

# Probabilistic Robotics Course

## Modeling Dependencies Bayesian Networks

**Giorgio Grisetti**

`grisetti@diag.uniroma1.it`

Department of Computer Control and Management Engineering  
Sapienza University of Rome

# Outline

- Considerations about dimensions
- Role of Conditional Independence
- Modeling Phenomena: Bayesian Networks
- Inference on Bayesian Networks

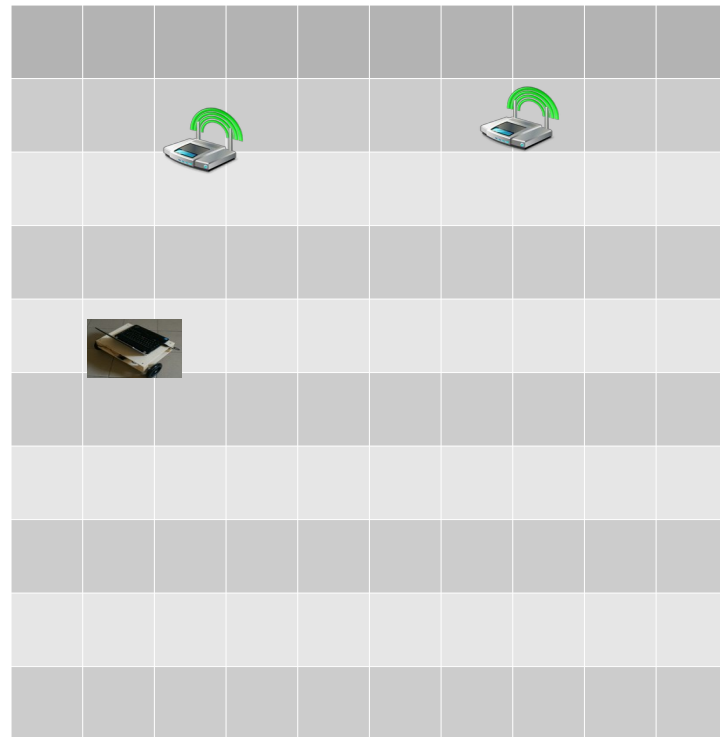
# Orazio!

Let us go back to our robot. The scene was

- 2 access points (AP)
- orazio moving
- one elevator

## Quantities

- signal strengths from the 2 APs
- floor of the elevator
- location of Orazio



# Dimensionality

Having

- 4 possible levels of strength per access point
- 2 access points
- 6 floors
- 100 locations

How many possible disjoint outcomes of events do I have?

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$$4*4*6*100 = \mathbf{9600}$$

# Dimensionality

What if we gift Orazio a monochromatic camera with resolution “poorVGA” (10x10), and 16 levels of gray

- number of disjoint events becomes:

$$4*4*6*100*16^{(10*10)} = 2.4790e+124$$

# Dimensionality

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Enough to challenge some good computer

# Queries

To perform a query of the type

$$p(S^A | S^B_2)$$

we can start from a joint probability distribution

$$P(S^A, S^B_2, I^{1,1}, \dots, I^{10,10}, L, E)$$

eliminate all variables that are non relevant through *marginalization*

$$P(S^A, S^B_2, \cancel{I^{1,1}}, \dots, \cancel{I^{10,10}}, \cancel{L}, \cancel{E})$$

and use the *Bayes' Rule* to get the answer




# Conditional Independence

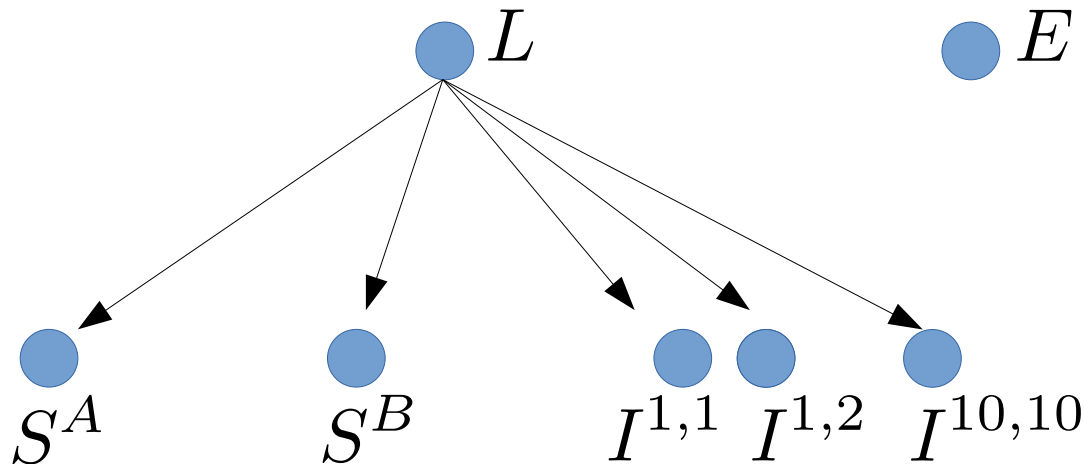
Using our wisdom we can notice some astonishing facts:

- The signal strength of an access point depends only on the location
- The pixels in the image depend only on the location
  - if nothing but Orazio move
  - if the light conditions do not change
- The elevator lives in his own world (as long as Orazio does not take it)

# Making Order



We can represent this phenomena as a *Bayesian Network*

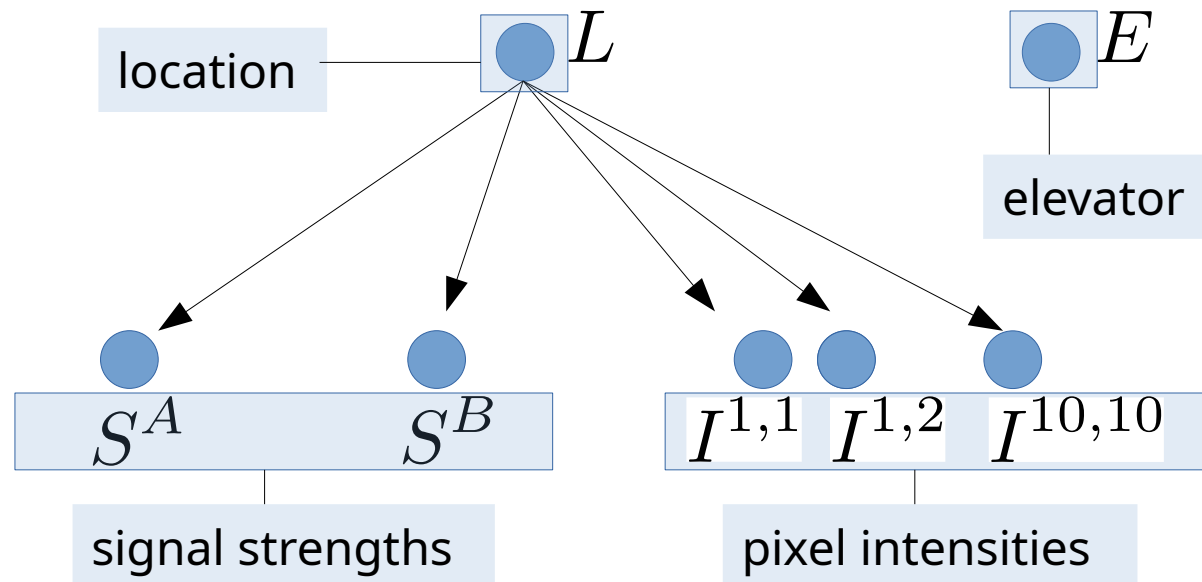
- One  per variable
- One  $\longrightarrow$  if the source influences the destination
- No loops, it's a directed acyclic graph!



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# Making Order

For storing the conditional probabilities we need:

- 100 numbers for robot locations
- $4 \times 100$  numbers for signal strength A
- $4 \times 100$  numbers for signal strength B
- $16 \times 100$  numbers for each pixel, as they are independent GIVEN the location
- 6 numbers for the elevator

$$100 + 2 \times 4 \times 100 + 100 \times 16 \times 100 + 6 = 160906$$

Much less than before

# Using the Order

By storing the conditional probability we need much less numbers

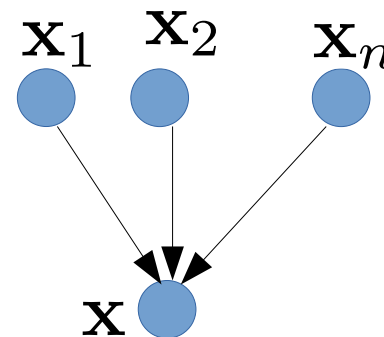
We can anyway recover the joint distribution and perform *inference* on that, by using the chain rule

Bayesian Networks are a formalism to highlight independence between variables, without loss of information

## Each node

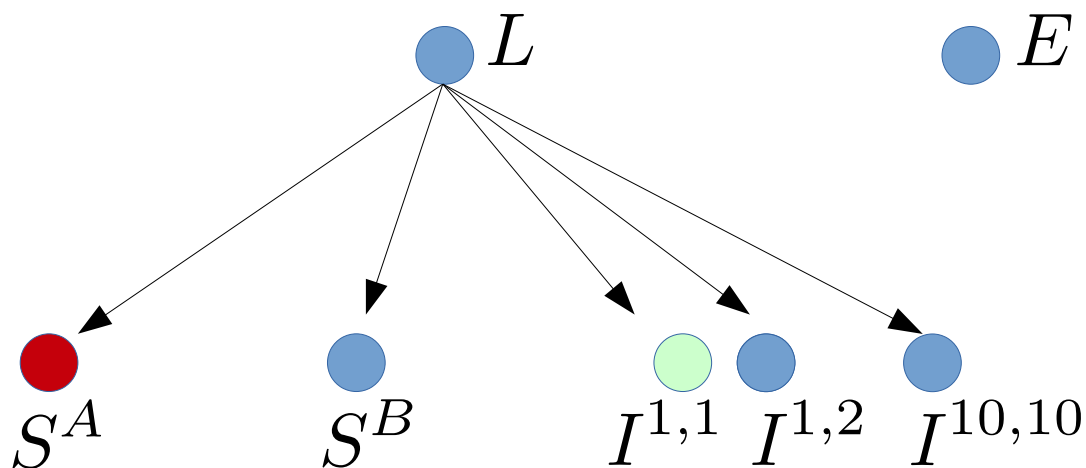
- represents a variable  $\mathbf{x}$
- stores a conditional probability:

$$p(\mathbf{x} \mid \mathbf{x}_1, \mathbf{x}_2, \dots \mathbf{x}_n)$$



# Example of Inference

What is the probability distribution of  $S^A$ ,  
given the pixel 1,1 has intensity 7 [ $I^{1,1} = 7$ ]



# Example of Inference

We want  $P(S^A | I_7^{1,1}) = ?$

- compute the joint

$$P(S^A, S^B, I_7^{1,1}, \dots, I^{10,10}, L, E) = P(S^A | L) P(S^B | L) P(I_7^{1,1} | L) \dots P(I^{10,10} | L) P(L) P(E)$$

it is the product of all distributions

- marginalize out the variables we don't need

$$P(S^A, S^B, I_7^{1,1}, \dots, I^{10,10}, L, E)$$

sum over all values of variables to suppress

to get a joint distribution over  $S^A, I_7^{0,0}$

$$P(S^A, I_7^{1,1}) = \sum_{S^B} \sum_{I^{0,1}} \dots \sum_{I^{10,10}} \sum_L \sum_E P(S^A, S^B, I_7^{1,1}, \dots, I^{10,10}, L, E)$$

- use conditioning to get the answer

$$P(S^A | I_7^{1,1}) = \frac{P(S^A, I_7^{1,1})}{\sum_{S^A} P(S^A, I_7^{1,1})}$$

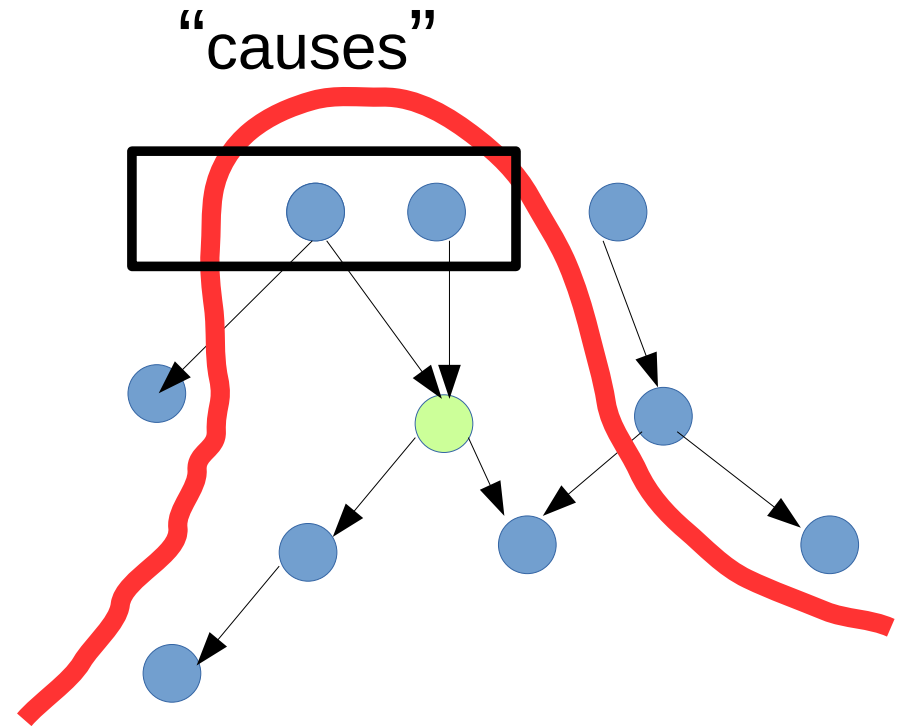
# Considerations

- Would we get the same answer if the elevator was not in the domain?
- Knowing the intensity of another pixel would provide us with more information?
- Does the same hold if we know the location?



# Local Semantics

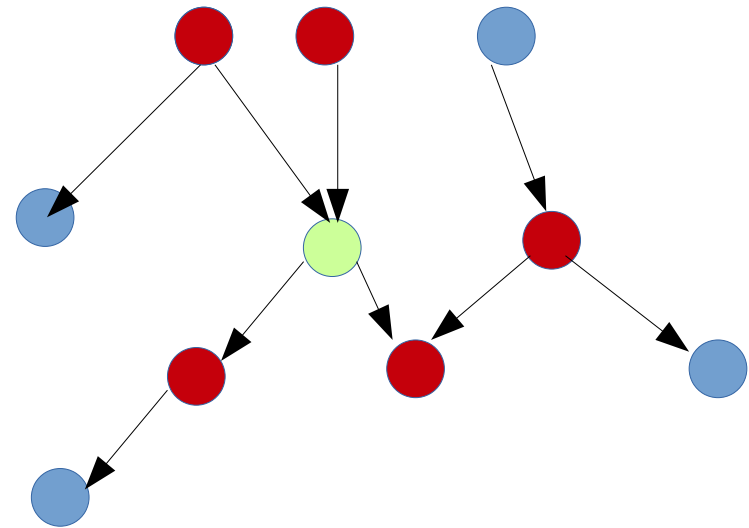
Each node is  
conditionally  
independent from its  
non-descendants  
given its parents



# Global Semantics

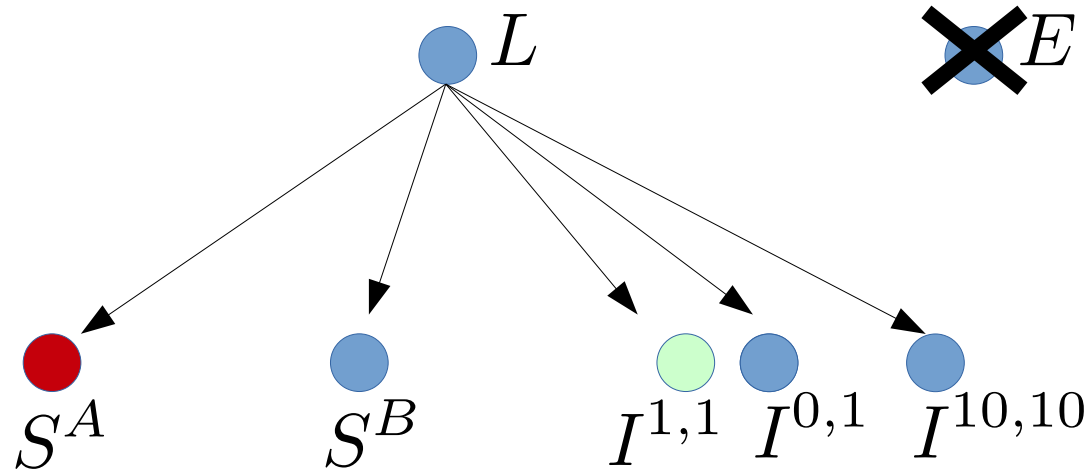
Each node is independent from the rest, given

- its parents
- its children
- the parents of its children



# Using Semantics

Elevator is disconnected, thus independent 

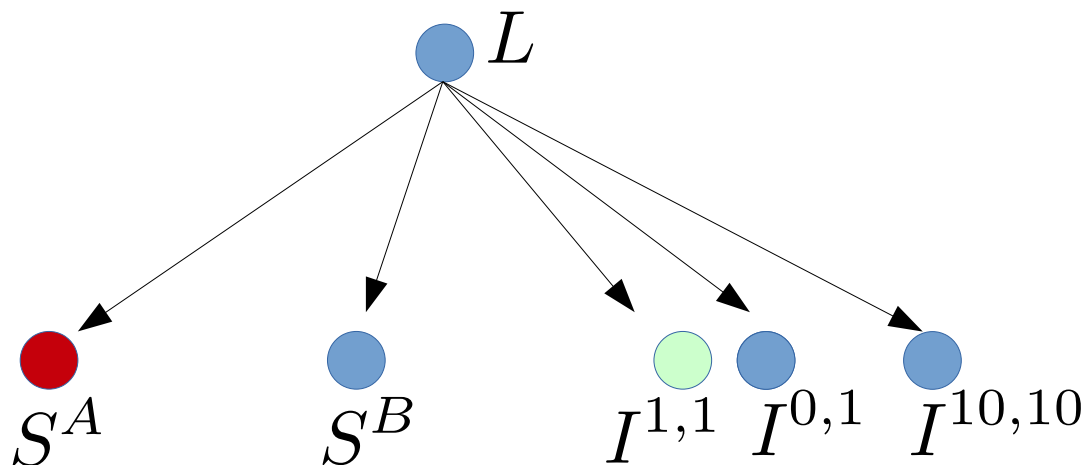


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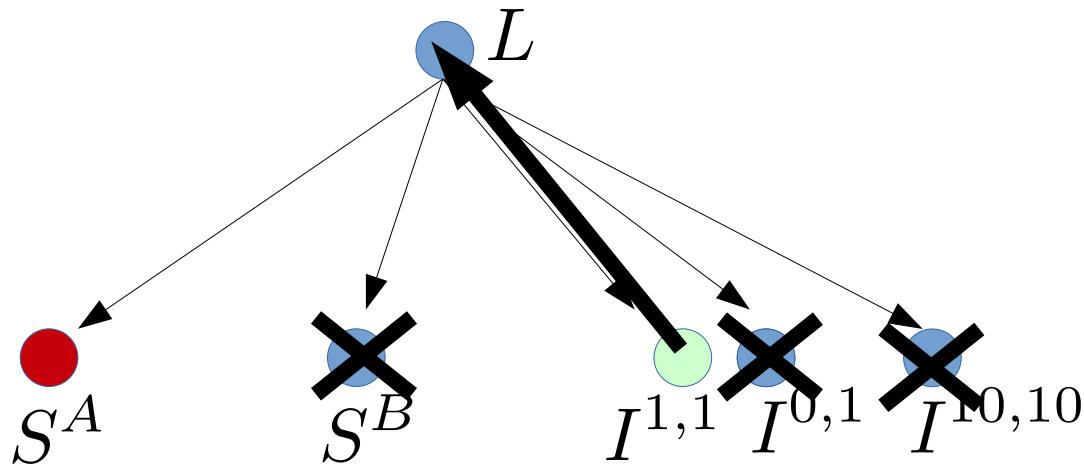
If we would know the location, the task would be easy

The only consequence of the location that we can observe is  $I^{1,1}$



# Using Semantics

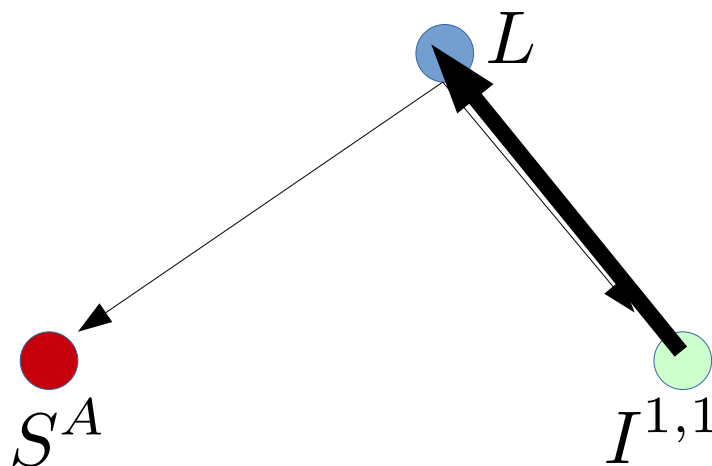
We can compute  $P(L \mid I_7^{1,1})$  while ignoring all the rest



# Using Semantics

We can compute  $P(L \mid I_7^{1,1})$  while ignoring all the rest

This will alter our prior about the location, incorporating the knowledge of the intensity



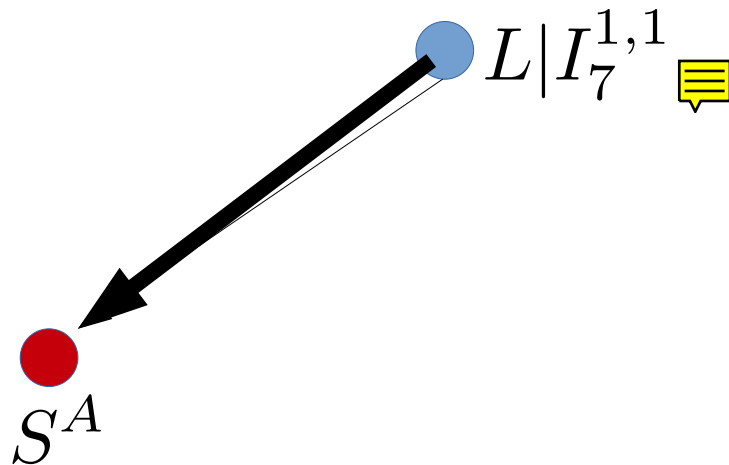
HOW:

- chain rule on  $P(I^{1,1} \mid L)$  and  $P(L)$  to get  $P(L, I)$
- conditioning on  $P(L, I)$ , to get  $P(L \mid I)$

# Using Semantics

We can compute  $P(L \mid I_7^{1,1})$  while ignoring all the rest

Finally, we determine the signal strength, from the improved location estimate



HOW:

- chain rule on  $P(S^A \mid L)$  and  $P(L \mid I)$

# References

An excellent tutorial on Bayes networks and graphical models:

<http://www.cs.ubc.ca/~murphyk/Bayes/bnintro.html>