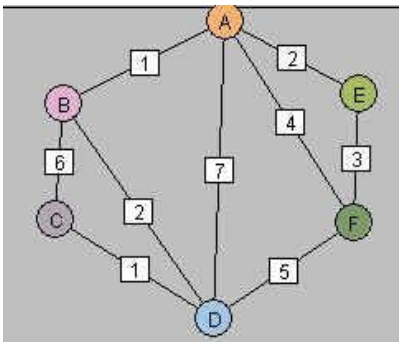
Lab 11B

1. Carry out the steps of Kruskal's algorithm for the following weighted graph, using the tree-based DisjointSets data structure to represent clusters. Keep track of edges as they are added to T and show the state of representing trees through each iteration of the main while loop.



2. Suppose G = (V, E) is an undirected (unweighted) simple graph. A subset U of V is called a *base* for G is every edge in G has at least one endpoint in U. Do the following:
    a. Given G = (V,E) is it true that V itself is a base for G? Explain
    b. Is there a graph G having a base that is the empty set? If so, give an example.
    c. Give an example of a graph having 8 vertices and having a base of size 1.
    d. Give an example of a graph G having 2n vertices with the property that every base for G has size at least n.
    e. Devise an algorithm to solve the Smallest Base Decision Problem: Given G = (V,E) and a nonnegative integer k, is there a base U for G having size $\leq$ k? What is the running time of your algorithm?

3. Suppose G = (V,E) is an undirected (unweighted) simple graph. A *spanning cycle* for G is a simple cycle in G that contains every vertex of G.
    a. Give an example of a connected graph having three or more vertices which has no spanning cycle.
    b. The Spanning Cycle Decision Problem is the following: Given a graph G = (V,E), does G contain a spanning cycle? Devise an algorithm to solve the Spanning Cycle Decision Problem. What is the running time of your algorithm? *Hints.* You may wish to use the following mathematical fact (which you should *prove* if you want to use it):
        A graph G is itself a simple cycle if and only if G is connected and every vertex in G has degree 2.