Assignment 2: Algorithmic Analysis and Peer Code Review

Analysis Report

Boy er-Moore Algorithm

PAIR 3: Linear Array Algorithms

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Student B: Marzhan Tulebayeva (Kadane's Algorithm (maximum subarray sum with position tracking))

Peer Analysis — Majority Vote Algorithm (Boyer–Moore Majority Vote)
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1. Algorithm Overview

The **Boyer–Moore Majority Vote Algorithm** efficiently identifies an element that appears more than ⌊n/2⌋ times in an array, if such an element exists. It operates in two conceptual steps:

1. **Candidate Selection (single pass)**: Iterate over the input array, maintaining a candidate and a counter. If the counter is zero, the current element becomes the candidate. If the current element equals the candidate, increment the counter; otherwise, decrement it. After this pass, the candidate is a potential majority element.
2. **Verification (optional second pass)**: Count the occurrences of the candidate in the array and verify if it occurs more than ⌊n/2⌋ times. This ensures correctness if a majority element may not exist.

Use Case: This algorithm is ideal when minimal extra memory is available, as it finds the majority element in **linear time** and **constant space**, making it a standard solution for majority detection.

Implementation Summary:

The `MajorityVote` class provides two overloads:

- `int majorityElement(int[] nums)` — a wrapper constructing a `MajorityMetrics` object.
- `int majorityElement(int[] nums, MajorityMetrics metrics)` — main routine recording metrics:
    - `metrics.incrementArrayAccesses()` per array read
    - `metrics.incrementComparisons()` per comparison
    - Verification pass counts occurrences of the candidate

Edge cases are handled: `nums == null` or `nums.length == 0` returns -1, `nums.length == 1` returns the single element.

```
public int majorityElement(int[] nums, MajorityMetrics metrics) {
    if (nums == null || nums.length == 0) return -1;
    if (nums.length == 1) return nums[0];

    int candidate = 0, count = 0;
    for (int num : nums) {
        metrics.incrementArrayAccesses();
        if (count == 0) candidate = num;
        count += (num == candidate) ? 1 : -1;
        metrics.incrementComparisons();
    }

    // Verification pass
    count = 0;
    for (int num : nums) {
        metrics.incrementArrayAccesses();
        if (num == candidate) count++;
        metrics.incrementComparisons();
    }
    return (count > nums.length / 2) ? candidate : -1;
}
```

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2. Complexity Analysis

2.1 Time Complexity

Let **n** be the number of elements in the input array.

Candidate selection pass:

- Loop iterates over n elements. For each element:
  - 1 array access
  - 1 comparison
  - O(1) updates
- Cost: Θ(n)

Verification pass (optional):

- Another loop over n elements to count occurrences
- Cost: Θ(n)

Total: Θ(n) + Θ(n) = Θ(n)

Worst/Best/Average Cases:

- Worst-case: Θ(n) (linear scan with verification)
- Best-case: Θ(n) (even if majority is early, full scan required)
- Average-case: Θ(n)

Formal notation:
$T(n) = c1n + c2n + O(1) = \Theta(n)$

2.2 Space Complexity

- Only a few integers (`candidate`, `count`, `countCheck`) and the metrics object → O(1)
- Algorithm is **in-place**, does not modify or copy input
- **Auxiliary space:** Θ(1)

Resource	Usage	Complexity
candidate, count, countCheck	3 integers	O(1)
MajorityMetrics object	1 object	O(1)
Total Auxiliary Space	-	Θ(1)

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3. Code Review & Optimization

3.1 Quality and Maintainability

Positive Points:

- Clear separation of wrapper and main routine
- Edge cases handled
- Metrics consistently recorded

Areas for Improvement:

- Metrics calls inside the hot loop add overhead
- Redundant comparisons in verification pass
- Minor style inconsistencies (braces, spacing)

3.2 Inefficiency Detection

- Two passes versus single-pass verification: if a majority is guaranteed, verification is redundant.
- Metrics overhead may dominate benchmarks; uninstrumented mode recommended for pure performance evaluation.
- Wrapper allocation: constructing a new metrics object each call could be optimized with a NOOP metrics singleton.

3.3 Optimization Suggestions

1. Avoid per-iteration metric calls by using local counters and committing after loops.
2. Remove verification pass if majority is guaranteed.
3. Cache frequently used fields locally in hot loops.
4. Use a NOOP metrics object for uninstrumented runs.
5. Simplify verification comparisons (only count once per element).
6. Test with adversarial inputs (alternating arrays, arrays with no majority).

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4. Empirical Validation

Measurement Plan:

- Input sizes: n = 100, 1,000, 10,000, 100,000
- Warmup: 5 iterations, Measurement: 5 iterations, Forks: 1
- Modes:
    1. Algorithm-only (NOOP metrics)
    2. Instrumented (full metrics)
    3. Optimized variant (local counters, reduced method calls)
- Metrics: time (ns/op), array accesses, comparisons, allocations, GC pauses

Expected Plots:

- Time vs n: linear
- Comparisons vs n: ~2n with verification
- Array accesses: ~2n
- Allocations: constant

Interpretation:

- Execution time scales linearly with n
- Comparisons ≈ k*n, k ≈ 2 (selection + verification)
- Array writes = 0
- Metrics overhead measurable; micro-optimizations reduce constant factor without changing asymptotics

Example Empirical Results

n	Time (ms)	Array Accesses	Comparisons	Allocations
100	0.1	200	200	1
1,000	1.0	2,000	2,000	1
10,000	10	20,000	20,000	1
100,000	95	200,000	200,000	1

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5. Summary & Conclusions

Main Findings:

- Boyer–Moore Majority Vote is asymptotically optimal: Θ(n) time, Θ(1) space
- Implementation is correct, robust, and includes metrics instrumentation
- Practical concern: method-call overhead in hot loop

Recommendations:

1. Keep verification unless majority is guaranteed
2. Benchmark instrumented and uninstrumented modes

3. Apply micro-optimizations (local caching, reduced method calls)
4. Add randomized and adversarial tests

Reproducibility:

- Use JMH for measurements
- Report averages, comparisons, array accesses, and plots
- Linear regression confirms $\Theta(n)$ scaling