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This paper deals with an economic production quantity inventory model for non-instantaneous deteriorating items under inflationary conditions considering customer returns. We adopt a price- and time-dependent demand function. Also, the customer returns are considered as a function of both price and demand. The effects of time value of money are studied using the Discounted Cash Flow approach. The main objective is to determine the optimal selling price, the optimal replenishment cycles, and the optimal production quantity simultaneously such that the present value of total profit is maximized. An efficient algorithm is presented to find the optimal solution. Finally, numerical examples are provided to solve the presented inventory model using our proposed algorithm, which is further clarified through a sensitivity analysis. The results of analysing customer returns provide important suggestions to financial managers who use price as a control to match the quantity sold to inventory while maximizing revenues. The paper ends with a conclusion and an outlook to future studies.

Keywords: inventory; price- and time-dependent demand; customer returns; non-instantaneous deteriorating items; inflation; efficient algorithm

AMS Subject Classification: 90B05; 78M50; 91B24

1. Introduction

For nearly three decades, the lodging industry has used revenue management practices and theory to extensively enhance inventory optimization and revenue generation. According to Chen and Bell [1], 'Revenue management (*RM*) has its source in the North American airline industry following deregulation in 1979 and has now been applied in many service industries and increasingly for manufactured products'. McGill and VanRyzin [2], Bitran and Caldentey [3], and Elmaghraby and Keskinocak [4] presented reviews of *RM*. Optimal pricing is an important revenue-enhancing business practice within *RM* that is often combined with inventory ordering policy. Federgruen and Heching [5] discussed the simultaneous determination of price and inventory replenishment strategies in a multi-period problem with stochastic demand. They considered both finite and infinite horizon models assuming that prices can either be adjusted arbitrarily (upward or downward) or that they can only be reduced.

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Recently, many researchers have studied the problem of joint pricing and inventory control for deteriorating items. Generally, deterioration is defined as decay, damage, spoilage, evaporation and loss of utility of the product. Most physical goods undergo decay or deterioration over time such as medicines, volatile liquids, blood banks and others.[6] The first attempt to describe optimal ordering policies for deteriorating items was made by Ghare and Schrader [7]. Later, Covert and Philip [8] derived the model with variable deteriorating rate of two-parameter Weibull distribution. Goyal and Giri [9] presented a detailed review of deteriorating inventory literatures. Abad [10,11] considered a pricing and lot-sizing problem for a perishable good under exponential decay and partial backlogging. Dye [12] proposed the joint pricing and ordering policies for a deteriorating inventory with price-dependent demand and partial backlogging. Dye et al. [13] developed an inventory and pricing strategy for deteriorating items with shortages when demand and deterioration rates are continuous and differentiable function of price and time, respectively. Chang et al. [14] introduced a deteriorating inventory model with log-concave demand and partial backlogging. Tsao and Sheen [15] proposed the problem of dynamic pricing and replenishment for deteriorating items under the supplier's trade credit and the retailer's promotional effort. Shi et al. [16] presented an optimal ordering and pricing policy with supplier quantity discounts and price-dependent stochastic demand. Sarkar [17] investigated a production-inventory model with probabilistic deterioration in two-echelon supply chain management. Sarkar and Sarkar [18] developed an inventory model with partial backlogging, time-varying deterioration and stock-dependent demand. Sarkar [19] presented an economic order quantity (EOQ) model with delay in payments, where demand and deterioration rate are both time-dependent. Sarkar [20] studied an EOQ model with delay in payments and stock-dependent demand in an imperfect production system. Sett et al. [21] proposed a two-warehouse inventory model with quadratically increasing demand and time-varying deterioration. Sarkar and Sarkar [22] developed an inventory model with inventory-dependent demand function and time-varying deterioration rate.

In most of the inventory models for deteriorating items in the literature, it is assumed that the deterioration occurs as soon as the commodities arrive in inventory. However, in real life, most goods would have a span of maintaining quality or original condition, i.e. during that period, there is no deterioration occurring. Wu et al. [23] defined the phenomenon as 'non-instantaneous deterioration'. They considered an inventory model for non-instantaneous deteriorating items with stock-dependent demand and permissible delay in payments. This type of phenomena can be commonly observed in food stuffs, fruits, green vegetables and fashionable goods, which have a span of maintaining fresh quality, and during that period there is almost no spoilage. For these kinds of items, the assumption that the deterioration starts from the instant of arrival in stock may cause retailers to make an inappropriate replenishment policy due to overvaluing of the total annual relevant inventory cost. Thus, it is necessary to consider the inventory problems for non-instantaneous deteriorating items. Ouyang et al. [24] proposed an inventory model for non-instantaneous deteriorating items with permissible delay in payments. Chang et al. [25] developed optimal replenishment policies for non-instantaneous deteriorating items with stock-dependent demand. Yang et al. [26] proposed an inventory and pricing strategy for non-instantaneous deteriorating items with price-dependent demand. In their model, shortages are allowed and partially backlogged. Geetha and Uthayakumar [27] proposed EOQ-based model for non-instantaneous deteriorating items with permissible delay in payments. In this model, demand and price are constant and shortages are allowed and partially backlogged. Musa and Sani [28] proposed a mathematical model for inventory control of non-instantaneous deteriorating items with permissible delay in payments. Maihami and Nakhai [29] presented a joint pricing and inventory control for non-instantaneous deteriorating items. In their model, the demand rate is a function of price and time simultaneously and shortages are allowed and partially backlogged. Also, Maihami and Nakhai [30] extended the mentioned model under permissible delay in payments.

In all the above-mentioned models, the inflation and the time value of money were disregarded, but most of the countries suffered from large-scale inflation and sharp decline in the purchasing power of money during years. As a result, while determining the optimal inventory policies, the effects of inflation and time value of money cannot be ignored. First, Buzacott [31] presented the EOQ model with inflation. Following Buzacott [31], several researchers (Misra [32], Jolai et al. [33], etc.) extended their approaches to distinguish the inventory models by considering the time value of money, the different inflation rates for the internal and external costs, finite replenishment, shortages, etc. Park [34] derived the EOQ in terms of purchasing credit. Datta and Pal [35] discussed a model with shortages and time-dependent demand rates to study the effects of inflation and time value of money on a finite time horizon. Goal et al. [36] developed the model economic discount value for multiple items with restricted warehouse space and the number of orders under inflationary conditions. Hall [37] presented a new model with the increasing purchasing price over time. Sarker and Pan [38] surveyed the effects of inflation and the time value of money on the optimal ordering quantities and the maximum allowable shortage in a finite replenishment inventory system. Hariga and Ben-Daya [39] presented time-varying lot-sizing models with a time-varying demand pattern and taking into account the effects of inflation and time value of money. Horowitz [40] discussed an EOQ model with a normal distribution for the inflation. Moon and Lee [41] developed an EOQ model under inflation and discounting with a random product life cycle. Mirzazadeh and Sarfaraz [42] presented a multiple-items inventory system with a budget constraint and the uniform distribution function for the external inflation rate. Dey et al. [43] developed the model for deteriorating items with time-dependent demand rate and interval valued lead-time under inflationary conditions. Mirzazadeh et al. [44] considered stochastic inflationary conditions with variable probability density functions (pdfs) over the time horizon and the demand rate is dependent on the inflation rates. Sarkar and Moon [45] developed a production inventory model for stochastic demand with inflation in an imperfect production system. Sarkar et al. [46] presented an economic manufacturing quantity (EMQ) model for time-varying demand with inflation in an imperfect production process. Wee and Law [47] developed a replenishment and pricing policy for deteriorating items taking into account the time value of money. In their model, shortages are allowed and the demand is considered as a function of price. Hsieh and Dye [48] presented pricing and inventory control model for deterioration items taking into account the time value of money. In their model, shortage is allowed and partially backlogged and the demand is considered as a function of price and time. Dye et al. [49] developed inventory and pricing strategies for deteriorating items taking into account time value of money. In their model, demand and deterioration rate are continuous and differentiable function of price and time, respectively, and shortages are allowed. Hou and Lin [50] presented an EOQ model for deteriorating items with price- and stock-dependent selling rates under inflation and time value of money. Hou and Lin [51] developed optimal pricing and ordering policies for deteriorating items under inflation and permissible delay in payments where demand rate is a linear function of price and decreases negative exponentially with time. Ghoreishi et al. [52] studied the problem of joint pricing and inventory control model for deteriorating items taking into account the time-value of money and customer returns. In this model, shortage is allowed and partially backlogged and the demand is deterministic and depends on time and price simultaneously. Sarkar and Sarkar [53] presented an EMQ model with deterioration and exponential demand under the effect of inflation and time value of money, where the production rate is a dynamic variable (varying with time) in a production system. Sarkar [54] proposed an EMQ model with price and advertising demand pattern in an imperfect production process under the effect of inflation, where the development cost, production cost and material costs are dependent on the reliability parameter. Sarkar et al. [55] considered an EMQ model for time-dependent (quadratic) demand pattern in an imperfect production process under the effect of inflation and time value of money. Sarkar and Moon [56] developed a production inventory model for stochastic demand with the effect of inflation. Sarkar et al. [57] discussed a finite replenishment model with time-varying demand under inflation and permissible delay in payments. Taheri-Tolgari et al. [58] proposed an inventory model for imperfect items under inflationary conditions with considering inspection errors. Mirzazadeh [59] presented an optimal inventory control problem with inflation-dependent demand, where the inflation and time horizon, both are random in nature. In this model, shortages are allowed and partially backlogged.

In the traditional economic production quantity (EPQ) model, customer returns are not considered, while in supply chain retailers can return some or all unsold items at the end of the selling season to the manufacturer and receive a full or partial refund. Hess and Mayhew [60] studied the problem of customer return by using regression methods to model the returns for a large direct market. Anderson et al. [61] found that the quantity sold has a strong positive linear relationship with number of returns. The same holds true for Hess and Mayhew; they used regression models to show that as the price increases, both the number of returns and the return rate increase. These empirical investigations provide evidence to support the view that customer returns increase with both the quantity sold and the price set for the product. Chen and Bell [1] considered the customer returns as a function of price and demand simultaneously. Pasternack [62] studied the newsvendor problem framework for a seasonal product where a percentage of the order quantity could be returned from the retailers to the manufacturer. Zhu [63] presented a single-item periodic-review model for the joint pricing and inventory replenishment problem with returns and expediting. Yet, only a few authors investigated the effect of customer returns on joint pricing and inventory control.

In this paper, we develop an appropriate pricing and inventory model for non-instantaneous deteriorating items under inflation and customer returns. There are a few models on pricing and inventory control with considering non-instantaneous deteriorating items. But, in the real world, the majority deteriorating items would have a span of maintaining quality or original condition, namely, during that period, there is no deterioration occurring. For this type of items, the assumption that the deterioration starts from the instant of arrival in stock may cause retailers to make inappropriate replenishment policies due to overvaluing the total annual relevant inventory cost.

Therefore, in the field of inventory management, it is necessary to consider the inventory problems for non-instantaneous deteriorating items. On the other hand, the coordination of price decisions and inventory control means optimizing the system rather than its individual elements. Thus, the optimal pricing combined with inventory ordering policy can yield considerable revenue increase. Also, in the previous research that considers non-instantaneous deteriorating items on pricing and inventory control, the effect of time value of money is not considered. However, in order to address the realistic circumstances, the effect of time value of money should be considered. In addition, in nearly all papers that consider the impact of customer returns on pricing and inventory control, the return functions are dependent on price or demand, separately. But, the empirical findings of Anderson et al. [61] provide evidence to support the view that customer returns increase with both the quantity sold and the price set for the product. Moreover, in the majority of papers that study pricing and inventory control for non-instantaneously deteriorating items, the demand functions are simple and dependent on price, stock or time, separately. But in the real world, the demand may increase when the price decreases, or it may vary through time. Therefore, in order to incorporate the realistic conditions, the price and the time should be considered simultaneously. Thus, in this work, we develop a finite planning horizon inventory model for non-instantaneous deteriorating items with price- and time-dependent demand rate. In addition, we consider the effects of customer returns and time value of money on replenishment policy and financial performance. We assume that the customer returns increase with both the quantity sold and the product price. An optimization algorithm is presented to derive the optimal length of the production period, selling price and the number of production cycles during the time horizon and then obtain the optimal production quantity when the total present value of profits is maximized. Therefore, the replenishment and price policies are appropriately developed. Numerical examples are provided to illustrate the proposed model, which is further clarified through a sensitivity analysis.

The rest of the paper is organized as follows. In Section 2, assumptions and notations throughout this paper are presented. In Section 3, we establish the mathematical model. Next, in Section 4, an algorithm is presented to find the optimal selling price and inventory control variables. In Section 5, we use numerical examples to illustrate the proposed model. Then a sensitivity analysis over a wide range of problem parameters is performed in Section 6. Finally, we give a summary and some suggestions for future studies in Section 7.

2. Notation and assumptions

The following notation and assumptions are used throughout the paper:

2.1. *Notation*

R	production rate for the item (units/unit time)
p	selling price per unit, where $p > c_2$ (decision variable)
σ	deteriorating rate of the items ($0 < \sigma < 1$)
K	set up cost per set up
c_1	holding cost per unit per unit time
c_2	purchasing price (or the production cost) per unit

T	duration of inventory cycle (decision variable)
t_p	length of the production period in an inventory cycle (decision variable)
t_d	length of time in which the product exhibits no deterioration
Q	production quantity
H	length of planning horizon
N	number of production cycles during the time horizon H
S	salvage value per unit
r	constant representing the difference between the discount (cost of capital) and the inflation rate
T^*	optimal length of the replenishment cycle time
Q^*	optimal production quantity
t_p^*	optimal length of the production period in an inventory cycle
p^*	optimal selling price per unit
I_0	maximum inventory level
$I_1(t)$	inventory level at time $t \in [0, t_d]$
$I_2(t)$	inventory level at time $t \in [t_d, t_p]$
$I_3(t)$	inventory level at time $t \in [t_p, T]$
$f(p, t_p, T; N)$	present value of total profit over the time horizon

2.2. *Assumptions*

- I The planning horizon is finite.
- II The initial and final inventory levels are both zero.
- III A single non-instantaneous deteriorating item is assumed.
- IV The production rate, which is finite, is higher than the demand rate.
- V Delivery lead time is zero.
- VI The demand rate, $D(p,t) = (a - bp)e^{\lambda t}$ (where $a, b > 0$) is a linearly decreasing function of the price and decreases (increases) exponentially with time if $\lambda < 0$ ($\lambda > 0$), respectively. [15]
- VII Shortages are not allowed.
- VIII Following the empirical findings of Anderson et al. [61], we assume that customer returns increase with both the quantity sold and the price. We use the general form: $RC(p,t) = \alpha D(p,t) + \beta p$ $(\beta \geq 0, 0 \leq \alpha < 1)$ that is presented by Chen and Bell [1]. Customers are assumed to return $RC(p,t)$ products during the period for full credit and these units are available for resale in the following period. We assume that the salvage value of the product at the end of the last period is S per unit.
- IX The length of the production period is larger than or equal to the length of time in which the product exhibits no deterioration, i.e. $t_p \geq t_d$.

3. The model formulation

Here, we considered a production inventory system for non-instantaneous deteriorating items, which will be described as follows. During the interval [0, t_d], the inventory level increases due to production as the production rate is much greater than demand rate. At time t_d, deterioration starts and, thus, the inventory level increases due to production

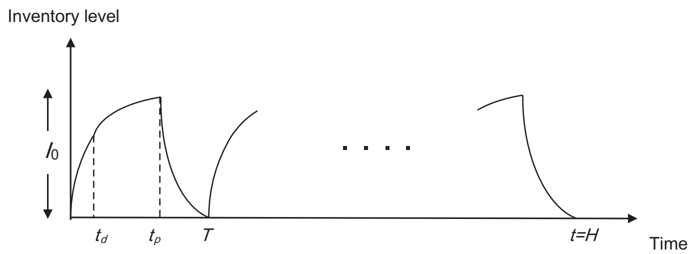

Figure 1. Graphical representation of an inventory system.

rate which is greater than the demand and the deterioration until the maximum inventory level is reached at $t = t_p$. During the interval $[t_p, T]$, there is no production and the inventory level decreases due to demand and deterioration until the inventory level becomes 0 at $t = T$. The graphical representation of the model is shown below in Figure 1. In this illustration, the demand rate increases exponentially with time (i.e. $\lambda > 0$).

During the time interval $[0, t_d]$, the system is subject to the effect of production and demand. Therefore, the change of the inventory level at time t, $I_1(t)$ is governed by

$$\frac{dI_1}{dt} = R - D(t,p). \tag{1}$$

With the condition $I_1(0) = 0$, solving Equation (1) yields

$$I_1(t) = \frac{(-a+bp)e^{\lambda t} - bp + Rt\lambda + a}{\lambda} \quad (0 \le t \le t_d). \tag{2}$$

In the time interval $[t_d, t_p]$, the system is affected by the combination of the production, demand and deterioration. Hence, the change of the inventory level at time t, $I_2(t)$, is governed with

$$\frac{dI_2}{dt} + \theta I_2(t) = R - D(t,p). \tag{3}$$

With the condition $I_2(t_p) = I_0$, Equation (3) yields

$$I_2(t) = \frac{(\theta(a-bp)(e^{t_p(\theta+\lambda)} - e^{t(\theta+\lambda)}) + ((I_0\theta - R)e^{t_p\theta} + Re^{\theta t})(\theta+\lambda))e^{-\theta t}}{\theta(\theta+\lambda)} \quad (t_d \le t \le t_p). \tag{4}$$

In the third interval $[t_p, T]$, the inventory level decreases due to demand and deterioration. Thus, the differential equation below represents the inventory status:

$$\frac{dI_3}{dt} + \theta I_3(t) = -D(t,p). \tag{5}$$

By the condition $I_3(T) = 0$, the solution of Equation (5) is

$$I_3(t) = \frac{(a-bp)(-e^{t(\theta+\lambda)} + e^{T(\theta+\lambda)})e^{-\theta t}}{\theta+\lambda} \quad (t_p \le t \le T). \tag{6}$$

Furthermore, in this interval with the condition $I_3(t_p) = I_0$, the maximum inventory level (I_0) yields the following value:

$$I_0 = \frac{(a - bp)(-e^{t_p(\theta+\lambda)} + e^{T(\theta+\lambda)})e^{-\theta t_p}}{\theta + \lambda}. \quad (7)$$

Note that the production occurs in continuous time-spans $[0,\ t_p]$. Hence, the lot size in this problem is given by

$$Q = R \cdot t_p. \quad (8)$$

Now, we can obtain the present-value inventory costs and sales revenue for the first cycle, which consists of the following elements:

(a) SR: the present value of the sales revenue for the first cycle:

$$SR = p\left(\int_0^T D(p,t) \cdot e^{-r \cdot t} dt\right). \quad (9)$$

(b) PC: The present value of production cost for the first cycle:

$$PC = c_2(R \cdot t_p). \quad (10)$$

(c) K: Since production set-up in each cycle is done at the beginning of each cycle, the present value of set-up cost for the first cycle is K, which is a constant value.

(d) HC: The present-value of inventory carrying cost for the first cycle:

$$HC = c_1\left(\int_0^{t_d} I_1(t) \cdot e^{-r \cdot t} dt + e^{-r \cdot t_d} \int_{t_d}^{t_p} I_2(t) \cdot e^{-r \cdot t} dt + e^{-r \cdot t_p} \int_{t_p}^{T} I_3(t) \cdot e^{-r \cdot t} dt\right). \quad (11)$$

(e) We assume that returns from period $i - 1$ are available for resale at the beginning of period i (except the first period in which there is no cycle previous to it). Also, it is assumed that the salvage value of product at the end of the last period ($i = N$) is S. Therefore, the present value of return cost and resale revenue for each cycle is obtained as follows:

$$PRC_i = \begin{cases} p\int_0^T (\alpha D(p,t) + \beta p)e^{-r.t}dt, & \text{for } i = 1, \\ PRC = p\int_0^T (\alpha D(p,t) + \beta p)e^{-r.t}dt - c_2\int_0^T (\alpha D(p,t) + \beta p)dt, & \text{for } i = 2,\ldots,N-1, \\ p\int_0^T (\alpha D(p,t) + \beta p)e^{-r.t}dt - c_2\int_0^T (\alpha D(p,t) + \beta p)dt & \\ \quad - Se^{-r.T}\int_0^T (\alpha D(p,t) + \beta p)dt, & \text{for } i = N. \end{cases} \quad (12)$$

Consequently, the *present value of total profit*, denoted by $f(p, t_p, T; N)$, is given by:

$$f(p, t_p, T;\ N) = \sum_{i=0}^{N-1}(SR - PC - K - HC - PRC)e^{-r\cdot i\cdot T} + S\cdot e^{-r\cdot H}\int_0^T(\alpha D(p,t) + \beta p)dt$$
$$- c_2\int_0^T(\alpha D(p,t) + \beta p)dt, \tag{13}$$

which we want to maximize subject to the following constraints:

$$p > 0,\ 0 < t_p < T, N \in \mathbb{N}.$$

The value of the variable T can be replaced by the Equation $T = H/N$, for some constant $H > 0$, and we will use Maclaurin's approximation for $\sum_{i=0}^{N-1} e^{-r\cdot i\cdot T} \cong (1 - e^{-r\cdot N\cdot T})/(1 - e^{-r\cdot T})$. Thus, the objective of this paper is to determine the values of t_p, p and N that maximize $f(p, t_p, T; N)$ subject to $p > 0$ and $0 < t_p < T$, where N is a discrete variable and p and t_p are continuous variables, can be reduced to maximizing $f(p, t_p, H/N; N)$. However, since $f(p, t_p, T; N)$, and still $f(p, t_p, H/N; N)$, is a very complicated function due to high-power expressions in the exponential function, it is difficult to show analytically the validity of the sufficient conditions. Hence, if more than one local maximum value exists, we would attain the largest of the local maximum values over the regions subject to $p > 0$ and $0 < t_p < T$. The largest value is referred to as the global maximum value of $f(p, t_p, T; N)$. So far, the procedure is to locate the optimal values of p and t_p when N is fixed. Since N is a discrete variable, the following algorithm can be used to determine the optimal values of p, t_p and N for the proposed model. We may refer to $f(p, t_p, H/N; N)$ and, for the sake of convenience, just denote it by $f(p, t_p, N)$.

4. The optimal solution procedure

The objective function has three variables. The number of replenishments (N) is a discrete variable and the production period in an inventory cycle (t_p) and the selling price per unit (p) are continuous variables. The following algorithm is used to obtain the optimal amount of t_p, p and N:

Step 1 let $N = 1$.

Step 2 Take the partial derivatives of $f(p, t_p, N)$ with respect to p and t_p, and equate the results to zero, the necessary conditions for optimality are

$$\frac{\partial}{\partial p} f(p, t_p, N) = 0 \tag{14}$$

and

$$\frac{\partial}{\partial t_p} f(p, t_p, N) = 0. \tag{15}$$

In Appendix A, we use the formula of $f(p, t_p, T; N)$ from Equation (13), inserted into Equations (14) and (15).

Step 3 For different integer N values, derive t_p^* and p^* from Equations (14) and (15). Substitute (p^*, t_p^*, N^*) to Equation (13) to derive $f(p^*, t_p^*, N^*)$.

Step 4 Add one unit to N and repeat Steps 2 and 3 for the new N. If there is no increasing in the last value of $f(p, t_p, N)$, then show the previous one which has the maximum value.

The point (p^*, t_p^*, N^*) and the value $f(p^*, t_p^*, N^*)$ constitute the optimal solution and satisfy the following conditions:

$$\Delta f(p^*, t_p^*, N^*) < 0 < \Delta f(p^*, t_p^*, N^* - 1), \tag{16}$$

where

$$\Delta f(p^*, t_p^*, N^*) = f(p^*, t_p^*, N^* + 1) - f(p^*, t_p^*, N^*). \tag{17}$$

We substitute (p^*, t_p^*, N^*) into Equation (8) to derive the Nth production lot size.

If the objective function was strictly concave, the following *sufficient* conditions must be satisfied:

$$\left(\frac{\partial^2 f}{\partial p \partial t_p}\right)^2 - \left(\frac{\partial^2 f}{\partial t_p^2}\right)\left(\frac{\partial^2 f}{\partial p^2}\right) < 0, \tag{18}$$

and any one of the following conditions:

$$\frac{\partial^2 f}{\partial t_p^2} < 0, \quad \frac{\partial^2 f}{\partial p^2} < 0. \tag{19}$$

Since f is a very complicated function due to high-power expression of the exponential function, it is unlikely to show analytically the validity of the above sufficient conditions. Our optimization problem is even more complex by that one of the variables, N, is an integer. However, it can be assessed numerically in the following illustrative examples.

5. Numerical examples

To illustrate the solution procedure and the results, let us apply the proposed algorithm to solve the following numerical examples. The results can be found by applying the Maple 13.

Example 1 This example is based on the following parameters and functions. R = 500 units/per unit time, c_1 = \$8/per unit/per unite time, c_2 = \$10/per unit, t_d = 0.04 unit time,

Table 1. Optimal solution of Example 1.

		Time interval			
N	p	t_p	T	Q	f
21	55.97656	0.94569	1.90476	472.84520	9358.184253
22*	55.92422*	0.94555*	1.81818*	472.77838*	9435.394724*
23	55.87688	0.94533	1.73913	472.66768	9434.405265

*Optimal solution.

K = \$250/per production run, σ = 0.08, r = 0.08, a = 200, b = 0.5, λ = −0.02, H = 40 unit time, α = 0.5, β = 0.7, S = \$3/per unit.

Using the solution procedure described above, the related results are shown in Table 1 and all the given conditions in Equations (18) and (19) are satisfied. In this example, the maximum present value of the total profit is found when the number of cycle (N) is 22. With 22 replenishments, the optimal solution is as follows:

$$p^* = 55.92422,\ t_p^* = 0.94555,\ T^* = 1.81818,\ f^* = 9435.394724,\ Q^* = 472.77838$$

Example 2 This example is based on the following data. R = 700 units/per unit time, c_1 = \$5/per unit/per unit time, c_2 = \$3/per unit, t_d = 1/12 unit time, K = \$250/per production run, σ = 0.08, r = 0.08, a = 200, b = 0.5, λ = −0.02, H = 40, α = 0.2, β = 0.875, S = \$1/per unit.

According to the computational results shown in Table 2, the optimal solution is as follows:

$$p^* = 63.59471,\ t_p^* = 0.69464,\ T^* = 1.42857,\ f^* = 45629.96427,\ Q^* = 486.24675.$$

6. Sensitivity analysis

First, we obtain the results of Example 1 for analysing the impact of customer returns on the optimal solutions and financial performance (Tables 3). The results illustrate that when returns are proportional to the quantity sold only (i.e. β = 0), the firm should raise the price and reduce the production quantity but if returns are proportional to price only (i.e. α = 0) the firm should decrease the price and increase the production quantity. The results confirm that when returns increase with the product price (when purchase costs are constant), the firm should set a lower price to the no-returns case (in order to discourage returns). Increasing α and/or β reduces the firm's profit.

Table 2. Optimal solution of Example 2.

		Time interval			
N	p	t_p	T	Q	F
27	63.59674	0.69455	1.48148	486.18623	45615.15453
28*	63.59471*	0.69464*	1.42857*	486.24675*	45629.96427*
29	63.59293	0.69468	1.37931	486.274309	45621.16343

*Optimal solution.

Table 3. The impact of customer returns on the optimal solutions of Example 1.

α, β	p^*	t_p^*	T^*	Q^*	f^*
α = 0.5, β = 0.7	55.92422	0.94555	1.81818	472.77838	9435.394724
α = 0, β = 0.7	86.52641	0.94486	1.42857	472.43026	70677.88368
α = 0.5, β = 0	204.82234	0.94195	2.22222	470.97935	176561.56649
α = 0, β = 0	204.25977	0.94218	2.10526	471.09078	208358.46520

Table 4. Sensitivity analysis results.

	Parameter change (%)	−%50	−%20	−%10	+%10	+%20	+%50
R							
	N	36	31	29	27	26	21
	t_p	0.69727	0.69553	0.69508	0.69424	0.69386	0.69166
	p	63.58495	63.58997	63.59294	63.59674	63.59906	63.61749
	f^*	49721.39324	47156.64293	46379.01178	44906.73240	44206.51149	39233.09699
	f^* change (%)	+8.9	+3.35	+1.64	−1.58	−3.12	−14.1
K							
	N	30	29	28	28	27	27
	t_p	0.69468	0.69468	0.69464	0.69464	0.69455	0.69455
	p	63.59136	63.59293	63.59472	63.59472	63.59674	63.59674
	f^*	46776.29252	46080.23991	45852.01643	45407.91227	45186.02154	44542.32208
	f^* change (%)	+2.51	+0.98	+0.48	−0.48	−0.97	−2.38
σ							
	N	28	28	28	28	28	29
	t_p	0.70239	0.69771	0.69616	0.69311	0.69161	0.68718
	p	63.58329	63.59011	63.59240	63.59703	63.59937	63.60377
	f^*	45699.90390	45658.47074	45644.30614	45615.44460	45600.74681	45557.54999
	f^* change (%)	+0.15	0	0	0	0	−0.15
r							
	N	28	28	28	28	28	28
	t_p	0.68167	0.68937	0.69199	0.69731	0.70001	0.70827
	p	63.63239	63.61048	63.60267	63.58664	63.57852	63.55412
	f^*	76694.21429	55090.68515	49990.89089	41876.57263	38625.48769	31116.99342
	f^* change (%)	+68.08	+20.73	+9.56	−8.23	−15.35	−31.81
a							
	N	21	25	26	30	31	36
	t_p	0.69176	0.69357	0.69407	0.69503	0.69536	0.69574
	p	38.83705	51.18680	57.38908	69.80974	76.03027	94.72272
	f^*	9997.744804	25312.86010	34847.98728	57643.07881	70888.00974	117967.57681
	f^* change (%)	−78.09	−44.53	−23.63	+26.32	+55.35	+158.53

Table 4. (*Continued*).

	Parameter change (%)	−%50	−%20	−%10	+%10	+%20	+%50
b							
	N	30	29	28	28	28	27
	t_p	0.69495	0.69477	0.69468	0.69459	0.69455	0.69435
	p	75.01590	67.70194	65.58007	61.72902	59.97248	55.28591
	f^*	56381.45661	49496.39081	47499.49394	43873.19570	42219.29192	37803.14853
	f^* change (%)	+23.56	+8.47	+4.09	−3.85	−7.47	−17.15
λ							
	N	29	28	28	28	28	27
	t_p	0.69464	0.69462	0.69463	0.69464	0.69464	0.69457
	p	63.30295	63.47445	63.53455	63.65493	63.71522	63.90955
	f^*	44994.44404	45370.66792	45500.14077	45760.13904	45890.66633	46290.21543
	f^* change (%)	−1.39	−0.57	−0.28	+0.29	+0.57	+1.47
c_1							
	N	24	26	27	29	30	32
	t_p	0.84845	0.76297	0.73389	0.65983	0.63876	0.48997
	p	63.45631	63.53133	63.55784	63.62874	63.64872	63.80809
	f^*	45711.89023	45690.27201	45670.21366	45579.27676	45541.98259	45433.42647
	f^* change (%)	+0.17	+0.13	0	−0.11	−0.12	−0.43
c_2							
	N	40	32	30	26	25	21
	t_p	0.38744	0.57171	0.63317	0.75613	0.81767	1.00241
	p	63.15891	63.42239	63.50871	63.68042	63.76372	64.01287
	f^*	50099.98208	47581.02306	46627.30443	44593.23085	43527.01351	40155.76549
	f^* change (%)	+9.79	+4.28	+2.19	−2.27	−4.61	−11.99
t_d							
	N	30	29	29	28	28	27
	t_p	0.64893	0.67343	0.68222	0.69977	0.70857	0.73491
	p	63.63672	63.61401	63.60527	63.58962	63.58090	63.55669
	f^*	46082.22962	45841.66587	45752.00185	45578.51265	45488.60382	45221.15257
	f^* change (%)	+0.99	+0.46	+0.27	−0.13	−0.31	−0.89

Table 4. (*Continued*).

Parameter change (%)	−%50	−%20	−%10	+%10	+%20	+%50
S						
N	28	28	28	28	28	28
t_p	0.69464	0.69464	0.69463	0.69463	0.69463	0.69464
p	63.59082	63.59316	63.59393	63.59549	63.59627	63.59862
f^*	45616.39510	45624.53655	45627.25033	45632.67810	45635.39201	45643.53389
f^* change (%)	0	0	0	0	0	0

Second, we carry out the sensitivity analysis of various parameters based on Example 2. The change in the values of parameters can take place due to uncertainties in any decision-making situation. In order to examine the implications of these changes, the sensitivity analysis will be of great help in a decision-making process. The values of the system which are considered here are N, t_p, p and f. Results of the sensitivity analysis are shown in Table 4. The main conclusions which one can draw from the sensitivity analysis are as follows:

(1) There is an increase (decrease) in the f value when a is increased (decreased).
(2) There is an increase (decrease) in the f value when R, K, r, b or c_2 are decreased (increased).
(3) t_p is less sensitive but p and N are moderately sensitive.
(4) All other changes in parameters do not affect the f significantly.

7. Conclusion and outlook

In real life, most goods would have a span of maintaining quality or original condition, i.e. during that period, there is no deterioration occurring. For this type of items, the assumption that the deterioration starts from the instant of arrival in stock may cause retailers to adopt inappropriate replenishment policies due to overvaluing the total annual relevant inventory cost. Therefore, in the field of inventory management, it is necessary to consider the inventory problems for non-instantaneous deteriorating items. On the other hand, the coordination of price decisions and inventory control means optimizing the system rather than its individual elements. Thus, the optimal pricing combined with inventory ordering policy can yield considerable revenue increase. Also, if we ignored inflation and time value of money the optimal present value of total profit is overstated. The overstatement of profits will lead to the wrong management decision. As a result, it is important to consider the effects of inflation and the time value of money on inventory policy. Moreover, the empirical findings of Anderson et al. [61] provide evidence to support the view that customer returns increase with both the quantity sold and the price set for the product. Consequently, in order to address the realistic circumstances, the customer returns should be considered as a function of both the quantity sold and the price. Moreover, in the majority of papers that study pricing and inventory control for non-instantaneously deteriorating items, the demand functions are simple and dependent on price, stock or time, separately. But in the real world, the demand may increase when the price decreases, or it may vary through time. Therefore, in order to incorporate the realistic conditions, the price and the time should be considered simultaneously.

In this work, we addressed the problem of joint pricing and inventory control model for non-instantaneous deteriorating items taking into account the time value of money and customer returns. The demand is deterministic and depends on time and price simultaneously. Also, the customer returns assumed as a function of both the quantity sold and the price. An algorithm is presented for deriving the optimal replenishment and pricing policy that wants to maximize the present value of total profit. Finally, numerical examples are solved and the sensitivity of the solution to changes in the values of different parameters is discussed. The results show that the present value of total profit is sensitive to changes in c_2, r, a, b and R. Hence, it is important to consider the effects of inflation and the time value of money on inventory policy and financial performance.

To the best of our knowledge, this is the first work that focuses on the optimal pricing and inventory control policy for non-instantaneously deteriorating items with the finite replenishment rate considering time- and price-dependent demand, customer returns and time value of money.

Our results suggest the following managerial insights:

- Since the customer returns have an effect on the income statement, balance sheet and cash flow statement, it will ultimately have an effect on the ratios used by financial managers to measure and compare a company's profitability, liquidity, activity and solvency. Therefore, the results of analysing customer returns provide important findings to financial managers who use the price as a control to match the quantity sold to inventory while maximizing revenues. A cost of raising the price will be an increase in returns and this cost needs to be taken into account when optimizing prices. Therefore, a firm facing customer returns that depend on the quantity of product sold should increase price and decrease production quantity to mitigate the loss in profit resulting from the customer returns. On the other hand, when customer returns increase with price, the firm should reduce price and increase the production quantity leading to a fewer returns. If the quantity of returns depends on both price and quantity sold, the firm may set a higher or lower price depending on which returns form is dominant.
- The result is intuitive, easy to implement and provides managerial insights of the effect of the change in the values of parameters shown in the sensitivity analysis table. These changes can take place due to uncertainties in any decision-making situation. Thus, in order to examine the implications of these changes, the sensitivity analysis will be of great help in a decision-making process.
- It can be seen that the present value of total profit in the instantaneous deterioration items case decrease. This implies the insight that the present value of total profit could be increased by changing the instantaneously to non-instantaneously items using the improved stock condition.
- Our model provides a decision support system fostered by Operational Research that could be implemented in management sciences, business administration and economics.

This paper can be extended in several ways. For instance, the constant deterioration rate could be extended to a time-dependent function. Furthermore, the deterministic deterioration may be extended to the stochastic deterioration. Also, the deterministic demand function could be extended to the stochastic demand function. Finally, we plan to extend the model to incorporate some more realistic features such as quantity discounts, two warehouse, allowable shortage and permissible delay in payments.

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Appendix A

For a given value of N, the necessary conditions for finding the optimal values of p^* and t_p^* are given as follows:

$$
\begin{aligned}
\frac{\partial}{\partial p} f(p, t_p, T) = &-\frac{1}{(-\lambda+r)(\sigma+r)(\sigma+\lambda)r\lambda N(-1+e^{-\frac{rH}{N}})} \\
&\times (Nbr\lambda c_1 e^{-rt_d} e^{(-\sigma-r)t_p-rt_d}(e^{-rH}-1)(-\lambda+r)e^{\frac{((t_p-t_d)\sigma+rt_p)N+(\sigma+\lambda)H}{N}} \\
&+ Nbrc_1 e^{-rt_p} e^{-\frac{rH}{N}} \lambda (e^{-rH}-1)(-\lambda+r)e^{\frac{(\sigma+\lambda+r)H-Nt_p(\sigma+r)}{N}} \\
&- Nbrc_1 e^{-rt_p} e^{-\frac{rH}{N}} \lambda (e^{-rH}-1)(\sigma+r)e^{\frac{-t_p(-\lambda+r)N+rH}{N}} \\
&- Nbr\lambda c_1 e^{-rt_d} e^{(-\sigma-r)t_p-rt_d}(e^{-rH}-1)(-\lambda+r)e^{\frac{(\sigma+\lambda)H+rt_dN}{N}} \\
&+ Nbr\lambda c_1 e^{-rt_d}(-e^{t_p(\sigma+r)+\lambda rt_d}+e^{t_p(\sigma+r)+rt_d})(e^{-rH}-1) \\
&\times (\sigma+r)e^{(-\sigma-r)t_p-rt_d} + (\sigma+\lambda)(-Nr\lambda(e^{-rH}-1)(\sigma+r)(-2bp+a)e^{\frac{-H(-\lambda+r)}{N}} \\
&+ (-Nr((-c_1be^{-rt_p}\lambda + \alpha(\sigma+r)(Sbr-((-2p+S)b+a)\lambda))e^{-rH} \\
&+ c_1be^{-rt_p}\lambda + (-bc_2r+((c_2-2p)b+a)\lambda)\alpha(\sigma+r)e^{\frac{\lambda H}{N}} \\
&+ (-N((\alpha a-2\alpha bp+2\beta p)r-2\beta p\lambda)\lambda(e^{-rH}-1)e^{\frac{rH}{N}} \\
&+ (-\lambda+r)((S(\beta H\lambda+\alpha Nb)r+2\lambda N\beta p)e^{-rH} - c_2(\beta H\lambda+\alpha Nb)r \\
&- 2\lambda N\beta p))(\sigma+r))e^{-\frac{rH}{N}} - (-Nbr\alpha e^{-rH}(-\lambda+r)(S-c_2)e^{\frac{\lambda H}{N}} \\
&+ (-Nbc_1(-r-e^{rt_d}\lambda+re^{\lambda t_d}+\lambda)e^{-rt_d} + r((S-c_2)(\beta H\lambda+\alpha Nb)r \\
&- (H\beta(S-c_2)\lambda + N(((S-c_2)\alpha-2p)b+a))\lambda))e^{-rH} \\
&+ N(bc_1(-r-e^{rt_d}\lambda+re^{\lambda t_d}+\lambda)e^{-rt_d}+\lambda r(-2pb+a))(\sigma+r)) = 0
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial}{\partial t_p} f(p, t_p, T) = &-\frac{1}{(-\lambda+r)(\sigma+r)(\sigma+\lambda)(-1+e^{-\frac{rH}{N}})} \\
&\times ((-2e^{-\frac{rH}{N}}(-\lambda+r)c_1e^{-rt_p}\left(r+\frac{1}{2}\sigma\right)(a-bp)e^{\frac{(\sigma+\lambda+r)H-Nt_p(\sigma+r)}{N}} \\
&+ 2e^{-\frac{rH}{N}}c_1(\sigma+r)e^{-rt_p}(a-bp)\left(r-\frac{1}{2}\lambda\right)e^{\frac{t_p(-\lambda+r)N+rH}{N}} \\
&+ c_1e^{-rt_d}e^{(-\sigma-r)t_p-rt_d}(-\lambda+r)(\sigma+r)(a-bp)e^{\frac{(\sigma+\lambda)H+rt_dN}{N}} + (-R(\sigma+\lambda)e^{(2\sigma+r)t_p-\sigma t_d} \\
&- (\sigma+r)(a-bp)e^{t_p(\sigma+\lambda)+rt_d} + Re^{rt_d+\sigma t_p}(\sigma+\lambda))(-\lambda+r)e^{-rt_d}c_1e^{(-\sigma-r)t_p-rt_d} \\
&+ (-rc_1e^{-rt_p}e^{\frac{\lambda H}{N}}(a-bp)e^{-\frac{rH}{N}} + Rc_2(-\lambda+r)(\sigma+r))(\sigma+\lambda))(e^{-rH}-1)) = 0.
\end{aligned}
$$

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Optimal pricing and lot-sizing policies for an economic production quantity model with non-instantaneous deteriorating items, permissible delay in payments, customer returns, and inflation

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Abstract

This article deals with an economic production quantity inventory model for non-instantaneous deteriorating items under inflationary conditions, permissible delay in payments, customer returns, and price- and time-dependent demand. The customer returns are assumed as a function of demand and price. The effects of time value of money are studied using the Discounted Cash Flow approach. The main objective is to determine the optimal selling price, the optimal length of the production period, and the optimal length of inventory cycle simultaneously such that the present value of total profit is maximized. An efficient algorithm is presented to find the optimal solution of the developed model. Finally, a numerical example is extracted to solve the presented inventory model using our proposed algorithm, and the effects of the customer returns, inflation, and delay in payments are also discussed.

Keywords

Inventory, permissible delay in payments, economic production quantity, non-instantaneous deteriorating items, customer returns, inflation, pricing

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Introduction

In the past few years, the deteriorating inventory systems have been studied considerably. Deterioration refers to the spoilage, damage, dryness, vaporization, and loss of utility of the products, such as vegetables, foodstuffs, meat, fruits, alcohol, radioactive substances, gasoline, and so on. The first authors who investigated the inventory models for deteriorating items were Ghare and Schrader.[1] Following Ghare and Schrader,[1] several efforts have been made on developing the inventory systems for deteriorating items, for example, in studies by Covert and Philip,[2] Hariga,[3] Heng et al.,[4] Jaggi et al.,[5] Moon et al.,[6] Sarker et al.,[7] and Wee.[8] Goyal and Giri[9] provided a detailed survey of deteriorating inventory literatures.

Optimal pricing is an important revenue enhancing business practice that is often combined with inventory control policy. Therefore, several researchers have studied the pricing and inventory control problems of deteriorating items. Shi et al.[10] developed the optimal pricing and ordering strategies with price-dependent stochastic demand and supplier quantity discounts. Dye[11] considered the optimal pricing and ordering policies for deteriorating items with partial backlogging and price-dependent demand. Heng et al.[4] and Abad[12] discussed the pricing and lot-sizing inventory model for a perishable good allowing shortage and partial backlogging. Dye et al.[13] investigated the optimal pricing and inventory control policies for deteriorating items with shortages and price-dependent demand. Chang

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et al.[14] developed the inventory model for deteriorating items with partial backlogging and log-concave demand. Samadi et al.[15] proposed the pricing, marketing, and service planning inventory model with shortages in fuzzy environment. In this model, the demand is considered as a power function of price, marketing expenditure, and service expenditure. Tsao and Sheen[16] considered the problem of dynamic pricing, promotion, and replenishment for deteriorating items under the permissible delay in payments.

In the real world, the majority of products, such as fruits, foodstuffs, green vegetables, and fashionable goods, would have a span of maintaining original state or quality, that is, there is no deterioration occurring during that period. Wu et al.[17] introduced the phenomenon as "non-instantaneous deterioration." For these types of items, the assumption that the deterioration starts from the instant of arrival in stock may lead to an unsuitable replenishment policy due to overstating relevant inventory cost. Thus, it is necessary to consider the inventory problems for non-instantaneous deteriorating items.

Moreover, in the traditional inventory model, it was implicitly assumed that the payment must be made to the supplier for items immediately after receiving the items. However, in real life, the supplier could encourage the retailer to buy more by allowing a certain fixed period for settling the account, and there is no charge on the amount owed during this period.

Recently, some researchers have studied the problem of joint pricing and inventory control for non-instantaneously deteriorating items under permissible delay in payments. Ouyang et al.[18] presented the inventory model for non-instantaneous deteriorating items considering permissible delay in payments. Chang et al.[19] investigated the inventory model for non-instantaneous deteriorating items with stock-dependent demand. Yang et al.[20] developed the optimal pricing and ordering strategies for non-instantaneous deteriorating items with partial backlogging and price-dependent demand. Geetha and Uthayakumar[21] considered the economic order quantity (EOQ) inventory model for non-instantaneous deteriorating items with permissible delay in payments and partial backlogging. Musa and Sani[22] discussed the inventory model for non-instantaneous deteriorating items with permissible delay in payments. Maihami and Nakhai Kamalabadi[23] presented the joint pricing and inventory control model for non-instantaneous deteriorating items with price- and time-dependent demand and partial backlogging. In addition, the mentioned model was extended by Maihami and Nakhai Kamalabadi[24] under permissible delay in payments.

In all of the models mentioned above, the inflation and the time value of money were ignored. It has happened mostly because most of decision makers believe that inflation does not have considerable influence on the inventory policy and thus do not consider the effect of inflation on the inventory system. But, today, inflation has become a perpetual feature of the economy. As a result, it is important to consider the effect of inflation and time value of money on the inventory policy and financial performance. The first author who considered the effect of inflation and time value of money on an *EOQ* model was Buzacott.[25] Following Buzacott,[25] several efforts have been made by researchers to reformulate the optimal inventory management policies taking into account inflation and time value of money, for example, in studies by Misra,[26] Park,[27] Datta and Pal,[28] Goal et al.,[29] Hall,[30] Sarker and Pan,[31] Hariga and Ben-Daya,[32] Horowitz,[33] Moon and Lee,[34] Mirzazadeh et al.,[35] Sarkar and Moon,[36] Sarkar et al.,[37] Taheri-Tolgari et al.,[38] and Gholami-Qadikolaei et al.[39] Wee and Law[40] presented a joint pricing and inventory control model for deteriorating items under inflation and price-dependent demand. Hsieh and Dye[41] developed the pricing and inventory control problem for deterioration considering price- and time-dependent demand and time value of money. Hou and Lin[42] presented the optimal pricing and ordering strategies for deteriorating items under inflation and permissible delay in payments. Ghoreishi et al.[43] proposed the joint pricing and inventory control model for deteriorating items taking into account inflation and customer returns. In this model, shortage is allowed and partially backlogged, and the demand is a function of both time and price. Ghoreishi et al.[44] addressed the problem of joint pricing and inventory control model for non-instantaneous deteriorating items under time value of money and customer returns. In this model, shortages are not allowed and the demand is deterministic and depends on time and price simultaneously.

Returns of product from customers to retailers are a significant problem for many direct marketers. Hess and Mayhew[45] used regression models to show that the number of returns has a strong positive linear relationship with the quantity sold. Anderson et al.[46] conducted empirical investigations that show that customer returns increase with both the quantity sold and the price set for the product. Chen and Bell[47] investigated the pricing and order decisions when the quantity of returned product is a function of both the quantity sold and the price. Zhu[48] considered the joint pricing and inventory control problem in a random and price-sensitive demand environment with return and expediting.

In this article, we develop an appropriate pricing and inventory control model for an economic production quantity (EPQ) model with non-instantaneous deteriorating items, permissible delay in payments, inflation, and customer returns. In the traditional inventory model, it was assumed that the payment must be made to the supplier for items immediately after receiving the items. However, in real life, the supplier could encourage the retailer to buy more by allowing a certain fixed period for settling the account, and there is no charge on the amount owed during this period. Therefore, in order to incorporate the realistic conditions, the delay in payment should be considered. Moreover, in practice, the majority of deteriorating items would have a span, in which there is no deterioration. For this type of items, the assumption that the deterioration starts from the instant of arrival in stock may lead to make inappropriate replenishment policies due to overvaluing the relevant inventory cost. As a result, in the field of inventory management, it is necessary to incorporate the inventory problems for non-instantaneous deteriorating items. On the other hand, the combination of price decisions and inventory control can yield considerable revenue increase due to optimizing the system rather than its individual elements. Also, the empirical findings of Anderson et al.[46] show that customer returns increase with both the quantity sold and the price set for the product. Moreover, in order to address the realistic circumstances, the effect of time value of money should be considered. Thus, a finite planning horizon inventory model for non-instantaneous deteriorating items with price- and time-dependent demand rate is developed. In addition, the effects of permissible delay in payments, customer returns, and time value of money on replenishment policy are also considered. We assume that the customer returns increase with both the quantity sold and the product price. An optimization algorithm is presented to derive the optimal length of the production period, selling price, and the number of production cycles during the time horizon, and then the optimal production quantity is obtained when the total present value of the total profit is maximized. Thus, the replenishment and price policies are appropriately developed. A numerical example is provided to illustrate the proposed model. The results of this example are used to analyze the impact of customer returns, inflation, and delay in payments on the optimal solution.

Following this, in section "Analysis method and assumptions," the analysis method and assumptions used are presented. In section "The model formulation," we establish the mathematical model. Next, in section "The optimal solution procedure," an algorithm is presented to find the optimal selling price and inventory control variables. In section "A numerical example," we give a numerical example and, finally, we provide a summary and some suggestions for future work in section "Conclusion and outlook."

Analysis method and assumptions

Analysis method

In this article, we develop a mathematical model that provides a decision support system fostered by Operational Research that could be implemented in management sciences, business administration, and economics. Therefore, we investigate an appropriate pricing and inventory control model for an EPQ model with non-instantaneous deteriorating items, permissible delay in payments, inflation, and customer returns. The notations used in this article are defined in Appendix 1.

Assumptions

1. A single non-instantaneous deteriorating item is assumed.
2. The initial and final inventory levels both are zero.
3. The production rate, which is finite, is higher than the demand rate.
4. Delivery lead time is zero.
5. The planning horizon is finite.
6. The demand rate, $D(p,t) = (a - bp)e^{\lambda t}$ (where $a, b > 0$), is a linearly decreasing function of the price and decreases (increases) exponentially with time if $\lambda < 0$ ($\lambda > 0$), respectively.[16]
7. Shortages are not allowed.
8. The length of the production period is larger than or equal to the length of time in which the product exhibits no deterioration, that is, $t_p \geqslant t_d$.
9. Following the empirical findings of Anderson et al.,[46] we assume that customer returns increase with both the quantity sold and the price. We use the general form $RC(p,t) = \alpha D(p,t) + \beta p$ ($\beta \geqslant 0, 0 \leqslant \alpha < 1$) that is presented by Chen and Bell.[47] Customers are assumed to return $RC(p,t)$ products during the period for full credit, and these units are available for resale in the following period. We assume that the salvage value of the product at the end of the last period is S per unit.

The model formulation

Here, we considered a production inventory system for non-instantaneous deteriorating items, which will be described as follows. During the interval [0, t_d], the inventory level increases due to production as the production rate is much greater than the demand rate. At time t_d, deterioration starts, and thus, the inventory level increases due to the production rate which is greater than the demand and the deterioration until the maximum inventory level is reached at $t = t_p$. During the interval [t_p, T], there is no production and the inventory level decreases due to demand and deterioration until the inventory level becomes 0 at $t = T$. The graphical representation of the model is shown in Figure 1. In this illustration, the demand rate increases exponentially with time (i.e. $\lambda > 0$).

During the time interval [0, t_d], the system is subject to the effect of production and demand. Therefore, the change of the inventory level at time t, $I_1(t)$ is governed by

$$\frac{dI_1(t)}{dt} = R - D(t,p) \tag{1}$$

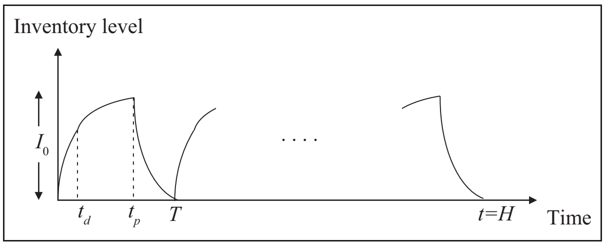

Figure 1. Graphical representation of an inventory system.

With the condition $I_1(0) = 0$, solving equation (1) yields

$$I_1(t) = \frac{(-a + bp)e^{\lambda t} - bp + Rt\lambda + a}{\lambda} \quad (0 \leqslant t \leqslant t_d) \quad (2)$$

In the time interval $[t_d, t_p]$, the system is affected by the combination of the production, demand, and deterioration. Hence, the change of the inventory level at time t, $I_2(t)$, is governed with

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = R - D(t,p) \quad (3)$$

With the condition $I_2(t_p) = I_0$, equation (3) yields

$$I_2(t) = \frac{\left(\theta(a - bp)\left(e^{t_p(\theta+\lambda)} - e^{t(\theta+\lambda)}\right) + ((I_0\theta - R)e^{t_p\theta} + Re^{\theta t})(\theta + \lambda)\right)e^{-\theta t}}{\theta(\theta + \lambda)}, \quad (t_d \leqslant t \leqslant t_p) \quad (4)$$

In the third interval $[t_p, T]$, the inventory level decreases due to demand and deterioration. Thus, the differential equation below represents the inventory status

$$\frac{dI_3(t)}{dt} + \theta I_3(t) = -D(t,p) \quad (5)$$

By the condition $I_3(T) = 0$, the solution of equation (5) is

$$I_3(t) = \frac{(a - bp)\left(-e^{t(\theta+\lambda)} + e^{T(\theta+\lambda)}\right)e^{-\theta t}}{\theta + \lambda} \quad (t_p \leqslant t \leqslant T) \quad (6)$$

Furthermore, in this interval with the condition $I_3(t_p) = I_0$, the maximum inventory level (I_0) yields the following value

$$I_0 = \frac{(a - bp)\left(-e^{t_p(\theta+\lambda)} + e^{T(\theta+\lambda)}\right)e^{-\theta t_p}}{\theta + \lambda} \quad (7)$$

Note that the production occurs in continuous timespans $[0, t_p]$. Hence, the lot size in this problem is given by

$$Q = R \cdot t_p \quad (8)$$

Now, we can obtain the present value of inventory costs and sales revenue for the first cycle, which consists of the following elements:

1. *SR*. The present value of the sales revenue for the first cycle

$$SR = p\left(\int_0^T D(p,t) \cdot e^{-r.t}dt\right) \quad (9)$$

2. *PC*. The present value of production cost for the first cycle

$$PC = c_2\left(R \cdot t_p\right) \quad (10)$$

3. *K*. Since production setup in each cycle is done at the beginning of each cycle, the present value of setup cost for the first cycle is *K*, which is a constant value.
4. *HC*. The present value of inventory carrying cost for the first cycle

$$HC = c_1\left(\int_0^{t_d} I_1(t) \cdot e^{-r \cdot t}dt + e^{-r \cdot t_d} \int_{t_d}^{t_p} I_2(t) \cdot e^{-r \cdot t}dt + e^{-r \cdot t_p}\int_{t_p}^{T} I_3(t) \cdot e^{-r \cdot t}dt\right) \quad (11)$$

5. The present value of return cost for each cycle.

We assume that returns from period $i - 1$ are available for resale at the beginning of period i (except the first period in which there is no cycle previous to it). Also, it is assumed that the salvage value of the product at the end of the last period ($i = N$) is S. Therefore, the present value of return cost and resale revenue for each cycle is obtained as follows

$$PRC_i = \begin{cases} p\int_0^T (\alpha D(p,t) + \beta p)e^{-r \cdot t}dt, & \text{for } i = 1, \\ PRC = p\int_0^T (\alpha D(p,t) + \beta p)e^{-r \cdot t}dt - c_2\int_0^T (\alpha D(p,t) + \beta p)dt, & \text{for } i = 2, \ldots, N-1, \\ p\int_0^T (\alpha D(p,t) + \beta p)e^{-r \cdot t}dt - c_2\int_0^T (\alpha D(p,t) + \beta p)dt - Se^{-r \cdot t}\int_0^T (\alpha D(p,t) + \beta p)dt, & \text{for } i = N \end{cases} \quad (12)$$

6. The present value of interest payable for the first cycle.

For each cycle, we need to consider cases where the length of the credit period is longer or shorter than the length of time in which the product exhibits no deterioration (t_d) and the length of the production period (t_p). Thus, we calculate the present value of

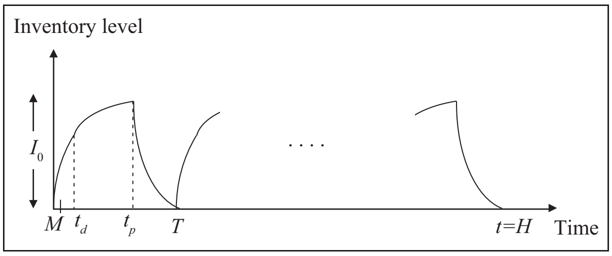

Figure 2. $0 < M \leqslant t_d$ (case 1).

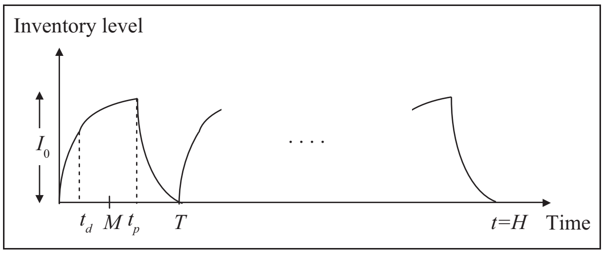

Figure 3. $t_d < M \leqslant t_p$ (case 2).

interest payable for the items kept in stock under the following three cases.

Case 1. The delay time of payments occurs before deteriorating time or $0 < M \leqslant t_d$ (see Figure 2).

In this case, payment for items is settled and the retailer starts paying the interest charged for all unsold items in inventory with rate I_p. Thus, the present value of interest payable for the first cycle is given as follows

$$IP_1 = c_2 I_p \left\{ e^{-r \cdot M} \int_M^{t_d} I_1(t) \cdot e^{-r \cdot t} dt + e^{-r \cdot t_d} \int_{t_d}^{t_p} I_2(t) \cdot e^{-r \cdot t} dt + e^{-r \cdot t_p} \int_{t_p}^{T} I_3(t) \cdot e^{-r \cdot t} dt \right\} \quad (13)$$

Case 2. The delay time of payments occurs after deteriorating time and before production period time; that is, $t_d < M \leqslant t_p$ (see Figure 3).

The conditions of this case are similar to those for case 1. Thus, the present value of interest payable for the first cycle is given as follows

$$IP_2 = c_2 I_p \left\{ e^{-r \cdot M} \int_M^{t_p} I_2(t) \cdot e^{-r \cdot t} dt + e^{-r \cdot t_p} \int_{t_p}^{T} I_3(t) \cdot e^{-r \cdot t} dt \right\} \quad (14)$$

Case 3. The delay time of payments occurs after production period time and before duration of inventory cycle or $t_p < M \leqslant T$ (see Figure 4).

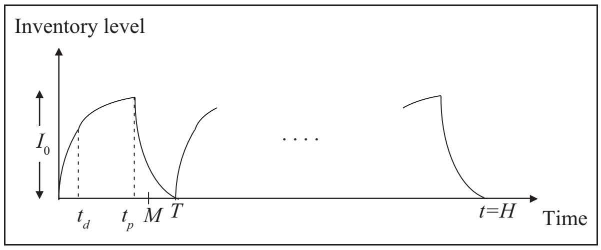

Figure 4. $t_p < M \leqslant T$ (case 3).

In this case, the retailer starts paying the interest for the items in stock from time M to T with rate I_p. Hence, the present value of interest payable for the first cycle is as follows

$$IP_3 = c_2 I_p \left\{ e^{-r \cdot M} \int_M^{T} I_3(t) \cdot e^{-r \cdot t} dt \right\} \quad (15)$$

7. The present value of interest earned for the first cycle.

There are different ways to tackle the interest earned. Here, we use the approach used in the study by Geetha and Uthayakumar.[21] We assume that during the time when the account is not settled, the retailer sells the goods and continues to accumulate sales revenue and earns interest with rate I_e. Therefore, the present value of the interest earned for the first cycle is as given below for the three different cases.

Case 1. The delay time of payments occurs before deteriorating time or $0 < M \leqslant t_d$

$$IE_1 = IE = pI_e \int_0^M t \cdot D(t) \cdot e^{-r \cdot t} dt \quad (16)$$

Case 2. The delay time of payments occurs after deteriorating time and before production period time; that is, $t_d < M \leqslant t_p$

$$IE_2 = IE = pI_e \int_0^M t \cdot D(t) \cdot e^{-r \cdot t} dt \quad (17)$$

Case 3. The delay time of payments occurs after production period time and before duration of inventory cycle or $t_p < M \leqslant T$

$$IE_3 = IE = pI_e \int_0^M t \cdot D(t) \cdot e^{-r \cdot t} dt \quad (18)$$

Consequently, the *present value of total profit*, denoted by $f(p, t_p, N)$, is given by

$$f(p,t_p,N)=\begin{cases} f_1(p,t_p,N)=\sum_{i=0}^{N-1}(SR-PC-K-HC-PRC+IE-IP_1)e^{-r\cdot i\cdot T} \\ +S\cdot e^{-r\cdot H}\int_0^T(\alpha D(p,t)+\beta p)dt-c_2\int_0^T(\alpha D(p,t)+\beta p)dt, \\ \qquad 0<M\leqslant t_d, \\ f_2(p,t_p,N)=\sum_{i=0}^{N-1}(SR-PC-K-HC-PRC+IE-IP_2)e^{-r\cdot i\cdot T} \\ +S\cdot e^{-r\cdot H}\int_0^T(\alpha D(p,t)+\beta p)dt-c_2\int_0^T(\alpha D(p,t)+\beta p)dt, \\ \qquad t_d<M\leqslant t_p \\ f_3(p,t_p,N)=\sum_{i=0}^{N-1}(SR-PC-K-HC-PRC+IE-IP_3)e^{-r\cdot i\cdot T} \\ +S\cdot e^{-r\cdot H}\int_0^T(\alpha D(p,t)+\beta p)dt-c_2\int_0^T(\alpha D(p,t)+\beta p)dt, \\ \qquad t_p<M\leqslant T \end{cases} \tag{19}$$

which we want to maximize, subject to the following constraints

$$p>0, 0<t_p<T, N\in\mathbb{N}$$

The value of the variable T can be replaced by the equation $T=H/N$ for some constant $H>0$, and we will use Maclaurin's approximation for $\sum_{i=0}^{N-1} e^{-r.i.T}\cong(1-e^{-r.N.T})/(1-e^{-r.T})$. Thus, the objective of this article is to determine the values of t_p, p, and N that maximize $f(p,t_p,N)$ subject to $p>0$ and $0<t_p<T$, where N is a discrete variable and p and t_p are continuous variables. However, since $f(p,t_p,N)$ is a very complicated function due to high-power expressions in the exponential function, it is difficult to show analytically the validity of the sufficient conditions. Hence, if more than one local maximum value exists, we would attain the largest of the local maximum values over the regions subject to $p>0$ and $0<t_p<T$. The largest value is referred to as the global maximum value of $f(p,t_p,N)$. So far, the procedure is to locate the optimal values of p and t_p when N is fixed. Since N is a discrete variable, the following algorithm can be used to determine the optimal values of p, t_p, and N.

The optimal solution procedure

The objective function has three variables. The number of production cycles (N) is a discrete variable, and the production period in an inventory cycle (t_p) and the selling price per unit (p) are continuous variables. We use the following algorithm for case 1, $0<M\leqslant t_d$, to obtain the optimal amount of t_p, p, and N.

Step 1. Let $N=1$.

Step 2. Take the partial derivatives of $f_1(p,t_p,N)$ with respect to p and t_p, and equate the results to zero; then the necessary conditions for optimality are

$$\frac{\partial}{\partial p}f_1(p,t_p,N)=0 \tag{20}$$

and

$$\frac{\partial}{\partial t_p}f_1(p,t_p,N)=0 \tag{21}$$

In Appendix 2, we use the formula of $f_1(p,t_p,N)$ from the first part of equation (19) and insert into equations (20) and (21).

Step 3. For different integer N values, derive t_p^* and p^* from equations (20) and (21). Substitute (p^*,t_p^*,N^*) into $f_1(p,t_p,N)$ from the first part of equation (19) to derive $f_1(p^*,t_p^*,N^*)$.

Step 4. Add one unit to N and repeat steps 2 and 3 for the new N. If there is no increase in the last value of $f_1(p,t_p,N)$, then consider the previous one which has the maximum value.

The point (p^*,t_p^*,N^*) and the value $f_1(p^*,t_p^*,N^*)$ constitute the optimal solution and satisfy the following conditions

$$\Delta f_1\left(p^*,t_p^*,N^*\right)<0<\Delta f_1\left(p^*,t_p^*,N^*-1\right) \tag{22}$$

where

$$\Delta f_1\left(p^*,t_p^*,N^*\right)=f_1\left(p^*,t_p^*,N^*+1\right)-f_1\left(p^*,t_p^*,N^*\right) \tag{23}$$

We substitute (p^*,t_p^*,N^*) into equation (8) to derive the Nth production lot size.

If the objective function was strictly concave, the following *sufficient* conditions must be satisfied

$$\left(\frac{\partial^2 f_1}{\partial p\partial t_p}\right)^2-\left(\frac{\partial^2 f_1}{\partial t_p^2}\right)\left(\frac{\partial^2 f_1}{\partial p^2}\right)<0 \tag{24}$$

Table 1. Optimal solution of the example.

N	p	Time interval		Q	f_1
		t_p	T		
22	56.243	1.110	1.818	555.384	3497.970
23[a]	56.182[a]	1.109[a]	1.739[a]	554.934[a]	3523.379[a]
24	56.126	1.108[a]	1.666	554.469	3468.388

[a]Optimal solution.

Table 2. The impact of customer returns on the optimal solutions of the example.

α, β	p^*	t_p^*	T^*	Q^*	f_1^*
$\alpha = 0.5, \beta = 0.7$	56.182	1.109	1.739	554.934	3523.379
$\alpha = 0, \beta = 0.7$	86.760	1.108	1.818	554.284	65,131.976
$\alpha = 0.5, \beta = 0$	206.672	1.105	2.222	552.718	91,398.947
$\alpha = 0, \beta = 0$	205.073	1.106	2.105	553.249	204,014.787

Table 3 The impact of parameter λ on the optimal solutions of the example.

λ	p^*	t_p^*	T^*	Q^*	f_1^*
0.04	58.395	1.117	1.818	558.599	5620.736
0.02	57.526	1.110	1.739	555.246	5008.264
−0.02	56.182	1.109	1.739	554.934	3523.379
−0.04	55.524	1.109	1.739	554.934	2819.001

and any one of the following conditions

$$\frac{\partial^2 f_1}{\partial t_p^2} < 0, \quad \frac{\partial^2 f_1}{\partial p^2} < 0 \tag{25}$$

It is difficult to show the validity of the above sufficient conditions, analytically, due to involvement of a high-power expression of the exponential function. However, it can be assessed numerically in the following example.

A numerical example

To illustrate the solution procedure and the results, let us apply the proposed algorithm to solve the following numerical example. The results can be found by applying Maple 13. This example is based on the following parameters and functions

$R = 500$ units/unit time, $c_1 =$ US\$8/unit/unite time, $c_2 =$ US\$10/unit, $t_d = 0.04$ unit time, $K =$ US\$250/production run, $\sigma = 0.08$, $r = 0.08$, $a = 200$, $b = 0.5$, $\lambda = -0.02$, $H = 40$ unit time, $\alpha = 0.5$, $\beta = 0.7$, $S =$ US\$3/unit, $M = 0.02$ unit time, $I_p = 0.15$/US\$/unit time, and $I_e = 0.12$/US\$/unit time.

Using the solution procedure described above, the related results are shown in Table 1, and all the given conditions in equations (24) and (25) are satisfied. In this example, the maximum present value of the total profit is found when the number of cycle (N) is 23. With 23 replenishments, the optimal solution is as follows

$$p^* = 56.182, t_p^* = 1.109, T^* = 1.739,$$
$$f_1^* = 3523.379, Q^* = 554.934$$

We obtain the results of this example for analyzing the impact of customer returns on the optimal solution and financial performance (Table 2). The results illustrate that when returns are proportional to the quantity sold only (i.e. $\beta = 0$), the firm should raise the price and reduce the production quantity, but if returns are proportional to price only (i.e. $\alpha = 0$), the firm should decrease the price and increase the production quantity. The results confirm that when returns increase with the product price (when production costs are constant), the firm should set a lower price to the no-returns case (in order to discourage returns). Increasing α and/or β reduces the firm's profit.

Moreover, if we ignored inflation and time value of money (i.e. $r = 0$), the optimal present value of total profit (f_1^*) is overstated by 24,295.241. The overstatement of profits will lead to the wrong management decision. Therefore, it is important to consider the effects of inflation and the time value of money on inventory policy.

Also, when the supplier does not provide a credit period (i.e. $M = 0$), the optimal present value of retailer total profit can be found as follows: $f_1^* = 2930.440$. It can be seen that the optimal present value of total profit decreases. Thus, retailers should try to get credit periods for their payments if they wish to increase their profit.

Conclusion and outlook

In this article, we study the effects of delay in payments, customer returns, and inflation on joint pricing and inventory control model for an EPQ model with non-instantaneous deteriorating items and price- and time-dependent demand. The customer returns are assumed as a function of price and demand simultaneously. To the best of our knowledge, this is the first model in pricing and inventory control models that considers EPQ model, delay in payments, inflation, non-instantaneously deteriorating items, and time- and price-dependent demand. The mathematical models are derived to determine the optimal selling price, the optimal length of inventory cycle time, and the optimal production quantity simultaneously. An optimization algorithm is presented to derive the optimal decision variables. Finally, a numerical example is solved and the effects of the customer returns, inflation, and delay in payments are also discussed.

The following inferences can be made from the results obtained.

- The results of analyzing customer returns provide the following insights (Table 2). A company facing customer returns that depend on the price set for the product could decrease returns by reducing price and increasing the production quantity. On the other hand, when customer returns increase with quantity of product sold, the company could mitigate the loss in profit resulting from the customer returns by increasing the price and decreasing the production quantity. If the quantity of returns depends on the price and quantity sold simultaneously, the company could set a higher or lower price based on dominant returns form.
- It can be seen that there is an improvement in the optimal present value of total profit when the discount rate of inflation is ignored (i.e. $r = 0$). The overstatement of profits will lead to the wrong management decision. Therefore, it is important to consider the effects of inflation and the time value of money on inventory policy.
- The results show that when a delay in payments is allowed, the optimal present value of total profit for the retailer does enhance. Thus, retailers should try to get credit periods for their payments if they wish to increase their profit.

The proposed model can be extended in numerous ways for future research. For example, we could incorporate (1) stochastic demand function, (2) two warehouse, (3) quantity discount, (4) deteriorating cost, and (5) shortages.

Declaration of conflicting interests

The authors declare that there is no conflict of interest.

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Appendix I

Notation

c_1	holding cost per unit time
c_2	purchasing price (or the production cost) per unit
$f(p, t_p, N)$	present value of total profit over the time horizon
H	length of planning horizon
I_0	maximum inventory level
$I_1(t)$	inventory level at time $t \in [0, t_d]$
$I_2(t)$	inventory level at time $t \in [t_d, t_p]$
$I_3(t)$	inventory level at time $t \in [t_p, T]$
I_e	interest earned per dollar per unit time

I_p interest charged per dollar per unit time
K setup cost per setup
M trade credit period
N number of production cycles during the time horizon H
p selling price per unit, where $p > c_2$ (decision variable)
p^* optimal selling price per unit
Q production quantity
Q^* optimal production quantity
R production rate for the item (units/unit time)
r constant representing the difference between the discount (cost of capital) and the inflation rate
S salvage value per unit
T duration of inventory cycle (decision variable)
T^* optimal length of inventory cycle
t_d length of time in which the product exhibits no deterioration
t_p length of the production period in an inventory cycle (decision variable)
t_p^* optimal length of the production period in an inventory cycle
σ deteriorating rate of the items ($0 < \sigma < 1$)

Appendix 2

For a given value of N, the necessary conditions for finding the optimal values of p^* and t_p^* are given as follows

$$
\begin{aligned}
&\frac{\partial}{\partial p} f_1(p, t_p, N) \\
&= -\frac{1}{(-\lambda + r)^2(\theta + \lambda)cr(\sigma + \lambda)(\theta + r)\lambda\Bigl(-1 + e^{-\frac{rH}{N}}\Bigr)}((-rb\lambda c_1 e^{-rt_d} e^{(-r-\sigma)t_p - rt_d}(\theta + \lambda) \\
&(-\lambda + r)^2(\theta + r)e^{\frac{((-t_d + t_p)\sigma + rt_p)N + (\sigma + \lambda)H}{N}} \\
&- rb\lambda c_1 e^{-\frac{rH}{N}} e^{-rt_p}(\theta + \lambda)(-\lambda + r)^2(\theta + r)e^{\frac{(\sigma + \lambda + r)H - Nt_p(r + \sigma)}{N}} \\
&+ rbe^{-\frac{rH}{N}} e^{-rt_p} c_1\lambda(\theta + \lambda)(-\lambda + r)(\theta + r)(\sigma + r)e^{\frac{-t_p(-\lambda + r)N + rH}{N}} \\
&+ rb\lambda c_1 e^{-rt_d} e^{(-r-\sigma)t_p - rt_d}(\theta + \lambda)(-\lambda + r)^2(\theta + r)e^{\frac{(\sigma + \lambda)H - rt_d N}{N}} \\
&- rb\lambda c_2 I_p e^{-rt_p}(\sigma + \lambda)(-\lambda + r)^2(\sigma + r)e^{-(-r-\theta)t_p + (\theta + \lambda)T} \\
&- rb\lambda c_2 I_p e^{-rt_d} e^{-(-r-\theta)t_p - rt_d}(\sigma + \lambda)(-\lambda + r)^2(\sigma + r)e^{(t_p - t_d + T)\theta + T\lambda + rt_p} \\
&- rb\lambda c_1 e^{-rt_d}(\theta + \lambda)(-\lambda + r)(\theta + r)\Bigl(e^{(\sigma + \lambda)t_p + rt_d} - e^{(\sigma + r)t_p + \lambda t_d}\Bigr)(r + \sigma)e^{(-\sigma-r)t_p - rt_d} \\
&+ (\lambda((-r - \theta)e^{(\theta + \lambda)t_p + rt_d} + (\theta + r)e^{(\theta + r)t_p + \lambda t_d} \\
&+ e^{(\theta + \lambda)T + rt_d}(-\lambda + r))e^{-rt_d}(-\lambda + r)(\sigma + r)c_2 r I_p b e^{(-\theta-r)t_p - rt_d} \\
&+ r(\theta + \lambda)\lambda(-\lambda + r)(\theta + r)(\sigma + r)(-2bp + a)e^{-\frac{H(-\lambda + r)}{N}}\lambda(-\lambda + r)(\theta + \lambda) \\
&\quad\Biggl(\Biggl(\Biggl(-2\Bigl(-\frac{1}{2}c_2 + p\Bigr)\alpha b - \beta c_2 + 2p\beta + a\alpha\Biggr)r - 2\beta\Bigl(-\frac{1}{2}c_2 + p\Bigr)\lambda\Biggr) \\
&\quad(\sigma + r)e^{\frac{rH}{N}} - (c_1 be{-}^{rt_p} + \alpha(r + \sigma)((-2p + c_2)b + a))re\frac{\lambda H}{N} \\
&-2(-\lambda + r)\beta\Bigl(-\frac{1}{2}c_2 + p\Bigr)(r + \sigma)\Biggr)(\theta + r)e^{\frac{rH}{N}} \\
&+ (r + \theta)\Bigl(rI_e\lambda(\theta + \lambda)(\theta + r)(-2bp + a)(rM - \lambda M + 1)e^{-M(-\lambda + r)}\Bigr) \\
&+ rb\lambda e^{-rt_p} I_p c_2(-\lambda + r)e^{-T(-\lambda + r)} + rbe^{-r(M + t_d)}e^{-rM}c_2(-\lambda + r)e^{rM + \lambda t_d} \\
&- rbe^{-r(M + t_d)}e^{-rM}I_p c_2(-\lambda + r)e^{rt_d + \lambda M} \\
&- c_2 b I_p e^{-rM}(-\lambda + r)^2\bigl(-e^{rt_d} + e^{rM}\bigr)e^{-r(M + t_d)} \\
&+ bc_1(-\lambda + r)\bigl(-r + re^{\lambda t_d} + \lambda - e^{rt_d}\lambda\bigr)e^{-rt_d} \\
&+ r\lambda(-\lambda + r + I_e)(-2bp + a))(\theta + r))))(\sigma + \lambda))\bigl(-1 + e^{-rH}\bigr)) \\
&+ \frac{Se^{-rH}(\alpha Nb - \alpha e^{\frac{H\lambda}{N}}Nb + \beta H\lambda}{\lambda N} - \frac{c_2(\alpha Nb - \alpha e^{\frac{H\lambda}{N}}Nb + \beta H\lambda}{\lambda N} = 0
\end{aligned}
$$

and

$$
\begin{aligned}
&\frac{\partial}{\partial t_p} f_1\left(p, t_p, N\right) \\
&= \frac{1}{(\sigma+\lambda)(-\lambda+r)(\theta+r)(\sigma+r)(\theta+\lambda)\left(-1+e^{-\frac{rH}{N}}\right)}\left(\left(-1+e^{-rH}\right)\left(-2\left(r+\frac{1}{2}\sigma\right)\right.\right. \\
&(a-bp)(-\lambda+r)(\theta+r)e^{-rt_p}e^{-\frac{rH}{N}}(\theta+\lambda)c_1 e^{-\frac{H(\sigma+\lambda+r)-Nt_p(\sigma+r)}{N}} \\
&+2\left(-\frac{1}{2}\lambda+r\right)(a-bp)(\theta+r)e^{-rt_p}(\sigma+r)e^{-\frac{rH}{N}}(\theta+\lambda)c_1 e^{\frac{-t_p(-\lambda+r)N+rH}{N}} \\
&+c_1 e^{-rt_d}e^{(-\sigma-r)t_p-rt_d}(\theta+\lambda)(-\lambda+r)(\theta+r)(\sigma+r)(a-bp)e^{-\frac{H(\sigma+\lambda)+Nt_d r}{N}} \\
&+(-\lambda+r)e^{-rt_d}(\theta+r)\left(-R(\sigma+\lambda)e^{(2\sigma+r)t_p-\sigma t_d}-(\sigma+r)\right. \\
&(a-bp)e^{(\sigma+\lambda)t_p+rt_d}+Re^{rt_d+\sigma t_p}(\sigma+\lambda))(\theta+\lambda)c_1 e^{(-\sigma-r)t_p-rt_d} \\
&+(\sigma+\lambda)\left(c_2(-\lambda+r)I_p e^{-rt_d}(\sigma+r)\right. \\
&\left(-R(\theta+\lambda)e^{(2\theta+r)t_p-\theta t_d}-(\theta+r)(a-bp)e^{(\theta+\lambda)t_p+rt_d}+(\theta+r)(a-bp)e^{(\theta+\lambda)T+rt_d}+Re^{rt_d+\theta t_p}(\theta+\lambda)\right) \\
&e^{(-\theta-r)t_p-rt_d}-re^{\frac{\lambda H}{N}}e^{-rt_p}c_1(\theta+\lambda)(\theta+r)(a-bp)e^{-\frac{rH}{N}}+c_2(\sigma+r)) \\
&\left.\left.(2\left(-\frac{1}{2}\lambda+r\right)(a-bp)I_p(\theta+r)e^{-rt_p}e^{-t_p(-\lambda+r)}+\left(-re^{-rt_p}I_p(a-bp)e^{-T(-\lambda+r)}+R(-\lambda+r)(\theta+r)(\theta+\lambda)\right)\right)\right)=0
\end{aligned}
$$

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Research Article

Joint Optimal Pricing and Inventory Control for Deteriorating Items under Inflation and Customer Returns

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This paper studies the effect of inflation and customer returns on joint pricing and inventory control for deteriorating items. We adopt a price and time dependent demand function, also the customer returns are considered as a function of both price and demand. Shortage is allowed and partially backlogged. The main objective is determining the optimal selling price, the optimal replenishment cycles, and the order quantity simultaneously such that the present value of total profit in a finite time horizon is maximized. An algorithm has been presented to find the optimal solution. Finally, we solve a numerical example to illustrate the solution procedure and the algorithm.

1. Introduction

Recently, many researchers have studied the problem of joint pricing and inventory control for deteriorating items. Generally, deterioration is defined as decay, damage, spoilage, evaporation, and loss of utility of the product. Most physical goods undergo decay or deterioration over time such as medicines, volatile liquids, blood banks, and others [1]. The first attempt to describe optimal ordering policies for deteriorating items was made by Ghare and Schrader [2]. Later, Covert and Philip [3] derived the model with variable deteriorating rate of two-parameter Weibull distribution. Goyal and Giri [4] presented a detailed review of deteriorating inventory literatures. Abad [5, 6] considered a pricing and lot-sizing problem for a perishable good under exponential decay and partial backlogging. Dye [7] proposed the joint pricing and ordering policies for a deteriorating inventory with price-dependent demand and partial backlogging. Dye et al. [8] developed an inventory and pricing strategy for deteriorating items with shortages when demand and deterioration rate are continuous and differentiable function of price and time, respectively. Chang et al. [9] introduced a deteriorating inventory model with price-time-dependent demand and partial backlogging. Nakhai and Maihami [10] developed the joint pricing and ordering policies for deteriorating items with partial backlogging where the demand is considered as a function of both price and time. Tsao and Sheen [11] proposed the problem of dynamic pricing and replenishment for a deteriorating item under the supplier's trade credit and the retailer's promotional effort. Sarkar [12] extended the model with finite replenishment rate, stock-dependent demand, imperfect production, and delay in payments with two progressive periods. Sarkar [13] proposed an EOQ (economic order quantity) model for finite replenishment rate with delay in payments. In this model, deterioration and demand of the item have been considered as a time-dependent function. Sett et al. [14] considered a two-warehouse inventory model with quadratic increasing demand and time-varying deterioration. This model is derived with a finite replenishment rate and unequal length of the cycle time. Sarkar et al. [15] developed an economic production quantity model with stochastic demand in an imperfect production system. This model is derived for both continuous and discrete random demands.

In all the above models, the inflation and the time value of money were disregarded, but most of the countries have suffered from large-scale inflation and sharp decline in the purchasing power of money during years. As a result, while determining the optimal inventory policies, the effects of inflation and time value of money cannot be ignored. First, Buzacott [16] presented the EOQ model with inflation.

Following Buzacott [16], several researchers (Misra [17], Jolai et al. [18], etc.) have extended their approaches to distinguish the inventory models by considering the time value of money, the different inflation rates for the internal and external costs, finite replenishment, shortages, and so forth. Park [19] derived the economic order quantity in terms of purchasing credit. Datta and Pal [20] discussed a model with shortages and time-dependent demand rates to study the effects of inflation and time value of money on a finite time horizon. Goel et al. [21] developed the model economic discount value for multiple items with restricted warehouse space and the number of orders under inflationary conditions. Hall [22] presented a new model with the increasing purchasing price over time. Sarker and Pan [23] surveyed the effects of inflation and the time value of money on the optimal ordering quantities and the maximum allowable shortage in a finite replenishment inventory system. Hariga and Ben-Daya [24] have presented time-varying lot-sizing models with a time-varying demand pattern, taking into account the effects of inflation and time value of money. Horowitz [25] discussed an EOQ model with a normal distribution for the inflation. Moon and Lee [26] developed an EOQ model under inflation and discounting with a random product life cycle. Mirzazadeh and Sarfaraz [27] presented a multiple-item inventory system with a budget constraint and the uniform distribution function for the external inflation rate. Dey et al. [28] developed the model for a deteriorating item with time-dependent demand rate and interval-valued lead time under inflationary conditions. Mirzazadeh et al. [29] considered stochastic inflationary conditions with variable probability density functions (pdfs) over the time horizon and the demand rate is dependent on the inflation rates. Wee and Law [30] developed a deteriorating inventory model taking into account the time value of money for a deterministic inventory system with price-dependent demand. Hsieh and Dye [31] presented pricing and inventory control model for deterioration items taking into account the time value of money. In their model, shortage was allowed and partially backlogged and the demand was assumed as a function of price and time. Sarkar and Moon [32] developed a production inventory model for stochastic demand with inflation in an imperfect production system. Sarkar et al. [33] presented an EMQ (economic manufacturing quantity) model for time varying demand with inflation in an imperfect production process. Sarkar et al. [34] considered an economic order quantity model for various types of deterministic demand patterns in which the delay periods and different discount rates on purchasing cost are offered by the supplier to the retailers in the presence of inflation.

In the classical EOQ models, customer returns have not been considered, while in supply chain retailers can return some or all unsold items at the end of the selling season to the manufacturer and receive a full or partial refund. Hess and Mayhew [35] studied the problem of customer return by using regression methods to model the returns for a large direct market. Anderson et al. [36] found that the quantity sold has a strong positive linear relationship with number of returns. Same as Hess and Mayhew, they used regression models to show that as the price increases, both the number of returns and the return rate increase. These empirical investigations provide evidence to support the view that customer returns increase with both the quantity sold and the price set for the product. Chen and Bell [37] considered the customer returns as a function of price and demand simultaneously. Pasternack [38] studied the newsvendor problem framework for a seasonal product where a percentage of the order quantity could be returned from the retailers to the manufacturer. Zhu [39] presented a single-item periodic-review model for the joint pricing and inventory replenishment problem with returns and expediting. Yet, only a few authors have investigated the effect of customer returns on joint pricing and inventory control.

In the previous research that considered the impact of customer returns on pricing and inventory control for deteriorating items, the effect of time value of money has not been considered. However, in order to consider the realistic circumstances, the effect of time value of money should be considered. On the other hand, in nearly all papers that consider the impact of customer returns on pricing and inventory control, the return functions are dependent on price or demand, separately. But the empirical findings of Anderson et al. [36] provide evidence to support the view that customer returns increase with both the quantity sold and the price set for the product. The present paper studies the effect of inflation and customer returns on the joint pricing and inventory control for deteriorating items. We assume that the customer returns increase with both the quantity sold and the product price. The demand is deterministic and depends on time and price simultaneously. Shortages are allowed and partially backlogged. An optimization procedure is presented to derive the optimal time with positive inventory, selling price, and the number of replenishments and then obtains the optimal order quantity when the total present value of profits is maximized. Thus, the replenishment and price policies are appropriately developed. Numerical examples are provided to illustrate the proposed model.

The rest of the paper is organized as follows. In Section 2, assumptions and notations throughout this paper are presented. In Section 3, we establish the mathematical model. Next, in Section 4, an algorithm is presented to find the optimal selling price and inventory control variables. In Section 5, we use a numerical example and, finally, summary and some suggestions for the future are presented in Section 6.

2. Notations and Assumptions

The following notations and assumptions are used throughout the paper.

Notations

A: Constant purchasing cost per order

c: Purchasing cost per unit

c_1: Holding cost per unit per unit time

c_2: Backorder cost per unit per unit time

c_3: Cost of lost sale per unit

p: Selling price per unit, where $p > c$

θ: Constant deterioration rate

r: Constant representing the difference between the discount (cost of capital) and the inflation rate

Q: Order quantity

T: Length of replenishment cycle time

t_1: Length of time in which there is no inventory shortage

SV: Salvage value per unit

H: Length of planning horizon

N: Number of replenishments during the time horizon H

T^*: Optimal length of the replenishment cycle time

Q^*: Optimal order quantity

t_1^*: Optimal length of time in which there is no inventory shortage

p^*: Optimal selling price per unit

$I_1(t)$: Inventory level at time $t \in [0, t_1]$

$I_2(t)$: Inventory level at time $t \in [t_1, T]$

I_0: Maximum inventory level

S: Maximum amount of demand backlogged

PWTP(p, t_1, T): The present-value of total profit over the time horizon.

Assumptions. In this paper, the following assumptions are considered.

(1) There is a constant fraction of the on-hand inventory deteriorates per unit of time and there is no repair or replacement of the deteriorated inventory.

(2) The replenishment rate is infinite and the lead time is zero.

(3) The demand rate, $D(p,t) = (a - bp)e^{\lambda t}$ (where $a > 0$, $b > 0$) is a linearly decreasing function of the price and decreases (increases) exponentially with time when $\lambda < 0$ ($\lambda > 0$) [11].

(4) Shortages are allowed. The unsatisfied demand is backlogged, and the fraction of shortage backordered is $\beta(x) = k_0\, e^{-\delta x}$, ($\delta > 0$, $0 < k_0 \le 1$), where x is the waiting time up to the next replenishment, δ is a positive constant, and $0 \le \beta(x) \le 1$, $\beta(0) = 1$ [5].

(5) The time horizon is finite.

(6) Following the empirical findings of Anderson et al. [36], we assume that customer returns increase with both the quantity sold and the price using the following general form: $R(p,t) = \alpha D(p,t) + \beta p$ ($\beta \ge 0$, $0 \le \alpha < 1$).

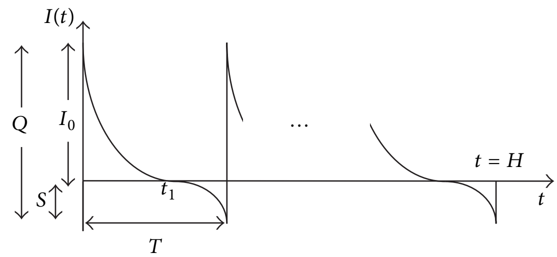

Figure 1: Graphical representation of inventory system.

3. Model Formulation

We use Nakhai and Maihami inventory shortage model [10]. According to this model; the inventory system is as follows: I_0 units of an item arrive at the inventory system at the beginning of each cycle and during the time interval $[0, t_1]$, drop to zero due to demand and deterioration. Finally, a shortage occurs due to demand and partial backlogging during the time interval $[t_1, T]$ (see Figure 1).

The equation representing the inventory status in system for the first interval [10] is as follows:

$$\frac{dI_1(t)}{dt} + \theta dI_1(t) = -D(p,t) \quad 0 \le t \le t_1, \tag{1}$$

when $I_1(t_1) = 0$, inventory level in the $(0, t_1)$ interval yields as the following:

$$I_1(t) = \frac{(a-bp)\,e^{-\theta t}}{\lambda + \theta}\left[e^{(\lambda+\theta)t_1} - e^{(\lambda+\theta)t}\right], \quad 0 \le t \le t_1. \tag{2}$$

Also in this interval with the condition $I_1(0) = I_0$, the maximum inventory level (I_0) yields as follows:

$$I_0 = \frac{(a-bp)}{\lambda+\theta}\left[e^{(\lambda+\theta)t_1} - 1\right]. \tag{3}$$

In the second interval (t_1, T), shortage is partially backlogged according to fraction $\beta(T-t)$. Therefore, the inventory level at time t is obtained by the following:

$$\frac{dI_2}{dt} = -D(p,t)\,\beta(T-t) = \frac{-D(p,t)}{e^{\delta(T-t)}}, \quad t_1 \le t \le T. \tag{4}$$

The solution of the above differential equation, after applying the boundary conditions $I_2(t_1) = 0$, is

$$I_2(t) = \frac{(a-bp)\,e^{-\delta T}\left(e^{(\delta+\lambda)t_1} - e^{(\lambda+\delta)t}\right)}{\lambda+\delta}, \quad t_1 \le t \le T. \tag{5}$$

If we put $t = T$ into $I_2(t)$, the maximum amount of demand backlogging (S) will be obtained:

$$S = -I_2(T) = -\frac{(a-bp)\,e^{-\delta T}\left(e^{(\delta+\lambda)t_1} - e^{(\lambda+\delta)T}\right)}{\lambda+\delta}. \tag{6}$$

Order quantity per cycle (Q) is the sum of S and I_0, that is:

$$Q = S + I_0 = \frac{(a-bp)\, e^{-\delta T}\left(e^{(\delta+\lambda)t_1} - e^{(\lambda+\delta)T}\right)}{\lambda+\delta} + \frac{(a-bp)}{\lambda+\theta}\left[e^{(\lambda+\theta)t_1} - 1\right]. \tag{7}$$

Now, we can obtain the present value of inventory costs and sales revenue for the first cycle, which consists of the following elements.

(1) Since replenishment in each cycle has been done at the start of each cycle, the present value of replenishment cost for the first cycle will be A, which is a constant value.

(2) Inventory occurs during period t_1; therefore, the present value of holding cost (HC) for the first cycle is

$$\mathrm{HC} = c_1\left(\int_0^{t_1} I_1(t)\cdot e^{-r\cdot t}dt\right). \tag{8}$$

(3) The present value of shortage cost (SC) due to backlog for the first cycle is

$$\mathrm{SC} = c_2\left(e^{-r\cdot t_1}\int_{t_1}^{T} -I_2(t)\cdot e^{-r\cdot t}dt\right). \tag{9}$$

(4) The present value of opportunity cost due to lost sales (OC) for the first cycle is

$$\mathrm{OC} = c_3\left(e^{-r\cdot t_1}\int_{t_1}^{T} D(p,t)\left(1-\beta(T-t)\right)\cdot e^{-r\cdot t}dt\right). \tag{10}$$

(5) The present value of purchase cost (PC) for the first cycle is

$$\mathrm{PC} = c\left(I_0 + Se^{-r\cdot T}\right). \tag{11}$$

(6) The present value of return cost (RC) for the first cycle is

$$\mathrm{RC} = (p-\mathrm{SV})\int_0^{t_1}\left(\alpha D(p,t) + \beta p\right)e^{-r\cdot t}dt. \tag{12}$$

(7) The present value of sales revenue (SR) for the first cycle is

$$\mathrm{SR} = p\left(\int_0^{t_1} D(p,t)\cdot e^{-r\cdot t}dt + S\cdot e^{-r\cdot T}\right). \tag{13}$$

There are N cycles during the planning horizon. Since inventory is assumed to start and end at zero, an extra replenishment at $t = H$ is required to satisfy the backorders of the last cycle in the planning horizon. Therefore, the total number of replenishment will be $N+1$ times; the first replenishment lot size is I_0, and the 2nd, 3rd, ..., Nth replenishment lot size is as follows:

$$Q = S + I_0. \tag{14}$$

Finally, the last or $(N+1)$th replenishment lot size is S.

Therefore, the present value of total profit during planning horizon (denoted by $\mathrm{PWTP}(p,t_1,T)$) is derived as follows:

$$\begin{aligned}\mathrm{PWTP}(p,t_1,T) &= \sum_{i=0}^{N-1}(\mathrm{SR} - A - \mathrm{HC} - \mathrm{SC} - \mathrm{OC} - \mathrm{PC} - \mathrm{RC})\, e^{-r.i\cdot T} \\ &\quad - A\cdot e^{-r\cdot H}.\end{aligned} \tag{15}$$

The value of the variable T can be replaced by the equation $T = H/N$ which uses Maclaurin's approximation for $\sum_{i=0}^{N-1} e^{-r\cdot i\cdot T} \cong 1 - e^{-r\cdot N\cdot T}/1 - e^{-r\cdot T}$. Thus, the objective of this paper is determining the values of t_1, p, and N that maximize $\mathrm{PWTP}(p,t_1,T)$ subject to $p > 0$ and $0 < t_1 < T$, where N is a discrete variable and p and t_1 are continuous variables. For a given value of N, the necessary conditions for finding the optimal p^* and t_1^* are given as follows:

$$\begin{aligned}\frac{\partial \mathrm{PWTP}}{\partial p}(p,t_1,N) &= -\frac{1}{(-\lambda+r)(\delta+\lambda)\, r\left(-1+e^{-(rH/N)}\right)} \\ &\quad\times\Big(\Big(-re^{-(\delta H/N)}e^{-(rH/N)}(-\lambda+r)(a-bp)\,e^{((\delta+\lambda)H)/N} \\ &\quad + re^{-(\delta H/N)}e^{(\delta+\lambda)t_1}(-\lambda+r)(a-bp)\,e^{-(rH/N)} \\ &\quad + (\delta+\lambda)\Big(r(a-bp)\,e^{-t_1(-\lambda+r)} \\ &\quad + \Big(\Big(\left((-\mathrm{SV}+2p)\beta + \alpha(a-bp)\right)r \\ &\quad -2\left(-\frac{1}{2}\mathrm{SV}+p\right)\lambda\beta\Big)e^{rt_1} \\ &\quad - r\alpha(a-bp)\,e^{\lambda t_1} \\ &\quad -2\left(-\frac{1}{2}\mathrm{SV}+p\right)(-\lambda+r)\beta\Big)e^{-rt_1} \\ &\quad - r(a-bp)\Big)\Big)\left(-1+e^{-rH}\right)\Big) = 0,\end{aligned} \tag{16}$$

$$\begin{aligned}\frac{\partial \mathrm{PWTP}}{\partial t_1}(p,t_1,N) &= \frac{1}{(r-\delta-\lambda)(-\lambda+r)(\theta+r)(\delta+\lambda)\, r\left(-1+e^{-(rH/N)}\right)} \\ &\quad\times\Big(\Big(-2(\delta+\lambda)\, r(\theta+r)\, e^{-rt_1}(a-bp)\, c_3\left(r-\frac{1}{2}\delta-\frac{1}{2}\lambda\right) \\ &\quad\times(-\lambda+r)\, e^{-(rH/N)}e^{(-t_1(r-\delta-\lambda)N+H(r-\delta))/N}\end{aligned}$$

Table 1: Optimal solution of the example.

N	p	Time interval t_1	Time interval T	Q	PWTP
10	257.5664	2.5798	4	391.0126	32372.5220
11*	**254.8477***	**2.7126***	**3.63***	**392.2260***	**35919.3928***
12	694.1629	0.2142	3.33	107.4832	−161403.4113

$$\begin{aligned}&+\Bigg(2(\delta+\lambda)\left(r-\frac{1}{2}\delta-\frac{1}{2}\lambda\right)e^{(t_1(\delta+\lambda)N+rH)/N}\\&\quad-r^2e^{((\delta+\lambda)H+rt_1N)/N}+e^{t_1(r+\delta+\lambda)}(r-\delta-\lambda)^2\Bigg)\\&\times(\theta+r)\,e^{-rt_1}\,(a-bp)\,(-\lambda+r)\,c_2\\&\times e^{(H(-r-\delta)-rt_1N)/N}+(\delta+\lambda)\\&\times\Bigg(2\,(r-\delta-\lambda)\,(\theta+r)\\&\quad\times\left(\frac{1}{2}\lambda+r\right)e^{-rt_1}\,(a-bp)\\&\quad\times c_3\,e^{-(rH/N)}e^{(-t_1(-\lambda+r)N+rH)/N}\\&\quad+\Big(e^{t_1(\delta+\lambda)}\,(-\lambda+r)\\&\qquad\times(r-\delta-\lambda)\,(-p+c)\\&\qquad\times e^{-(\delta H/N)}+r\delta c_3e^{-rt_1}e^{H\lambda/N}\Big)\\&\quad\times(\theta+r)(a-bp)\,e^{-(rH/N)}-(r-\delta-\lambda)\\&\quad\times\Big(-(rp+p\theta+c_1)\,(a-bp)\,e^{-t_1(-\lambda+r)}\\&\quad+(a-bp)\,(c\theta+c_1+rc)\,e^{t_1(\theta+\lambda)}\\&\quad+(p-\mathrm{SV})\left(\alpha\,(a-bp)\,e^{\lambda t_1}+\beta p\right)\\&\qquad\times(\theta+r)\,e^{-rt_1}\Big)(-\lambda+r)\Bigg)r\Bigg)\\&\times\left(-1+e^{-rH}\right)\Bigg)=0.\end{aligned}\tag{17}$$

4. Optimal Solution Procedure

The objective function has three variables. The number of replenishments (N) is a discrete variable, the length of time in which there is no inventory shortage (t_1), and the selling price per unit (p). The following algorithm is used to obtain the optimal amount of t_1, p, N [30].

Step 1. let $N = 1$.

Step 2. For different integer N values, derive t_1^* and p^* from (16) and (17). Substitute (p^*, t_1^*, N^*) to (15) to derive $\mathrm{PWTP}(p^*, t_1^*, N^*)$.

Step 3. Add one unit to N and repeat Step 2 for new N. If there is no increase in the last PWPT, then show the last one.

The (p^*, t_1^*, N^*) and $\mathrm{PWTP}(p^*, t_1^*, N^*)$ values constitute the optimal solution and satisfy the following conditions:

$$\Delta\mathrm{PWTP}\left(p^*, t_1^*, N^*\right) < 0 < \Delta\mathrm{PWTP}\left(p^*, t_1^*, N^*-1\right),\tag{18}$$

where

$$\begin{aligned}\Delta\mathrm{PWTP}\left(p^*, t_1^*, N^*\right) = {}&\mathrm{PWTP}\left(p^*, t_1^*, N^*+1\right)\\&-\mathrm{PWTP}\left(p^*, t_1^*, N^*\right).\end{aligned}\tag{19}$$

Substitute (p^*, t_1^*, N^*) to (7) to derive the Nth replenishment lot size.

If the objective function is concave, the following sufficient conditions must be satisfied:

$$\left(\frac{\partial^2\mathrm{PWTP}}{\partial p\partial t_1}\right)^2-\left(\frac{\partial^2\mathrm{PWTP}}{\partial t_1^2}\right)\left(\frac{\partial^2\mathrm{PWTP}}{\partial p^2}\right)<0,\tag{20}$$

and any one of the following:

$$\frac{\partial^2\mathrm{PWTP}}{\partial t_1^2}<0,\qquad\frac{\partial^2\mathrm{PWTP}}{\partial p^2}<0.\tag{21}$$

Since PWTP is a very complicated function due to high-power expression of the exponential function, it is not possible to show analytically the validity of the above sufficient conditions. Thus, the sign of the above quantity in (21) is assessed numerically. The computational results are shown in the following illustrative example.

5. Numerical Example

To illustrate the solution procedure and the results, let us apply the proposed algorithm to solve the following numerical examples. The results can be found by applying Maple 13. This example is based on the following parameters and functions:

$D(p,t) = (500 - 0.5p)e^{-0.98t}$, c_1 = \$50/per unit/per unit time, c_2 = \$30/per unit/per unit time, c_3 = \$30/per unit, c = \$100/per unit, A = \$30/per order, $\theta = 0.08$, $\beta(x) = e^{0.2x}$, $r = 0.12$, $R(p,t) = 0.2 * D(p,t) + 0.3p$, SV = \$200/per unit, and $H = 40$ unit time.

From Table 1, if all the conditions and constraints in (15)–(21) are satisfied, optimal solution can be derived. In this example, the maximum present value of total profit is found

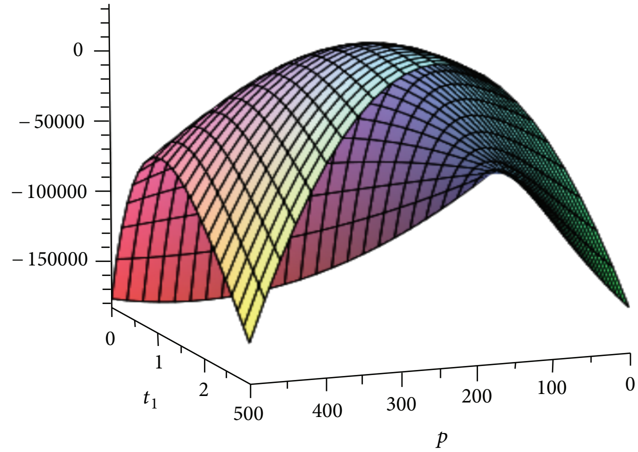

FIGURE 2: The graphical representation of the concavity of the present worth of total profit function $\mathrm{PWTP}(p, t_1, 11)$.

in the 11th cycle. The total number of order is therefore $(N+1)$ or 12. With twelve orders, the optimal solution is as follows:

$$\begin{gathered} p^* = 254.8477, \qquad t_1^* = 2.7126, \\ T^* = 3.63, \qquad \mathrm{PWTP}^* = 35919.3928, \\ Q^* = 392.2260. \end{gathered} \tag{22}$$

By substituting the optimal values of N^*, p^*, and t_1^* to (21), it will be shown that PWTP is strictly concave (Figure 2):

$$\frac{\partial^2 \mathrm{PWTP}}{\partial t_1^2} = -11764.95287, \qquad \frac{\partial^2 \mathrm{PWTP}}{\partial p^2} = -5.7433. \tag{23}$$

6. Conclusion

In this work, we addressed the problem of joint pricing and inventory control model for deteriorating items taking into account the time value of money and customer returns. The demand is deterministic and depends on time and price simultaneously. Also, the customer returns assumed as a function of both the quantity sold and the price. Shortage is allowed and partially backlogged. An algorithm is presented for deriving the optimal replenishment and pricing policy that wants to maximize the present value of total profit. Finally, a numerical example is provided to illustrate the algorithm and the solution procedure.

This paper can be extended in several ways. For instance, the constant deterioration rate could be extended to a time-dependent function. Also, the deterministic demand function could be extended to the stochastic demand function. Finally, we could extend the model to incorporate some more realistic features such as quantity discounts and permissible delay in payments.

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Joint Pricing and Replenishment Decisions for Non-instantaneous Deteriorating Items with Partial Backlogging, Inflation- and Selling Price-Dependent Demand and Customer Returns

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ABSTRACT. This paper develops an Economic Order Quantity (EOQ) model for non-instantaneous deteriorating items with selling price- and inflation-induced demand under the effect of inflation and customer returns. The customer returns are assumed as a function of demand and price. Shortages are allowed and partially backlogged. The effects of time value of money are studied using the Discounted Cash Flow approach. The main objective is to determine the optimal selling price, the optimal length of time in which there is no inventory shortage, and the optimal replenishment cycle simultaneously such that the present value of total profit is maximized. An efficient algorithm is presented to find the optimal solution of the developed model. Finally, a numerical example is extracted to solve the presented inventory model using the proposed algorithm and the effects of the customer returns, inflation, and non-instantaneous deterioration are also discussed. The paper ends with a conclusion and outlook to future studies.

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Key words and phrases. Inventory, non-instantaneous deteriorating items, partial backlogging, Inflation- and selling price-dependent demand, customer returns, pricing.

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1. Introduction. In the last decade, the inflation and time value of money ruin the global economy. As a result, while determining the optimal inventory policy, the effect of inflation should not be ignored. The first who considered the effect of inflation and time value of money on an EOQ model was Buzacott [5]. Following [5], several efforts have been made by researchers to reformulate the optimal inventory management policies taking into account inflation and time value of money, such as Misra [35], Park [41], Datta and Pal [10], Goal et al. [20], Hall [23], Sarker and Pan [45], Hariga and Ben-Daya [25], Horowitz [28], Moon and Lee [36], Mirzazadeh et al. [34], Sarkar and Moon [43], Sarkar et al. [44], Taheri-Tolgari et al. [47], and Gholami-Qadikolaei et al. [15], Wee and Law [50], Hsieh and Dye [30], Hou and Lin [29], Ghoreishi et al. [16], Guria et al. [22], Ghoreishi et al. [17], Ghoreishi et al. [18], and Gilding [19].

Deterioration refers to the spoilage, change, damage, vaporization, dryness, pilferage, and loss of utility of the product, such as vegetables, foodstuffs, meat, fruits, alcohol, radioactive substances, gasoline, and etc. The first authors who studied the inventory models for deteriorating items were Ghare and Schrader [14]. Following [14], several efforts have been made on developing the inventory systems for deteriorating items, such as Covert and Philip [9], Hariga [24], Heng et al. [26], Jaggi et al. [31], Moon et al. [37], Sarker et al. [45], and Wee [49]. Goyal and Giri [21] provided a detailed survey of deteriorating inventory literatures. Bhunia et al. [4] studied a two warehouse inventory model with partially backlogged hortages for single deteriorating item considering permissible delay in payments.

In the real world, the majority of products would have a span of maintaining original state or quality, i.e., there is no deterioration occurring during that period, such as fruits, food stuffs, green vegetables, and fashionable goods. Wu et al. [51] introduced the phenomenon as "non-instantaneous deterioration" and developed a replenishment policy for non-instantaneous deteriorating items with stock-dependent demand. For these types of items the assumption that the deterioration starts from the instant of arrival in stock may lead to make an unsuitable replenishment policy due to overstating relevant inventory cost. As a result, it is necessary to consider the inventory problems for non-instantaneous deteriorating items. Ouyang et al. [39] considered the inventory model for non-instantaneous deteriorating items considering permissible delay in payments. Chang et al. [6] developed the inventory model for non-instantaneous deteriorating items with stock-dependent demand. Yang et al. [52] investigated the optimal pricing and ordering strategies for non-instantaneous deteriorating items with partial backlogging and price-dependent demand. Geetha and Uthayakumar [14] presented the EOQ inventory model for non-instantaneous deteriorating items with permissible delay in payments and partial backlogging. Musa and Sani [38] developed the inventory model for non-instantaneous deteriorating items with permissible delay in payments. Maihami and Nakhai [32] proposed the joint pricing and inventory control model for non-instantaneous deteriorating items with price- and time-dependent demand and partial backlogging. In addition, the mentioned model was extended by Maihami and Nakhai [33] under permissible delay in payments.

Pricing strategy is one of the major policies for sellers or retailers to obtain its maximum profit that is often combined with inventory control policy. Shi et al. [46] proposed the optimal pricing and ordering strategies with price-dependent stochastic demand and supplier quantity discounts. Dye [11] considered the optimal pricing and ordering policies for deteriorating items with partial backlogging and price-dependent demand. Abad [1, 2] studied the pricing and lot-sizing inventory model for a perishable good allowing shortage and partial backlogging. Dye et al. [12] considered the optimal pricing and inventory control policies for deteriorating items with shortages and price-dependent demand. Chang et al. [7] presented the inventory model for deteriorating items with partial backlogging and log-concave demand. Samadi et al. [42] developed the pricing, marketing and service planning inventory model with shortages in fuzzy environment. In this model, the demand is considered as a power function of price, marketing expenditure and service expenditure. Tsao and Sheen [48] discussed the problem of dynamic pricing, promotion and replenishment for deteriorating items under the permissible delay in payments. Zhang et al. [53] considered an inventory model for simultaneously determining the optimal pricing and the optimal preservation technology investment policies for deteriorating items. Ouyang et al. [40] studied the joint pricing and ordering policies for deteriorating item with retail price-dependent demand in response to announced supply price increase.

Chen and Bell [8] showed that customer returns affect the firm's pricing and inventory decisions. They developed the pricing and order decisions when the quantity of returned product is a function of both the quantity sold and the price. Hess and Mayhew [27] used regression models to show that the number of returns has a strong positive linear relationship with the quantity sold. Anderson et al. [3] showed that customer returns increase with both the quantity sold and the price set for the product. Zhu [54] proposed the joint pricing and inventory control problem in a random and price-sensitive demand environment with return and expediting.

TABLE 1. Major characteristics of inventory models on selected articles

Author(s)	Pricing	Replenishment rate	Inflation and selling price dependent demand	Non –instantaneous deterioration	Partial backlogging shortage	Inflation	Customer returns
Abad [2]	Yes	Infinite	No	No	Yes	No	No
Chang et al. [7]	Yes	Infinite	No	No	Yes	No	No
Covert and Philip [9]	No	Infinite	No	No	No	No	No

Datta and Pal [10]	No	Infinite	No	No	No	Yes	No
Dye [11]	Yes	Infinite	No	No	Yes	No	No
Dye et al. [12]	Yes	Infinite	No	No	No	Yes	No
Ghoreishi et al. [17]	Yes	Finite	No	Yes	No	Yes	Yes
Guria et al. [22]	No	Infinite	Yes	No	No	Yes	No
Hou and Lin [29]	Yes	Infinite	No	No	No	Yes	No
Hsieh and Dye [30]	Yes	Finite	No	No	Yes	Yes	No
Jaggi et al. [31]	No	Infinite	No	No	No	Yes	No
Maihami and Nakhai Kamalabadi [32]	Yes	Infinite	No	Yes	Yes	No	No
Mirzazadeh et al. [34]	No	Finite	No	No	No	Yes	No
Moon and Lee [36]	No	Infinite	No	No	No	Yes	No
Moon et al. [37]	No	Infinite	No	No	Yes	Yes	No
Tsao and Sheen [48]	Yes	Infinite	No	No	No	No	No
Yang et al. [49]	Yes	Infinite	No	Yes	Yes	No	No
Wee and Law [50]	Yes	Infinite	No	No	No	Yes	No
Wu et al. [51]	No	Infinite	No	Yes	Yes	No	No
Zhang et al. [53]	Yes	Infinite	No	No	No	No	No
Present study	Yes	Infinite	Yes	Yes	Yes	Yes	Yes

The major assumptions mentioned in the selected articles are summarized in Table 1.

2. Motivation section

In practice, the majority of deteriorating items would have a span, in which there is no deterioration. For this type of items, the assumption that the deterioration starts from the instant of arrival in stock may lead to make inappropriate replenishment policies due to overvaluing the relevant inventory cost. As a result, in the field of inventory management, it is necessary to incorporate the inventory problems for non-instantaneous deteriorating items. On the other hand, the coordination of price decisions and inventory control means optimizing the system rather than its individual elements. Thus, the optimal pricing combined with inventory ordering policy can yield considerable revenue increase. Moreover, inflation plays a significant role for the optimal order policy and affects the demand of certain products. As inflation increases, the value of money goes down and erodes the future worth of saving and forces one for more current spending. Usually, these spending are on peripherals and luxury items that give rise to demand of these items. Consequently, the effect of inflation and time value of the money cannot be ignored for determining the optimal inventory policy. Also, in the real world customer returns could increase with both the quantity sold and the price set for the product.

In this study, a finite planning horizon inventory model for non-instantaneous deteriorating items with price- and inflation-dependent demand rate and partial backlogging is developed. In addition, the effects of customer returns and time value of money on replenishment policy are also considered. We assume that the customer returns increase with both the quantity sold and the product price. Also, Inflation affects the demand of certain products. As inflation increases, the value of money goes down and erodes the future worth of saving and forces one for more current spending. Therefore, inflation has a major effect on the demand of the goods, especially for fashionable goods for middle and higher income groups. Besides, the selling price of an item influences the demand of that item, i.e., whenever the selling price of an item increases, the demand of that decreases. As a result, here, we considered a price- and inflation-dependent demand function. An optimization algorithm is presented to derive selling price, the optimal length of time in which there is no inventory shortage, and the optimal replenishment cycle during the time horizon and then obtain the optimal order quantity when the present value of total profit is maximized. Thus, the replenishment and price policies are appropriately developed. A numerical example is provided to illustrate the proposed model. The results of this example are used to analyze the impact of customer returns, inflation, and non-instantaneous deterioration on the optimal solution.

To the best of our knowledge, this is the first model in pricing and inventory control models that considers price- and inflation-induced demand, non-instantaneously deteriorating items, and customer returns. In this model shortages are allowed and partially backlogged. The backlogging rate is variable and dependent on the time of waiting for the next replenishment. The main objective is determining the optimal selling price, the optimal length of time in which there is no inventory shortage, and the optimal replenishment cycle simultaneously such that the present value of total profit is maximized. This is the first work that follows the above assumptions.

3. Notation and assumptions. The following notation and assumptions are used throughout the paper:

3.1. **Notation**.

A: constant purchasing cost per order,

c: purchasing cost per unit,

c_1: holding cost per unit per unit time,

c_2: backorder cost per unit per unit time,

c_3: cost of lost sale per unit,

p: selling price per unit, where $p > c$ (decision variable),

θ: constant deterioration rate,

r: constant representing the difference between the discount (cost of capital) and the inflation rate,

Q: order quantity,

T: length of replenishment cycle time (decision variable),

t_1: length of time in which there is no inventory shortage (decision variable),

t_d: length of time in which the product exhibits no deterioration,

SV: salvage value per unit,

H: length of planning horizon,

N: Number of replenishments during the time horizon H,

T^*: optimal length of the replenishment cycle time,

Q^*: optimal order quantity,

t_1^*: optimal length of time in which there is no inventory shortage,

p^*: optimal selling price per unit,

$I_1(t)$: inventory level at time $t \in [0, t_d]$,

$I_2(t)$: inventory level at time $t \in [t_d, t_1]$,

$I_3(t)$: inventory level at time $t \in [t_1, T]$,

I_0: maximum inventory level,

S: maximum amount of demand backlogged,

$PWTP(p, t_1, T; N)$: present value of total profit over the time horizon.

3.2. **Assumptions**

In this paper, the following assumptions are considered:

1. There is a constant fraction of the on-hand inventory deteriorates per unit of time and there is no repair or replacement of the deteriorated inventory.
2. A single non-instantaneous deteriorating item is assumed.
3. The replenishment rate is infinite and the lead time is zero.
4. Demand is inflation rate and selling price dependent, i.e., $D(t) = (a - bp)e^{krt}$ (where $0 < k < 1$, $a > 0$, $b > 0$).

5. Shortages are allowed. The unsatisfied demand is backlogged, and the fraction of shortage backordered is $\beta(x) = k_0\, e^{-\delta x}$ $(\delta > 0,\ 0 < k_0 \le 1)$, where x is the waiting time up to the next replenishment and δ is a positive constant and $0 \le \beta(x) \le 1,\ \beta(0) = 1$ [1].

6. Following the empirical findings of Anderson et al. [3], we assume that customer returns increase with both the quantity sold and the price. We use the general form: $R(p,t) = \alpha D(p,t) + \beta p$ $(\beta \ge 0,\ 0 \le \alpha < 1)$ that is presented by Chen and Bell [8]. Customers are assumed to return $R(p,t)$ products during the period for full credit and these units are available for resale in the following period. We assume that the salvage value of the product at the end of the last period is SV per unit.

6. The time horizon is finite.

4. Model formulation. We use the same inventory shortage model as in Yang et al. [52]. Base on this model; the inventory system is as follows: I_0 units of item arrive at the inventory system at the beginning of each cycle. During the time interval $[0, t_d]$, the inventory level decreases due to demand only. Afterwards the inventory level drops to zero due to both demand and deterioration during the time interval $[t_d, t_1]$. Finally, a shortage occurs due to demand and partial backlogging during the time interval $[t_1, T]$ (see Fig. 1).

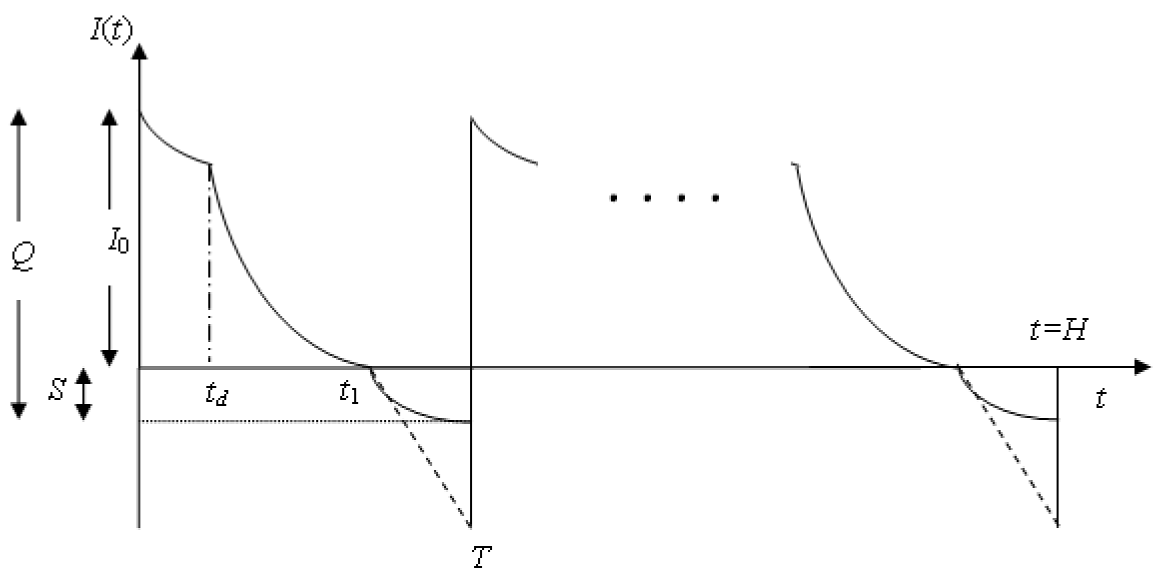


FIGURE 1. Graphical representation of the inventory system

The equation representing the inventory status in system for the first interval:
During the time interval [0, t_d], the differential equation representing the inventory status is given by

$$\frac{dI_1(t)}{dt} = -D(t) = -(a - bp)e^{krt}. \qquad (1)$$

With the condition $I_1(0) = I_0$, solving Equation (1) yields

$$I_1(t) = \frac{(-a + bp)e^{krt} - bp + I_0 kr + a}{kr} \qquad (0 \le t \le t_d). \qquad (2)$$

In the second interval [t_d, t_1], the inventory level decreases due to demand and deterioration. Thus, the differential equation below represents the inventory status:

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -D(t). \qquad (3)$$

By the condition $I_2(t_1) = 0$, the solution of Equation (3) is

$$I_2(t) = -\frac{(a-bp)(e^{t(kr+\theta)} - e^{t_1(kr+\theta)})e^{-\theta t}}{kr+\theta} \qquad (t_d \le t \le t_1). \qquad (4)$$

It is clear from Fig. 1 that $I_1(t_d) = I_2(t_d)$ therefore, the maximum inventory level I_0 can be obtained

$$I_0 = \frac{1}{(kr+\theta)kr}\left((a - bp)\left(e^{-\theta t_d} kre^{t_1(kr+\theta)} - e^{-\theta t_d} kre^{t_d(kr+\theta)} + (-1 + e^{krt_d})(kr + \theta)\right)\right). \qquad (5)$$

In the third interval (t_1, T), shortage is partially backlogged according to fraction $\beta(T - t)$. Therefore, the inventory level at time t is obtained by the following equation:

$$\frac{dI_3(t)}{dt} = -D(t)\beta(T - t) = \frac{-D(t)}{e^{\delta(T-t)}} \qquad (t_1 \le t \le T). \qquad (6)$$

The solution of the above differential equation after apply the boundary conditions $I_3(t_1) = 0$, is

$$I_3(t) = -\frac{(a-bp)(e^{(-T+t)\delta+krt} - e^{(kr+\delta)t_1-\delta T}}{kr+\delta} \qquad (t_1 \le t \le T). \tag{7}$$

If we put t=T into $I_3(t)$, the maximum amount of demand backlogging (S) will be obtained:

$$S = -I_3(T) = \frac{(a-bp)(e^{krT} - e^{(kr+\delta)t_1-\delta T})}{kr+\delta}. \tag{8}$$

Order quantity per cycle (Q) is the sum of S and I_0, i.e.,

$$\begin{aligned} Q = S + I_0 = \frac{1}{(kr+\theta)kr(kr+\delta)}\Bigg((a-bp)\Big((-k^2r^2 - \theta kr)e^{(kr+\delta)t_1-\delta T} \\ + ke^{-\theta t_d}r(kr+\delta)e^{t_1(kr+\theta)} - ke^{-\theta t_d}r(kr+\delta)e^{t_d(kr+\theta)} \\ + \Big((kr+\delta)e^{krt_d} - kr + kre^{krT} - \delta\Big)(kr+\theta)\Big)\Bigg). \end{aligned} \tag{9}$$

Now, we can obtain the present value inventory costs and sales revenue for the first cycle, which consists of the following elements:

1) Since replenishment in each cycle has been done at the start of each cycle, the present value of replenishment cost for the first cycle will be A, which is a constant value.

2) Inventory occurs during period t_1, therefore, the present value of holding cost (HC) for the first cycle is

$$HC = c_1\left(\int_0^{t_d} I_1(t)\cdot e^{-r\cdot t}\mathrm{dt} + e^{-r\cdot t_d}\int_{t_d}^{t_1} I_2(t)\cdot e^{-r\cdot t}dt\right). \tag{10}$$

3) The present value of shortage cost (SC) due to backlog for the first cycle is

$$SC = c_2\left(e^{-r\cdot t_1}\int_{t_1}^{T} -I_3\,(t)\cdot e^{-r\cdot t}dt\right). \tag{11}$$

4) The present value of opportunity cost due to lost sales (OC) for the first cycle is

$$OC = c_3\left(e^{-r\cdot t_1}\int_{t_1}^{T} D(t)(1-\beta(T-t))\cdot e^{-r\cdot t}dt\right). \tag{12}$$

5) The present value of purchase cost (PC) for the first cycle is

$$PC = c(I_0 + Se^{-r\cdot T}). \tag{13}$$

6) The present value of return cost for each cycle.

We assume that returns from period $i-1$ are available for resale at the beginning of period i (except the first period in which there is no cycle previous to it). It is also assumed that the salvage value of product at the end of the last period (i=N) is SV. Therefore, the present value of return cost and resale revenue for each cycle is obtained as follows:

$$RC_i = \begin{cases} p\int_0^{t_1}(\alpha D(t)+\beta p)e^{-r.t}dt, & \text{for } i=1, \\ RC = p\int_0^{t_1}(\alpha D(t)+\beta p)e^{-r.t}dt - c\int_0^{t_1}(\alpha D(t)+\beta p)dt, & \\ & \text{for } i=2,\dots,N-1, \\ p\int_0^{t_1}(\alpha D(t)+\beta p)e^{-r.t}dt - c\int_0^{t_1}(\alpha D(t)+\beta p)dt - SVe^{-r\cdot T}\int_0^{t_1}(\alpha D(t)+\beta p)dt, & \\ & \text{for } i=N. \end{cases} \tag{14}$$

7) The present value of sales revenue (SR) for the first cycle is

$$SR = p\left(\int_0^{t_1} D(t)\cdot e^{-r\cdot t}\,dt + S\cdot e^{-r\cdot T}\right). \tag{15}$$

There are N cycles during the planning horizon. Since inventory is assumed to start and end at zero, an extra replenishment at t=H is required to satisfy the backorders of the last cycle in the planning horizon. Therefore, the total number of replenishment will be $N+1$ times; the first replenishment lot size is I_0, and the 2^{nd}, 3^{rd}, … , N^{th} replenishment lot size is as follows:

$$Q = S + I_0.$$

Finally, the last or $(N+1)^{th}$ replenishment lot size is S.

Therefore, the present value of total profit during planning horizon, denoted by $PWTP(p, t_1, T; N)$, is derived as follows:

$$\begin{aligned} &PWTP(p, t_1, T; N) \\ &\quad = \sum_{i=0}^{N-1}(SR - A - HC - SC - OC - PC - RC)e^{\,-r.i\cdot T} + SV \\ &\quad\cdot e^{-r.H}\int_0^{t_1}(\alpha D(t)+\beta p)dt - c\int_0^{t_1}(\alpha D(t)+\beta p)dt \quad - A\cdot e^{-r\cdot H}, \end{aligned} \tag{16}$$

which we want to maximize subject to the following constraints:

$$p > 0,\ 0 < t_1 < T,\ N \in \mathbb{N}.$$

The value of the variable T can be replaced by the equation $T = H/N$, for some constant $H > 0$, and we will use Maclaurin's approximation for $\sum_{i=0}^{N-1} e^{-r\cdot i\cdot T} \cong (1 - e^{-r\cdot N\cdot T})/(1 - e^{-r\cdot T})$. Thus, the problem is to obtain optimal values of t_1, p and N that maximize $PWTP(p, t_1, T)$ subject to $p > 0$ and $0 < t_1 < T$, where N is a discrete variable and p and t_1 are continuous variables. However, since $PWTP(p, t_1, T; N)$, and still $PWTP(p, t_1, H/N; N)$, is a very complicated function due to high-power expressions in the exponential function, it is difficult to show analytically the validity of the sufficient conditions. Hence, if more than one local maximum value exists, we would attain the largest of the local maximum values over the regions subject to $p > 0$ and $0 < t_1 < T$. The largest value is referred to as the global maximum value of $PWTP(p, t_1, T; N)$. So far, the procedure is to locate the optimal values of p and t_1 when N is fixed. Since N is a discrete variable, the following algorithm can be used to determine the optimal values of p, t_1 and N of the proposed model. We may refer to $PWTP(p, t_1, H/N; N)$ and, for the sake of convenience, just denote it by $PWTP(p, t_1, N)$.

5. The optimal solution procedure. The objective function has three variables. The number of replenishments (N) is a discrete variable, the length of time in which there is no inventory shortage (t_1) and the selling price per unit (p) are continuous variables. The following algorithm is used to obtain the optimal amount of t_1, p and N:

Step 1: let $N = 1$.

Step 2: Take the partial derivatives of $PWTP(p, t_1, N)$ with respect to p and t_1, and equate the results to zero, the necessary conditions for optimality are

$$\frac{\partial}{\partial p} PWTP(p, t_1, N) = 0 \tag{17}$$

and

$$\frac{\partial}{\partial t_1} PWTP(p, t_1, N) = 0. \tag{18}$$

In Appendix A, we use the formula of $PWTP_1(p, t_1, T; N)$ from Equation (16), inserted into Equations (17) and (18).

Step 3: For different integer N values, derive t_1^* and p^* from Equations (17) and (18). Substitute $(p^*,\ t_1^*, N^*)$ to $PWTP(p, t_1, T; N)$ from Equation (16) to derive $PWTP(p^*, t_1^*, N^*)$.

Step 4: Add one unit to *N* and repeat step 2 and 3 for the new *N*. If there is no increasing in the last value of $PWTP(p, t_1, N)$, then show the previous one which has the maximum value.

The point (p^*, t_1^*, N^*) and the value $PWTP(p^*, t_1^*, N^*)$ constitute the optimal solution and satisfy the following conditions:

$$\Delta PWTP(p^*, t_1^*, N^*) < 0 < \Delta PWTP(p^*, t_1^*, N^* - 1), \tag{19}$$

where

$$\Delta PWTP(p^*, t_1^*, N^*) = PWTP(p^*, t_1^*, N^* + 1) - PWTP(p^*, t_1^*, N^*). \tag{20}$$

We substitute (p^*, t_1^*, N^*) into Equation (9) to derive the N^{th} replenishment lot size.

If the objective function was strictly concave, the following *sufficient* conditions must be satisfied:

$$\left(\frac{\partial^2 PWTP}{\partial p \partial t_1}\right)^2 - \left(\frac{\partial^2 PWTP}{\partial t_1^2}\right)\left(\frac{\partial^2 PWTP}{\partial p^2}\right) < 0, \tag{21}$$

and any one of the following conditions:

$$\frac{\partial^2 PWTP}{\partial t_p^2} < 0, \qquad \frac{\partial^2 PWTP}{\partial p^2} < 0. \tag{22}$$

Since $PWTP$ is a very complicated function due to high-power expression of the exponential function, it is unlikely to show analytically the validity of the above sufficient conditions. Thus, the sign of the above quantity in Equation (22) is assessed numerically. The computational results are shown in the following illustrative example.

6. A numerical example. To illustrate the solution procedure and the results, let us apply the proposed algorithm to solve the following numerical examples. The results can be found by applying Maple 13.

Example 1 c = \$10per unit, c_1 = \$1/per unit/per unite time, c_2 = \$5/per unit/per unite time, c_3 = \$25/per unit, t_d = 0.08 unit time, A = \$250/per order run, $\theta = 0.08$, $r = 0.12$, $\delta = 0.1$, $H = 40$ unit time, $\alpha = 0.2$, $\beta = 0.3$, SV = \$3/per unit, $a = 200$, $b = 4$, $k = 0.03$.

From Table 2, the maximum present value of total profit is found in 35th cycle. The total number of order is therefore (N+1) or 36. With thirty six orders, the optimal solution is as follows:
$p^* = 30.138,\quad t_1^* = 0.429,\quad T^* = 1.142,\quad PWTP^* = 7892.824,\quad Q^* = 89.431.$

TABLE 2. Optimal solution of the example

N	p	Time interval		Q	$PWTP$
		t_1	T		
34	30.122	0.454	1.176	92.152	7891.722
35*	30.138*	0.429*	1.142*	89.431*	7892.824*
36	30.155	0.432	1.111	86.145	7892.321

*Optimal solution.

By substituting the optimal values of N^*, p^* and t_1^* to Equation (22), it will be shown that $PWTP$ is strictly concave (cf. Fig. 2):

$$\frac{\partial^2 PWTP}{\partial t_1^2} = -8021.195, \qquad \frac{\partial^2 PWTP}{\partial p^2} = -59.962.$$

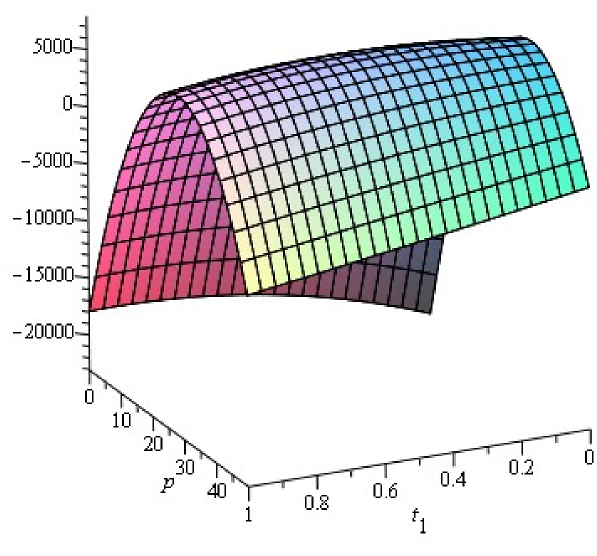


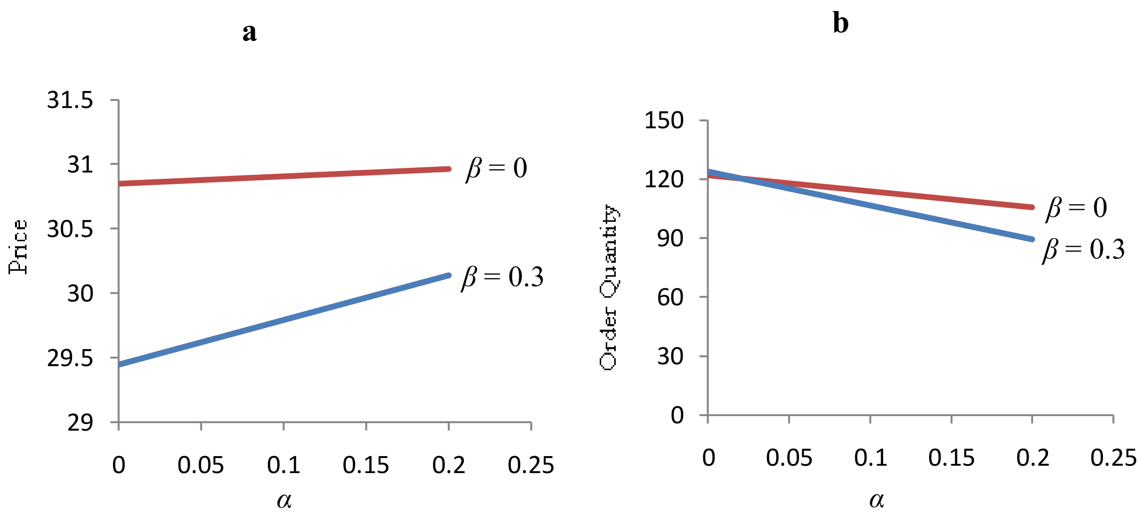
FIGURE 2. The graphical representation of the concavity of the present value of total profit function $PWTP(p, t_1, 35)$

We obtain the results of this example for investigating the impact of customer returns on the optimal solution (Table 3). The results show that when returns are dependent on the quantity sold only (i.e., β=0), the company should raise the price and decrease the order quantity, but if returns are dependent on price only (i.e., α=0) the company should reduce the price, and increase the order quantity. The results verify that when returns increase with the product price (when purchase costs are constant), the company should set a lower price (in order to discourage returns). Increasing α and/or β reduces the company's present value of total profit.

TABLE 3. The impact of customer returns on the optimal solution of the example

α, β	p^*	t_1^*	T^*	Q^*	$PWTP^*$
α=0, β=0	30.847	1.228	1.600	121.977	10633.511
α=0, β=0.3	29.447	0.985	1.481	123.865	9466.923
α=0.2, β=0	30.961	0.805	1.428	105.688	8702.702
α=0.2, β=0.3	30.138	0.429	1.142	89.431	7892.824

The numerical results of the Table 3 are summarized in Fig. 3a–c.



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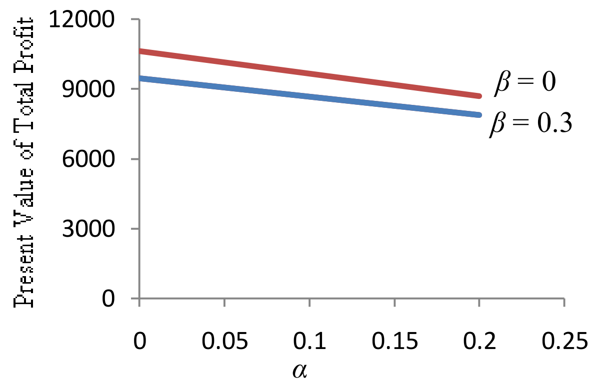

FIGURE 3. The impact of customer returns on price, order quantity and profit. (a) Impact of α and β on price. (b) Impact of α and β on order quantity. (c) Impact of α and β on present value of total profit

Moreover, as observed in Table 4, when the net discount rate of inflation (r) decreases then optimal cycle time, optimal order quantity, and the optimal present value of total profit increase. Therefore, the results confirm that when the discount rate of inflation decrease, the purchasing power will be raised, which will lead to an enhancement in order quantity. Thus, it is important to consider the effects of inflation and the time value of money on inventory policy.

TABLE 4. The impact of the discount rate of inflation on the optimal solution of the example

r	p^*	t_1^*	T^*	Q^*	$PWTP^*$
0.02	30.255	0.459	1.250	99.743	27852.076
0.06	30.190	0.455	1.212	97.161	14998.570
0.12	30.138	0.429	1.142	89.431	7892.824
0.16	30.126	0.398	1.081	82.356	5810.532

Also, as observed in Table 5, If t_d=0, then the model converts into the instantaneous deterioration items case, and the optimal present value of total profit can be found as follows: $PWTP$=7828.117. It can also be seen that the optimal present value of total profit in the instantaneous deterioration items case decrease. This implies that the optimal present value of total profit could be increased by changing the instantaneously to non-instantaneously items using the improved stock condition.

TABLE **5.** The results with instantaneous and non-instantaneous deteriorating models of the example

t_d	p^*	t_1^*	T^*	Q^*	$PWTP^*$
0	30.220	0.422	1.142	86.673	7828.117

0.08	30.138	0.429	1.142	89.431	7892.824
0.16	30.009	0.393	1.081	82.801	8044.555
0.24	29.797	0.308	0.952	73.779	8327.999

Example 2 c = \$15per unit, c_1 = \$1.5/per unit/per unite time, c_2 = \$8.5/per unit/per unite time, c_3 = \$37.5/per unit, t_d = 0.04 unit time, A = \$500/per order run, $\theta = 0.10$, $r = 0.12$, $\delta = 0.1$, $H = 40$ unit time, $\alpha = 0.3$, $\beta = 0.45$, SV = \$5/per unit, $a = 300$, $b = 6$, $k = 0.045$.

According to the computational results shown in Table 6, the optimal solution is as follows:

$p^* = 33.804,\quad t_1^* = 0.572,\quad T^* = 1.379,\quad PWTP^* = 8077.648,\quad Q^* = 141.361.$

TABLE 6. Optimal solution of the example

N	p	Time interval		Q	$PWTP$
		t_1	T		
28	33.796	0.607	1.429	141.900	8072.458
29*	33.804*	0.572*	1.379*	141.361*	8077.648*
30	33.817	0.539	1.333	140.806	8075.768

Example 3 c = \$5per unit, c_1 = \$0.5/per unit/per unite time, c_2 = \$2.5/per unit/per unite time, c_3 = \$12.5/per unit, t_d = 0.12 unit time, A = \$100/per order run, $\theta = 0.06$, $r = 0.10$, $\delta = 0.1$, $H = 20$ unit time, $\alpha = 0.1$, $\beta = 0.2$, SV = \$2/per unit, $a = 100$, $b = 2$, $k = 0.02$.

According to the computational results shown in Table 7, the optimal solution is as follows:

$p^* = 26.830,\quad t_1^* = 0.462,\quad T^* = 1.176,\quad PWTP^* = 6520.159,\quad Q^* = 56.757.$

TABLE **7.** Optimal solution of the example

N	p	Time interval		Q	$PWTP$
		t_1	T		
16	26.760	0.524	1.250	57.204	6519.311
17*	26.830*	0.462*	1.176*	56.757*	6520.159*
18	26.903	0.406	1.111	56.326	6519.177

7. Conclusion and outlook. In this paper, we investigate the effects of inflation and customer returns on joint pricing and inventory control model for non-instantaneous deteriorating items with inflation- and selling price-dependent demand and partial backlogging. The customer returns are assumed as a function of price and demand simultaneously. The backlogging rate is variable and dependent on the time of waiting for the next replenishment. The mathematical models are derived to determine the optimal selling price, the optimal length of time in which there is no inventory shortage, and the optimal replenishment cycle simultaneously. An optimization algorithm is presented to derive the optimal decision variables. Finally, a numerical example is solved and the effects of the customer returns, inflation, and non-instantaneous deteriorating items are also discussed.

From Table 3, it can be observed that when the customer returns depend on the quantity of product sold only (i.e., β=0), the price increase and order quantity decrease. On the other hand, when customer returns increase with price only (i.e., α=0), the price reduces and order quantity increases. Also, observed in Table 4, it can be seen that there is an improvement in the optimal cycle time, optimal order quantity, and the optimal present value of total profit when the discount rate of inflation decreases. Moreover, from Table 5, it can be observed that the optimal present value of total profit in the instantaneous deterioration items case decrease.

To the best of our knowledge, this is the first model in pricing and inventory control models that consider inflation- and selling price-dependent demand rate, partial backlogging, and customer returns for non-instantaneously deteriorating items. The proposed model can be extended in numerous ways for future research. For example, we could incorporate: (1) stochastic demand function (2) two warehouse (3) quantity discount (4) finite replenishment rate and (5) deteriorating cost.

Appendix A

For a given value of *N*, the necessary conditions for finding the optimal values p^* and t_1^* are given as follows:

$$
\begin{aligned}
&\frac{\partial}{\partial t_1} PWTP(p,t_1,T) \\
&\quad= (-(k-1)(r+\theta)e^{-rt_1}r(a-bp)(-1+e^{-rH})\big((-2+k)r+\delta\big)(kr \\
&\quad+\delta)c_3 e^{(\delta+(k-1)r)t_1-\delta T} - (k-1)r(\delta+(k-1)r)(a-bp)c_1(-1+e^{-rH})(kr \\
&\quad+\delta)e^{-rt_d}e^{t_1(kr+\theta)-t_d(r+\theta)} \\
&\quad+(k \\
&\quad-1)\Big(\big((-2+k)r+\delta\big)c_2\big((e^{-rt_1})\big)^2 - e^{-rT}(\delta+(k-1)r)c_2 e^{-rt_1} \\
&\quad+e^{-rT}r(kr+\delta)(c-p)\big)(r+\theta)(\delta+(k-1)r)(a-bp)(-1 \\
&\quad+e^{-rH})e^{(kr+\delta)t_1-\delta T} - (k-1)(r+\theta)e^{-rt_1}r(\delta+k-1)r\Big)(a-bp)(-1 \\
&\quad+e^{-rH})c_2(-2e^{-rt_1}+e^{rT})e^{(-T+t)\delta+krt} \\
&\quad+(-(k-1)(r+\theta)(\delta \\
&\quad+(k-1)r(a-bp)c_1(-1+e^{-rH})e^{-t_d(r+\theta)}e^{t_1(kr+\theta)+rt_d} \\
&\quad-(k-1)(r+\theta)r(\delta+(k-1)r)(\alpha(a-bp)e^{krt_1}+\beta p)(c-SVe^{-rH})e^{-\frac{rH}{N}} \\
&\quad-(k-1)(r+\theta)(\delta+(k-1)r)(e^{-\theta t_d}\,cr - e^{-t_d(r+\theta)}c_1)(a-bp)(-1 \\
&\quad+e^{-rH})e^{t_1(kr+\theta)} \\
&\quad+r\bigg(\Big(c_3(r+\theta)(-2+k)e^{-rt_1} \\
&\quad+(k-1)\big(e^{-rt_d}c_1+p(r+\theta)\big)(\delta+(k-1)r)(a-bp)(-1+e^{-rH})e^{-rt_1(k-1)} \\
&\quad+(r+\theta)\Big(\delta c_3 e^{-rt_1}(-1+e^{-rH})(a-bp)e^{rT(k-1)} \\
&\quad+(k-1)\big((-SV+c-pe^{-rt_1})e^{-rH}+pe^{-rt_1}\big)(\delta+(k-1)r)(\alpha(a-bp)e^{krt_1} \\
&\quad+\beta p)\Big)\Big)\bigg)(kr+\delta))/((k-1)(r+\theta)r(\delta+(k-1)r)(-1+e^{-\frac{rH}{N}})(kr+\delta)) \\
&\quad= 0
\end{aligned}
$$

and

$$\frac{\partial}{\partial p} PWTP(p,t_1,T)$$

$$= ((bkr^2c_3\, e^{-rt_1}(r+\theta)(k-1)(kr+\theta)(kr+\delta)e^{(\delta+(k-1)r)t_1-\delta T}$$
$$+ bkr^2c_1\, e^{-rt_d}(k-1)(kr+\delta)(-r+kr+\delta)e^{t_1(kr+\theta)-t_d(r+\theta)}$$
$$-(kr+\theta)(-r+kr+\delta)(k-1)(b\left((e^{-rt_1}))^2c_2 - c_2e^{-rt_1}be^{-rT}\right.$$
$$\left.+r\big((c-2p)b+a\big)e^{-rT}\right)r(r+\theta)ke^{(kr+\delta)t_1-\delta T}$$
$$-bkrc_2\, e^{-rt_1}(r+\theta)(k-1)(kr+\theta)(-e^{-rt_1}+e^{-rT})(-r+kr$$
$$+\delta)e^{(-T+t)\delta+krt}$$
$$+bkre^{-t_d(r+\theta)}c_1(k-1)(kr+\delta)(-r+kr+\delta)(r+\theta)e^{t_1(kr+\theta)+rt_d}$$
$$+b\theta e^{-t_d(r+\theta)}c_1(k$$
$$-1)(kr+\delta)(-r+kr+\delta)(r+\theta)e^{((1+k)r+\theta)t_d}bkr(k-1)(kr+\delta)(-r+kr$$
$$+\delta)(r+\theta)\big(e^{-\theta t_d}cr-e^{-t_d(r+\theta)}c_1\big)e^{t_1(kr+\theta)}$$
$$-\big(c_1(r+\theta)e^{-t_d(r+\theta)}+ce^{-\theta t_d}r^2(k-1\big)b(-r+kr+\delta)(r+\theta)k(kr$$
$$+\delta)e^{t_d(kr+\theta)}+(kr+\theta)(-r+kr+\delta)rk(-bc_3(r+\theta)e^{-rt_1}-e^{-rt_d}bc_1$$
$$+(r+\theta)(-2bp+a)(kr+\delta)e^{rt_1(k-1)}$$
$$+(bkrc_1e^{-rt_d}(kr+\delta)(-r+kr+\delta)e^{rt_d(k-11)}+kr$$
$$+\theta)(bc_1e^{t_d(r+\theta)}(kr+\delta)(-r+kr+\delta)e^{-t_d(r+\theta)}$$
$$+r(bkc_3e^{-rt_1}\delta(kr+\delta)e^{rT(k-1)}+(-r+kr$$
$$+\delta)((kr+\delta)k\Big(-\alpha(-2bp+a)e^{krt_1}+(-2\beta pk-2\alpha bp+2\beta p+\alpha a)e^{rt_1}$$
$$+2p\beta(k-1)\Big)e^{-rt_1}-bc\alpha(k-1)(kr+\delta)e^{krt_1}$$
$$+r\big((c-2p)b+a\big)ke^{krT}(k-1)e^{-rT}$$
$$-\Big(-bc(k-1)e^{krt_d}-\beta t_1crk^2+(\beta t_1cr+(-2p+c-c\alpha)b+a)k$$
$$+bc(-1+\alpha)\Big)(kr+\delta)))))(r+\theta))(-1+e^{-rH})/((kr+\theta)(k-1)r^2(kr$$
$$+\delta)k(r+\theta)(-r+kr+\delta)(-1+e^{-\frac{rH}{N}}))+\frac{SVe^{-rH}(\alpha b-\alpha be^{krt_1}+\beta t_1kr)}{kr}$$
$$+\frac{c(-\beta t_1kr+\alpha be^{krt_1}-\alpha b}{kr}=0.$$

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Frédéric Vanderhaegen
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Introduce a novel barrier classification

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Abstract: This paper aims to introduce a barrier modelling technique to use through the design processes of industrial systems. In this model, the barriers are classified according to their effects on the failures and resulted accident process. In this paper, the occupational accidents modeled. The basis of barrier analysis is used to define an accident as the impact of a hazardous agent on a target, as a result of failure of barriers. This definition is enhanced to introduce a novel accident model. The efficiency of barriers are compared by considering their position in the successive protection layers. This approach can be used in complex systems to develop accident models and to estimate the importance of barriers.

Keywords: Risk analysis, accident model, HAZOP, barriers, failures mode, barrier analysis

1. INTRODUCTION

Barrier analysis is a simple and straightforward method, which is performed by evaluating hazards and making proposals for improvements (Harms-Ringdahla 2003).

In this paper, an enhanced form of the barrier approach is used to model accidents scenario and to identify the most critical protective barriers. It presents related works in barrier analysis and the various definitions and classifications for barriers. The definition of an accident as the consequence of deviations, and the role of failure of barriers in this process is used to develop a novel accident model. The remainder of the paper is organized as follows. Section 2 provides a background for barrier definition and barrier analysis and related works. Section 3 explains the proposed accident model, including an accident progress diagram. Section 4 presents an example, and sections 5 and 6 present the discussion and conclusion.

2. RELATED WORKS

There are no universal and commonly accepted definitions for the term “barrier”, neither in literature, nor in regulations and standards (Harms-Ringdahl 2009). In the simplest way, it is a protector of a target from an HA impact (Livingston and Jackson2001; Bernard and Hasan 2002). Basnyat et al. (2007) represents several definitions for barriers among them obstacles, obstructions, or hindrances that prevent the occurrence of an action or an event, and also prevent or limit the consequences.

In accordance with further requirements, this concept is stretched to include the measures that prevent the accident, mitigate the accident consequences, or resurrect the target (Polet 2002; Zhang et al. 2004; Guldenmund et al. 2006). Using this definition, knowledge and skills, and supervision (Hollnagel 1999) and the distance between energy and target (Guldenmund et al. 2006) are also considered as barriers. Basnyat et al. (2007) even considered manufacturer guidelines, procedures, and training as (human related) barriers.

Many authors, among them Hollnagel (2008) and Hollnagel (1999), classify barriers as material, functional, symbolic, and immaterial.

To facilitate the evaluation of safety barrier management, De Dianous and Fiévez (2006) defined the following categories of safety barriers: (1) passive barriers, (2) activated barriers, (3) human actions, and (4) symbolic barriers. Duijm (2009) and De Dianous and Fiévez (2006) differentiate safety functions as technical or organizational actions, from barriers that are considered as objects or physical systems.

Hollnagel (1999) and De Dianous and Fiévez (2006) differentiated passive, active and procedural barriers.

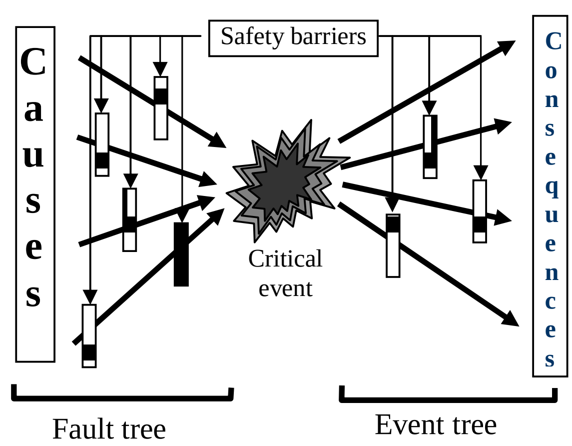

Fig. 1. Prevention and mitigation safety functions presented in the Bow-tie model (De Dianous and Fiévez 2006)

According to Duijm (2009) passive barriers implement the safety function by the mere presence of their elements, while active barriers perform an action in response to a certain state or condition always including a sequence of detection-

diagnosis-action. Hollnagel (1999) differentiates the preventive barriers from the protector barriers and defines a barrier as a piece of equipment, a construction, or rule that impedes the development of an accident. De Dianous and Fiévez (2006) use the same idea in the form of the bow-tie diagram, where the effects of the barriers are categorized into prevention and mitigation effects (Fig. 1).
Schupp et al. (2006) adds the inherent barriers or add-on barriers to this classification.
PSAN (2005) defines accidents as failure of control and defense barriers. A barrier failure is the termination of the ability of a barrier to perform its required function (Rausand 2005). As Fig. 2 illustrates, it defines control barriers as the barriers that mitigate hazards, and defense barriers as the protectors of the target from the hazards.

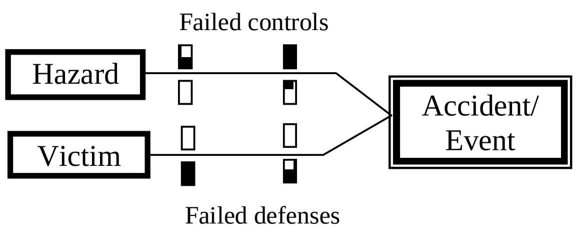

Fig. 2. Accident model showing breached barriers (PSAN 2005)

As illustrated in Fig. 3, Shahrokhi and Bernard (2009) categorize barriers based on the affected entities in the HAZOP information system.

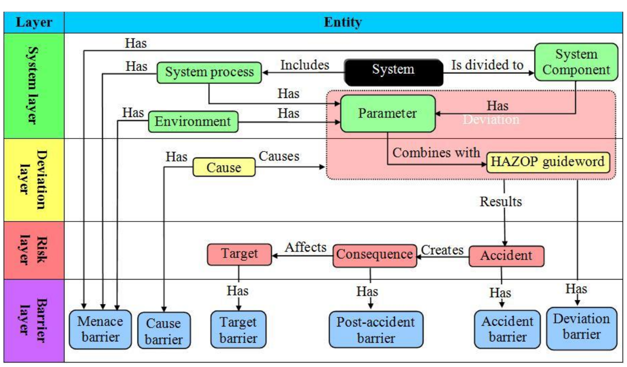

Fig. 3. Barrier classification

Sklet (2006) provides a hierarchical classification for barriers as passive or active barrier systems, and as physical, technical or human/operational barrier systems.
Dowell and Hendershot (2002) present the method “layers of protection approach (LOPA)” that systematically assesses the accident scenarios and propose a series of hierarchically organized protective layers. These protection layers can be passive (e.g. walls), active (e.g. an emergency shutdown system) or procedural (a sequence of steps carried out by the operator) (Schupp et al. 2006).
Hollnagel (2008) identifies the following attributes: (1) functionality/effectiveness, (2) reliability/availability, (3) response time, (4) robustness, and (5) triggering event or condition to characterize the safety barriers performance and presents several methods for evaluating the performance of barriers. According to Harms-Ringdahla (2003) the ‘‘efficiency’’ of a safety function is equivalent to the ‘‘probability of success’’ of a safety function, which is the probability that a safety device exists and performs its intended function, when needed. Through the implementation of ARAMIS, a European project, explained by Salvi and Debray (2006), the barriers reliability and efficiency are used to calculate the frequency or probability of specified accidental scenarios.
Many papers have studied systems which have a combination of several barriers (Made et al. 1997), among them Sklet (2006) who defined the concept of defense-in-depth as "successive layers of protection". However, Keller and Modarres (2005) expanded this concept and introduced a set of regulatory design and safety principles, namely:

1. Use of multiple barriers
2. Incorporation of large design margins
3. Application of quality assurance
4. Operation within predetermined safe design limits.
5. Continuous testing, inspections, and maintenance

The basic assumption for developing multiple layers of protection is that if one layer fails, the next will protect the system from failure (Möller and Hansson 2008). A barrier failure may result of an intentional violation related to a non-respect of a rule or a deactivation of a technical system (Vanderhaegen et al. 2009) by human or due to technical failures and insufficiencies. Polet (2002) developed a model the study the effect of various parameters on the human behavior related to remove safety barriers, and Caccavale et al. (2009) studied the most important failures and their causes in chemical plants, and introduced an analytical model to calculate the barriers effectiveness. Mohaghegh et al. (2009) also considered the effects of organizational impacts of organization and management on the safety performance of equipment and personnel and explored the feasibility of developing a set of principles on which models of organizational influences could be developed and validated. In the Swiss cheese model, the weaknesses in individual barriers are represented as the holes in the cheese slices, and when all of the holes in each of the slices momentarily align, permitting "a trajectory of accident opportunity", the hazard passes through all the holes in all the defenses, leading to failure (Lundberg et al. 2009).
Ale et al. (2008) distinguished two layers in the chain of events leading from an exposure to an accident: primary safety barriers and support safety barriers.
Basnyat et al. (2007) used a hazard-barrier-target diagram to show how the integration of barriers within the system model increases the entire system safety
The barrier concept is used by many authors to develop novel risk and accident analysis approaches. The various barrier classification methods are adapted for modeling the physical and behavioral factors that lead to the identification of the most critical elements in system protection. Safety-barrier and bow-tie diagrams have become popular methods in risk analysis and safety management (Duijm 2009) and are used by Ale et al. (2008) to develop applications for systematically developing and analyzing the scenarios. Bellamy et al. (2007) uses these applications to implement an occupational risk analysis case study.

The system layer includes the functional system components, processes and parameters that are the basic information used to comprehend and analyze the system design, constraints and intents.

As illustrated in Fig. 3, the HAZOP analysis entities are arranged in four layers, including system, deviation, risk and barrier layers. To carry out a HAZOP analysis, the information for these layers should be completed in the same sequence.

The deviation layer comprises the deviations (i.e. combination of the standard HAZOP guidewords and the system parameters) and deviation causes. As an example, for a liquid storage vessel, a "high level" deviation can be caused by the accumulation of dust in its level control instruments.

The risk layer characterizes the accidents and the consequences on the targets (i.e. damageable assets, humans and the environment). A deviation may result in several accidents and consequences. The accident resulting from exampled deviation can be a release of toxic materials with consequences on humans, the environment and the economy (e.g. the cleanup cost).

The barrier layer includes the barriers that limit accident probability and/or severity. Barriers are classified into six categories: unsafe-design barriers (MB), cause barriers (CB), deviation barriers (DB), accident barriers (AB), target barriers (TB) and post-accident barriers (PB). They are explained as follows:

- An unsafe-design barrier is a solution that removes the hazard agent, neutralizes it or replaces it with a safe agent. The unsafe-design barriers are not physical or procedural barriers; however, they are safe design alternatives that make design inherently safe and are often applicable in the earlier design phases. Use of low voltage electricity, removing cutting edges, and use of non-toxic substances are examples of unsafe-design barriers. The use of unsafe-design barriers is the most effective approach to deal with dangers, because they eliminate dangers. By using these measures, a deviation in the system parameters does not create dangerous situations.
- A cause barrier is a physical or organizational measure that limits or removes the reason of a deviation, by impeding physical and human failures and undesirable environment effects. An example of a cause barrier for a liquid level control system is an isolator that stops dust entering the control instruments and therefore it impedes a "high level" liquid deviation.
- A deviation barrier impedes parameters from reaching abnormal values. The control and monitoring instrumentations are the most widely used deviation barriers. Other examples are cooling systems (that impede "high temperatures") and enforcing structures.
- An accident barrier blocks the accident process by splitting the accident events sequence chain. It prevents a recognized deviation from producing an out-of-control dangerous situation. The alarm systems, emergency safety procedures and emergency shutdown systems are some examples of accident barriers.
- An exposure barrier protects a target from dangerous agents. It may be a physical barrier like an individual or a group protection measure or a procedural barrier that separates the target by time and/or space.
- The consequence barriers are used to limit the range of damages of an accident by applying the rehabilitation procedures and facilities. Medical facilities are among these barriers.

The following conceptual relation can be considered for barrier effectiveness:

$$E(MB) \succ E(CB) \succ E(DB) \succ E(AB) \succ E(TB) \succ E(PB) \quad (1)$$

Where letter E notes the effectiveness of a kind of barrier. It emphasizes early prevention, by removing the origins of undesirable phenomena.

A barrier may play several protection roles simultaneously, and then become part of several barrier categories.

The barrier effects are detailed in [40, 41, 42], through the development of risk analysis applications in CAD platforms.

This paper provides a new classification for barriers by defining the accident process stages and distinguishing the preventive roles of barriers in these stages.

3. ACCIDENT PROCESS MODEL

Failures are result of two different factors: unsafe-design (resulted from mal-constructed theories) and human and technical failures (resulted technical failures and inconvenient environment conditions). Unsafe-design might be result of the following factors:

- Unreal assumptions and mal-estimation of operational conditions
- Mal-constructed theories
- Modeling mistakes
- Calculation and data exchange mistakes

Technical and human failures could be classified as:

- Prevention failure
- Control failure

Protection failure

- Rehabilitation failure

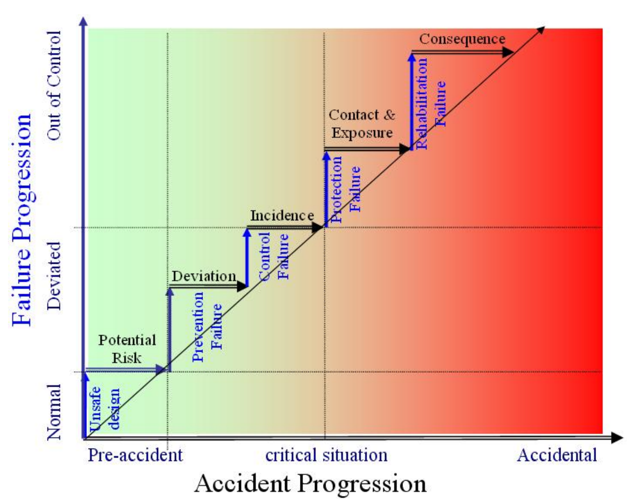

Fig. 3. The proposed accident model

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Introducing a structured HAZOP analysis

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***Abstract*- This paper introduces an information configuration for the HAZOP analysis entities, leading to a new barriers classification. The HAZOP analysis entities are formed in a tree data structure to facilitate data sharing and to provide a more comprehensive analysis. As an illustration, this data structure is used to analyze the risk for a gas furnace. This methodology can be applied to analyze the accident risk in the preliminary design stages and to enrich the risk information, during the implementation and operation phases.**

I. INTRODUCTION

This paper proposes a structured information system for performing HAZOP analysis, in the graphical platform. A tree data structure for risk analysis is proposed to interlink the data of system components and parameters to their potential parameters' deviation and relevant barriers. To apply this method, barriers are classified and the specifications of each kind of barrier are explained. This method is exemplified for a simple gas furnace.

The rest of this paper is organized as follows: section 2 explains the related work, section 3 introduces the proposed methodology, section 4 provides a comparison between the proposed method and conventional HAZOP method, and section 5 presents a conclusion.

II. RELATED WORKS

A Hazard and Operability (HAZOP) study is a systematic approach to determine the effects of possible functional and operational deviations and failures and to propose appropriate protection measures [1, 2, 3].

It is a comprehensive qualitative technique [4] based on guidewords to identify the risks and to propose the appropriate barriers, during a set of brainstorm meetings of a multi-disciplinary team [1].

To increase the analysis influence, the HAZOP study should be carried in the early design phases [1]. However, it is also applicable to identify modifications that should be implemented to reduce risk and operability problems in the existing facilities or as a final check when the detailed design has been completed [1].

Generally, the analysis results are summarized in a worksheet, describing the deviations (ways in which the process conditions may depart from their design/process intent), their consequences and barriers (the facilities that help to reduce the deviation frequency or consequences) [1].

The deviations are described by combining the HAZOP guidewords, such as: "no", "more", "less" with the process parameters and relevant variables [1, 2, 3].

According to [5], HAZOP analysis focuses on functions (e.g. start-up, normal operation, normal shutdown, emergency stop, etc.), however [1] states that the analysis also can be applied to the processes, procedures and software risk analysis and there are special HAZOP for analyzing the human errors.

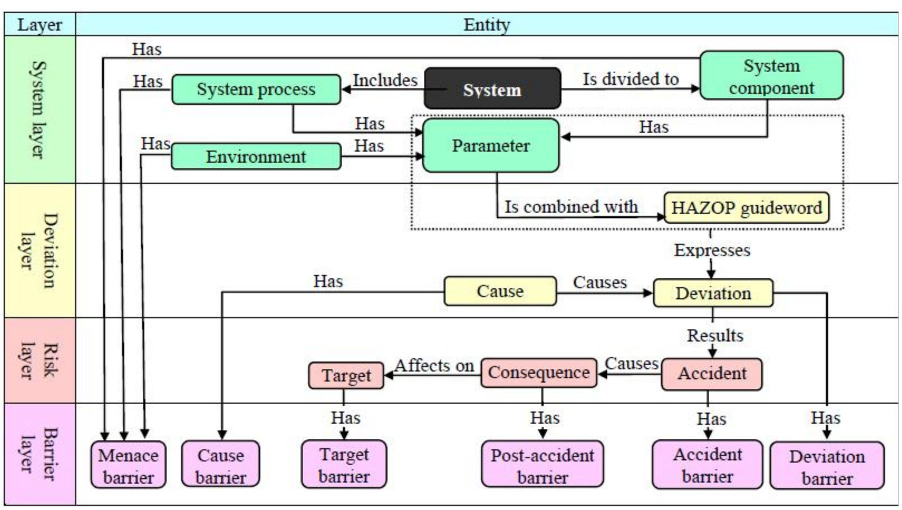

Fig. 1. The HAZOP entity diagram

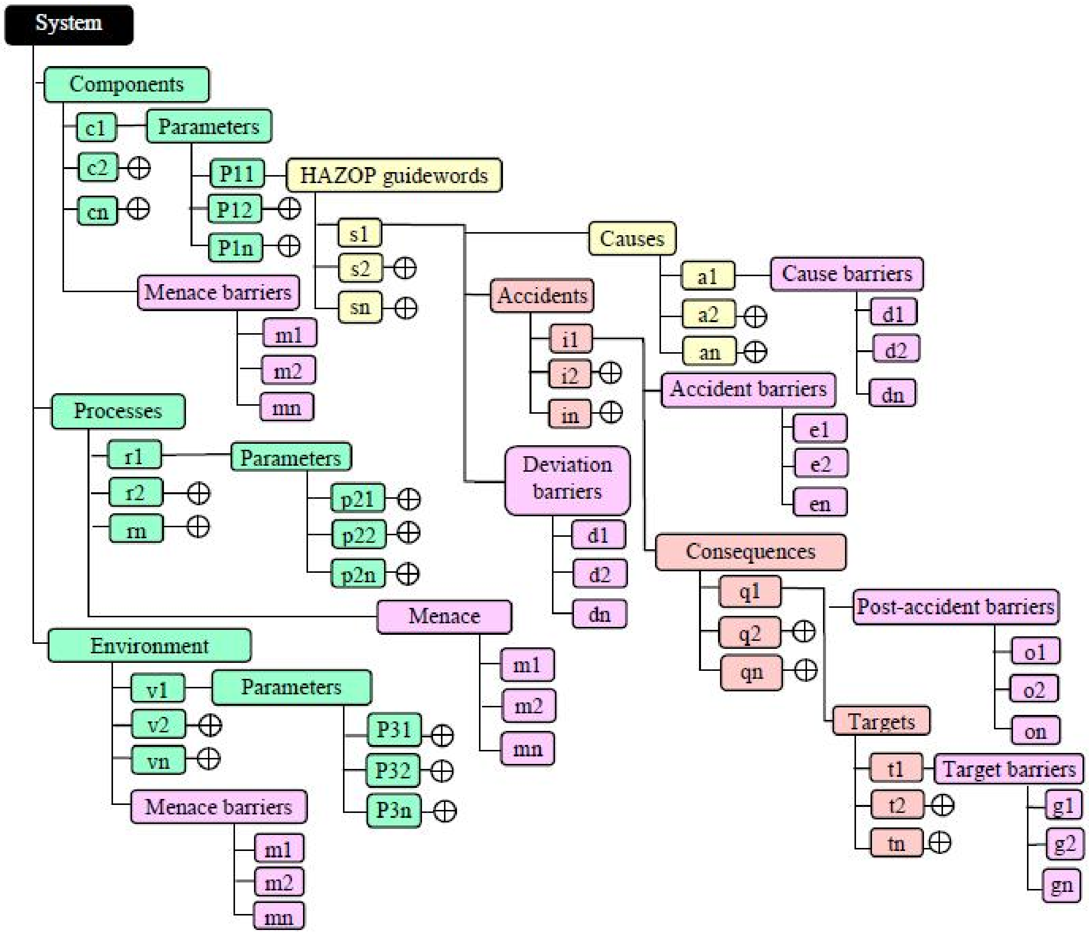

Fig. 2. The HAZOP data tree structure

Several efforts are made previously to integrate the risk analysis in the design model by using the tree data structures [6,7,8]. This paper introduces a new structured HAZOP analysis methodology, illustratable by the graphical interfaces.

The objective is to provide the more entities visibility and to encourage development and sharing of new ideas through the brainstorming meetings.

The method is exemplified by analyzing the risks for a gas furnace and the results are illustrated by the Mindjet LLC `s Mind Manager graphical interface.

This model can be used to analyze the complex systems, during the design phases and revisions.

III. METHODOLOGY

As is illustrated in Figure 1, in this paper a four-layer information system is used to organize the HAZOP analysis entities, including the system, deviation, risk and barrier layers. During HAZOP analysis, the information for these layers should be determined in the same sequence.

The First layer, the system layer includes the functional system workings and their components, processes and parameters. It characterizes the basic information for analyzing the risk, including the system characteristics, constrains and intents that are used during HAZOP analysis.

The deviation layer includes the parameters deviations and their causes. Deviations are expressed by combining the standard HAZOP guidewords and the system parameters.

As an example, for an LPG (liquid petroleum gas) container, an important parameter is the amount of gas leak. A fast gas leak deviation can be a deviation caused by porosity in the welded container parts.

The risk layer characterizes the accidents, their consequences and the targets. Targets are the potential damageable properties, human and the environment.

A deviation may results several accidents and consequences. The accident resulted by the exampled deviation can be inflame of the released gas and explosion with the consequences of damage to the human, properties and the environment damages.

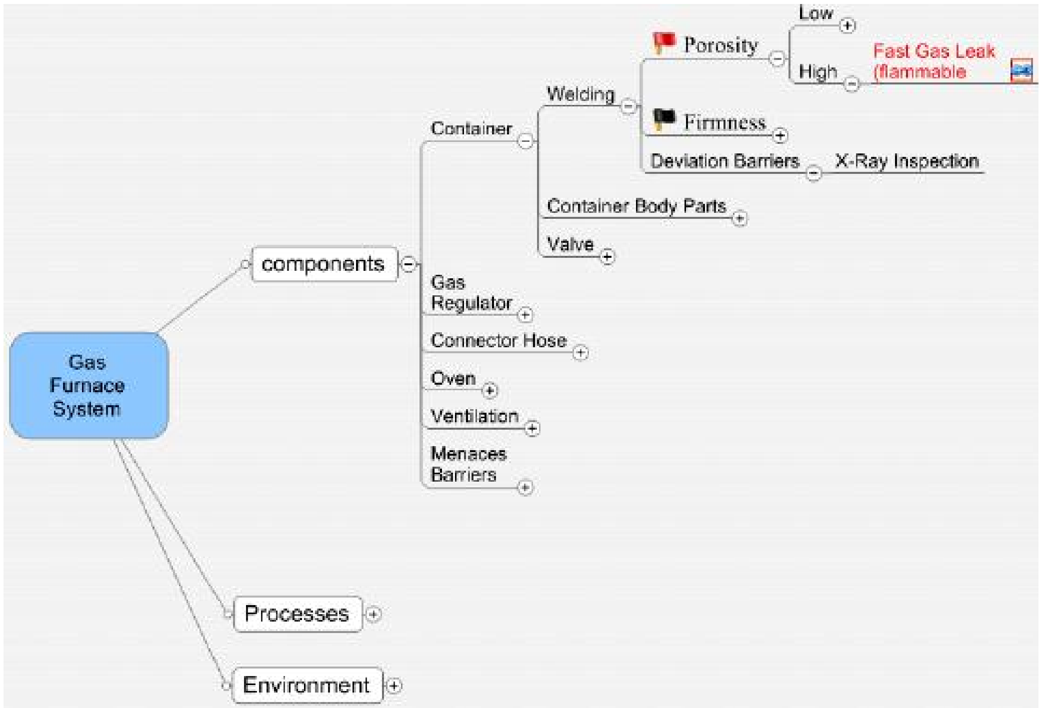

Fig. 3. An exemplified structured HAZOP analysis

The barrier layer includes six kinds of existing or proposed barriers that limit the accidents probability and/or severity.

Based on the introduced entities definition for the HAZOP analysis, barriers are classified as menace barriers (MB), cause barriers (CB), deviation barriers (DB), accident barriers (AB), target barriers (TB) and post-accident barriers (PB).

A menace barrier impedes the creation, neutralizes or annihilates the dangerous agents, by chemical and physical techniques. It also may impede impact of the external threats on the system. The additives that decrease the materials inflammability are examples of the menace barrier. A cause barrier is a physical or organizational measure that limits or removes the causes of deviations and physical and human failures. For example, an anti-humidity insulator is a cause barrier to impede malfunction of a control system.

An accident barrier blocks the accident process by splitting the accident events sequence chain. It impedes that a created deviation produce an out of control dangerous situation. The alarm systems and emergency shutdown systems are some examples of accident barrier.

The post-accident barriers are used to limit the range of damages of a realized accident by applying the emergency and rehabilitation procedures and facilities. Neutralization and firefighting facilities are among the post-accident barriers.

A target barrier protects a target from the dangerous agents. It may be a physical barrier like an individual or a group protection measure or a procedural barrier that separates the target by time and/or space.

IV. Discussion

This paper proposes a structured HAZOP analysis with predefined barrier classification. This fully-graphical approach provides an interface to interlink the system components to the entities of HAZOP risk analysis. Use of this interface, in place of traditional textual reports, facilitates communication during the brainstorming meetings and further reviews. The volume of information is reduced by using the common references and hyperlinks. The related figures and other documents can be visualized in the risk model and the supplemented information can be attached to the risk analysis entities, in this graphical platform. Figure 2 illustrates the HAZOP analysis entities in a tree data structure. In this structure by removing or displacing a parent entity, all children entities and related data will be removed or displaced, coherently.

Figure 3 shows the details of an exemplified risk analysis for components of a gas furnace system. The accidents are marked by the Map Markers to illustrate their severity (expected or worst results) and importance (the production of the risk severity and probability). The specified marks are used to define various hazards, like radiation, toxic and flammable materials. By using the hyperlinks document is summarized in a cellular and manageable format. This is illustrated, in the figure 4, by linking the fast leak accident to the unique detailed consequences and barriers.

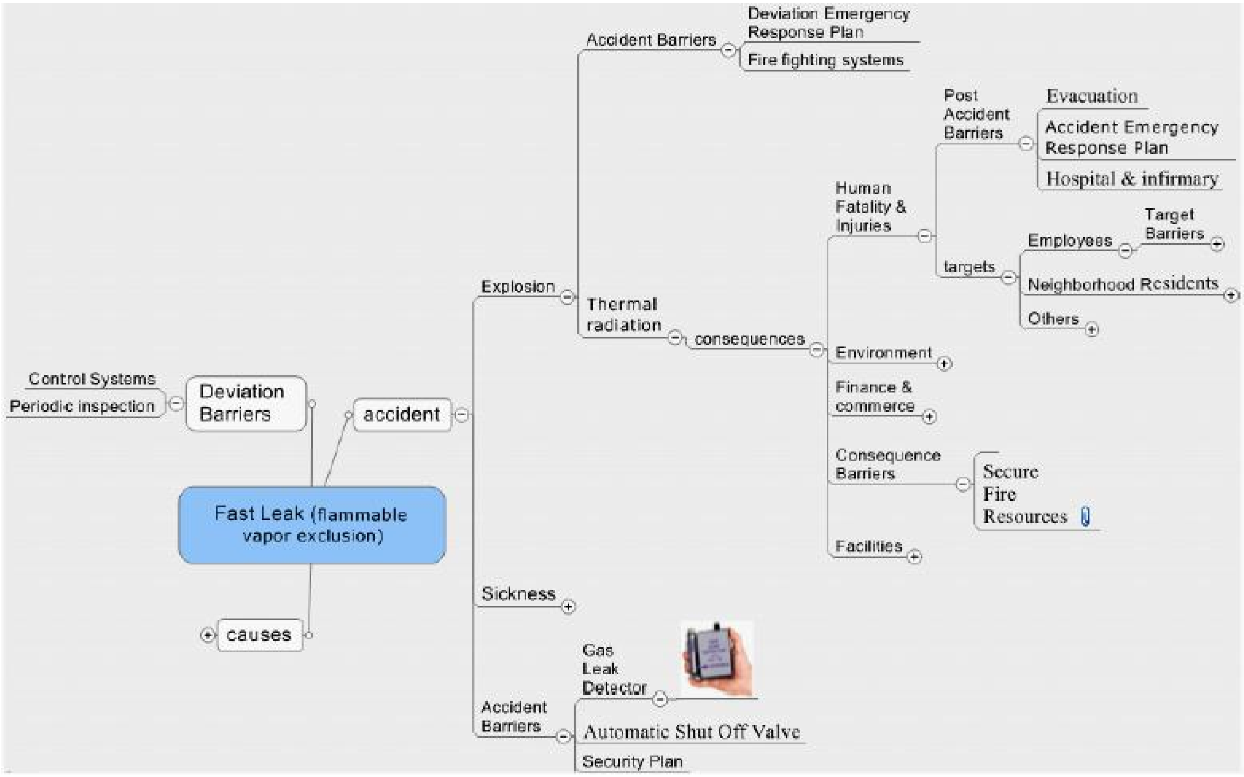

Fig. 4. An illustration of the detailed information related to the accident "Fast flammable (gas) leak"

During the system exploitation and operation, the model can be upgraded according to the latest information (e.g. maintenance and operation records, and accidents/incidents analysis).

As the document becomes an easy to read risk information source, it can be used in a more effective way to aid to make the decisions, during the accidents and in the critical moments.

V. Conclusion

In this paper, we have presented an overview of HAZOP analysis and related work. A fully-graphical approach is proposed to perform risk analysis during the design of systems. In this approach, a four-layer data structure for HAZOP analysis is introduced which classifies barriers into six categories. It is transferred to a tree structure and is applied in a graphical environment to perform the risk analysis for the defined system components. A pattern model is developed to interlink the HAZOP analysis information to the related study nodes of the 3D design model.

This approach is exemplified for a simplified plant and its capabilities and constraints are discussed.

In the future, using the tree data structure, the HAZOP analysis entities will be interlinked to the relevant objects in the platform of the 3D design applications to provide the risk analysis through the concurrent design and operation.

VI.

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