Champagne Method for Source Localization in MEG

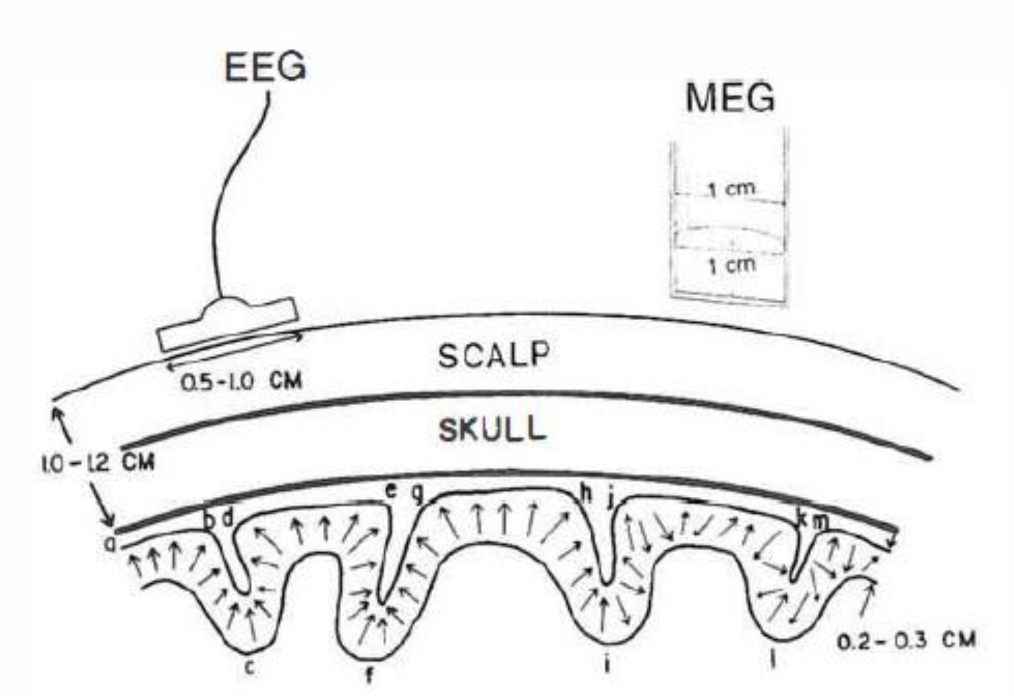
Content

- Forward Model & Inverse Problem
- Champagne: High-Level Overview
- Background: Bayesian inference
- Champagne algorithm
- Evaluation

Forward problem

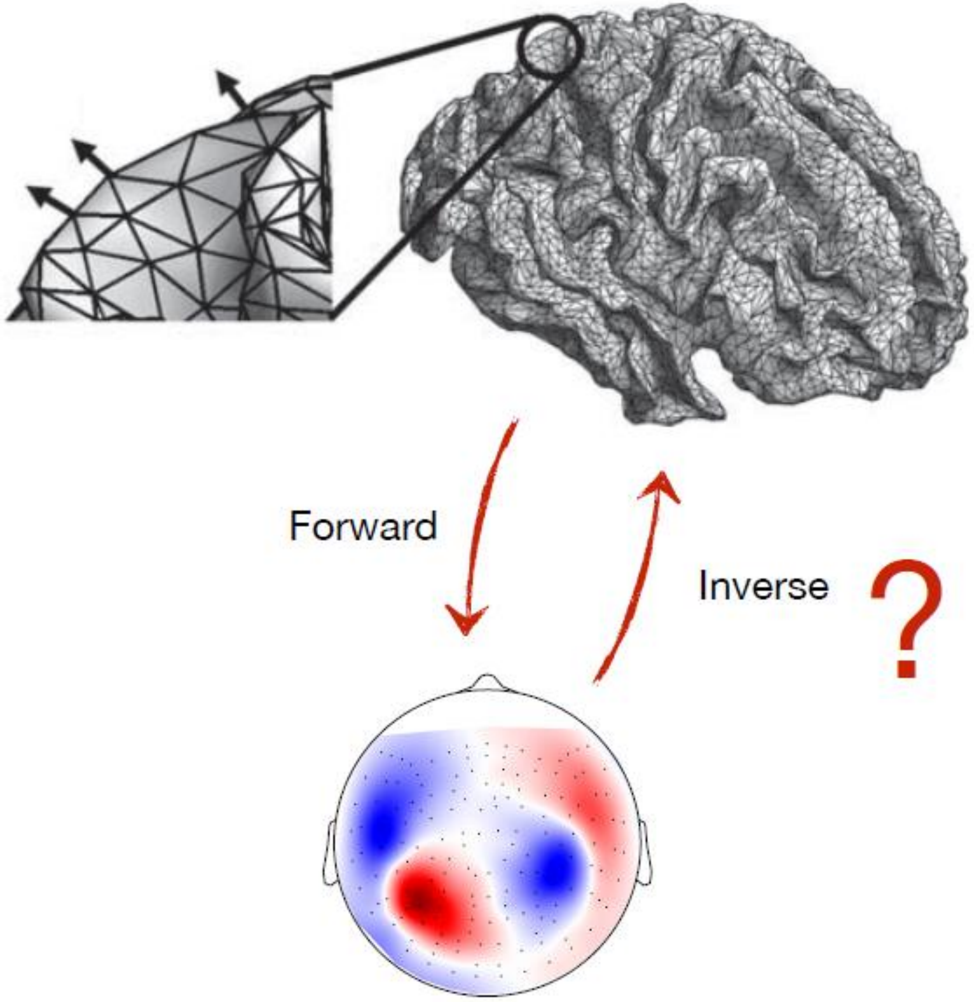
- M/EEG sensor response is calculated as linear function of source dipole current
- Effects from different dipoles combine linearly
  - B: M/EEG signal (n_sensors × n_times)
  - L: Lead-field matrix (n_sensor × n_sources)
  - S: Neural Sources (n_sources: d_s × n_times)
  - ε: Noise

$$B = \sum_{i=1}^{d_s} L_i S_i + \varepsilon$$



Inverse problem

- Sensor measurements with unknown source currents
- Ill-posed ($d_s \gg d_b$)
  - Massively underdetermined (~5000 dipoles,~150 magnetic sensors)
  - No unique solution
- Solved by adding constraints to the solution
  - Many different approaches
  - Currently there is no "right" or "best" solution



What is the Champagne Method?

- A Bayesian approach to solving the MEG inverse problem.
- Estimates the number, location, and time course of brain sources.

- Main advantages:
  - Handles correlated neural sources effectively
  - Suppresses noise and interference adaptively

Understanding Bayes' Theorem

- Bayes' Rule is a fundamental theorem in probability and statistics.
- It allows us to update our belief about an event based on new evidence.

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

P(A|B): Probability of event A occurring given event B (**posterior probability**).

P(B|A): Probability of event B occurring given event A (**likelihood**).

P(A): Initial probability of event A (**prior probability**).

P(B): Overall probability of event B (**marginal probability**): can be omitted because it remains constant for all values of A.

$$P(A|B) \propto P(B|A) \cdot P(A)$$

Probability Density Function (PDF)

- Gaussian distribution for multi-variates:

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{d/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T\Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu})\right)$$

- X is a vector of variables of size d
- μ is a mean vector
- Σ is covariance matrix

Champagne

- Assumes brain sources follow Gaussian distribution.
- Likelihood function:

$$p(B|S) \propto \exp\left(-\frac{1}{2}\left\|B - \sum_{i=1}^{d_s} L_i S_i\right\|_{\Sigma_e^{-1}}^2\right),$$

where $||X||_W$ denotes the weighted matrix norm $\sqrt{\text{trace}[X^T W X]}$.

- Goal is to find Si that maximize likelihood function.

Type II- Maximum likelihood

- Define hyperparameter Γ such that Γi is covariance of each Si.
- Marginal likelihood:

$$p(B|\Gamma) = \int p(B|S)p(S|\Gamma)dS \propto |\Sigma_b|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}B^T \Sigma_b^{-1} B\right),$$

Covariance matrix of estimated brain signals

$$\Sigma_b \triangleq \Sigma_\varepsilon + L\Gamma L^T.$$

- This is equivalent to minimizing the cost function:

$$\mathrm{trace}\left[C_b \Sigma_b^{-1}\right] + \log|\Sigma_b|$$

Covariance matrix of brain signals

How it works?

- Starts with initial covariance matrix for sources (prior probability)
  - For example: $\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$

- Estimates new covariance matrix that minimize cost function iteratively, until convergence.

- For example $\begin{matrix} 1.2 & 0.05 & 0.3 \\ 0.05 & 0.02 & 0.1 \\ 0.3 & 0.1 & 1 \end{matrix}$

Different versions of Champagne

Champ S: simple version by considering two constraints on the parameterization of Γ that lead to much less complex updates:

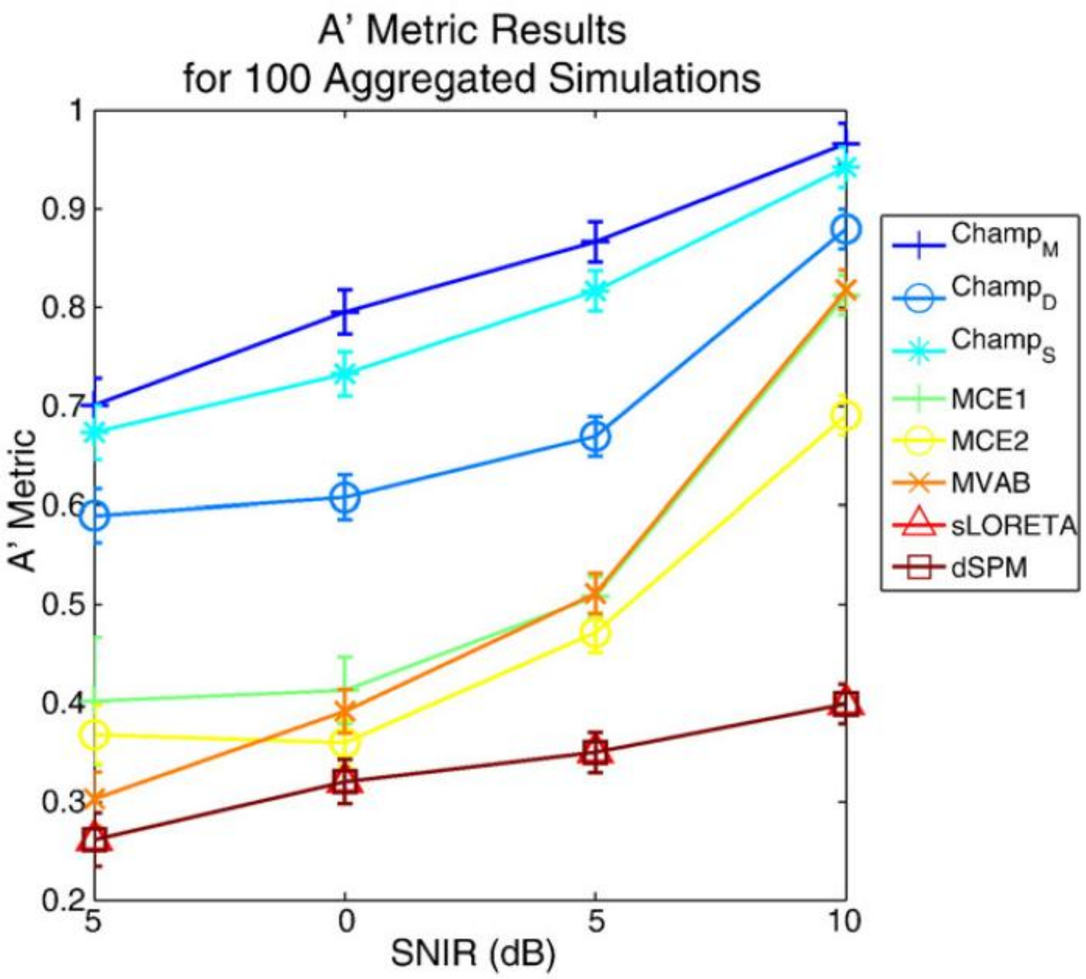
- Zero off-diagonal terms
- Equal diagonal terms

$$\begin{array}{ccc} \mathrm{CHAMP_S} & \mathrm{CHAMP_D} & \mathrm{CHAMP_M} \\ \Gamma_i = \begin{bmatrix} \gamma_i & 0 \\ 0 & \gamma_i \end{bmatrix} & \Gamma_i = \begin{bmatrix} \gamma_{i_1} & 0 \\ 0 & \gamma_{i_2} \end{bmatrix} & \Gamma_i = \begin{bmatrix} \gamma_{i_{11}} & \gamma_{i_{12}} \\ \gamma_{i_{12}} & \gamma_{i_{22}} \end{bmatrix}. \end{array}$$

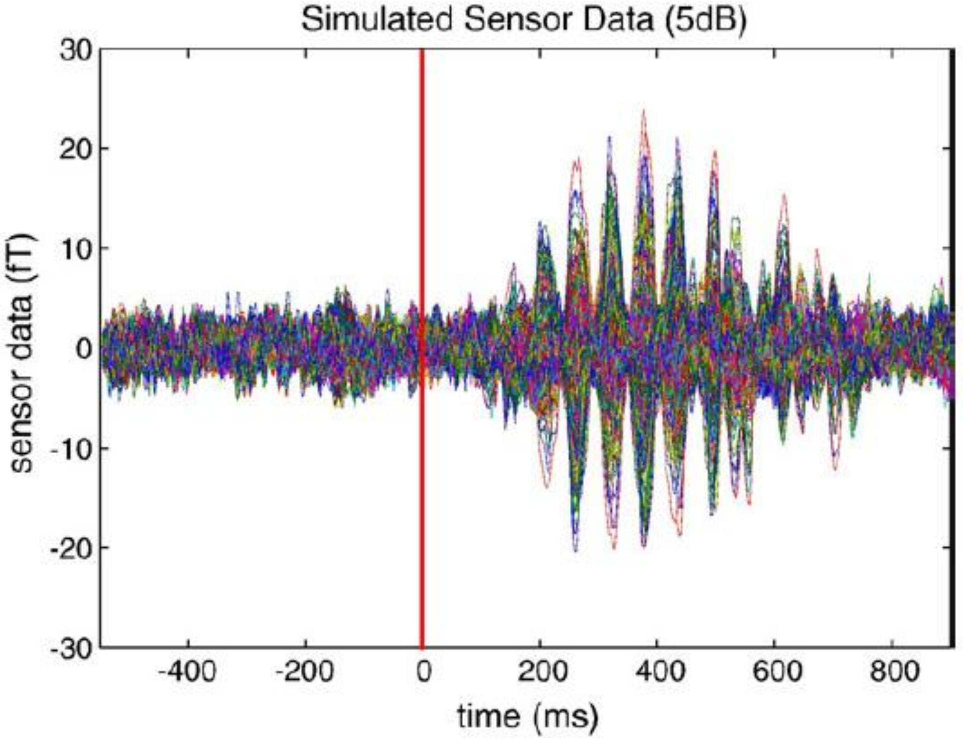
Evaluation

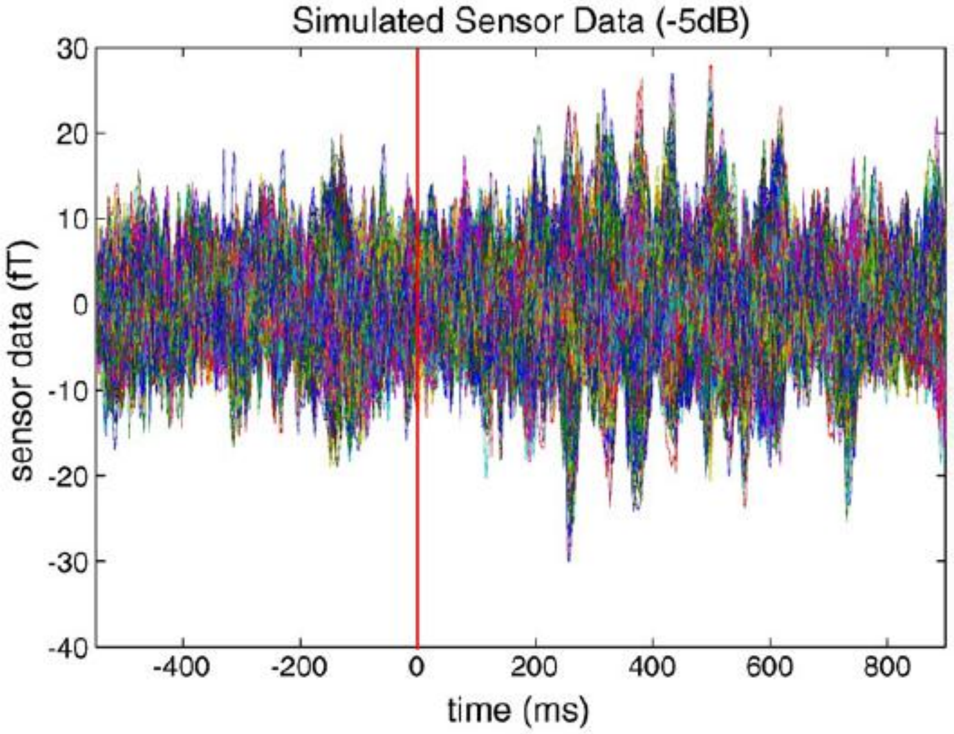
- Localization accuracy:

A' metric= hit/(hit+false positive)

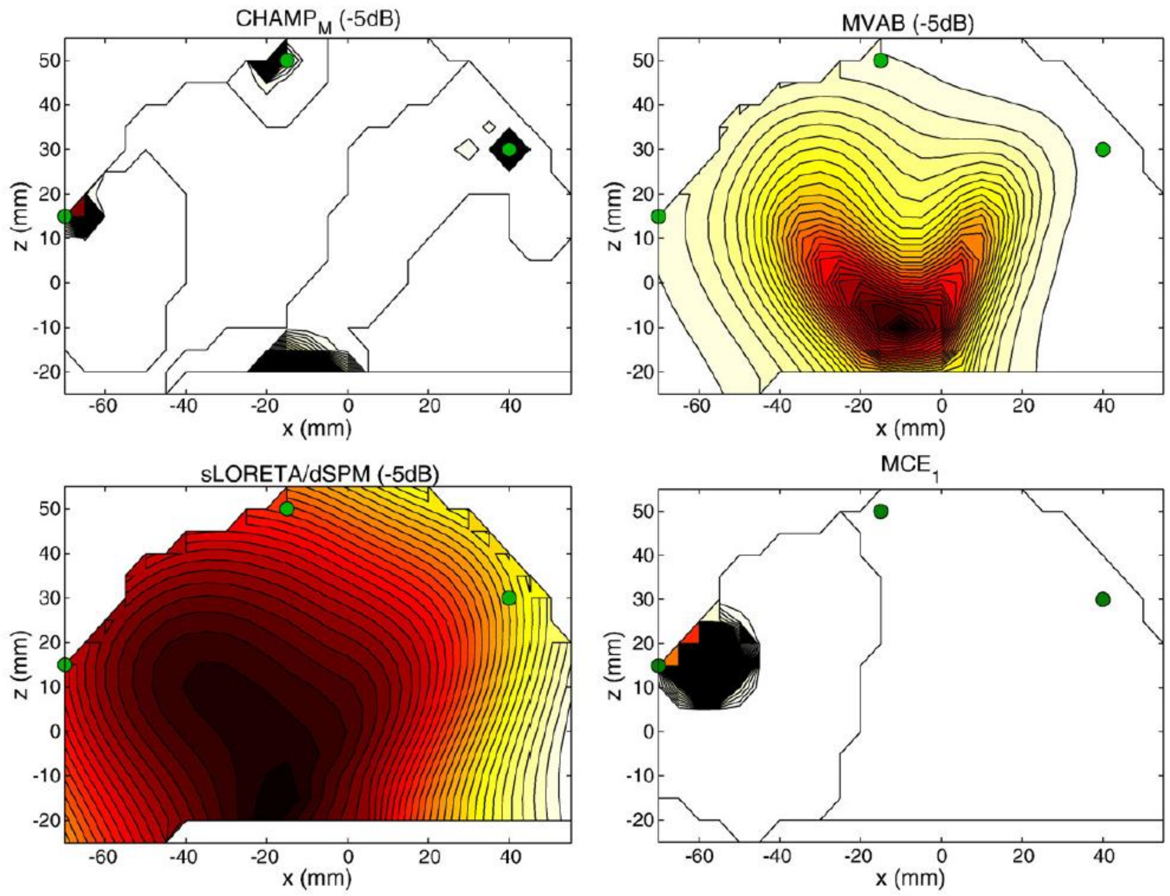


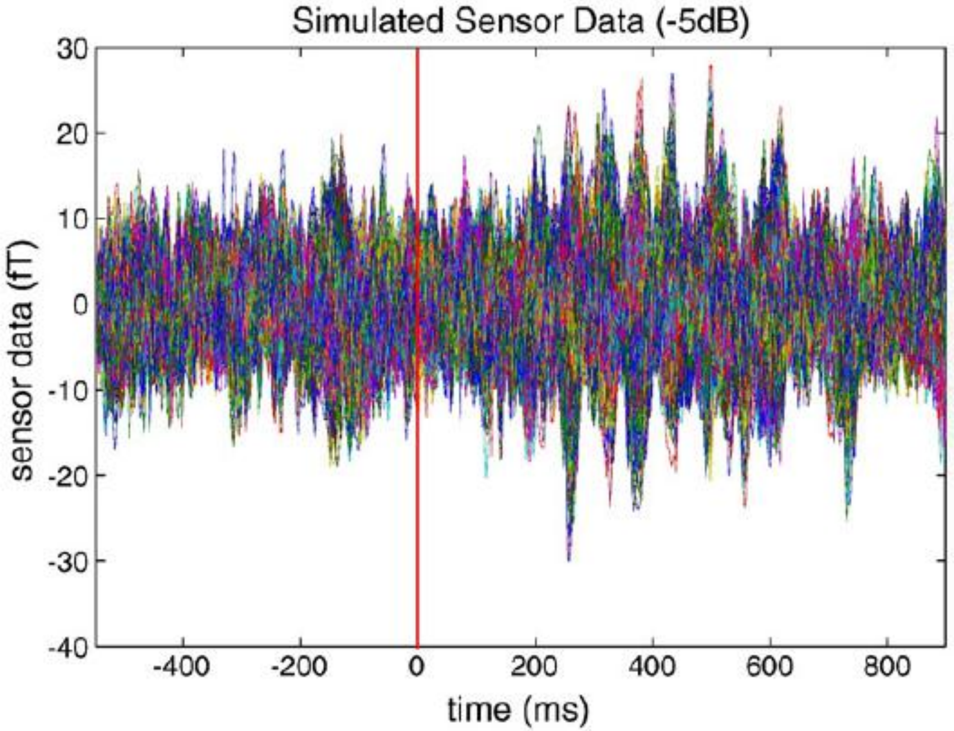
Robust to noise: SNIR=-5.0 dB





Robust to noise: SNIR=-5.0 dB





Conclusion and Discussion

- Computational Cost: it is quadratic in number of sensors, and cubic in number of sources , but these quantities are relatively small.

- Handles multiple correlated sources with unknown orientations

- Robust interference suppression