Lab 01: Units and Orders of Magnitude

Astronomy asks us to ponder structures and systems on scales way outside the realm of human experience. From itty-bitty atoms to massive clusters of galaxies, from the speed of light to the age of stars, the sheer extremity of the numbers we toss around sometimes allows them to lose their meaning. If I tell you that Jupiter has a mass of 1,898,000,000,000,000,000,000,000,000 kg, it probably won't mean much to you. However, a working understanding of numbers, units, and especially *orders of magnitude* is a vital tool of scientific literacy. The skills we will develop today allow us to rescue numbers from mathematical abstraction and root them in a familiar context.

1 A Review on Numbers and Units

Please answer in your lab notebooks, showing all work. No calculators, electronics, or asking your neighbor (just for now).

a. Write down the metric (or Système International, SI) prefixes and abbreviations for:

$$10^{-9} \qquad 10^{-6} \qquad 10^{-3} \qquad 10^{0} \qquad 10^{3} \qquad 10^{6} \qquad 10^{9}$$

b. Sketch a graph and plot the following data points:

$$(1.1, 300) \quad (11, 120) \quad (100, 30) \quad (900, 10)$$

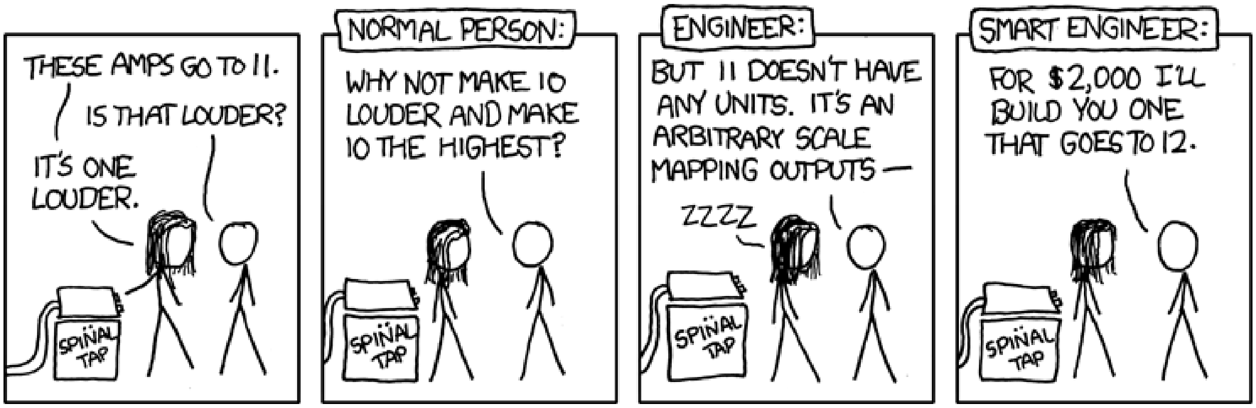
c. What is $\left(3 \times 10^{14}\ \mathrm{m}\right) \times \left(3 \times 10^{-8}\right) / \left(4.5 \times 10^{11}\right)$? Write your answer in scientific notation. Which SI prefix from above is closest to this value?

d. Astronomers (including you, now!) often use funny units because we must study phenomena on a wide range of scales. A Megaparsec (Mpc) is about the distance to the Andromeda Galaxy. A cubic Megaparsec is thus a good unit for, say, finding the density of galaxies in the universe.

What is 1 Mpc^3 in m^3? Note that 1 parsec = 3.1×10^{16} m.

e. A radio message is broadcast to Voyager 2, which is 2.2×10^{13} m from Earth (as of 2019 Jan 29). Radio waves travel at the speed of light, $c = 3.0 \times 10^{8}$ m/s. How many hours will it take for the message to reach Voyager?

2 Units and Equations



Source: xkcd.com/670

Centimeters, miles, kilograms, and years are examples of *units*. Every number we will deal with in this lab represents *something*, and almost all such numbers will require a unit. Class is not 3 long – it is 3 *hours* long. The Brooklyn Bridge is not 1.13 across – it is 1.3 *miles* across. For dinner, you did not eat 2 – you ate 2 *apples*. Whenever you report a measurement, you must include the unit.

2.1 Converting Units

Your units must agree in all calculations. You can't add centimeters and meters until you convert them to the same units. You can't say $X = Y$ if X is in kilograms and Y is in seconds.

To convert units, multiply by a fraction that's equal to 1: $\frac{1\,\text{year}}{365\,\text{days}}$, $\frac{12\,\text{inches}}{1\,\text{foot}}$, etc.

For example, to convert 1 year to seconds:

1 year

Or, to convert 5 acres to m^2 (1 acre = 66 ft × 660 ft, and 1 ft = 0.3048 m):

5 acre

As a general rule, you can treat units like multiplicative constants.

Convert the following, showing your conversion steps explicitly. Note 1 mi = 1.609 km.

- Typical human height: 5'9" (5 feet and 9 inches) → meters
- One marathon: 26.2 miles → kilometers
- A speed limit: 65 miles per hour → kilometers per second

2.2 Simplifying Units

Many quantities have units that are some combination of mass, length, and time, multiplied or divided together. For example, acceleration has units m/s^2, length over time squared. One convention is to write the units of a quantity like so:

$$[a] = \left[\frac{\mathrm{m}}{\mathrm{s}^2}\right]$$

Write down the units of each quantity below using only meters, kilograms, and seconds (MKS units). The right-side variables are m = mass, a = acceleration, c = speed of light, t = time, and r = distance.

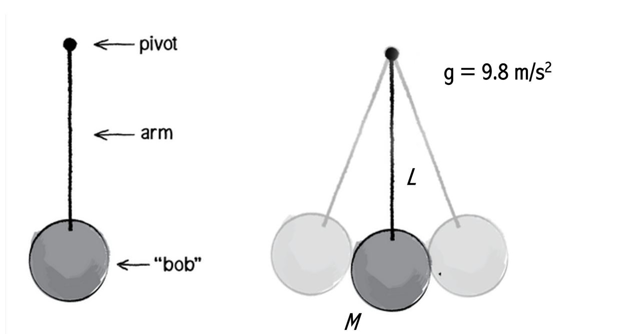
- Force: $F = ma$
- Energy: $E = mc^2$
- Power: $L = E/t$
- Flux: $F = L/(4\pi r^2)$

Some quantities have units that can't be reduced to just meters, kilograms, and seconds.

- Can you think of some examples? Give at least two.
- Can you think of, or make up, a quantity that has no units? Such a quantity is called *unitless* or *dimensionless*. Think of and define one such quantity, and write it down.

One type of value that won't have units is a ratio of like units: if someone is twice as old as you, we say they are "two times" older, and that two is "unitless". This is because if your age is 20 years and your friend's age is 40 years, the ratio of your ages is $\frac{40\,\text{years}}{20\,\text{years}} = 2$: the units cancel.

2.3 Detective Work with Units



Consider a pendulum of length L and mass M swinging in the Earth's surface gravitational field. The period T between successive swings can depend only on M, L, g. Which of the below options is the correct equation for pendulum's period? Explain. Hint: L is usually measured in metres (m) and M is usually measured in kilograms (kg). *Do not look up the formula online!*

$T = 2gM/L$	$T = 2\pi gM/L$
$T = 2\pi L^2/(Mg)$	$T = \pi g M^2/L^3$
$T = 2M\sqrt{g}$	$T = \sqrt{2Mg}$
$T = g\sqrt{L}$	$T = 2\pi L/M$
$T = 2\pi\sqrt{L/g}$	$T = 2\pi\sqrt{g/L}$

Suppose you are taking an exam in Prof. Appelgate's Earth, Moon, and Planets class. On the provided formula sheet you see two equations:

$$F = \frac{Gm_1m_2}{r^2}$$

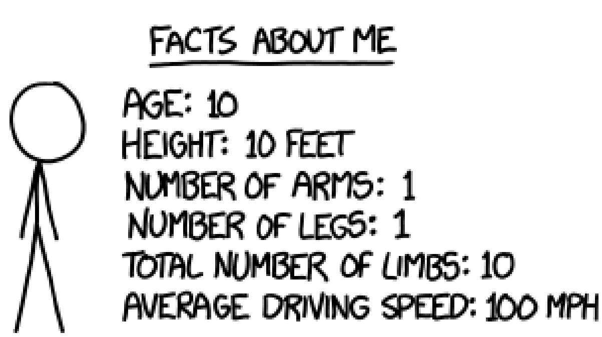
$$P = \frac{2\pi r^{3/2}}{(Gm)^{1/2}}$$

Pretend: you don't remember what these equations mean, but you do recall the following facts:

- F is a force
- r is a distance between two unspecified objects
- m_1, m_2, and m are masses of some unspecified objects
- G is Newton's gravitational constant, but you don't remember its value or units

What are the units of P and G?

3 Orders of Magnitude



Source: what-if.xkcd.com/84

The *order of magnitude* of a value is the "power of ten" that is closest to that value. To find the order of magnitude, write your value in scientific notation $m \times 10^n$ and round the coefficient m to either 1 or 10. You should end up with 10^x, where x is a positive or negative integer. For instance, 0.001, 1, and $1,000,000$ are orders of magnitude, though they are usually written in exponential form: 10^{-3}, 10^0, and 10^6.

Scientists use orders of magnitude when knowing the specific value of a number is not necessary or practical. If you want to compare the radii of the Sun and Earth, it often suffices to know if the Sun's radius is $0.01\times$, $100\times$, or $1,000,000\times$ the Earth's radius.

In this course, you will calculate many numbers. You should regularly ask yourself, "Is my order of magnitude reasonable?" For example, if you measure the height of Pupin Hall to be of order 10 cm, you'd immediately know that your answer is wrong.

Describe what an order of magnitude is, in your own words. Then determine the order of magnitude for the following quantities:

- 123456789.
- 0.00000768
- 994.
- 50 seconds.
- 373 m (Empire State Building's 102nd floor observatory height)

3.1 Powers of Ten

How good is your intuition for orders of magnitude? Place the following items on a nice big number line spanning 10^{-16} to 10^{24} meters. *Don't look anything up, though you may discuss with your peers! Give your best estimates.*

- Earth
- 1 light-second
- Proton
- Distance to the nearest star
- Milky Way
- Red blood cell
- Virgo Supercluster of galaxies
- Ant
- Sun
- Distance to Andromeda
- Human
- Length of Lake Michigan
- Distance to Pluto
- DNA nucleotide
- Moon
- Distance the Sun

Then we'll watch a short movie called "Powers of Ten": youtu.be/0fKBhvDjuy0
and a spoof thereof: youtu.be/FEuEx1jnt0M

- Compare your estimates to the actual sizes. How good were your estimates? For which objects was your intuition good, or not so good?
- How many Earths can you line up along the diameter of the sun? Give an answer to order-of-magnitude precision (so, an answer within $3\times$ or $1/3\times$ the correct value is good).

4 Conclusions

1. In general terms, when is the order-of-magnitude precision enough? When is it not? Give some examples and explain.
2. In the Simpsons spoof of "Powers of Ten", the universe was itself embedded within atoms of a greater universe. Do you think our universe could be embedded in another universe in the same way? If so, how could we tell?
3. What did you like or dislike about this lab? Any suggestions? Leave comments in your lab notebook.
4. What is one (or more!) remaining question(s) you have after completing this lab?

Lastly, if you liked thinking about orders-of-magnitude, you may also enjoy this visualization of what the solar system would look like if the Moon were 1 pixel wide:
joshworth.com/dev/pixelspace/pixelspace_solarsystem.html