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# JOINT PRICING AND REPLENISHMENT DECISIONS FOR NON-INSTANTANEOUS DETERIORATING ITEMS WITH PARTIAL BACKLOGGING, INFLATION- AND SELLING PRICE-DEPENDENT DEMAND AND CUSTOMER RETURNS

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Abstract. This paper develops an Economic Order Quantity (EOQ) model for non-instantaneous deteriorating items with selling price- and inflation-induced demand under the effect of inflation and customer returns. The customer returns are assumed as a function of demand and price. Shortages are allowed and partially backlogged. The effects of time value of money are studied using the Discounted Cash Flow approach. The main objective is to determine the optimal selling price, the optimal length of time in which there is no inventory shortage, and the optimal replenishment cycle simultaneously such that the present value of total profit is maximized. An efficient algorithm is presented to find the optimal solution of the developed model. Finally, a numerical example is extracted to solve the presented inventory model using the proposed algorithm and the effects of the customer returns, inflation, and non-instantaneous deterioration are also discussed. The paper ends with a conclusion and outlook to future studies.

1. **Introduction.** In the last decade, the ination and time value of money ruin the global economy. As a result, while determining the optimal inventory policy, the effect of inflation should not be ignored. The first who considered the effect of inflation and time value of money on an EOQ model was Buzacott [5]. Following [5], several efforts have been made by researchers to reformulate the optimal inventory management policies taking into account inflation and time value of money, such as Misra [35], Park [41], Datta and Pal [10], Goal et al. [20], Hall [23], Sarker and Pan [45], Hariga and Ben-Daya [25], Horowitz [28], Moon and Lee [36], Mirzazadeh et al. [34], Sarkar and Moon [43], Sarkar et al. [44], Taheri-Tolgari et al. [47], and

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Gholami-Qadikolaei et al. [15], Wee and Law [50], Hsieh and Dye [30], Hou and Lin [29], Ghoreishi et al. [16], Guria et al. [22], Ghoreishi et al. [17], Ghoreishi et al. [18], and Gilding [19].

Deterioration refers to the spoilage, change, damage, vaporization, dryness, pilferage, and loss of utility of the product, such as vegetables, foodstuffs, meat, fruits, alcohol, radioactive substances, gasoline, and etc. The first authors who studied the inventory models for deteriorating items were Ghare and Schrader [14]. Following [14], several efforts have been made on developing the inventory systems for deteriorating items, such as Covert and Philip [9], Hariga [24], Heng et al. [26], Jaggi et al. [31], Moon et al. [37], Sarker et al. [45], and Wee [49]. Goyal and Giri [21] provided a detailed survey of deteriorating inventory literatures. Bhunia et al. [4] studied a two warehouse inventory model with partially backlogged shortages for single deteriorating item considering permissible delay in payments.

In the real world, the majority of products would have a span of maintaining original state or quality, i.e., there is no deterioration occurring during that period, such as fruits, food stuffs, green vegetables, and fashionable goods. Wu et al. [51] introduced the phenomenon as non-instantaneous deterioration and developed a replenishment policy for non-instantaneous deteriorating items with stock-dependent demand. For these types of items the assumption that the deterioration starts from the instant of arrival in stock may lead to make an unsuitable replenishment policy due to overstating relevant inventory cost. As a result, it is necessary to consider the inventory problems for non-instantaneous deteriorating items. Ouyang et al. [39] considered the inventory model for non-instantaneous deteriorating items considering permissible delay in payments. Chang et al. [6] developed the inventory model for non-instantaneous deteriorating items with stock-dependent demand. Yang et al. [52] investigated the optimal pricing and ordering strategies for non-instantaneous deteriorating items with partial backlogging and price-dependent demand. Geetha and Uthayakumar [14] presented the EOQ inventory model for non-instantaneous deteriorating items with permissible delay in payments and partial backlogging. Musa and Sani [38] developed the inventory model for non-instantaneous deteriorating items with permissible delay in payments. Maihami and Nakhai [32] proposed the joint pricing and inventory control model for non-instantaneous deteriorating items with price- and time-dependent demand and partial backlogging. In addition, the mentioned model was extended by Maihami and Nakhai [33] under permissible delay in payments.

Pricing strategy is one of the major policies for sellers or retailers to obtain its maximum profit that is often combined with inventory control policy. Shi et al. [46] proposed the optimal pricing and ordering strategies with price-dependent stochastic demand and supplier quantity discounts. Dye [11] considered the optimal pricing and ordering policies for deteriorating items with partial backlogging and price-dependent demand. Abad [1, 2] studied the pricing and lot-sizing inventory model for a perishable good allowing shortage and partial backlogging. Dye et al. [12] considered the optimal pricing and inventory control policies for deteriorating items with shortages and price-dependent demand. Chang et al. [7] presented the inventory model for deteriorating items with partial backlogging and log-concave demand. Samadi et al. [42] developed the pricing, marketing and service planning inventory model with shortages in fuzzy environment. In this model, the demand is considered as a power function of price, marketing expenditure and service expenditure. Tsao and Sheen [48] discussed the problem of dynamic pricing, promotion and replenishment for deteriorating items under the permissible delay in payments. Zhang et al. [53] considered an inventory model for simultaneously determining the optimal pricing and the optimal preservation technology investment policies for deteriorating items. Ouyang et al. [40] studied the joint pricing and ordering policies for deteriorating item with retail price-dependent demand in response to announced supply price increase.

Chen and Bell [8] showed that customer returns affect the firms pricing and inventory decisions. They developed the pricing and order decisions when the quantity of returned product is a function of both the quantity sold and the price. Hess and Mayhew [27] used regression models to show that the number of returns has a strong positive linear relationship with the quantity sold. Anderson et al. [3] showed that customer returns increase with both the quantity sold and the price set for the product. Zhu [54] proposed the joint pricing and inventory control problem in a random and price-sensitive demand environment with return and expediting.

The major assumptions mentioned in the selected articles are summarized in Table 1 .

## 2. **Motivation section.**

In practice, the majority of deteriorating items would have a span, in which there is no deterioration. For this type of items, the assumption that the deterioration starts from the instant of arrival in stock may lead to make inappropriate replenishment policies due to overvaluing the relevant inventory cost. As a result, in the field of inventory management, it is necessary to incorporate the inventory problems for non-instantaneous deteriorating items. On the other hand, the coordination of price decisions and inventory control means optimizing the system rather than its individual elements. Thus, the optimal pricing combined with inventory ordering policy can yield considerable revenue increase. Moreover, inflation plays a significant role for the optimal order policy and affects the demand of certain products. As inflation increases, the value of money goes down and erodes the future worth of saving and forces one for more current spending. Usually, these spending are on peripherals and luxury items that give rise to demand of these items. Consequently, the effect of inflation and time value of the money cannot be ignored for determining the optimal inventory policy. Also, in the real world customer returns could increase with both the quantity sold and the price set for the product.

In this study, a finite planning horizon inventory model for non-instantaneous deteriorating items with price- and inflation-dependent demand rate and partial backlogging is developed. In addition, the effects of customer returns and time value of money on replenishment policy are also considered. We assume that the customer returns increase with both the quantity sold and the product price. Also, Inflation affects the demand of certain products. As inflation increases, the value of money goes down and erodes the future worth of saving and forces one for more current spending. Therefore, inflation has a major effect on the demand of the goods, especially for fashionable goods for middle and higher income groups. Besides, the selling price of an item inflluences the demand of that item, i.e., whenever the selling price of an item increases, the demand of that decreases. As a result, here, we considered a price- and inflation-dependent demand function. An optimization algorithm is presented to derive selling price, the optimal length of time in which there is no inventory shortage, and the optimal replenishment cycle during the time horizon and then obtain the optimal order quantity when the present value of total

TABLE 1. Major characteristics of inventory models on selected articles

| Author(s) | Pricing | Replenishment rate | Inflation and selling price dependent demand | Non instantaneous deterioration | Partial backlogging shortage | Inflation | Customer returns |
|---|---|---|---|---|---|---|---|
| Abad [2] | Yes | Infinite | No | No | Yes | No | No |
| Chang et al. [7] | Yes | Infinite | No | No | Yes | No | No |
| Covert and Philip [9] | No | Infinite | No | No | No | No | No |
| Datta and Pal [10] | No | Infinite | No | No | No | Yes | No |
| Dye [11] | Yes | Infinite | No | No | Yes | No | No |
| Dye et al. [12] | Yes | Infinite | No | No | No | Yes | No |
| Ghoreishi et al. [17] | Yes | Finite | No | Yes | No | Yes | Yes |
| Guria et al. [22] | No | Infinite | Yes | No | No | Yes | No |
| Hou and Lin [29] | Yes | Infinite | No | No | No | Yes | No |
| Hsieh and Dye [30] | Yes | Finite | No | No | Yes | Yes | No |
| Jaggi et al. [31] | No | Infinite | No | No | No | Yes | No |
| Maihami and N. Kamalabadi [32] | Yes | Infinite | No | Yes | Yes | No | No |
| Mirzazadeh et al. [34] | No | Finite | No | No | No | Yes | No |
| Moon and Lee [36] | No | Infinite | No | No | No | Yes | No |
| Moon et al. [37] | No | Infinite | No | No | Yes | Yes | No |
| Tsao and Sheen [48] | Yes | Infinite | No | No | No | No | No |
| Yang et al. [49] | Yes | Infinite | No | Yes | Yes | No | No |
| Wee and Law [50] | Yes | Infinite | No | No | No | Yes | No |
| Wu et al. [51] | No | Infinite | No | Yes | Yes | No | No |
| Zhang et al. [53] | Yes | Infinite | No | No | No | No | No |
| Present study | Yes | Infinite | Yes | Yes | Yes | Yes | Yes |

profit is maximized. Thus, the replenishment and price policies are appropriately developed. A numerical example is provided to illustrate the proposed model. The results of this example are used to analyze the impact of customer returns, inflation, and non-instantaneous deterioration on the optimal solution.

To the best of our knowledge, this is the first model in pricing and inventory control models that considers price- and inflation-induced demand, non-instantaneously deteriorating items, and customer returns. In this model shortages are allowed and partially backlogged. The backlogging rate is variable and dependent on the time of waiting for the next replenishment. The main objective is determining the optimal selling price, the optimal length of time in which there is no inventory shortage, and the optimal replenishment cycle simultaneously such that the present value of total profit is maximized. This is the first work that follows the above assumptions.

## 3. **Notation and assumptions.**

### 3.1. **Notation.**
In this paper, the following assumptions are used:

$A$: constant purchasing cost per order,
$c$: purchasing cost per unit,
$c_1$: holding cost per unit per unit time,
$c_2$: backorder cost per unit per unit time,
$c_3$: cost of lost sale per unit,
$p$: selling price per unit, where $p > c$ (decision variable),
$\theta$: constant deterioration rate,
$r$: constant representing the difference between the discount (cost of capital) and the inflation rate,
$Q$: order quantity,
$T$: length of replenishment cycle time (decision variable),
$t_1$: length of time in which there is no inventory shortage (decision variable),
$t_d$ : length of time in which the product exhibits no deterioration,
$SV$: salvage value per unit,
$H$: length of planning horizon,
$N$: Number of replenishments during the time horizon $H$,
$T^*$: optimal length of the replenishment cycle time,
$Q^*$: optimal order quantity,
$t_1^*$: optimal length of time in which there is no inventory shortage,
$p^*$: optimal selling price per unit,
$I_1(t)$: inventory level at time $t \in [0, t_d]$,
$I_2(t)$: inventory level at time $t \in [t_d, t_1]$,
$I_3(t)$: inventory level at time $t \in [t_1, T]$,
$I_0$: maximum inventory level,
$S$: maximum amount of demand backlogged,
$PWTP(p, t_1, T; N)$: present value of total profit over the time horizon.

### 3.2. **Assumptions.**
In this paper, the following assumptions are considered:

1. There is a constant fraction of the on-hand inventory deteriorates per unit of time and there is no repair or replacement of the deteriorated inventory.
2. A single non-instantaneous deteriorating item is assumed.
3. The replenishment rate is infinite and the lead time is zero.
4. Demand is inflation rate and selling price dependent, i.e., $D(t) = (a - bp)e^{krt}$ (where $0 < k < 1, a > 0, b > 0$).
5. Shortages are allowed. The unsatisfied demand is backlogged, and the fraction of shortage backordered is $\beta(x) = k_0 e^{-\delta x}$ $(\delta > 0, 0 < k_0 \leq 1)$, where x is the waiting time up to the next replenishment and $\delta$ is a positive constant and $0 \leq \beta(x) \leq 1, \beta(0) = 1$[1].
6. Following the empirical findings of Anderson et al. [3], we assume that customer returns increase with both the quantity sold and the price. We use the general form: $R(p, t) = \alpha D(p, t) + \beta p$ $(\beta \geq 0, 0 \leq \alpha < 1)$ that is presented by Chen and Bell [8]. Customers are assumed to return $R(p, t)$ products during the period for full credit and these units are available for resale in the following period. We assume that the salvage value of the product at the end of the last period is $SV$ per unit.
7. The time horizon is finite.

4. **Model formulation.** We use the same inventory shortage model as in Yang et al. [52]. Base on this model; the inventory system is as follows: $I_0$ units of item arrive at the inventory system at the beginning of each cycle. During the time interval $[0, t_d]$, the inventory level decreases due to demand only. Afterwards the inventory level drops to zero due to both demand and deterioration during the time interval $[t_d, t_1]$. Finally, a shortage occurs due to demand and partial backlogging during the time interval $[t_1, T]$ (see Fig. 1 ).

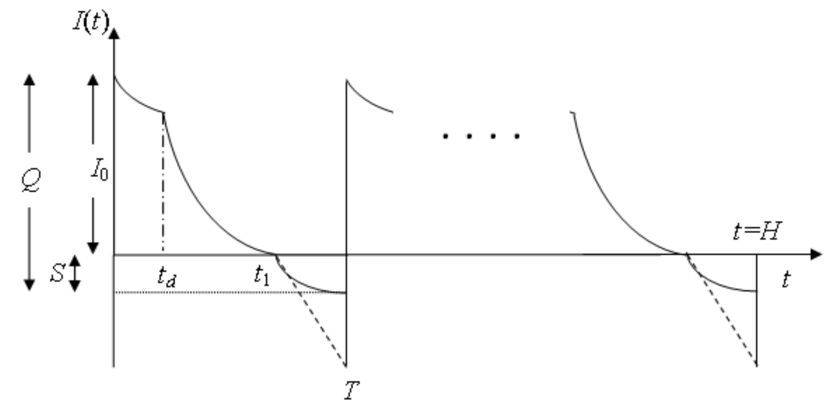


FIGURE 1. Graphical representation of the inventory system

The equation representing the inventory status in system for the first interval: During the time interval $[0, t_d]$, the differential equation representing the inventory status is given by

$$\frac{dI_1(t)}{dt} = -D(t) = -(a - bp)e^{krt}. \tag{1}$$

With the condition $I_1(0) = I_0$, solving Equation (1) yields

$$I_1(t) = \frac{(-a + bp)e^{krt} - bp + I_0kr + a}{kr} \quad (0 \le t \le t_d). \tag{2}$$

In the second interval $[t_d, t_1]$, the inventory level decreases due to demand and deterioration. Thus, the differential equation below represents the inventory status:

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -D(t). \tag{3}$$

By the condition $I_2(t_1) = 0$, the solution of Equation (3) is

$$I_2(t) = -\frac{(a - bp)(e^{t(kr+\theta)} - e^{t_1(kr+\theta)})e^{-\theta t}}{kr + \theta} \quad (t_d \le t \le t_1). \tag{4}$$

It is clear from Fig. 1 that $I_1(t_d) = I_2(t_d)$ therefore, the maximum inventory level $I_0$ can be obtained

$$I_0 = \frac{1}{(kr + \theta)kr}((a-bp)(e^{-\theta t_d}kre^{t_1(kr+\theta)} - e^{-\theta t_d}kre^{t_d(kr+\theta)} + (-1+e^{krt_d})(kr+\theta))). \tag{5}$$

In the third interval $(t_1, T)$, shortage is partially backlogged according to fraction $\beta(T-t)$. Therefore, the inventory level at time t is obtained by the following equation:

$$\frac{dI_3(t)}{dt} = -D(t)\beta(T-t) = \frac{-D(t)}{e^{\delta(T-t)}} \quad (t_1 \le t \le T). \tag{6}$$

The solution of the above differential equation after apply the boundary conditions $I_3(t_1) = 0$, is

$$I_3(t) = -\frac{(a-bp)(e^{(-T+t)\delta+krt} - e^{(kr+\delta)t_1-\delta T}}{kr+\delta} \quad (t_1 \le t \le T). \tag{7}$$

If we put $t = T$ into $I_3(t)$, the maximum amount of demand backlogging $(S)$ will be obtained:

$$S = -I_3(T) = \frac{(a-bp)(e^{krT} - e^{(kr+\delta)t_1-\delta T})}{kr+\delta}. \tag{8}$$

Order quantity per cycle $(Q)$ is the sum of $S$ and $I_0$, i.e.,

$$\begin{aligned} Q = S + I_0 = \frac{1}{(kr+\theta)kr(kr+\delta)}&((a-bp)((-k^2r^2-\theta kr)e^{(kr+\delta)t_1-\delta T} \\ &+ke^{-\theta t_d}r(kr+\delta)e^{t_1(kr+\theta)} - ke^{-\theta t_d}r(kr+\delta)e^{t_d(kr+\theta)} \\ &+((kr+\delta)e^{krt_d} - kr + kre^{krT} - \delta)(kr+\theta))). \end{aligned} \tag{9}$$

Now, we can obtain the present value inventory costs and sales revenue for the first cycle, which consists of the following elements:

1) Since replenishment in each cycle has been done at the start of each cycle, the present value of replenishment cost for the first cycle will be $A$, which is a constant value.

2) Inventory occurs during period $t_1$, therefore, the present value of holding cost $(HC)$ for the first cycle is

$$HC = c_1(\int_0^{t_d} I_1(t)e^{-rt}dt + e^{-rt_d}\int_{t_d}^{t_1} I_2(t)e^{-rt}dt). \tag{10}$$

3) The present value of shortage cost $(SC)$ due to backlog for the first cycle is

$$SC = c_2(e^{-rt_1}\int_{t_1}^{T} -I_3(t)e^{-rt}dt). \tag{11}$$

4) The present value of opportunity cost due to lost sales $(OC)$ for the first cycle is

$$OC = c_3(e^{-rt_1}\int_{t_1}^{T} D(t)(1-\beta(T-t))e^{-rt}dt). \tag{12}$$

5) The present value of purchase cost $(PC)$ for the first cycle is

$$PC = c(I_0 + Se^{-rT}). \tag{13}$$

6) The present value of return cost for each cycle. We assume that returns from period $i-1$ are available for resale at the beginning of period i (except the first period in which there is no cycle previous to it). It is also assumed that the salvage value of product at the end of the last period $(i = N)$ is $SV$. Therefore, the present

value of return cost and resale revenue for each cycle is obtained as follows:

$$RC_i = \begin{cases} p\int_0^{t_1}(\alpha D(t)+\beta p)e^{-rt}dt, & \text{for } i=1, \\ \\ RC = p\int_0^{t_1}(\alpha D(t)+\beta p)e^{-rt}dt \\ -c\int_0^{t_1}(\alpha D(t)+\beta p)dt, & \text{for } i=2,..,N-1, \\ \\ p\int_0^{t_1}(\alpha D(t)+\beta p)e^{-rt}dt - c\int_0^{t_1}(\alpha D(t)+\beta p)dt \\ -SVe^{-rT}\int_0^{t_1}(\alpha D(t)+\beta p)dt, & \text{for } i=N. \end{cases} \tag{14}$$

7) The present value of sales revenue ($SR$) for the first cycle is

$$SR = p(\int_0^{t_1} D(t)e^{-rt}dt + Se^{-rT}). \tag{15}$$

There are $N$ cycles during the planning horizon. Since inventory is assumed to start and end at zero, an extra replenishment at $t = H$ is required to satisfy the backorders of the last cycle in the planning horizon. Therefore, the total number of replenishment will be $N+1$ times; the first replenishment lot size is $I_0$, and the $2^{nd}$, $3^{rd}$, . . . , $N^{th}$ replenishment lot size is as follows:

$$Q = S + I_0.$$

Finally, the last or$(N+1)^{th}$ replenishment lot size is $S$. Therefore, the present value of total profit during planning horizon, denoted by $PWTP(p,t_1,T;N)$, is derived as follows:

$$\begin{aligned} PWTP(p,t_1,T;N) = &\sum_{i=0}^{N-1}(SR-A-HC-SC-OC-PC-RC)e^{-riT} \\ &+SVe^{-rH}\int_0^{t_1}(\alpha D(t)+\beta p)dt - c\int_0^{t_1}(\alpha D(t)+\beta p)dt \\ &-Ae^{-rH} = (SR-A-HC-SC-OC-PC-RC) \\ (\frac{1-e^{rNT}}{1-e^{-rT}}) + SVe^{-rH}&\int_0^{t_1}(\alpha D(t)+\beta p)dt - c\int_0^{t_1}(\alpha D(t)+\beta p)dt) - Ae^{-rH}, \end{aligned} \tag{16}$$

which we want to maximize subject to the following constraints:

$$p{>}0, 0 < t_1 < T, N \in \mathbb{N}.$$

The value of the variable $T$ can be replaced by the equation $T = H/N$, for some constant $H > 0$, thus, the problem is to obtain optimal values of $t_1$ , $p$ and $N$ that maximize $PWTP(p,t_1,T)$ subject to $p{>}0$ and $0 < t_1 < T$, where $N$ is a discrete variable and $p$ and $t_1$ are continuous variables. However, since $PWTP(p,t_1,T;N)$, and still $PWTP(p,t_1,H/N;N)$, is a very complicated function due to high-power expressions in the exponential function, it is difficult to show analytically the validity of the sufficient conditions. Hence, if more than one local maximum value exists, we would attain the largest of the local maximum values over the regions subject to $p > 0$ and $0 < t_1 < T$. The largest value is referred to as the global maximum value of $PWTP(p,t_1,T;N)$. So far, the procedure is to locate the optimal values of $p$ and $t_1$ when $N$ is fixed. Since $N$ is a discrete variable, the following algorithm can be used to determine the optimal values of $p$, $t_1$ and $N$ of the proposed model. We may refer to $PWTP(p, t_1, H/N; N)$ and, for the sake of convenience, just denote it by $PWTP(p, t_1, N)$.

## 5. **The optimal solution procedure.**

The objective function has three variables. The number of replenishments ($N$) is a discrete variable, the length of time in which there is no inventory shortage ($t_1$) and the selling price per unit ($p$) are continuous variables. The following algorithm is used to obtain the optimal amount of $t_1$, $p$ and $N$:

**Step 1.** let $N$=1.

**Step 2.** Take the partial derivatives of $PWTP(p, t_1, N)$ with respect to $p$ and $t_1$, and equate the results to zero, the necessary conditions for optimality are

$$\frac{\delta}{\delta p} PWTP(p, t_1, N) = 0. \tag{17}$$

and

$$\frac{\delta}{\delta t_1} PWTP(p, t_1, N) = 0. \tag{18}$$

In Appendix A, we use the formula of $PWTP_1(p, t_1, T; N)$ from Equation (16), inserted into Equations (17) and (18).

**Step 3.** For different integer $N$ values, derive ${t_1}^*$ and $p^*$ from Equations (17) and (18). Substitute $(p^*, {t_1}^*, N^*)$ to $PWTP(p, t_1, T; N)$ from Equation (16) to derive $PWTP(p^*, {t_1}^*, N^*)$.

**Step 4.** Add one unit to $N$ and repeat step 2 and 3 for the new $N$. If there is no increasing in the last value of $PWTP(p, t_1, N)$, then show the previous one which has the maximum value. The point $(p^*, {t_1}^*, N^*)$ and the value $PWTP(p^*, t_1^*, N^*)$ constitute the optimal solution and satisfy the following conditions:

$$\Delta PWTP(p^*, {t_1}^*, N^*) < 0 < \Delta PWTP(p^*, {t_1}^*, N^* - 1). \tag{19}$$

where

$$\Delta PWTP(p^*, {t_1}^*, N^*) = PWTP(p^*, {t_1}^*, N^* + 1) - PWTP(p^*, {t_1}^*, N^*). \tag{20}$$

We substitute $(p^*, t_1^*, N^*)$ into Equation(9) to derive the $N^{th}$ replenishment lot size.

If the objective function was strictly concave, the following sufficient conditions must be satisfied:

$$(\frac{\delta^2 PWTP}{\delta p \delta t_1})^2 - (\frac{\delta^2 PWTP}{\delta {t_1}^2})(\frac{\delta^2 PWTP}{\delta p^2}) < 0, \tag{21}$$

and any one of the following conditions:

$$\frac{\delta^2 PWTP}{\delta {t_1}^2} < 0, \quad \frac{\delta^2 PWTP}{\delta p^2} < 0, \tag{22}$$

Since $PWTP$ is a very complicated function due to high-power expression of the exponential function, it is unlikely to show analytically the validity of the above sufficient conditions. Thus, the sign of the above quantity in Equation (22) is assessed numerically. The computational results are shown in the following illustrative example.

6. **Numerical examples.** To illustrate the solution procedure and the results, let us apply the proposed algorithm to solve the following numerical examples. The results can be found by applying Maple 13.

**Example 1.** $c$ =\$ 10per unit, $c_1$ = \$ 1/per unit/per unite time, $c_2$ = \$ 5/per unit/per unite time, $c_3$ = \$ 25/per unit, $t_d = 0.08$ unit time, $A$ = \$ 250/per order run, $\theta = 0.08$,$r = 0.12$, $\delta = 0.1$, $H = 40$ unit time, $\alpha = 0.2$, $\beta = 0.3$, $SV$ = \$ 3/per unit, $a = 200$, $b = 4$, $k = 0.03$.

From Table 2 , the maximum present value of total profit is found in $35^{th}$ cycle. The total number of order is therefore $(N + 1)$ or 36. With thirty six orders, the optimal solution is as follows:
$p^*$=30.138, $t_1{}^*$=0.429, $T^*$=1.142, $PWTP^*$=7892.824, $Q^*$=89.431.

Table 2. Optimal solution of the example

| | | Time interval | | | |
|---|---|---|---|---|---|
| N | P | $t_1$ | T | Q | PWTP |
| 34 | 30.122 | 0.454 | 1.176 | 92.152 | 7891.722 |
| 35* | 30.138* | 0.429 | 1.142* | 89.431 | 7892.824* |
| 36 | 30.155 | 0.432 | 1.111 | 86.145 | 7892.321 |

*Optimal solution.

By substituting the optimal values of $N^*$,$p^*$ and $t_1^*$ to Equation (22), it will be shown that $PWTP$ is strictly concave (cf. Fig. 2): $\frac{\delta^2 PWTP}{\delta t_1^2} = -8021.195$, $\frac{\delta^2 PWTP}{\delta p^2}$ $= -59.962$.

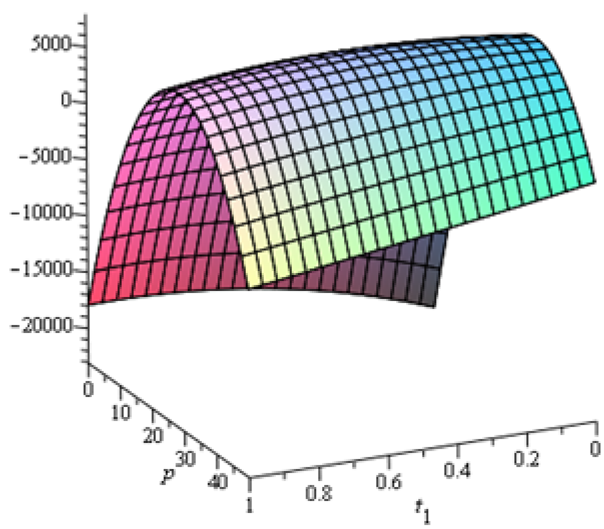


Figure 2. The graphical representation of the concavity of the present value of total profit function $PWTP(p, t_1, 35)$

We obtain the results of this example for investigating the impact of customer returns on the optimal solution (Table 3 ). The results show that when returns are dependent on the quantity sold only ($i.e., \beta = 0$), the company should raise the price and decrease the order quantity, but if returns are dependent on price only ($i.e., \alpha = 0$) the company should reduce the price, and increase the order quantity. The results verify that when returns increase with the product price (when purchase costs are constant), the company should set a lower price (in order to discourage returns). Increasing $\alpha$ and / or $\beta$ reduces the companys present value of total profit.

TABLE 3. The impact of customer returns on the optimal solution of the example

| $\alpha, \beta$ | $p^*$ | $t_1^*$ | $T^*$ | $Q^*$ | $PWTP^*$ |
|---|---|---|---|---|---|
| $\alpha = 0, \beta = 0$ | 30.847 | 1.228 | 1.600 | 121.977 | 10633.511 |
| $\alpha = 0, \beta = 0.3$ | 29.447 | 0.985 | 1.481 | 123.865 | 9466.923 |
| $\alpha = 0.2, \beta = 0$ | 30.961 | 0.805 | 1.428 | 105.688 | 8702.702 |
| $\alpha = 0.2$ , $\beta = 0.3$ | 30.138 | 0.429 | 1.142 | 89.431 | 7892.824 |

The numerical results of the Table 3 are summarized in Fig. 3 *a* - *c*.

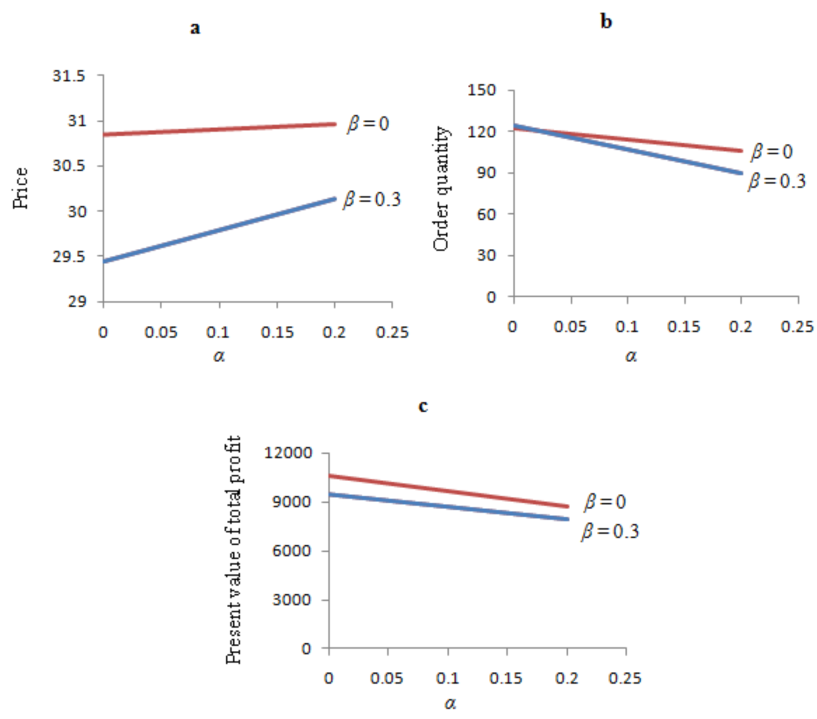


FIGURE 3. The impact of customer returns on price, order quantity and prot. (a) Impact of $\alpha$ and $\beta$ on price. (b) Impact of $\alpha$ and $\beta$ on order quantity. (c) Impact of $\alpha$ and $\beta$ on present value of total prot

Moreover, as observed in Table 4 , when the net discount rate of inflation ($r$) decreases then optimal cycle time, optimal order quantity, and the optimal present value of total profit increase. Therefore, the results confirm that when the discount rate of inflation decrease, the purchasing power will be raised, which will lead to an enhancement in order quantity. Thus, it is important to consider the effects of inflation and the time value of money on inventory policy.

TABLE 4. The impact of the discount rate of inflation on the optimal solution of the example

| r | $p^*$ | $t_1^*$ | $T^*$ | $Q^*$ | $PWTP^*$ |
|---|---|---|---|---|---|
| 0.02 | 30.255 | 0.459 | 1.250 | 99.743 | 27852.076 |
| 0.06 | 30.190 | 0.455 | 1.212 | 97.161 | 14998.570 |
| 0.12 | 30.138 | 0.429 | 1.142 | 89.431 | 7892.824 |
| 0.16 | 30.126 | 0.398 | 1.081 | 82.356 | 5810.532 |

Also, as observed in Table 5 , If $t_d$=0, then the model converts into the instantaneous deterioration items case, and the optimal present value of total profit can be found as follows: $PWTP$=7828.117. It can also be seen that the optimal present value of total profit in the instantaneous deterioration items case decrease. This implies that the optimal present value of total profit could be increased by changing the instantaneously to non-instantaneously items using the improved stock condition.

TABLE 5. The results with instantaneous and non-instantaneous deteriorating models of the example

| $t_d$ | $p^*$ | $t_1^*$ | $T^*$ | $Q^*$ | $PWTP^*$ |
|---|---|---|---|---|---|
| 0 | 30.220 | 0.422 | 1.142 | 86.673 | 7828.117 |
| 0.08 | 30.138 | 0.429 | 1.142 | 89.431 | 7892.824 |
| 0.16 | 30.009 | 0.393 | 1.081 | 82.801 | 8044.555 |
| 0.24 | 29.7976 | 0.308 | 0.952 | 73.779 | 8327.999 |

**Example 2.** $c$ = \$ 15per unit, $c_1$ = \$ 1.5/per unit/per unite time, $c_2$ = \$ 8.5/per unit/per unite time, $c_3$ = \$ 37.5/per unit, $t_d = 0.04$ unit time, $A$ = \$ 500/per order run, $\theta = 0.10$, $r = 0.12$, $\delta = 0.1$, $H = 40$ unit time, $\alpha = 0.3$, $\beta = 0.45$, $SV$ = \$ 5/per unit, $a = 300$, $b = 6$, $k = 0.045$.

According to the computational results shown in Table 6 , the optimal solution is as follows:

$$p^* = 33.804, t_1^* = 0.572, T^* = 1.379, PWTP^* = 8077.648, Q^* = 141.361.$$

TABLE 6. Optimal solution of the example

| | | Time interval | | | |
|---|---|---|---|---|---|
| $N$ | $P$ | $t_1$ | $T$ | $Q$ | $PWTP$ |
| 28 | 33.796 | 0.607 | 1.429 | 141.900 | 8072.458 |
| 29* | 33.8048* | 0.572* | 1.379* | 141.361 | 8077.648* |
| 30 | 33.817 | 0.539 | 1.333 | 140.806 | 8075.768 |

**Example 3.** $c$ =\$ 5per unit, $c_1$ =\$ 0.5/per unit/per unite time, $c_2$ = \$ 2.5/per unit/per unite time, $c_3$ = \$ 12.5/per unit, $t_d = 0.12$ unit time, $A$ =\$ 100/per order

run, $\theta = 0.06$, $r = 0.10$, $\delta = 0.1$, $H = 20$ unit time, $\alpha = 0.1$, $\beta = 0.2$, $SV =\$\ 2$/per unit, $a = 100$, $b = 2$, $k = 0.02$.

According to the computational results shown in Table 7 , the optimal solution is as follows:

$$p^* = 26.830, t_1^* = 0.462, T^* = 1.176, PWTP^* = 6520.159, Q^* = 56.757.$$

TABLE 7. Optimal solution of the example

| | | Time interval | | | |
|---|---|---|---|---|---|
| $N$ | $P$ | $t_1$ | $T$ | $Q$ | $PWTP$ |
| 16 | 26.760 | 0.524 | 1.250 | 57.204 | 6519.311 |
| 17* | 26.830* | 0.462* | 1.176* | 56.757* | 6520.159* |
| 18 | 26.903 | 0.406 | 1.111 | 56.326 | 6519.177 |

7. **Conclusion and outlook.** In this paper, we investigate the effects of inflation and customer returns on joint pricing and inventory control model for non-instantaneous deteriorating items with inflation- and selling price-dependent demand and partial backlogging. The customer returns are assumed as a function of price and demand simultaneously. The backlogging rate is variable and dependent on the time of waiting for the next replenishment. The mathematical models are derived to determine the optimal selling price, the optimal length of time in which there is no inventory shortage, and the optimal replenishment cycle simultaneously. An optimization algorithm is presented to derive the optimal decision variables. Finally, a numerical example is solved and the effects of the customer returns, inflation, and non-instantaneous deteriorating items are also discussed.

From Table 3, it can be observed that when the customer returns depend on the quantity of product sold only (i.e.,$\beta = 0$), the price increase and order quantity decrease. On the other hand, when customer returns increase with price only (i.e., $\alpha = 0$), the price reduces and order quantity increases. Also, observed in Table 4, it can be seen that there is an improvement in the optimal cycle time, optimal order quantity, and the optimal present value of total profit when the discount rate of inflation decreases. Moreover, from Table 5, it can be observed that the optimal present value of total profit in the instantaneous deterioration items case decrease.

To the best of our knowledge, this is the first model in pricing and inventory control models that consider inflation- and selling price-dependent demand rate, partial backlogging, and customer returns for non-instantaneously deteriorating items. The proposed model can be extended in numerous ways for future research. For example, we could incorporate: (1) stochastic demand function (2) two warehouse (3) quantity discount (4) finite replenishment rate and (5) deteriorating cost.

**Appendix A.** For a given value of $N$, the necessary conditions for finding the optimal values $p^*$ and $t_1^*$ are given as follows:

$$\frac{\delta}{\delta t_1} PWTP(p, t_1, T)$$
$$= (-(k-1)(r+\theta)e^{-rt_1} r(a-bp)(-1+e^{-rh})((-2+k)r+\delta)(kr+\delta)c_3$$
$$e^{(\delta+(k-1)r)t_1-\delta T} - (k-1)r(\delta+(k-1)r)(a-bp)c_1(-1+e^{-rH})(Kr+\delta)$$
$$e^{-rt_d} e^{t_1(kr+\theta)-t_d(r+\theta)}$$
$$+(k-1)(((-2+k)r+\delta)c_2(((e^{-rt_1})^2 - e^{-rT}(\delta+(k-1)r)c_2 e^{-rt_1}$$
$$+e^{-rT} r(kr+\delta)(c-p))(r+\theta)(\delta+(k-1)r)(a-bp)(-1$$

$$
\begin{aligned}
&+e^{-rH})e^{(kr+\delta)-\delta T}-(k-1)(r+\theta)e^{-rt_1}r(\delta+k-1)r)(a-bp)(-1\\
&+e^{-rH})c_2(-2e^{-rt_1}+e^{rT})e^{(-T+t)\delta+krt}+(-(k-1)(r+\theta)(\delta\\
&+(k-1)r(a-bp)c_1(-1+e^{-rH})e^{-t_d(r+\theta)}e^{t_1(kr+\theta)+rt_d}\\
&-(k-1)(r+\theta)r(\delta+(k-1)r)(\alpha(a-bp)e^{krt_1}+\beta p)(c-SVe^{-rH})e^{-\frac{rH}{N}}\\
&-(k-1)(r+\theta)(\delta+(k-1)r)(e^{-\theta t_d}cr-e^{-t_d(r+\theta)}c_1)(a-bp)(-1\\
&e^{-rH})e^{t_1(kr+\theta)}\\
&+r((c_3(r+\theta)(-2+k)e^{-rt_1}\\
&+(k-1)(e^{-rt_d}c_1+p(r+\theta))(\delta+(k-1)r)(a-bp)(-1+e^{-rH})e^{-rt_1(k-1)}\\
&+(r+\theta)(\delta c_3e^{-rt_1}(-1+e^{-rH})(a-bp)e^{rT(k-1)}\\
&(k-1)((-SV+c-pe^{-rt_1})e^{-rH}+pe^{-rt_1}(\delta+(k-1)r)(\alpha(a-bp)e^{krt_1}\\
&+\beta p))))(kr+\delta))/((k-1)(r+\theta)r(\delta+(k-1)r)(-1+e^{-\frac{rH}{N}})(kr+\delta))\\
&=0 \text{ and} \tfrac{\delta}{\delta p}PWTP(p,t_1,T)=((bkr^2c_3e^{(-rt_1)}\\
&(r+\theta)(k-1)(kr+\theta)(kr+\delta)e^{((\delta+(k-1)r)t_1-\delta T)}+bkr^2c_1e^{(-rt_d)}(k-1)(kr+\delta)(-r+kr+\delta)\\
&e^{(t_1(kr+\theta)-t_d(r+\theta))}-(kr+\theta)(-r+kr+\delta)(k-1)(b((e^{-rt_1})))^2c_2-c_2e^{(-rt_1)}be^{(-rT)}\\
&+r((c-2p)b+a)e^{(-rT)})r(r+\theta)ke^{((kr+\delta)t_1-\delta T)}-bkrc_2)e^{(-rt_1)}(r+\theta)(k-1)(kr+\theta)\\
&(-e^{(-rt_1)}+e^{(-rT)})(-r+kr+\delta)e^{((-T+t)\delta+krt)}+bkre^{(-t_d(r+\theta))}c_1(k-1)(kr+\delta)(-r+\\
&kr+\delta)\\
&(r+\theta)e^{(t_1(kr+\theta)+rt_d)}+b\theta e^{(-t_d(r+\theta))}c_1(k-1)(kr+\delta)(-r+kr+\delta)(r+\theta)\\
&e^{(((1+k)r+\theta)t_d)}bkr(k-1)(kr+\delta)(-r+kr+\delta)(r+\theta)(e^{(-\theta t_d)cr}-e^{(-t_d(r+\theta))}c_1)\\
&e^{(t_1(kr+\theta))}-(c_1(r+\theta)e^{(-t_d(r+\theta))}+ce^{(-\theta t_d)}r^2\ (k-1)b(-r+kr+\delta)(r+\theta)k(kr+\delta)\\
&e^{(t_d(kr+\theta))}+(kr+\theta)(-r+kr+\delta)rk(-bc_3(r+\theta)e^{(-rt_1)}-e^{(-rt_d)}bc_1+(r+\theta)(-2bp+\\
&a)(kr+\delta)\\
&e^{(rt_1(k-1))}+(bkrc_1e^{(-rt_d)}(kr+\delta)(-r+kr+\delta)e^{(rt_d(k-11))}+kr+\theta)(bc_1e^{(t_d(r+\theta))}(kr+\delta)\\
&(-r+kr+\delta)e^{(-t_d(r+\theta))}+r(bkc_3e^{(-rt_1)}\delta(kr+\delta)e^{rT(k-1)}+(-r+kr+\delta)((kr+\\
&\delta)k(-\alpha(-2bp+a)\\
&e^{(krt_1)}+(-2\beta pk-2\alpha bp+2\beta p+\alpha a)e^{(rt_1)}+2p\beta(k-1))e^{(-rt_1)}-bc\alpha(k-1)(kr+\\
&\delta)e^{(krt_1)}+r((c-2p)b+a)ke^{krT}\\
&(k-1)e^{(-rT)}-(-bc(k-1)e^{(krt_d)}-\beta t_1crk^2+(\beta t_1cr+(-2p+c-c\alpha)b+a)k+\\
&bc(-1+\alpha))(kr+\delta)))))(r+\theta))\\
&(-1+e^{(-rH)})/((kr+\theta)(k-1)r^2(kr+\delta)k(r+\theta)(-r+kr+\delta)(-1+e^{(-rH/N)}))+\\
&(SVe^{(-rH)}\\
&(\alpha b-\alpha be^{(krt_1)}+\beta t_1kr))/kr+(c(-\beta t_1kr+\alpha be^{(krt_1)}-\alpha b)/kr=0.
\end{aligned}
$$

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