Day / Date

FA18-BSM-022
Maryam Asif.

Strain - Displacement relation from cylindrical to spherical co-ordinate system.

$U_r = U_x \cos\theta + U_y \sin\theta$
$U_\theta = U_x r\cos\theta + U_y r\sin\theta$
$U_z = U_z$

$$U_R = \frac{\partial U_r}{\partial r} \cdot \frac{\partial r}{\partial R} + \frac{\partial U_r}{\partial \theta} \cdot \frac{\partial \theta}{\partial R} + \frac{\partial U_r}{\partial z} \cdot \frac{\partial z}{\partial R}$$

$x = R\sin\phi\cos\theta$
$r\cancel{\cos\theta} = R\sin\phi\cancel{\cos\theta}$
$r = R\sin\phi$
$y = R\sin\phi\sin\theta$
$r\sin\theta = R\sin\phi\sin\theta$
$r = R\sin\phi$
$z = R\cos\phi$

$$\frac{\partial U}{\partial r} = \frac{\partial}{\partial r}(U_x\cos\theta + U_y\sin\theta)$$
$$= \cos\theta \frac{\partial}{\partial r} U_x + \sin\theta \frac{\partial U_y}{\partial r}$$

$$r = \sqrt{x^2+y^2}$$

$$\theta = \tan^{-1}\frac{y}{x}$$

$$z = z$$

$$U_x = \frac{\partial U}{\partial x} = \frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial x} + \frac{\partial u}{\partial \theta}\cdot\frac{\partial \theta}{\partial x} + \frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial x}$$

$$= U_r\left(\frac{1}{2\sqrt{x^2+y^2}}(2x)\right) + U_\theta \frac{1}{1+\left(\frac{y}{x}\right)^2}(y)\left(\frac{1}{x^2}\right) + 0$$

$$U_y = U_r \cdot \frac{x}{\sqrt{x^2+y^2}} - \frac{y}{x^2+y^2} U_\theta$$

$$U_y = U_r \cdot \frac{y}{\sqrt{x^2+y^2}} + \frac{x}{x^2+y^2} U_\theta$$

$$\frac{\partial U_r}{\partial r} = \cos\theta \frac{\partial}{\partial r}\left[U_r \frac{x}{\sqrt{x^2+y^2}} - U_\theta \frac{y}{\sqrt{x^2+y^2}}\right]$$

$$+ \sin\theta \frac{\partial}{\partial r}\left[U_r \frac{y}{\sqrt{x^2+y^2}} + U_\theta \cdot \frac{x}{x^2+y^2}\right]$$