. The higher the ratio, the better the risk-adjusted returns.

DOWNSIDE DEVIATION FORMULA

Since DD is simply the deviation vis-à-vis a certain threshold return, the formula is very straightforward. First define Lt

$$L_t = min(0, r_t - MAR)$$

Then, downside deviation (DD) is defined as

$$DD = \sqrt{\frac{\sum_{t=1}^{n}(L_t)^2}{n}}$$

for a given level of MAR. DD is strongly related to the Sortino ratio. This measure is similar to the Sharpe ratio, but uses DD in the denominator.

10. d ratio

The d ratio is similar to the Bernado Ledoit ratio but inverted and taking into account the frequency of positive and negative returns.

```
function (R, ...)
{
    R = checkData(R)
    if (ncol(R) == 1 || is.null(R) || is.vector(R)) {
        R = na.omit(R)
        r1 = R[which(R > 0)]
        r2 = R[which(R < 0)]
        nd = length(r2)
        nu = length(r1)
        result = (-nd * sum(r2))/(nu * sum(r1))
        return(result)
    }
    else {
        result = apply(R, MARGIN = 2, DRatio, ...)
        result <- t(result)
        colnames(result) = colnames(R)
```

```
        rownames(result) = paste("d ratio", sep = "")
        return(result)
    }
}
```

11. Drawdown Deviation

```
function (R, ...)
{
    R = checkData(R)
    dd <- function(R) {
        R = na.omit(R)
        n = length(R)
        Dj = findDrawdowns(as.matrix(R))$return
        result = sqrt(sum((Dj[Dj < 0]^2)/n))
        return(result)
    }
    result = apply(R, MARGIN = 2, dd)
    dim(result) = c(1, NCOL(R))
    colnames(result) = colnames(R)
    rownames(result) = "Drawdown Deviation"
    return(result)
}
```

12. Information ratio

- (similar to sharpe ratio, insteadof absolute return, use excess return)

- information ratio of 0.5 is good, 0.75 is very good and 1.0 is exceptional.

These numbers certainly accord with my personal experience if sustained over a substantial period (3 to 5 years).

- a positive information ratio indicates outperformance and a negative information ratio indicates underperformance

simply the ratio of excess return and tracking error as follows:

$$IR = \frac{\text{Annualized excess return}}{\text{Annualized tracking error}}$$

13. Kelly Ratio

$$Kelly\ \% = W - \left[\frac{(1-W)}{R}\right]$$

where:

$Kelly\ \% =$ percent of investor's capital to put into a single trade

$W =$ historical win percentage of trading system

$R =$ trader's historical win/loss ratio

two key components to the formula for the Kelly criterion:

- the winning probability factor (W): The winning probability is the probability a trade will have a positive return.
- the win/loss ratio (R). The win/loss ratio is equal to the total positive trade amounts, divided by the total negative trading amounts.

The result of the formula will tell investors what percentage of their total capital that they should apply to each investment.

```
function (R, Rf = 0, method = "half") {
    R = checkData(R)
    if (!is.null(dim(Rf)))
        Rf = checkData(Rf)
    kr <- function(R, Rf, method) {
        xR = Return.excess(R, Rf)
        KR = mean(xR, na.rm = TRUE)/StdDev(R, na.rm = TRUE)^2
        if (method == "half") {
            KR = KR/2 }
        return(KR) }
    result = sapply(R, kr, Rf = Rf, method = method)
    dim(result) = c(1, NCOL(R))
    colnames(result) = colnames(R)
    rownames(result) = "Kelly Ratio"
    return(result)}
```

15. MartinRatio

- Sharpe-type statistic

Martin ratio (or ulcer performance index)

If the duration of drawdowns is a concern for investors the Martin ratio is similar to the modified Burke index but using the ulcer index in the denominator:

$$\text{Martin ratio } MR = \frac{r_P - r_F}{\sqrt{\sum_{i=1}^{i=n} \frac{D_i'^2}{n}}} \tag{4.57}$$

-refrence: modified burke ratio

Modified Burke ratio

For consistency with other Shape-type statistics it might be more appropriate to define the modified Burke ratio using drawdown deviation in the denominator as follows:

$$\text{Modified Burke ratio } MBR = \frac{r_P - r_F}{\sqrt{\sum_{j=1}^{j=d} \frac{D_j^2}{n}}} \tag{4.56}$$

Clearly, both the modified and standard Burke ratios will generate identical portfolio rankings if the number of drawdowns is not restricted to the largest drawdowns.

16. max drawdown

Maximum drawdown

The maximum drawdown (D_{Max}), not to be confused with the largest individual drawdown, is the maximum potential loss over a specific time period, typically 3 years. Maximum drawdown represents the maximum loss an investor can suffer in the fund buying at the highest point and selling at the lowest. Like any other statistic it is essential to compare performance over the same time period.

$$MDD = \frac{Trough\ Value - Peak\ Value}{Peak\ Value}$$

17. Mean absolute Deviation (Risk Measures

Mean absolute deviation

Clearly, if added together, the positive and negative differences of each return from the average return would cancel; however, using the absolute difference (i.e. ignore the sign) we are able to calculate the mean or average absolute deviation as follows:

$$\text{Mean absolute deviation} = \frac{\sum_{i=1}^{i=n} |r_i - \bar{r}|}{n} \tag{4.1}$$

where: n = number of observations
r_i = return in month i
$\bar{r}$ = mean return.

18. Omega (downside

The Omega ratio is a relative measure of the likelihood of achieving a given return, such as a minimum acceptable return (MAR) or a target return. The higher the omega value, the greater the probability that a given return will be met or exceeded. Omega represents a ratio of the cumulative probability of an investment's outcome above an investor's defined return level (a threshold level), to the cumulative probability of an investment's outcome below an investor's threshold level. The omega concept divides expected returns into two parts – gains and losses, or returns above the expected rate (the upside) and those below it (the downside). Therefore, in simple terms, consider omega as the ratio of upside returns (good) relative to downside returns (bad).

$$\Omega(r) = \frac{\int_r^b (1-F(x))dx}{\int_a^r F(x)dx}$$

Where

r is the threshold return, and

F is cumulative density function of returns.

Therefore, omega allows investors to visualize the trade-off between risk and return at different threshold levels for various investment choices.

Note that when the threshold is set to the mean of the distribution, the omega ratio is equal to 1.

Omega ratio (Ω)

In their article "A universal performance measure" Shadwick and Keating (2002) suggest a gain–loss ratio that captures the information in the higher moments of a return distribution as follows:

$$\text{Omega ratio } \Omega = \frac{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_i - r_T, 0)}{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_T - r_i, 0)} \tag{4.68}$$

Note:

$$\sum_{i=1}^{i=n} \max(r_T - r_i, 0) = -1 \times \sum_{i=1}^{i=n} \min(r_i - r_T, 0) \tag{4.69}$$

The omega ratio can be used as a ranking statistic; the higher the better. It equals 1 when r_T is the mean return.

The omega ratio implicitly adjusts for both skewness and kurtosis in the return distribution.

19. Pain Ratio (drawdown

Sharpe-type statistic

Pain ratio

The equivalent to the Martin ratio but using the pain index is the pain ratio:

$$\text{Pain ratio } PR = \frac{r_P - r_F}{\sum_{i=1}^{i=m} \frac{D'_i}{n}} \tag{4.58}$$

- reference: Martin ratio/ Pain Index

Pain index

If the drawdowns are not squared then the resulting pain index is very similar to the Zephyr pain index in discrete form as proposed by Thomas Becker in 2006:

$$\text{Pain index } PI = \sum_{i=1}^{i=n} \frac{|D'_i|}{n} \tag{4.50}$$

20. Prospect Ratio (used to penalise loss since most people feel loss greater than gain) (downside)

Prospect ratio

Watanabe notes that people have a tendency to feel loss greater than gain – a well-known phenomena described by prospect theory (Kahneman and Tversky, 1979). He suggests penalising loss as follows in the prospect ratio:

$$\text{Prospect ratio} = \frac{\frac{1}{n} \times \sum_{i=1}^{i=n} (\text{Max}(r_i, 0) + 2.25 \times \text{Min}(r_i, 0)) - r_T}{\sigma_D} \tag{4.86}$$

21. StdDev Sharpe (Rf=0%, p=95%)

The Sharpe ratio is simply the return per unit of risk (represented by variability). In the classic case, the unit of risk is the standard deviation of the returns.

$$SR = \frac{r_P - r_F}{\sigma_P}$$

where: r_P = portfolio return
r_F = risk-free rate
σ_P = portfolio risk (variability, standard deviation of return) normally annualised.

The higher the Sharpe ratio, the steeper the gradient and therefore the better combination of risk and return. The Sharpe ratio can be described as the return (or reward) per unit of variability (or risk).

Negative returns will generate negative Sharpe ratios, which despite the views of some commentators still retain meaning. Perversely for negative returns, it is better to be more variable not less! For those that think higher variability is always less desirable, negative Sharpe ratios are difficult statistics to interpret.

22. SkewnessKurtossRatio （downside）

Skewness–kurtosis ratio

Watanabe (2006) also explicitly adjusts for skewness and kurtosis by suggesting using the skewness–kurtosis ratio in conjunction with the Sharpe ratio, ranking portfolios using the addition of the two rather than the Sharpe ratio in isolation. Again, higher rather than lower ratios are preferred:

$$\text{Skewness} - \text{kurtosis ratio} = \frac{S}{K} \tag{4.85}$$

23. smoothing index

- provide a normalized measure of "liquidity risk."
- Interpretation of the resulting value is difficult. All we can say is that lower values appear to have autocorrelation structure like we might expect of "less liquid" instruments. Higher values appear "more liquid" or are poorly fit or mis-specified.

24. Sortino Ratio (downside

- Sharpe-type statistic
- more sensitive to downside or extreme risks than measures that use volatility

Sortino contends that risk should be measured in terms of not meeting the investment goal. This gives rise to the notion of "Minimum Acceptable Return" or MAR. All of Sortino's proposed measures include the MAR, and are more sensitive to downside or extreme risks than measures that use volatility(standard deviation of returns) as the measure of risk.

Sortino ratio

A natural extension of the Sharpe and Sharpe–omega ratios is suggested by Sortino and van der Meer (1991) (Figure 4.15) which uses downside risk in the denominator as follows:

$$\text{Sortino ratio} = \frac{(r_P - r_T)}{\sigma_D} \tag{4.77}$$

Clearly, investors should be seeking returns greater than the risk-free rate (why take any risk otherwise), therefore the minimum accepted return in most cases should be greater than the risk-free rate.

25. Ulcer Index (drawdown statistic

- it is designed as a measure of an instrument's volatility or risk
- similar with drawdown deviation
- take into account both depth and duration of drawdowns

Ulcer index

The ulcer index developed by Peter G. Martin in 1987 (Martin and McCann, 1987) (so called because of the worry suffered by both the portfolio manager and investor) is similar to drawdown deviation with the exception that the impact of the duration of drawdowns is incorporated by selecting the negative return for each period below the previous peak or high water mark. The impact of long, deep drawdowns will have a significant impact since the underperformance since the last peak is squared:

$$\text{Ulcer index } UI = \sqrt{\sum_{i=1}^{i=n} \frac{D_i'^2}{n}} \qquad (4.49)$$

where: D_i' = drawdown since pervious peak in period i.
This approach is clearly sensitive to the frequency of time period and clearly penalises managers that take time to recover to previous highs, taking into account both the depth and duration of drawdowns.

26. upside frequency (downside

the ratio or probability of returns more than the target compared to the total number of returns

UpsideFrequency(R, MAR) = length(subset of returns above MAR) / length(total returns)

27. VaR (Value at Risk)

VaR is an industry standard for measuring downside risk.

VaR measures the worst expected loss over a given time interval under normal market conditions at a given confidence level.