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# Overview

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# What you'll be able to do!

machine translation

"hello!"



"bonjour!"

document search

"Can I get a  
refund?"



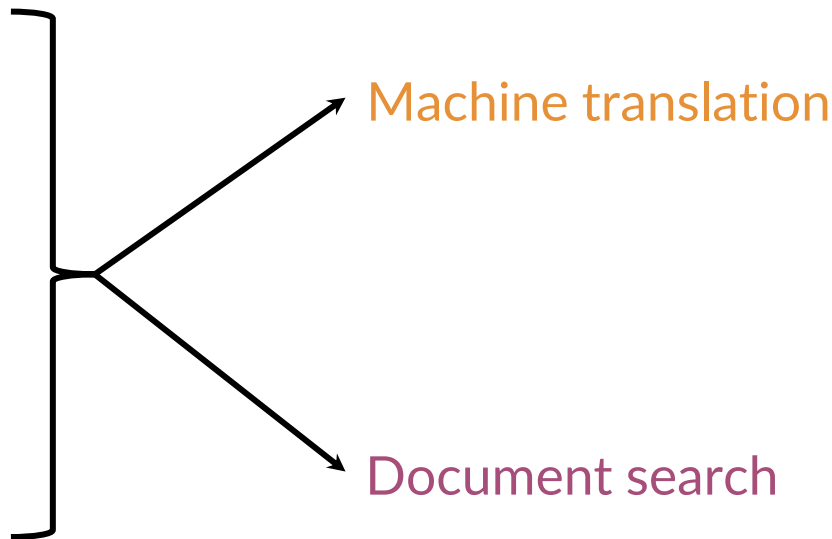
"What's your return  
policy?"

...

"May I get my money  
back?"

# Learning Objectives

- Transform vector
- “K nearest neighbors”
- Hash tables
- Divide vector space into regions
- Locality sensitive hashing
- Approximated nearest neighbors





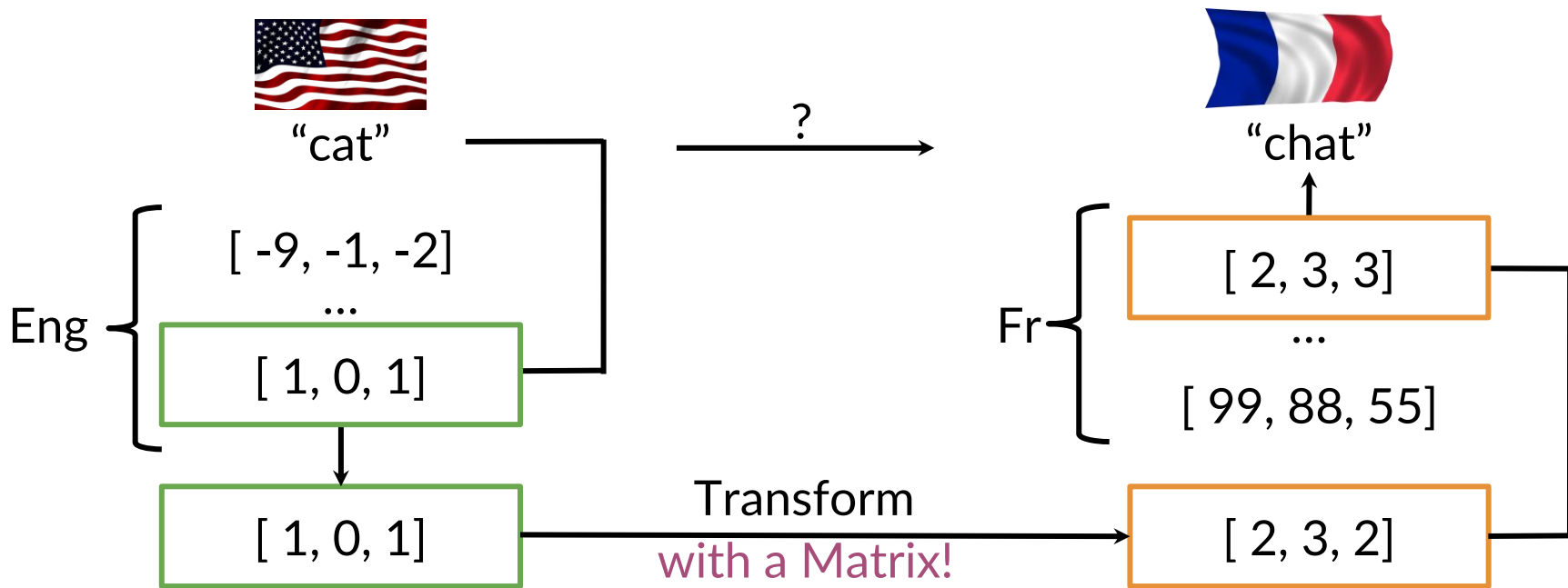
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# Transforming word vectors

# Outline

- Translation = Transformation
- How to get a good transformation

# Overview of Translation



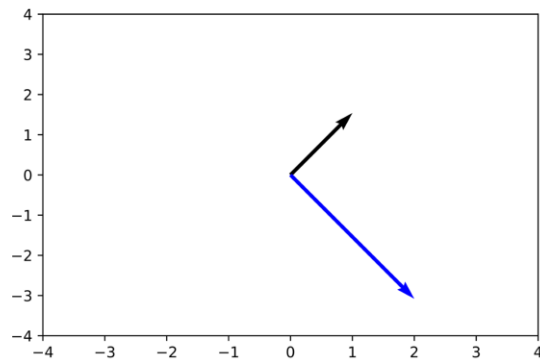
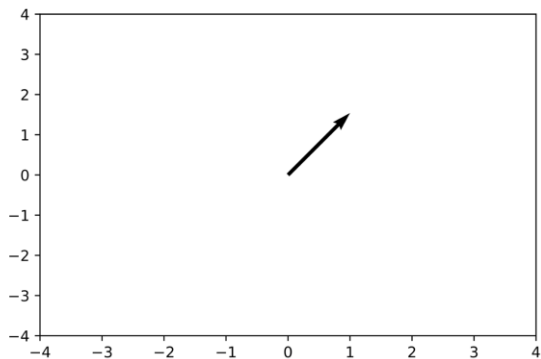
# Transforming vectors

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 2 & -2 \end{pmatrix}$$

X

R

Y





# Transforming vectors

```
R = np.array([[2,0],  
              - - -  
              [1,1]])  
x = np.array([[1,1]])  
np.dot(x,R)  
  
array([[2,-2]])
```

Try it  
yourself!

# Align word vectors

$$\begin{pmatrix} [\text{"cat" vector}] \\ [\dots \text{vector}] \\ [\text{"zebra" vector}] \end{pmatrix} \mathbf{X} \mathbf{R} \approx \mathbf{Y} \begin{pmatrix} [\text{"chat" vecteur}] \\ [\dots \text{vecteur}] \\ [\text{"z bresse" vecteur}] \end{pmatrix} \mathbf{Y}$$

subsets of the full  
vocabulary

# Solving for R

initialize R  
in a loop:

$$Loss = \| \text{our translation } \mathbf{XR} - \text{actual trans } \mathbf{Y} \|_F$$
$$g = \frac{d}{dR} Loss \quad \text{gradient}$$
$$R = R - \alpha g \quad \text{update}$$

# Frobenius norm

$$\| \mathbf{X}\mathbf{R} - \mathbf{Y} \|_F$$

$$\mathbf{A} = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$$

$$\|\mathbf{A}_F\| = \sqrt{2^2 + 2^2 + 2^2 + 2^2}$$

$$\|\mathbf{A}_F\| = 4$$

$$\|\mathbf{A}\|_F \equiv \sqrt{\sum_{i=1}^m \sum_{j=1}^n |a_{ij}|^2}$$

# Frobenius norm

```
A = np.array([[2,2],  
- - -  
A_squared = np.square(A)  
A_squared  
array([[4,4],
```

Try it yourself!

```
A_Frobenious = np.sqrt(np.sum(A_squared))  
A_Frobenious  
4.0
```

Frobenius norm squared

$$\|\mathbf{XR} - \mathbf{Y}\|_F^2$$

*easier  
to minimize*

$$\mathbf{A} = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$$

$$\|\mathbf{A}\|_F^2 = \left( \sqrt{2^2 + 2^2 + 2^2 + 2^2} \right)^2$$

$$\|\mathbf{A}\|_F^2 = 16$$

# Gradient

$$Loss = \|\mathbf{XR} - \mathbf{Y}\|_F^2$$

$$g = \frac{d}{dR} Loss = \frac{2}{m} (\mathbf{X}^T (\mathbf{XR} - \mathbf{Y}))$$

Implement in the assignment!

# Summary

- $\mathbf{X}\mathbf{R} \approx \mathbf{Y}$
- minimize  $\|\mathbf{X}\mathbf{R} - \mathbf{Y}\|_F^2$





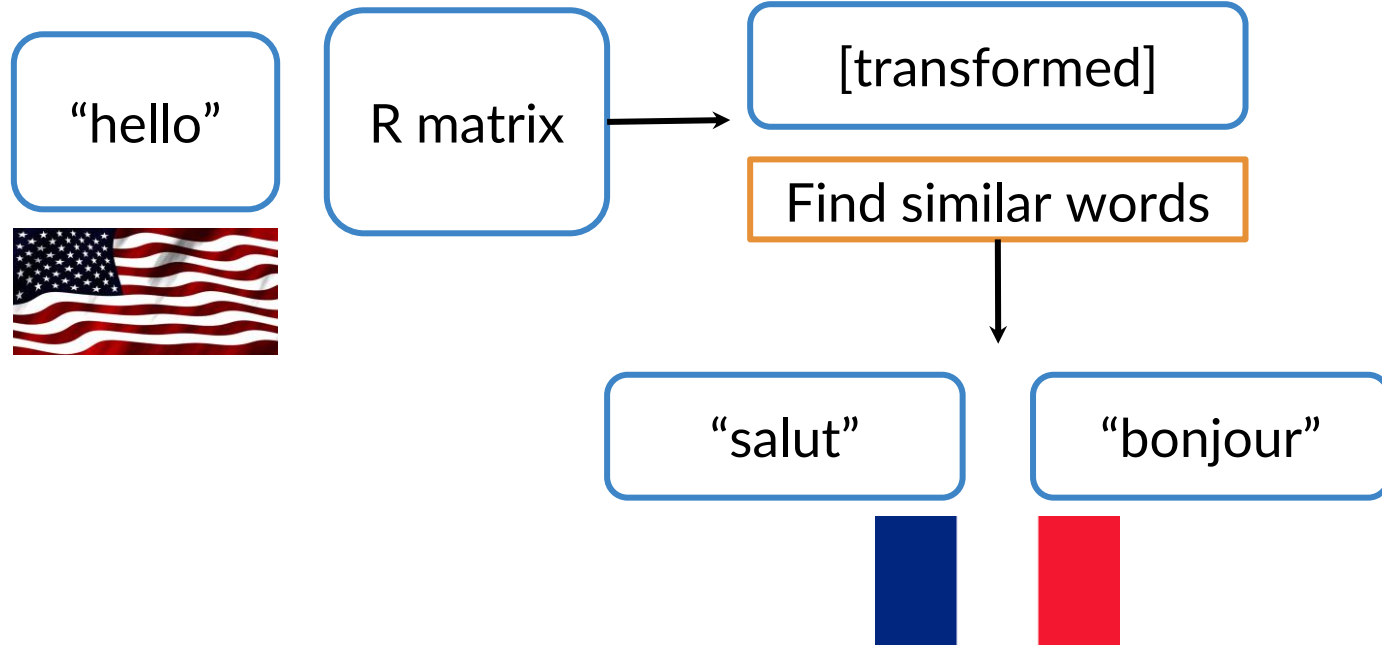
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# K-nearest neighbors

# Outline

- K closest matches → K-nearest neighbors

# Finding the translation



# Nearest neighbours



You  
San Francisco



Friend



Location

Shanghai

Nearest

2



Bangalore

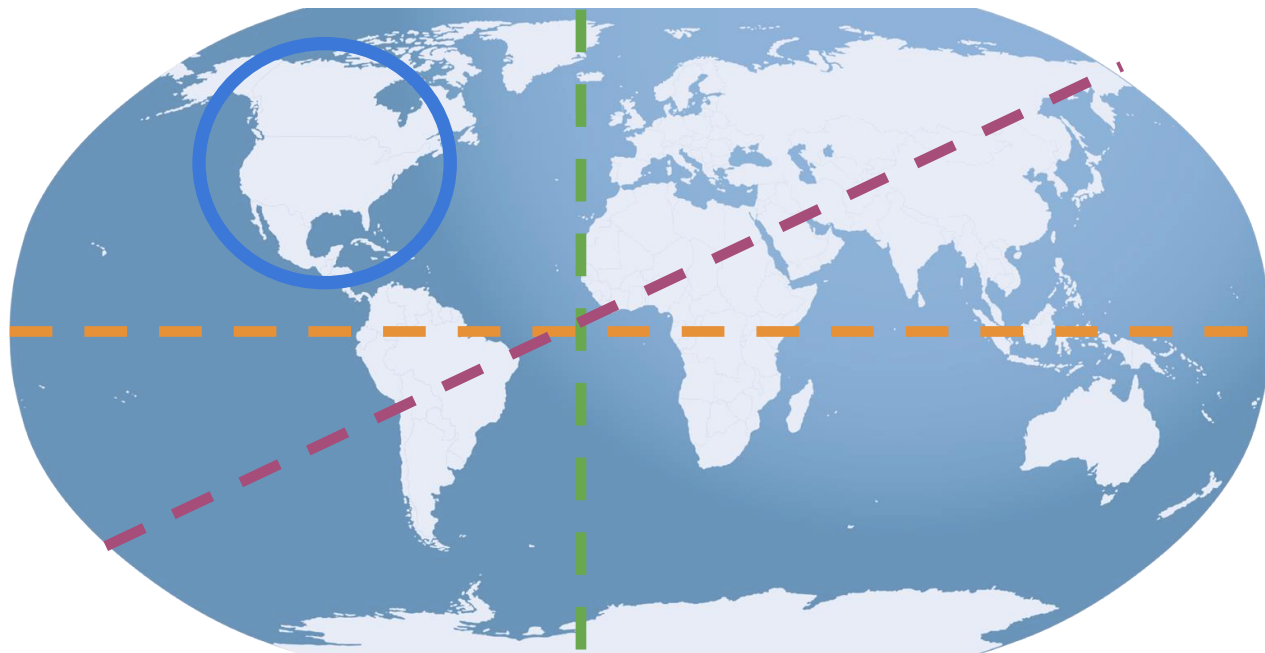
3



Los Angeles

1

# Nearest neighbors



Hash  
tables!



# Summary

- K-nearest neighbors, for closest matches
- Hash tables



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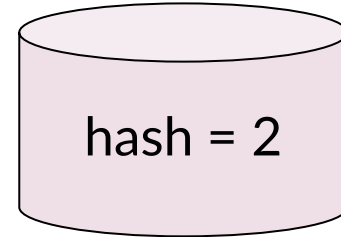
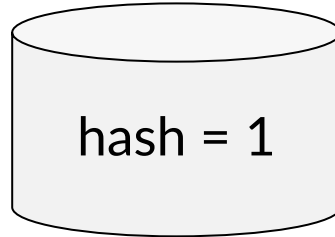
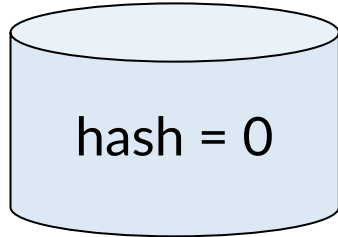
# Hash tables and hash functions

# Outline

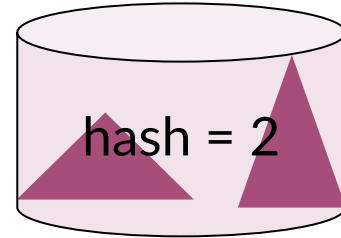
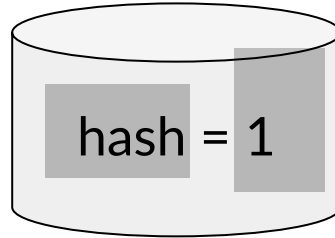
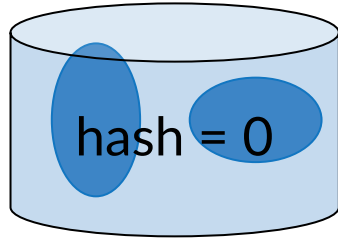
- Hash values
- Hash functions



# Hash tables



# Hash tables



# Hash function

|     |   |   |   |    |   |   |    |   |   |
|-----|---|---|---|----|---|---|----|---|---|
| 0   | 1 | 2 | 3 | 4  | 5 | 6 | 7  | 8 | 9 |
| 100 |   |   |   | 14 |   |   | 17 |   |   |
| 10  |   |   |   |    |   |   | 97 |   |   |

Hash function (vector)  $\longrightarrow$  Hash value

Hash value = vector % number of buckets

# Create a basic hash table

```
def basic_hash_table(value_l, n_buckets):  
    def hash_function(value_l, n_buckets):  
        return int(value_l) % n_buckets  
    hash_table = {i:[] for i in range(n_buckets)}  
    for value in value_l:  
        hash_value =  
hash_function(value, n_buckets)  
        return hash_table
```

# Hash function

|     |   |   |   |    |   |   |    |   |   |
|-----|---|---|---|----|---|---|----|---|---|
| 0   | 1 | 2 | 3 | 4  | 5 | 6 | 7  | 8 | 9 |
| 100 |   |   |   | 14 |   |   | 17 |   |   |
| 10  |   |   |   |    |   |   | 97 |   |   |

# Hash function by location?

|    |   |   |   |   |   |   |   |   |     |
|----|---|---|---|---|---|---|---|---|-----|
| 0  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9   |
| 14 |   |   |   |   |   |   |   |   | 100 |
| 10 |   |   |   |   |   |   |   |   | 97  |
| 17 |   |   |   |   |   |   |   |   |     |

Locality sensitive hashing, next!

# Summary

Hash function (vector)  $\longrightarrow$  Hash value



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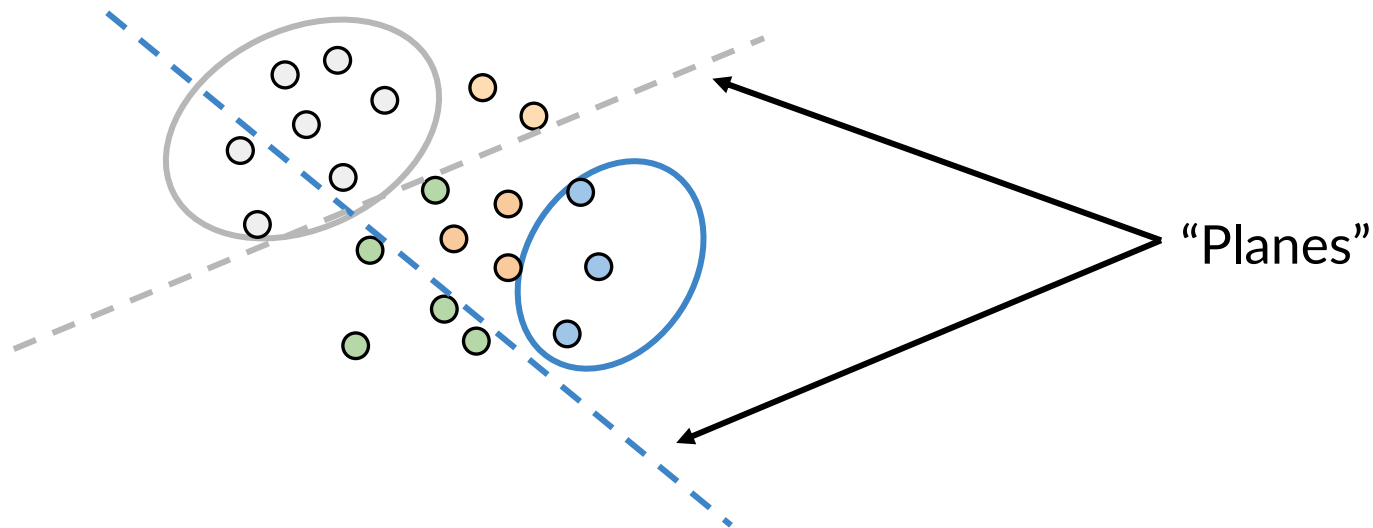
# Locality sensitive hashing



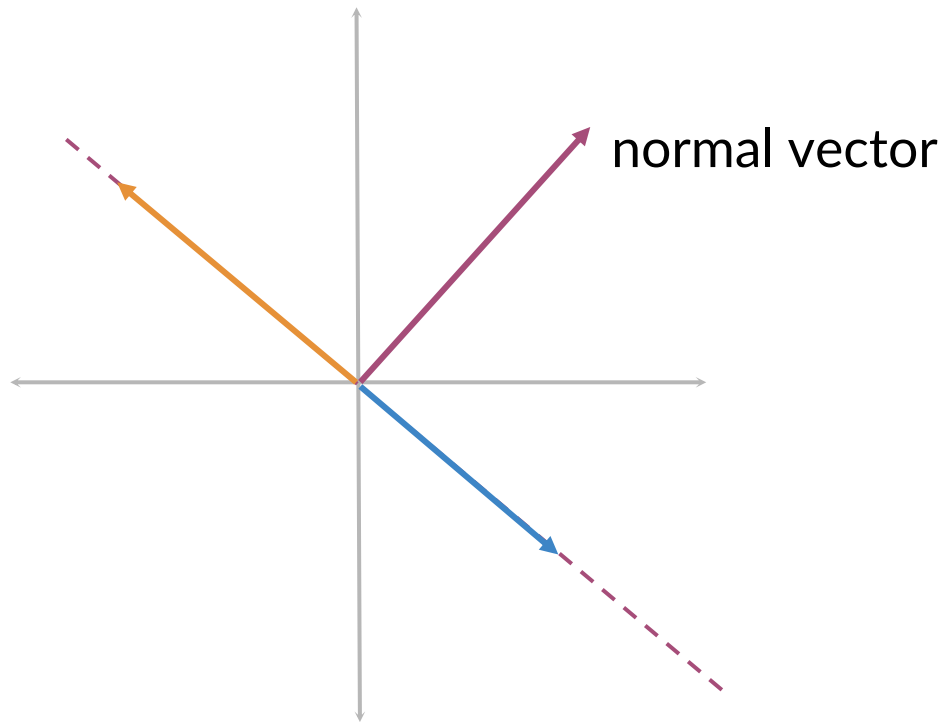
# Outline

- Locality sensitive hashing with planes in vector spaces

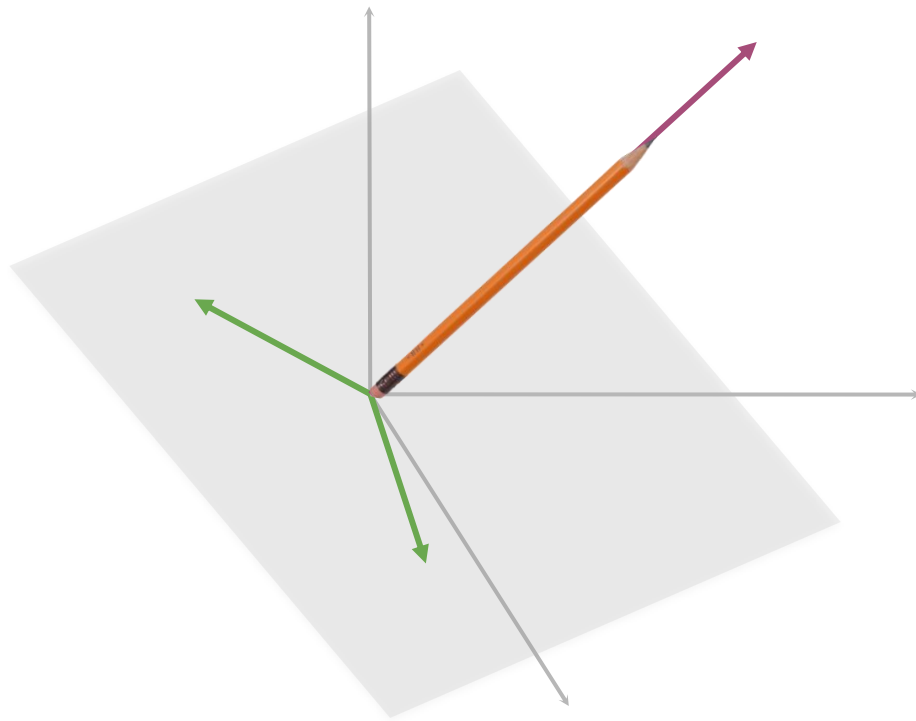
# Locality Sensitive Hashing



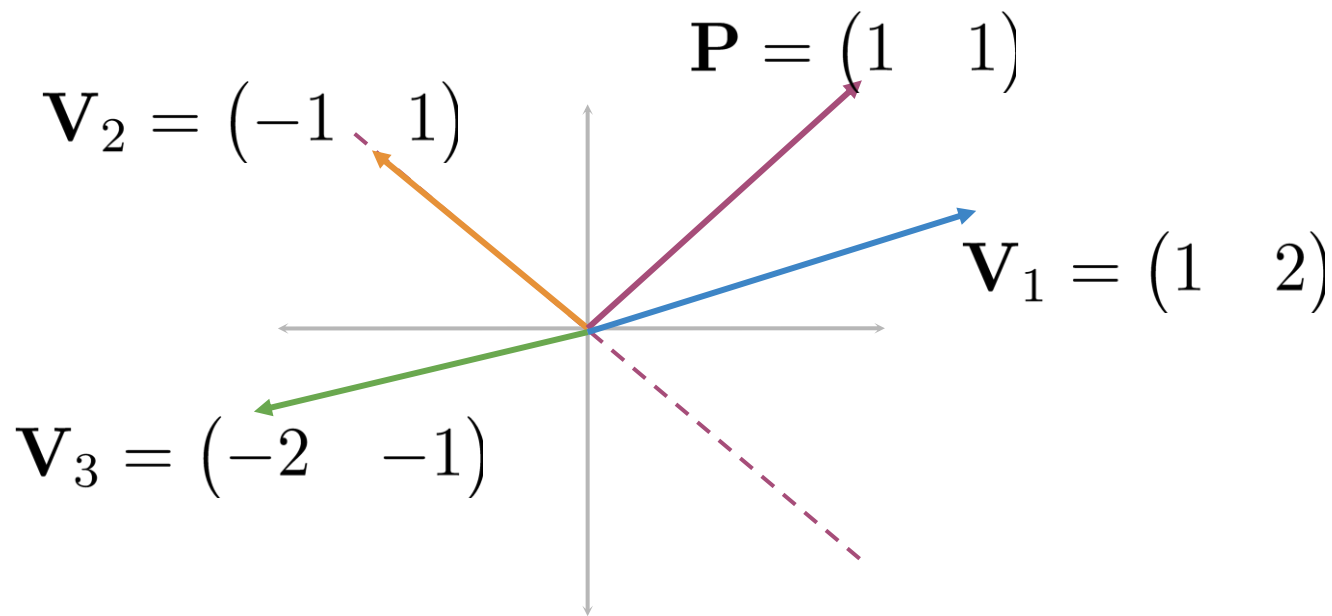
# Planes



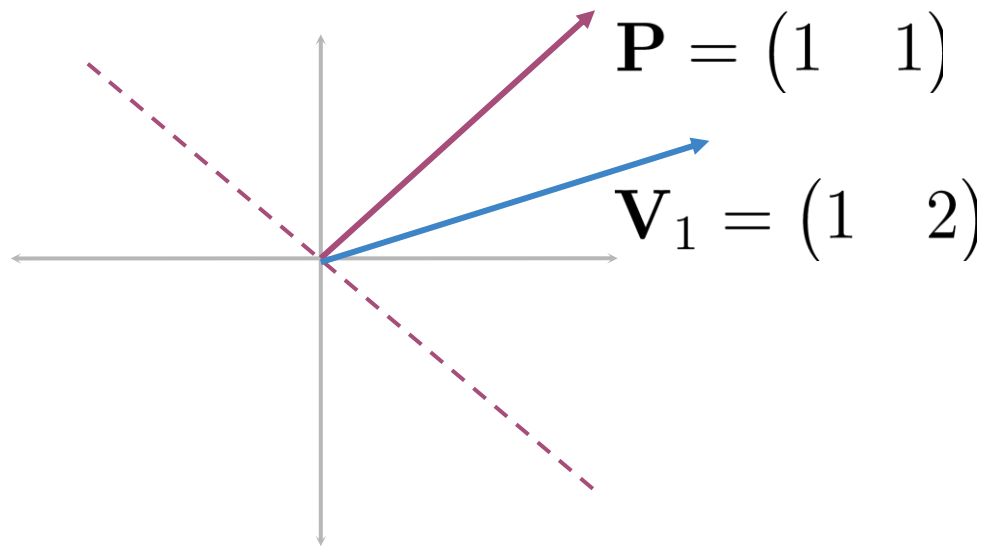
# Planes



Which side of the plane?



Which side of the plane?

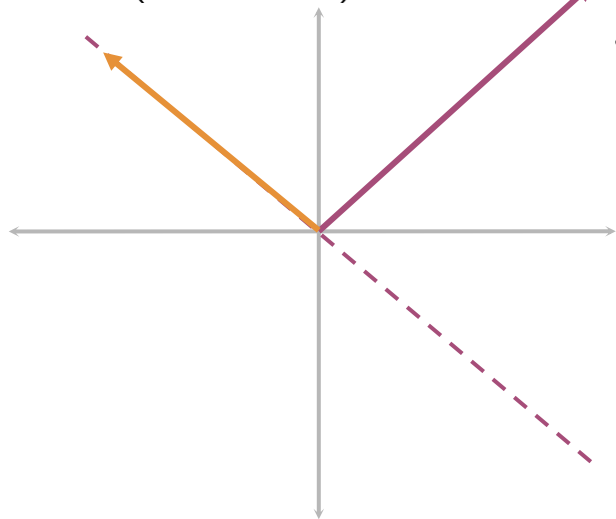


$$\mathbf{P}\mathbf{V}_1^T = 3$$

Which side of the plane?

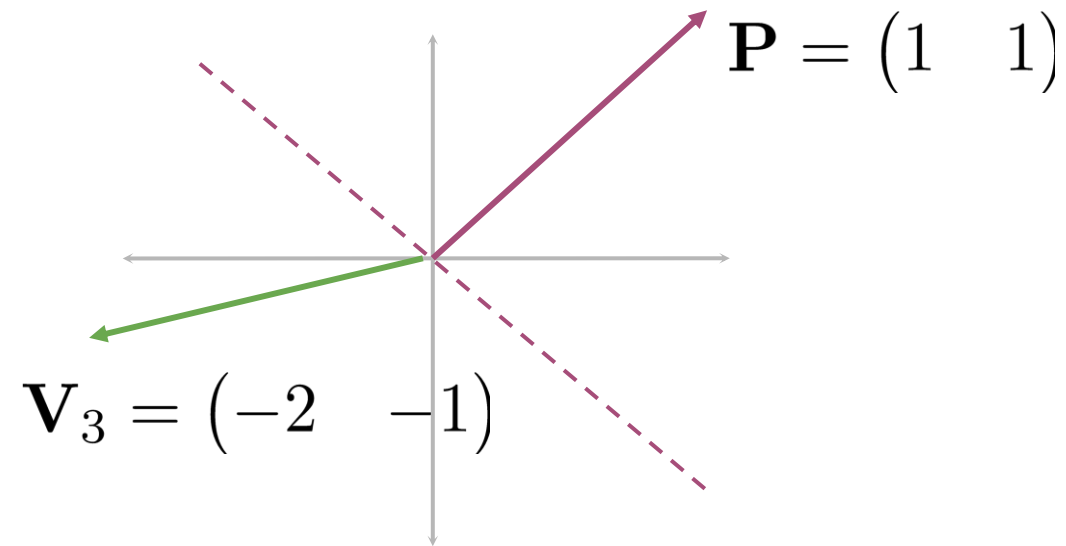
$$\mathbf{V}_2 = \begin{pmatrix} -1 & 1 \end{pmatrix}$$

$$\mathbf{P} = \begin{pmatrix} 1 & 1 \end{pmatrix}$$



$$\mathbf{P}\mathbf{V}_2^T = 0$$

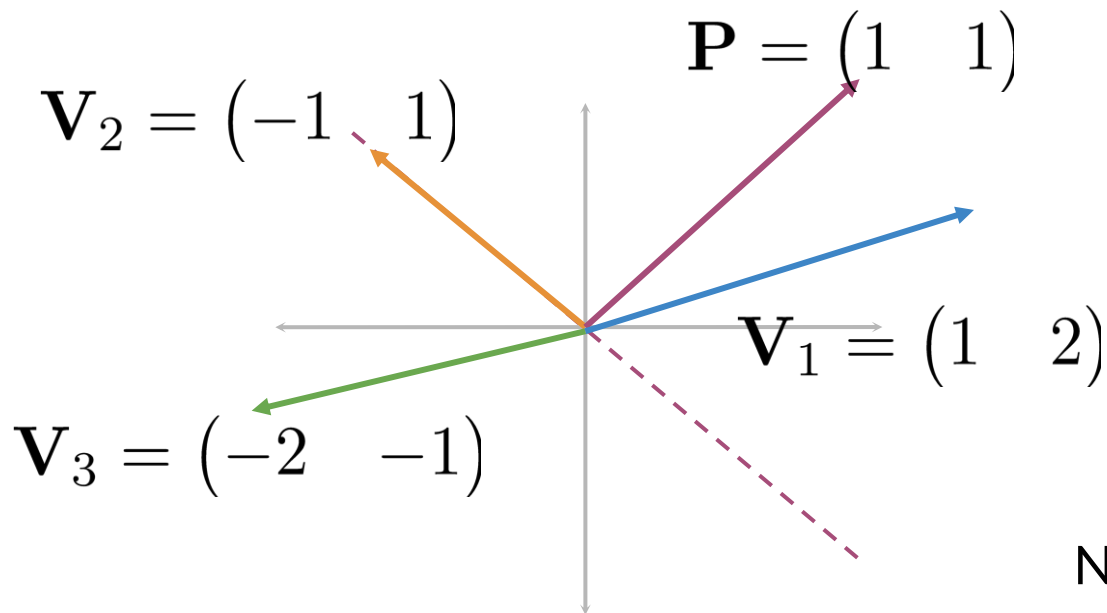
Which side of the plane?



$$\mathbf{P}\mathbf{V}_3^T = -3$$



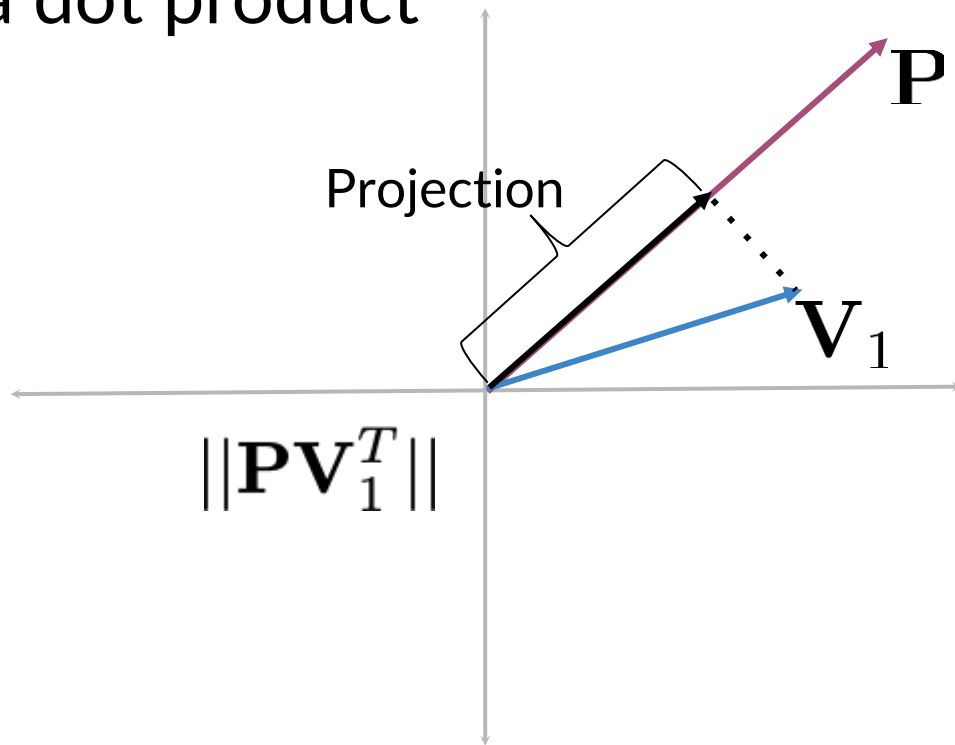
Which side of the plane?



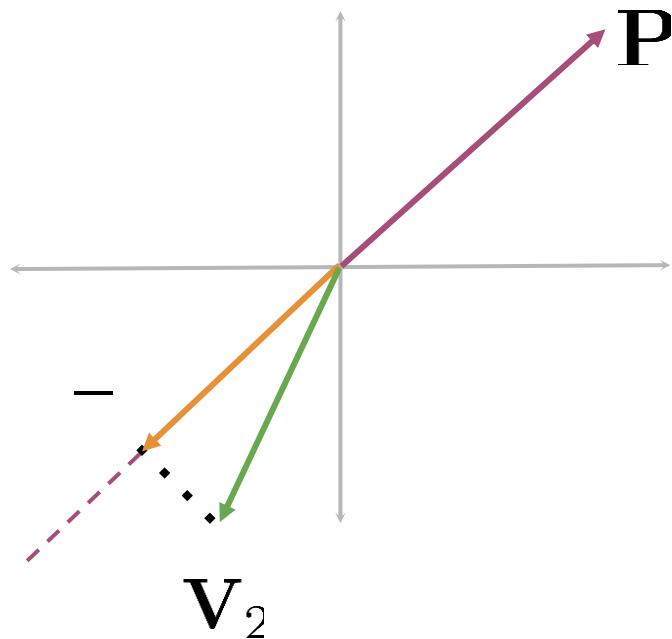
$$\begin{aligned}\mathbf{P}\mathbf{V}_1^T &= 3 \\ \mathbf{P}\mathbf{V}_2^T &= 0 \\ \mathbf{P}\mathbf{V}_3^T &= -3\end{aligned}$$

Notice the signs?

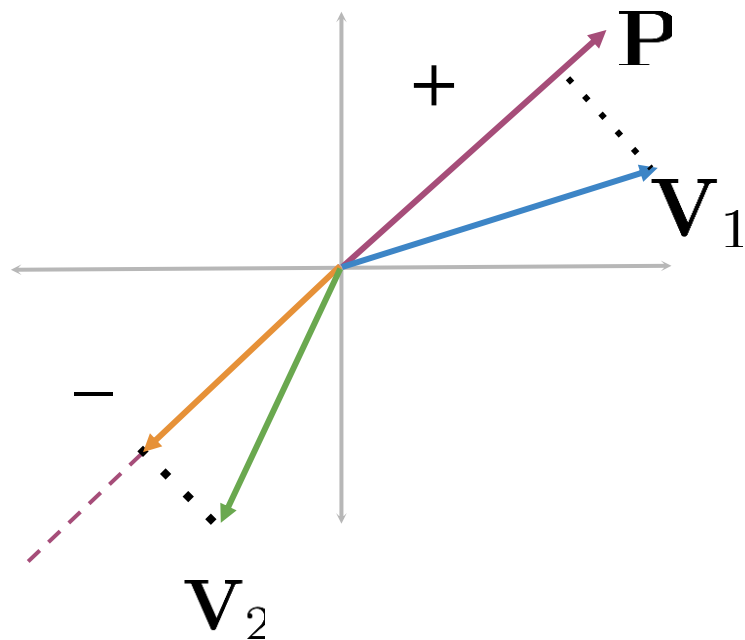
# Visualizing a dot product



# Visualizing a dot product



# Visualizing a dot product



Sign indicates direction

# Which side of the plane?

```
def side_of_plane(P,v):  
    dotproduct = np.dot(P,v.T)  
    sign_of_dot_product = np.sign(dotproduct)  
    sign_of_dot_product_scalar= np.asscalar(sign_of_dot_product)  
    return sign_of_dot_product_scalar
```

Try it!

# Summary

- Sign of dot product  $\longrightarrow$  Hash values



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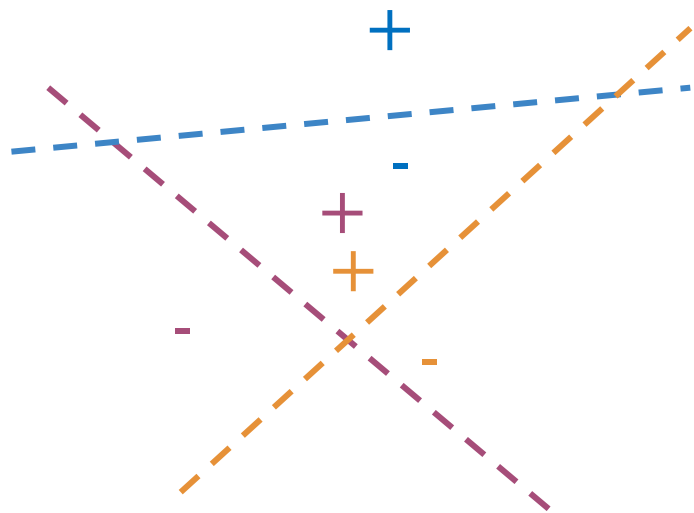
# Multiple Planes

# Outline

- Multiple planes  $\longrightarrow$  Dot products  $\longrightarrow$  Hash values

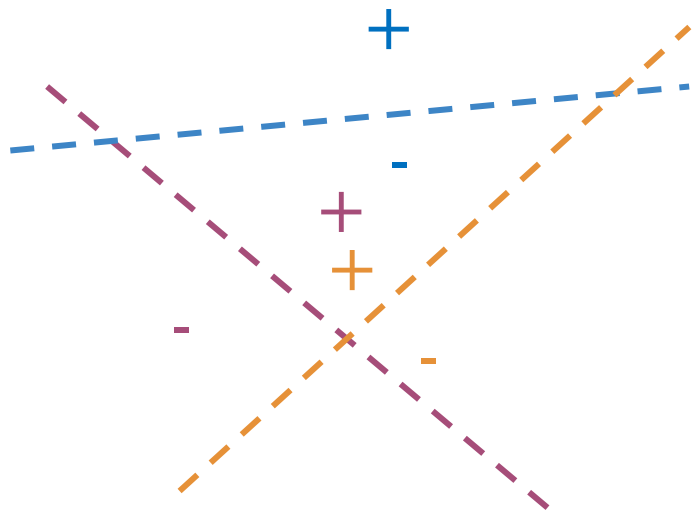


# Multiple planes



single hash value?

# Multiple planes, single hash value?



$$\mathbf{P}_1 \mathbf{v}^T = 3, \text{sign}_1 = +1, h_1 = 1$$

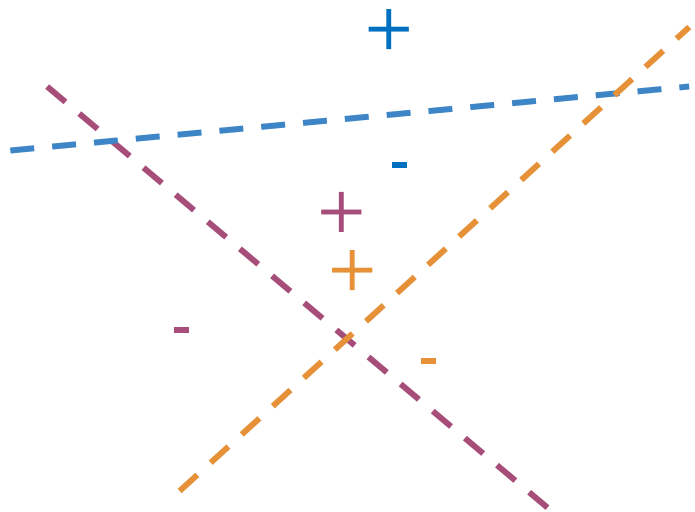
$$\mathbf{P}_2 \mathbf{v}^T = 5, \text{sign}_2 = +1, h_2 = 1$$

$$\mathbf{P}_3 \mathbf{v}^T = -2, \text{sign}_3 = -1, h_3 = 0$$

$$\begin{aligned} \text{hash} &= 2^0 \times h_1 + 2^1 \times h_2 + 2^2 \times h_3 \\ &= 1 \times 1 + 2 \times 1 + 4 \times 0 \end{aligned}$$

$$= 3$$

# Multiple planes, single hash value!



$$\text{sign}_i \geq 0, \rightarrow h_i = 1$$

$$\text{sign}_i < 0, \rightarrow h_i = 0$$

$$\text{hash} = \sum_i^H 2^i \times h_i$$

# Multiple planes, single hash value!!

```
def hash_multiple_plane(P_l,v):  
    hash_value = 0  
  
    for i, P in enumerate(P_l):  
        sign = side_of_plane(P,v)  
        hash_i = 1 if sign >=0 else 0  
        hash_value += 2**i * hash_i  
  
    return hash_value
```

Try it!

# Summary

- Planes  $\longrightarrow$  Sign of dot product  $\longrightarrow$  Hash values



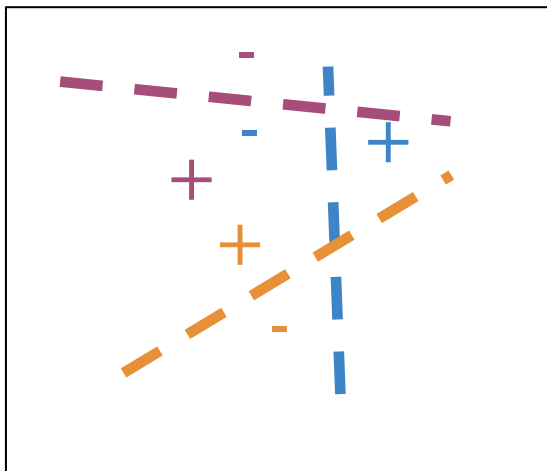
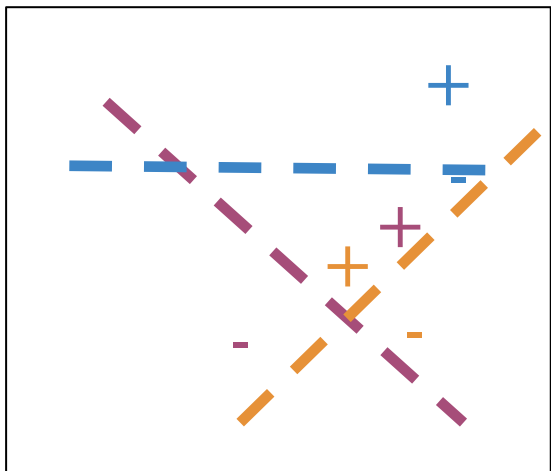
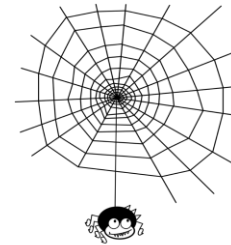
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# Approximate nearest neighbors

# Outline

- Multiple sets of planes for approximate K-nearest neighbors

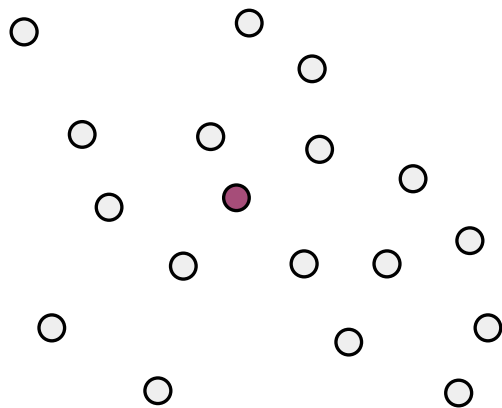
# Random planes



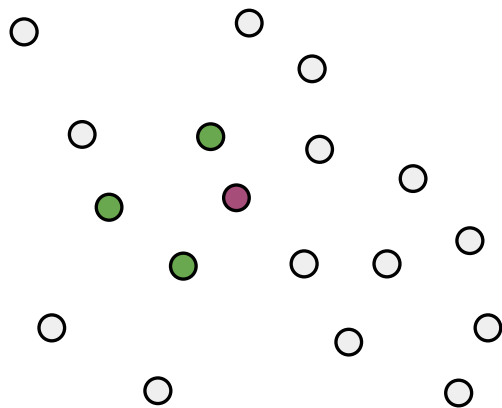
Cultural reference: Spider-Man: Into the Spider-Verse



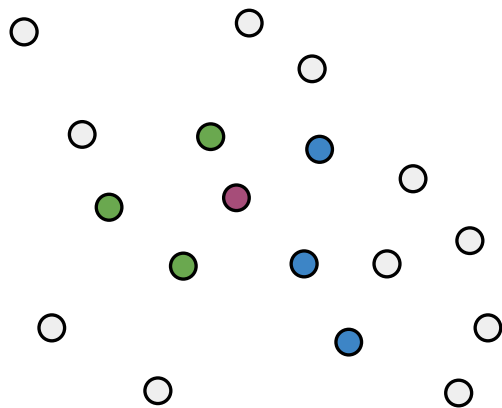
# Multiple sets of random planes



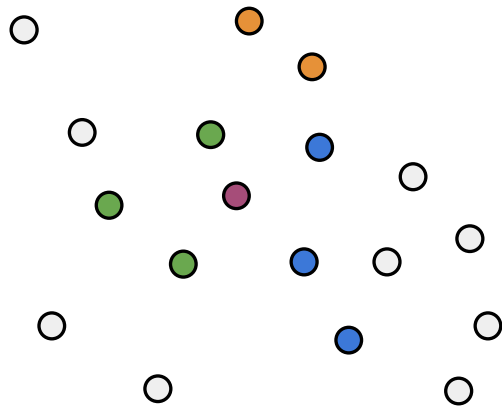
# Multiple sets of random planes



# Multiple sets of random planes



# Multiple sets of random planes



Approximate nearest  
(friendly) neighbors

# Make one set of random planes

```
num_dimensions = 2 #300 in assignment
num_planes = 3 #10 in assignment

random_planes_matrix = np.random.normal(
    size=(num_planes,
          num_dimensions))
```

```
array([[ 1.76405235  0.40015721]
       [ 0.97873798  2.2408932 ]
       [ 1.86755799 -0.97727788]])
```

```
v = np.array([[2,2]])
```

```
def side_of_plane_matrix(P,v):
    dotproduct = np.dot(P,v.T)
    sign_of_dot_product =
    np.sign(dotproduct)

    num_planes_matrix = side_of_plane_matrix(
        random_planes_matrix,v)
```

```
array([[1.]
       [1.]
       [1.]])
```

See notebook for calculating the hash value!

# Summary





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# Searching documents

# Outline

- Representation for documents
- Document search with K-nearest neighbors



# Document representation

I love learning!

[?, ?, ?]

I

[1, 0, 1]

love

+

[-1, 0, 1]

learning

+

[1, 0, 1]

I love learning!

=

[1, 0, 3]

Document Search

K-NN!

# Document vectors

```
word_embedding = {"I": np.array([1,0,1]),
                  "love": np.array([-1,0,1]),
                  "learning": np.array([1,0,1])}

words_in_document = ['I', 'love', 'learning']
document_embedding = np.array([0,0,0])

for word in words_in_document:
    document_embedding += word_embedding[word]

print(document_embedding)

array([1 0 3])
```

Try it!

# Revisit Learning Objectives

- Transform vector
- “K nearest neighbors”
- Hash tables
- Divide vector space into regions
- Locality sensitive hashing
- Approximated nearest neighbors

