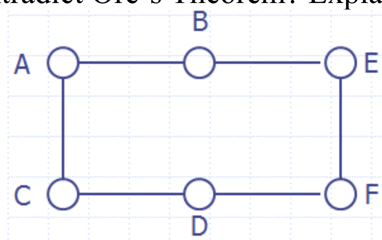
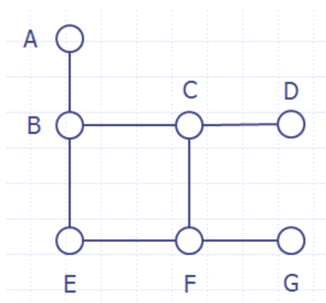
Lab 12A

1. Ore's Theorem implies that graphs with "many edges" tend to be Hamiltonian. Is it true that every dense graph is Hamiltonian? Prove your answer.

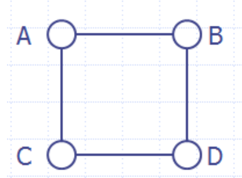
2. Answer the following questions about the graph G having n = 6 vertices, below.
   a. Is G Hamiltonian?
   b. Can you find two non-adjacent vertices the sum of whose degrees is less than 6?
   c. Do these facts contradict Ore's Theorem? Explain.



3. Consider the undirected graph G below.
   a. Is G bipartite? If so, exhibit the partition (X,Y) of the vertices, and re-draw the graph using this partition so that bipartiteness of the graph is obvious.
   b. Exhibit a maximum matching in G.
   c. Exhibit a minimum vertex cover for G.
   d. Is it true that the size of the matching is the same as the size of your vertex cover?



4. Illustrate the proof that the HamiltonianCycle problem is polynomial reducible to TSP by considering the following Hamiltonian graph—an instance of HamiltonianCycle—and transforming it to a TSP instance in polynomial time so that a solution to the HC problem yields a solution to the TSP problem, and conversely.



5. Show that TSP is NP-complete. (Hint: use the relationship between TSP and HamiltonianCycle discussed in the slides. You may assume that the HamiltonianCycle problem is NP-complete.)

6. Show that the worst case for VertexCoverApprox can happen by giving an example of a graph G which has these properties:
    a. G has a smallest vertex cover of size *s*
    b. VertexCoverApprox outputs size 2*s as its approximation to optimal size.