Probabilistic Robotics Course

Discrete Filtering: Localization

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Exercise

What if grid-orazio has also an *orientation* and its available controls change to:

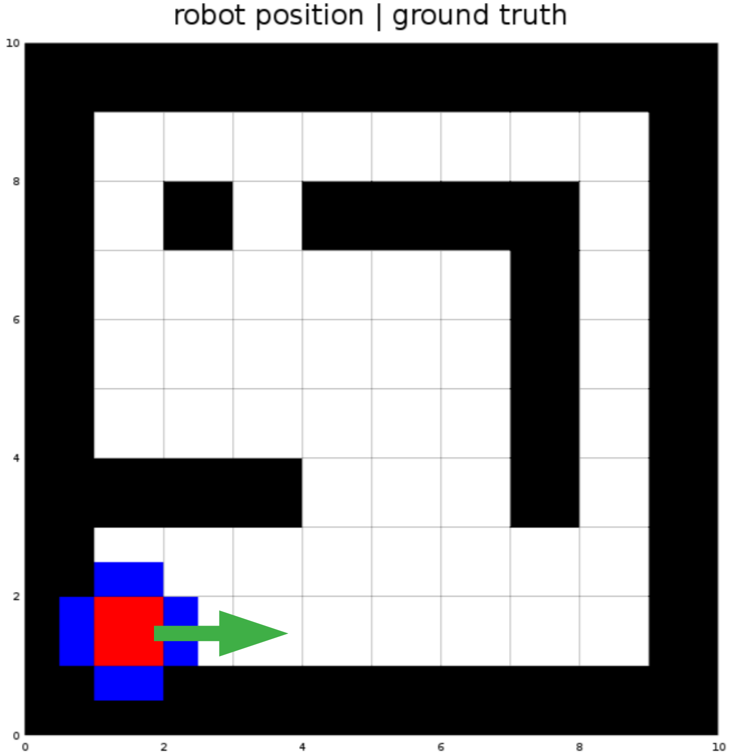
- MOVE_FORWARD
- MOVE_BACKWARD
- ROTATE_LEFT
- ROTATE_RIGHT

How does the state change?

What about the observation and transition model?

State representation

- The state of the robot is 3-dimensional
  - A) Position in the 2D grid (row, col)
  - B) Orientation which can be $0, \pi, \pi/2, -\pi$



State representation

- The state is 3-dimensional
- The belief state is a cube

```
#initialize robot states: position belief values over the complete grid
number_of_free_cells =  sum(sum(map < 0.01));
belief_initial_value = 1/(number_of_free_cells*THETA_VALUES);
global state_belief = zeros(rows(map), columns(map), THETA_VALUES);
for (theta = 1:THETA_VALUES)
  state_belief(:,:,theta) = (~map)*belief_initial_value;
endfor
global observations  = [0 0 0 0];
```

- Map orientation to indices of the 3rd dimension, CCW and CW rotations become

```
theta_to = mod(theta_from_, THETA_VALUES) + 1;

theta_to = mod(theta_from_ - 2, THETA_VALUES) + 1;
```

State representation (drawBelief.m)

- Marginalize out the orientation to obtain a 2D matrix
- In this way you can visualize the probability of being in a cell independently of the orientation

```
global THETA_VALUES
num_rows = rows(map_);
num_cols = columns(map_);
state_belief_2D = zeros(num_rows, num_cols);
#state_belief_2D
for (row = 1:num_rows)
    for (col = 1:num_cols)
        for (theta = 1:THETA_VALUES)
            state_belief_2D(row, col) += state_belief_(row, col, theta);
        endfor
    endfor
endfor
#invert belief value for plotting (0:black: 100% confidence, 1:white: 0% confidence)
plotted_state_belief_ = flipud(ones(size(state_belief_2D)) - state_belief_2D);
```

State representation (getNextState.m)

- In all possible values of the next state consider also the next possible orientation

```
#available motion range
min_row = state_ground_truth_(1)-1; #MOVE_UP
max_row = state_ground_truth_(1)+1; #MOVE_DOWN
min_col = state_ground_truth_(2)-1; #MOVE_LEFT
max_col = state_ground_truth_(2)+1; #MOVE_RIGHT
#over for the available motion range check if probability is higher than the extracted sample
for (i=1:THETA_VALUES)
  for (row = min_row:max_row)
    for (col = min_col:max_col)
      cumulative_probability += transition_probability(row, col,i);
      if(cumulative_probability > minimum_probability)
        #return with new position
        state_ground_truth = [row, col, i];
        return;
      endif
    endfor
  endfor
endfor
```

- Use of cumulative distribution to get random sample from prob. distribution (see PF slides)

A) Transition Model (transitionModel.m)

How to move grid-orazio? We need to implement a function in the form:

```
function transition_probability_matrix = transitionModel(map_, row_from_, col_from_, theta_from_, control_input_)
```

that given:

- a start state [row_from_, col_from_, theta_from_]
- a control input

```
#available robot controls (corresponding to keyboard key values)
global MOVE_FORWARD;
global MOVE_BACKWARD;
global ROTATE_LEFT;
global ROTATE_RIGHT;
```

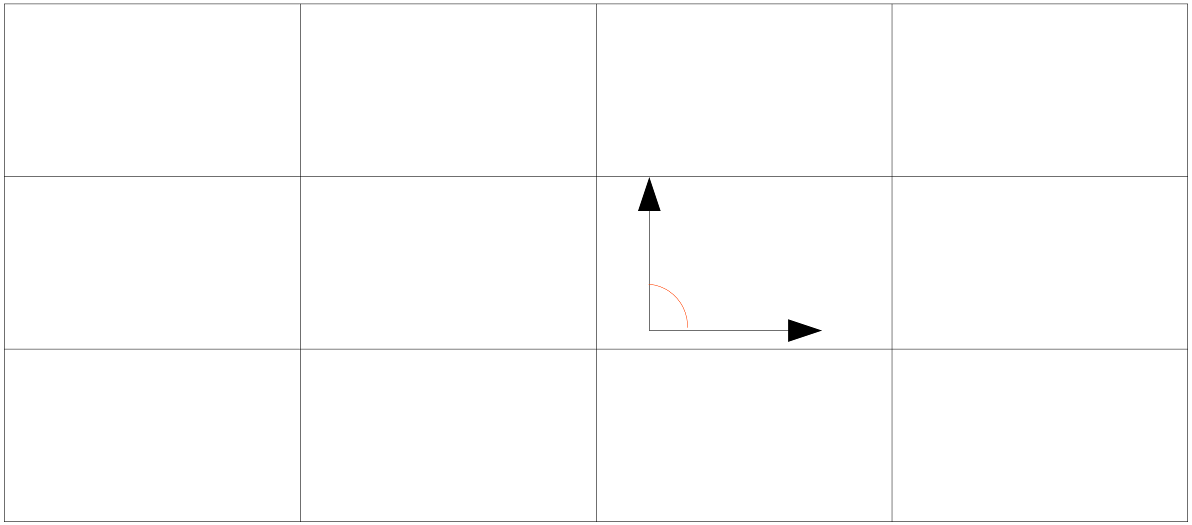
returns the probability of moving to any cell with any orientation in the map from the start state

A1) Transition Model: Without noise

We assume that the controls we issue to grid-orazio, have a deterministic effect (**no noise**).

- To a control ROTATE_LEFT, the robot will respond by rotating clockwise by $\pi/2$ with probability 1.0.

ROTATE_LEFT:

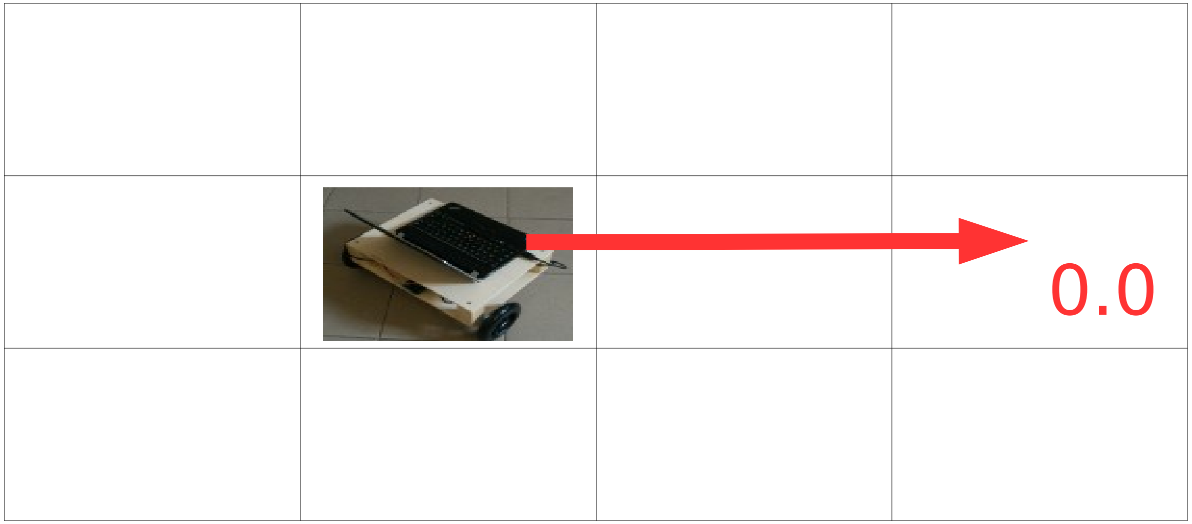


The behavior is symmetric for ROTATE_RIGHT

A1) Transition Model: Motion constraint

The robot can move only to adjacent cells, for farther cells the transition probability becomes 0.0.

MOVE_RIGHT:

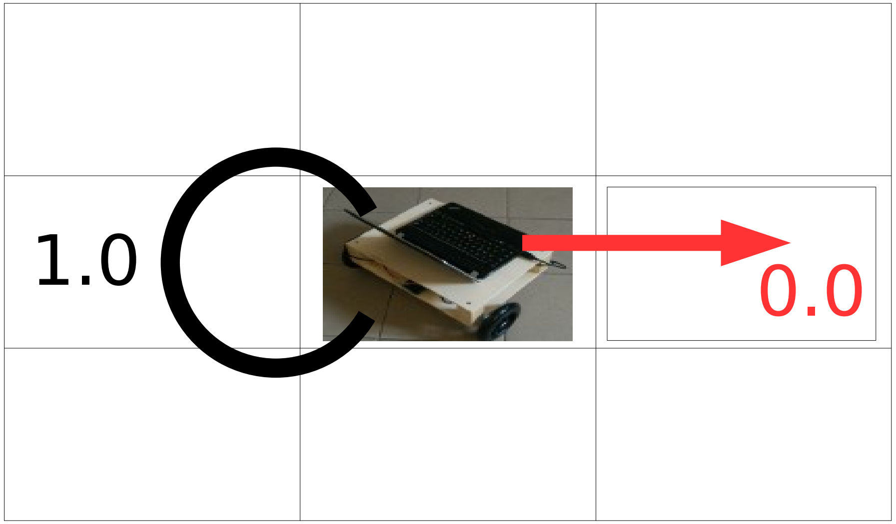


The behavior is symmetric for MOVE_LEFT

A1) Transition Model: Motion feasibility

If the target cell (**noise free** transition) is occupied, the robot will stay where it is with probability 1.0.

MOVE_RIGHT:



The behavior is symmetric for all 4 controls

A1) Transition Model: Motion constraint

Loop over the rows and columns of the map

The robot can move only to adjacent cells:

```
#compute resulting position difference
translation_rows = row_to - row_from_;
translation_cols = col_to - col_from_;

#allow only unit motions (1 cell): check if we have a bigger motion
if(abs(translation_rows) > 1 || abs(translation_cols) > 1)
  continue;
endif
```

If the two cells are farther away than 1, the transition probability remains 0.

A1) Transition Model: Next state

Retrieve the *noise free* next state based on the control input:

```
case MOVE_FORWARD
  switch theta_from_
    case 1
      # move right
      target_col++;
    case 2
      # move up
      target_row--;
    case 3
      # move left
      target_col--;
    case 4
      # move down
      target_row++;
    otherwise
      disp("Not a known angle")
  endswitch
```

```
case MOVE_BACKWARD
  switch theta_from_
    case 1
      # move left
      target_col--;
    case 2
      # move down
      target_row++;
    case 3
      # move right
      target_col++;
    case 4
      # move up
      target_row--;
    otherwise
      disp("Not a known angle")
  endswitch
```

A1) Transition Model: Motion feasibility

We have to check if the next state is feasible on our map (i.e. the cell is not occupied and we're not going over the border):

```
#check if the desired motion is infeasible
invalid_motion = false;
if (target_row < 1 || target_row > map_rows || target_col < 1 || target_col > map_cols) #if we're going over the border
  invalid_motion = true;
elseif (map_(target_row, target_col) == 1 || map_(row_to, col_to) == 1) #obstacle in the goal cell
  invalid_motion = true;
endif
if (invalid_motion)

  #if the desired translation is zero
  if (translation_rows == 0 && translation_cols == 0)
    transition_probability_matrix(row_to, col_to) = 1; #we stay with 100% probability (no motion has full confidence)
    continue;
  else
    continue; #we cannot move
  endif
endif
```

A1) Transition Model:

MOVE_FORWARD/BACKWARD

Set the probability of moving to a cell depending on the control input:

```
switch theta_from_
    case 1
      # move right
      if (translation_rows     ==  0 && translation_cols ==  1)
        transition_probability_matrix(row_to, col_to, theta_from_) = 1;
      endif
    case 2
      # move up
      if (translation_rows     ==  -1 && translation_cols ==  0)
        transition_probability_matrix(row_to, col_to, theta_from_) = 1;
      endif
    case 3
      # move left
      if (translation_rows     ==  0 && translation_cols ==  -1)
        transition_probability_matrix(row_to, col_to, theta_from_) = 1;
      endif
    case 4
      # move down
      if (translation_rows     ==  1 && translation_cols ==  0)
        transition_probability_matrix(row_to, col_to, theta_from_) = 1;
      endif
    otherwise
      disp("Not a known angle")
endswitch
```

Analogous for MOVE_BACKWARD

A1) Transition Model:

ROTATE_LEFT/RIGHT

- The robot cannot translate and rotate at the same time
- When it rotates, it can only remain in the same state with different orientation
- Modulo to always get admissible angle

```
case ROTATE_LEFT
  # theta_to = theta_from_ + pi/2;
  theta_to = mod(theta_from_, THETA_VALUES) + 1;
  if (translation_rows     ==  0 && translation_cols == 0)
    transition_probability_matrix(row_to, col_to, theta_to) = 1.0;
  endif
case ROTATE_RIGHT
  theta_to = mod(theta_from_ - 2, THETA_VALUES) + 1;
  if (translation_rows     ==  0 && translation_cols == 0)
    transition_probability_matrix(row_to, col_to, theta_to) = 1.0;
  endif
```

B) Observation Model (observationModel.m)

To retrieve the 4 observations around grid-orazio we use our observation model

```
current_probability = observationModel(map_,
                                       state_ground_truth_(1),
                                       state_ground_truth_(2),
                                       current_observations);
```

that given:

- a start state
- a observation sample (4 values)

returns the probability of observing the current observation sample

B) Observation Model: Modeling the Bumper

Given the location, each of the 4 bumpers is independent

A bumper gives a wrong measurement with probability 0.2

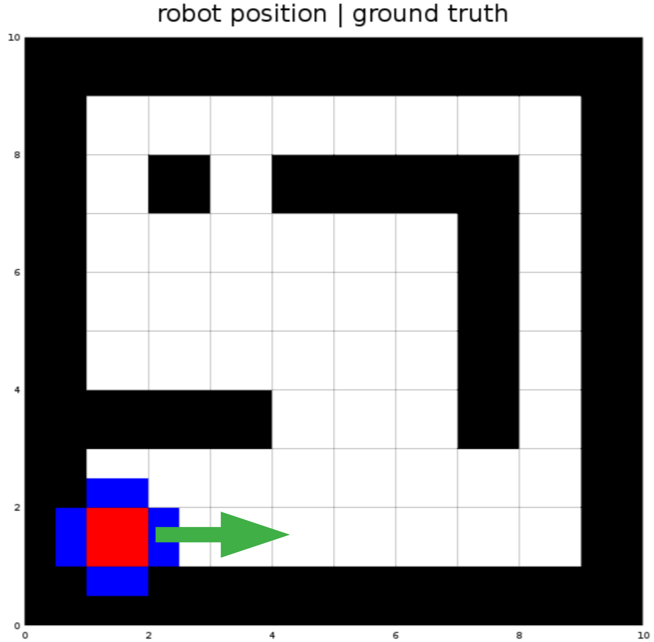
In this situation

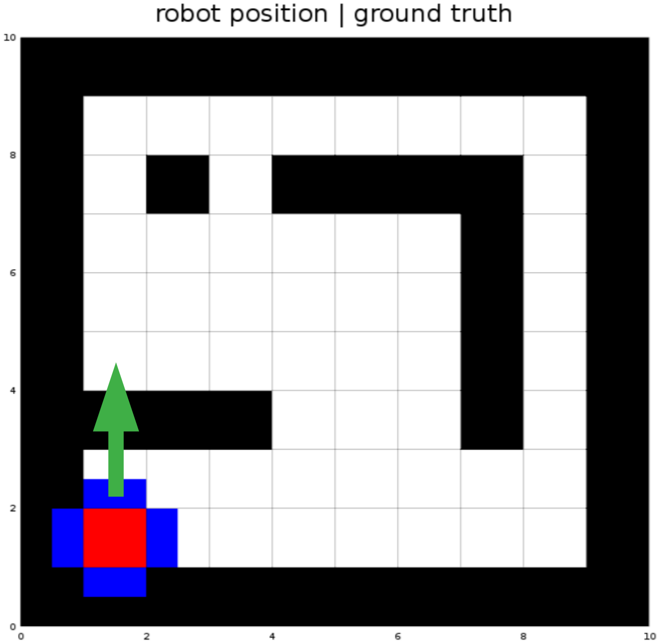
p($\mathbf{z}_{RIGHT}$=toggled)=0.8

During the synthesis of an observation model you **assume** you know **both** state and the measurement. The observation model tells you how likely the measurement is in the state

B) Observation Model: Modeling the Bumper

- Depending on the orientation of the robot, the bumpers capture different cells on the map
- Bumper 1 is in the cell up with orientation 0 *but* in cell left with orientation π





B) Observation Model: Modeling the Bumper

One possible strategy is to write a matrix encoding where the bumpers should be found depending on the orientation

```
orientation_to_cells_mapping(:,:,1) = [ -1 1 0 0;
                                         0 0 -1 1];
orientation_to_cells_mapping(:,:,2) = [ 0 0 1 -1;
                                        -1 1 0 0];
orientation_to_cells_mapping(:,:,3) = [ 1 -1 0 0;
                                         0 0 1 -1];
orientation_to_cells_mapping(:,:,4) = [ 0 0 -1 1;
                                         1 -1 0 0];
```

```
cell_bumper_west = cell_ + orientation_to_cells_mapping(:,1,theta_);
```

cell_up→cell_west, if the robot always points the north)

Observation Model

- Once you retrieved the cells corresponding to the bumpers compute the probability of the observation
- Using the convention robot always pointing north we have

```
#update probability depending on observations
if (cell_bumper_west_occupied == observations_(1))
  observation_probability *= .8;
else
  observation_probability *= .2;
endif
if (cell_bumper_est_occupied == observations_(2))
  observation_probability *= .8;
else
  observation_probability *= .2;
endif
if (cell_bumper_south_occupied == observations_(3))
  observation_probability *= .8;
else
  observation_probability *= .2;
endif
if (cell_bumper_north_occupied == observations_(4))
  observation_probability *= .8;
else
  observation_probability *= .2;
endif
```

Localizing Orazio

- Predict belief

```
#PREDICT robot position belief
state_belief_previous = state_belief;
state_belief = zeros(map_rows, map_cols, THETA_VALUES);
for (theta=1:THETA_VALUES)
  for row = 1:map_rows
    for col = 1:map_cols
      state_belief += transitionModel(map, row, col, theta, control_input)*state_belief_previous(row, col, theta);
    endfor
  endfor
endfor
```

- Update belief

```
#UPDATE robot position belief and COMPUTE the normalizer
disp("observations: ")
disp(observations)
inverse_normalizer = 0;
for (theta=1:THETA_VALUES)
  for row = 1:map_rows
    for col = 1:map_cols
      state_belief(row, col, theta) *= observationModel(map, row, col, observations);
      inverse_normalizer            += state_belief(row, col, theta);
    endfor
  endfor
endfor
```

Useful hints for debugging

- Create certainty by divide-and-conquer
  - A) Ensure that the noise-free model is correct
  - B) Ensure that predict step is working
  - C) Then add update step
  - D) Check that the inputs to all the main passages are correct
  - E) Try to visualize as much as possible