

Ordered Multi-Signatures with Public-Key Aggregation from SXDH Assumption

○Masayuki Tezuka

Keisuke Tanaka

Institute of Science Tokyo

Version 2025/11/26

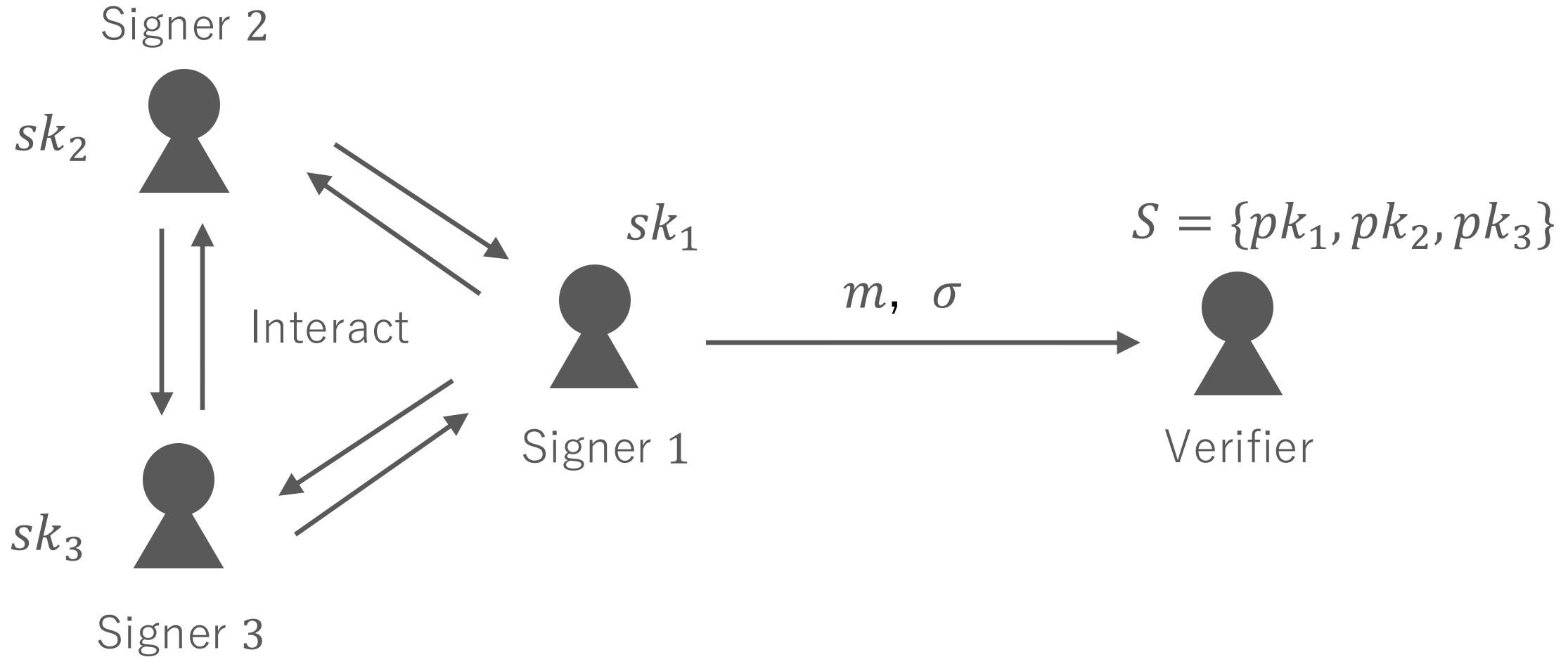
@ IWSEC 2025, Fukuoka, Japan

(Interactive) Multi-Signatures and Ordered Multi-Signatures

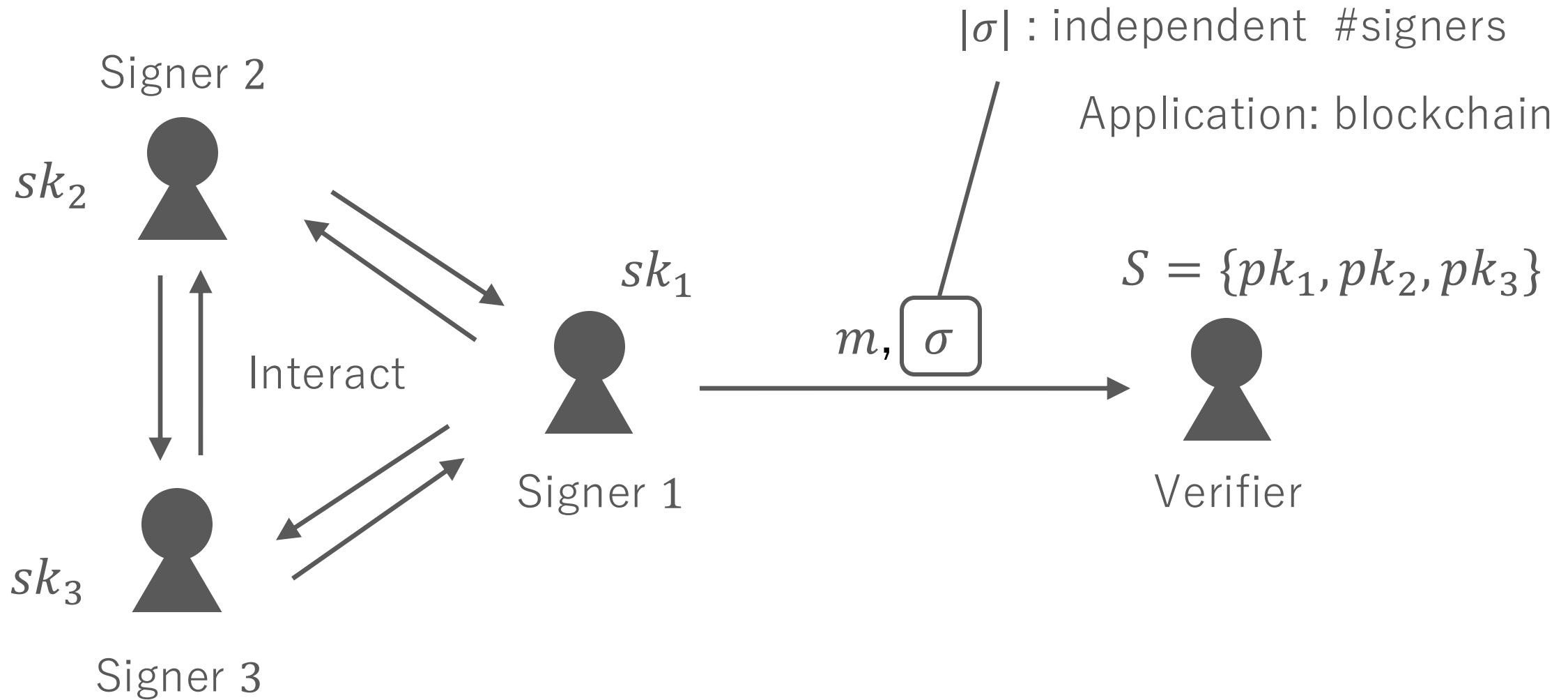
(Interactive) Multi-Signature (MS) [IN83]



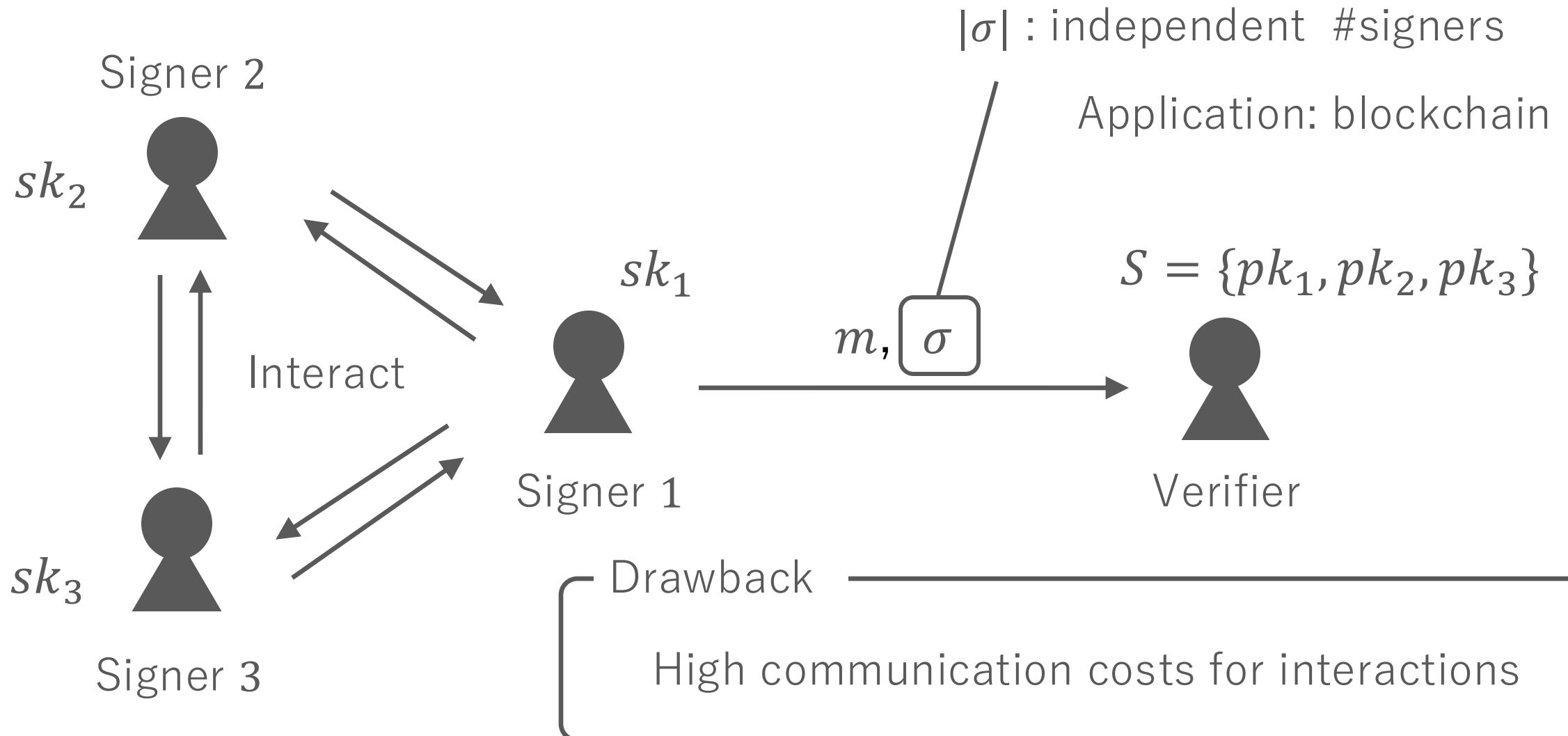
(Interactive) Multi-Signature (MS) [IN83]



(Interactive) Multi-Signature (MS) [IN83]



(Interactive) Multi-Signature (MS) [IN83]



Ordered Multi-Signature (OMS) [BGY07]

7

 sk_1 

Signer 1

 sk_2 

Signer 2

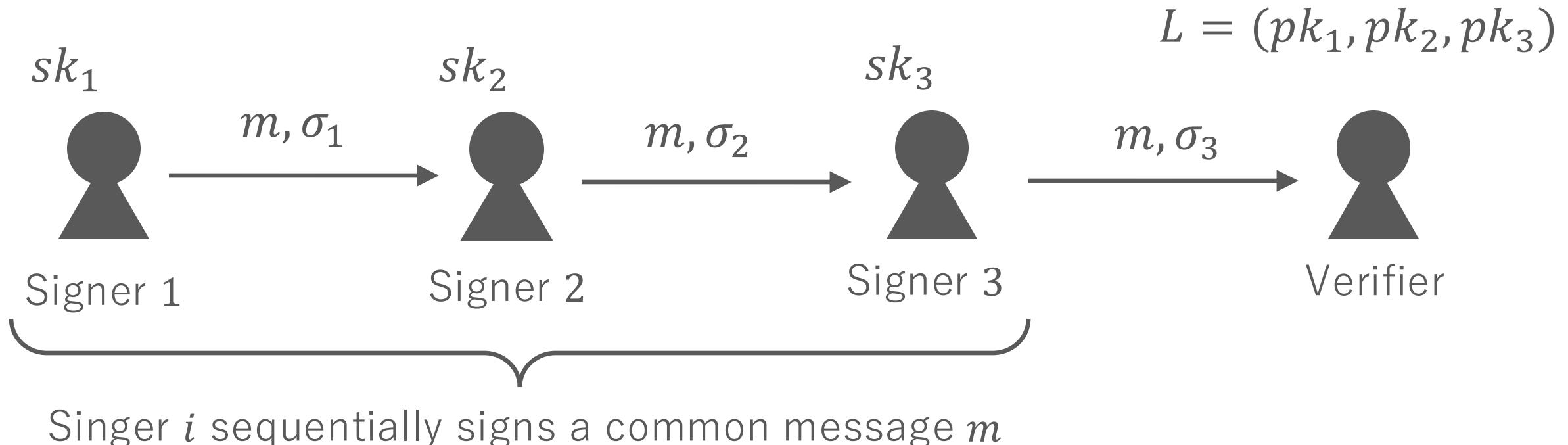
 sk_3 

Signer 3

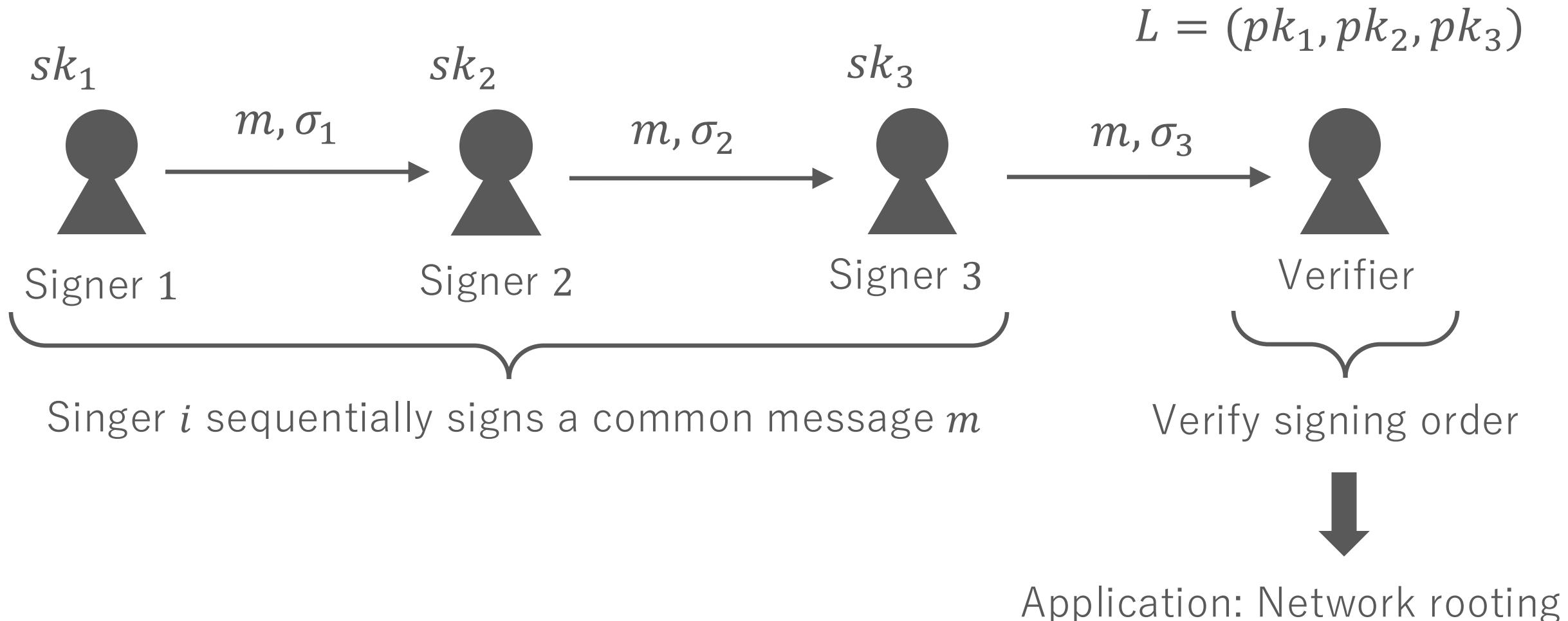
 $L = (pk_1, pk_2, pk_3)$ 

Verifier

Ordered Multi-Signature (OMS) [BG0Y07]

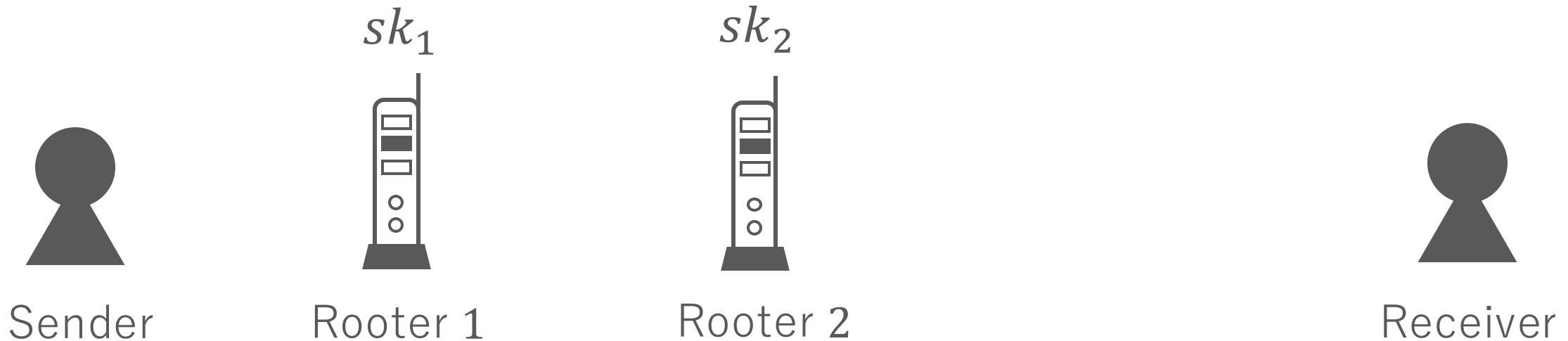


Ordered Multi-Signature (OMS) [BGY07]



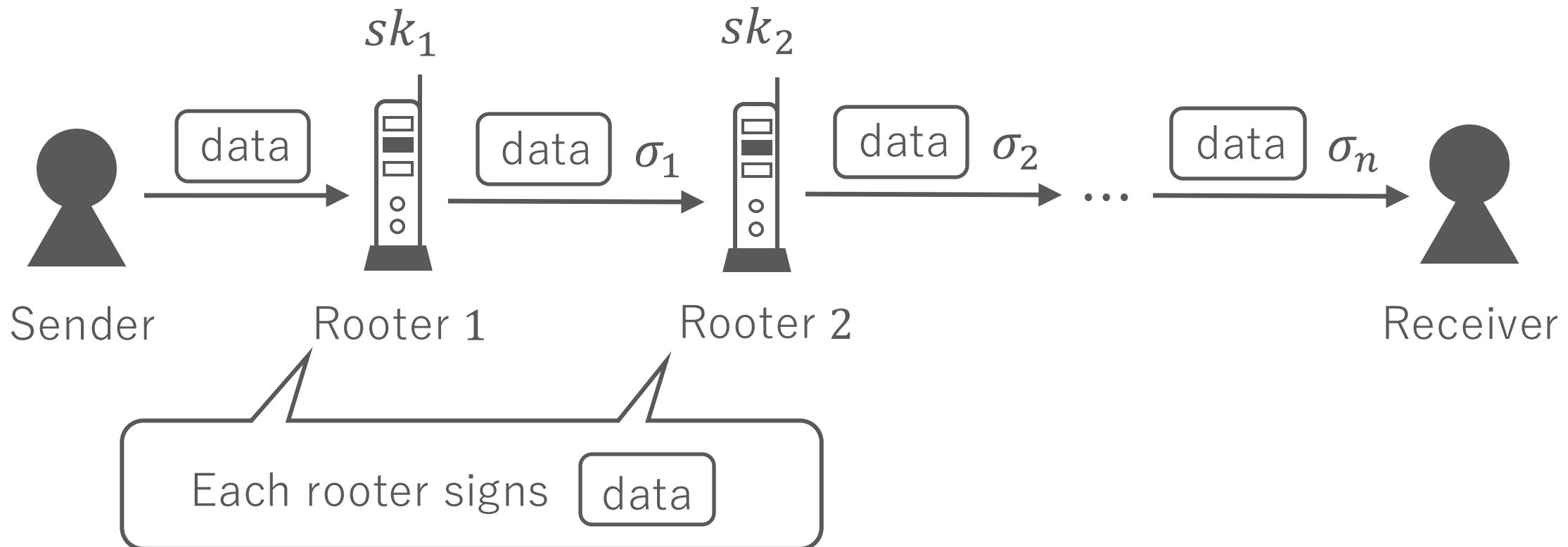
Application of OMS for Network Rooting

The sender sends the packet data **data** to the receiver.



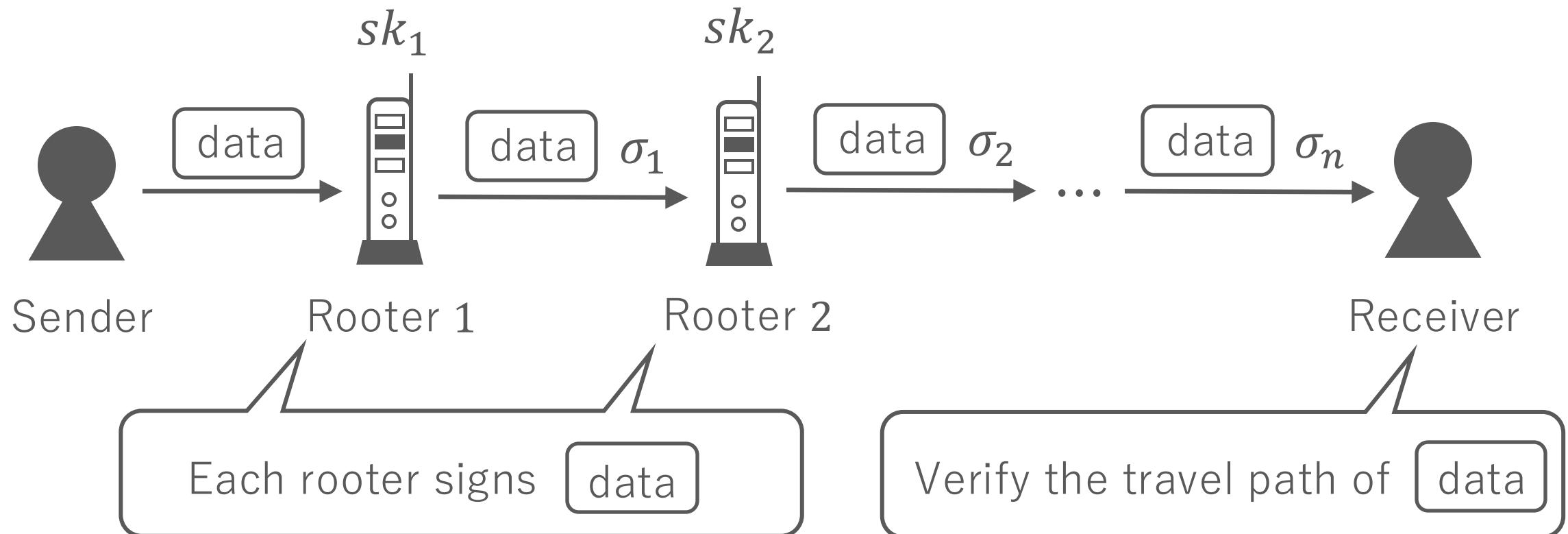
Application of OMS for Network Rooting

The sender sends the packet data **data** to the receiver.



Application of OMS for Network Rooting

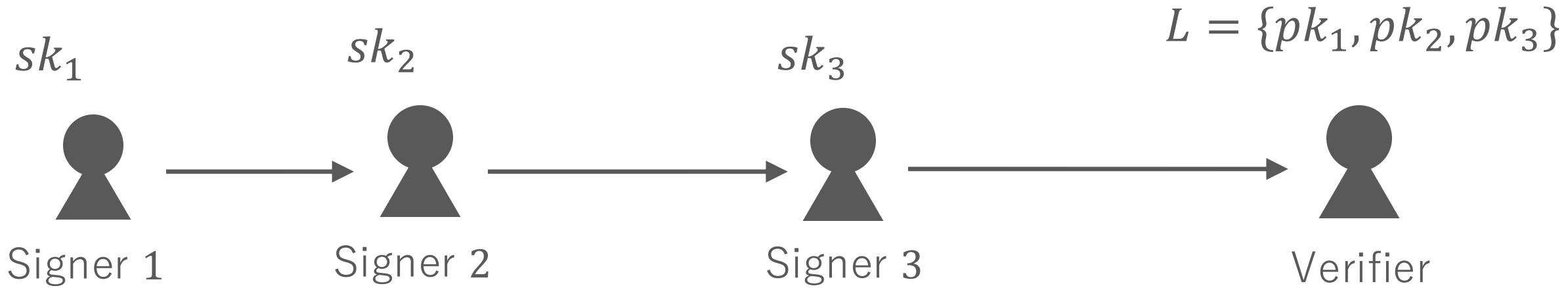
The sender sends the packet data data to the receiver.



Sequential Aggregate Signatures and Ordered Multi-Signatures

Sequential Aggregate Signature (SAS) [LMRS04]

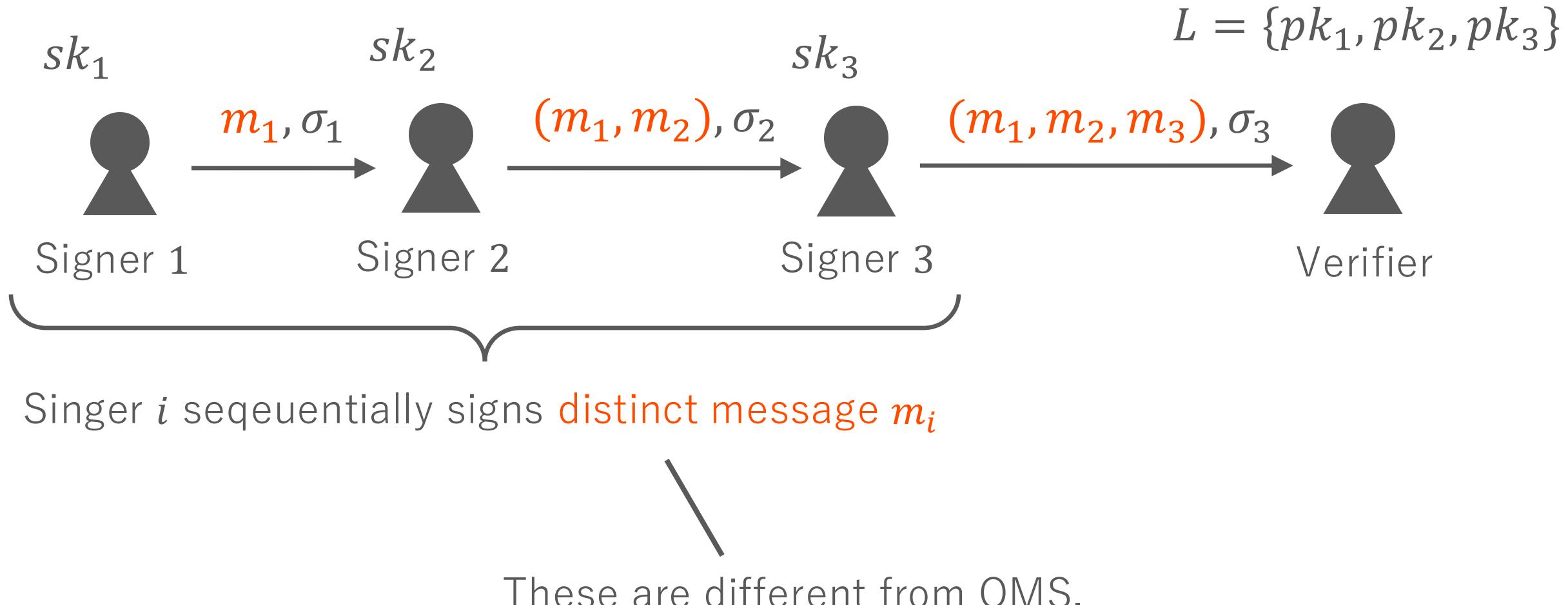
14



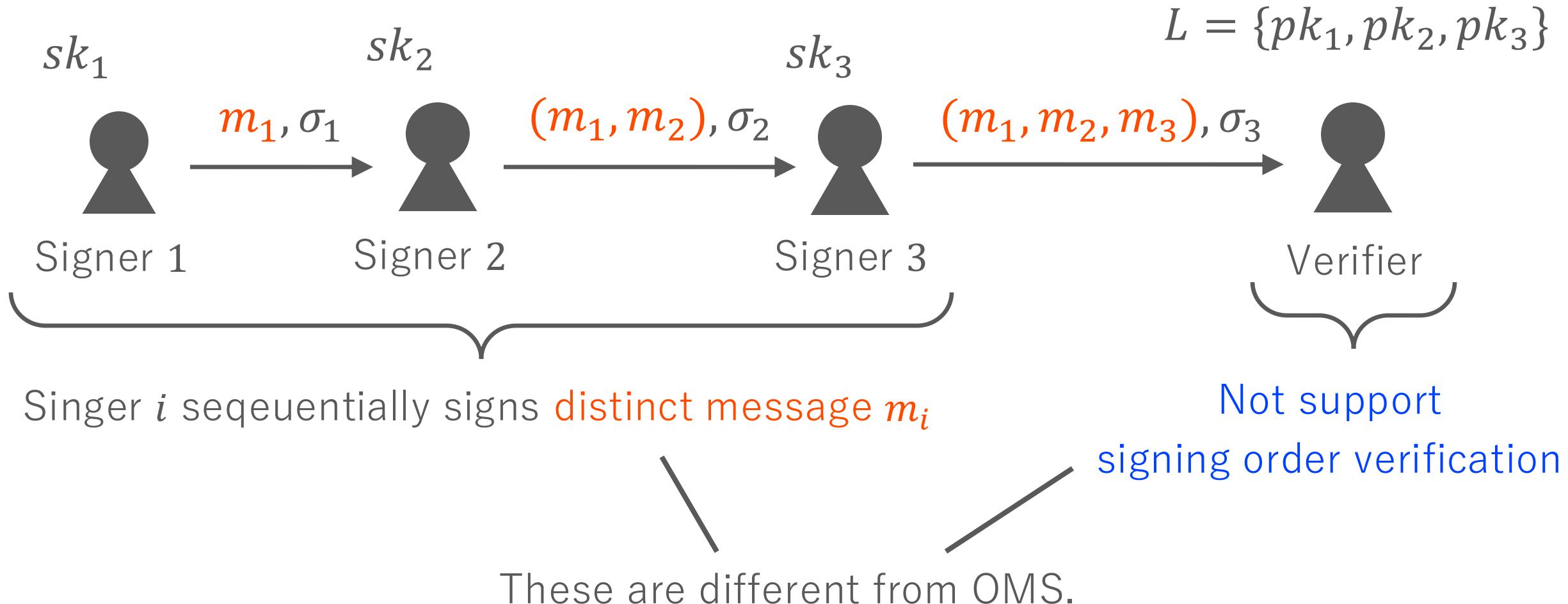
Two main difference between SAS and OMS !

Sequential Aggregate Signature (SAS) [LMRS04]

15



Sequential Aggregate Signature (SAS) [LMRS04]



Boldyreva et al.'s Transformation: From SAS to OMS

In SAS, simply signing a common message m does not yield OMS.

SAS does not support the signing order verification.

Boldyreva et al.'s Transformation: From SAS to OMS

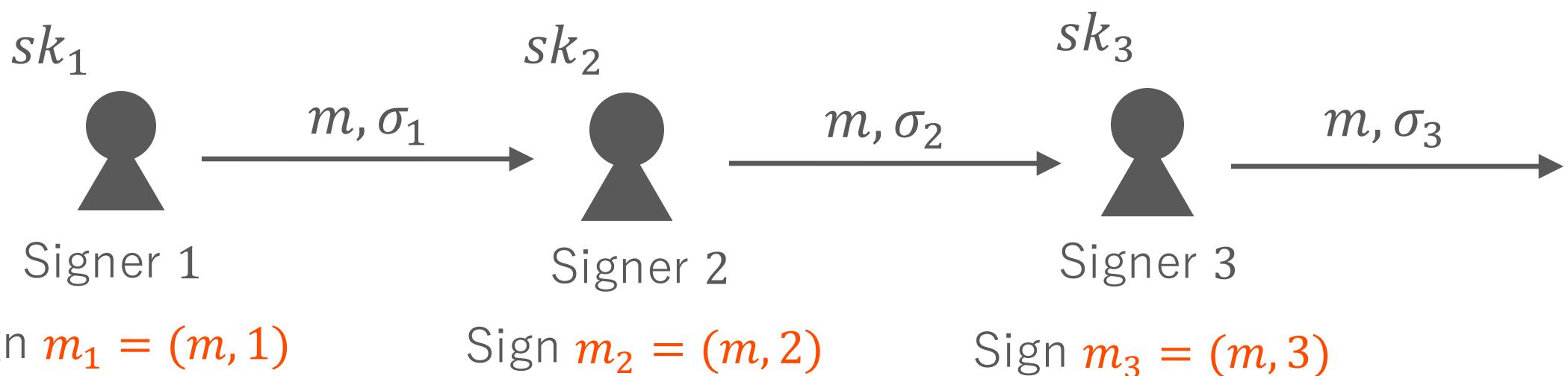
In SAS, simply signing a common message m does not yield OMS.

SAS does not support the signing order verification.

Boldyreva et al. [BGOY07]

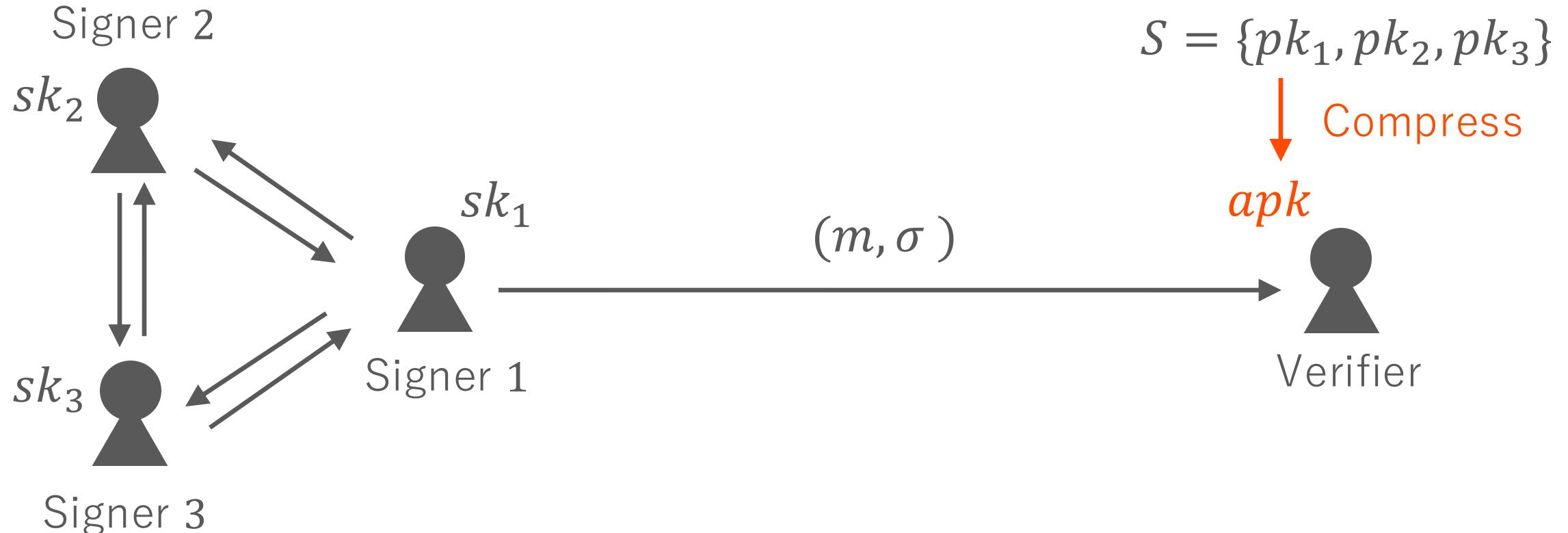
OMS is obtained from SAS !!

Idea : signer i signs $\textcolor{red}{m_i} = (m, i)$.



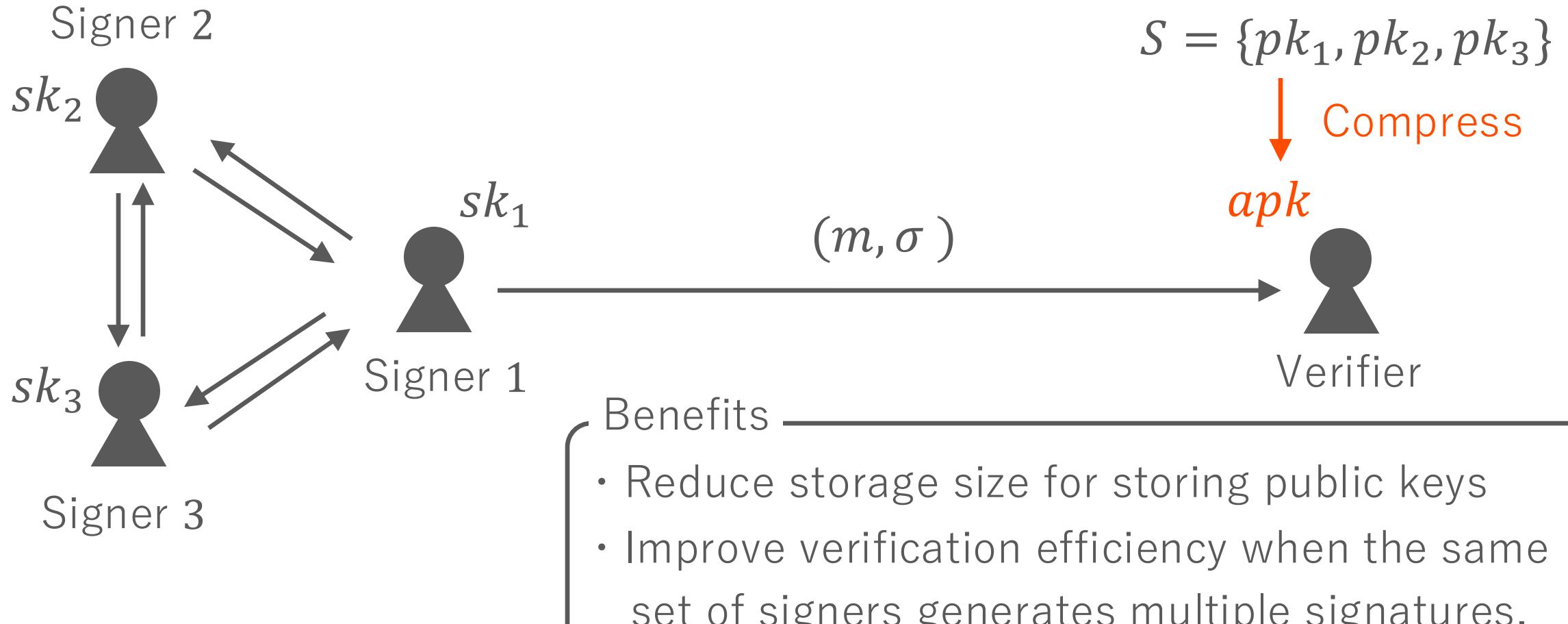
Public Key Aggregation for MS

Motivation: Public Key Aggregation in MS [MPSW19]

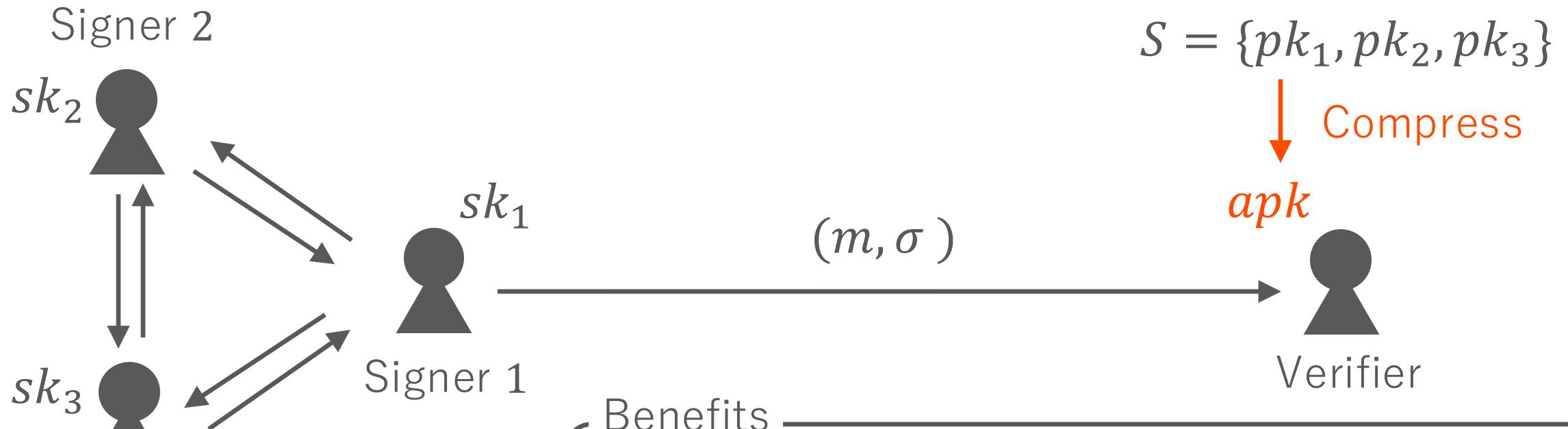


Motivation: Public Key Aggregation in MS [MPSW19]

21



Motivation: Public Key Aggregation in MS [MPSW19]



Benefits

- Reduce storage size for storing public keys
- Improve verification efficiency when the same set of signers generates multiple signatures.

In OMS, some schemes support public key aggregation.

Scheme	Assumption	pp size	pk size	sig size	PK Agg
OMS [YMO13]	CDH	$(m + 3) \mathbb{G} $	$3 \mathbb{G} $	$3 \mathbb{G} $	YES
SAS [LLY13]	LRSW (Interactive)	$2 \mathbb{G} $	$ \mathbb{G} $	$3 \mathbb{G} $	No
SAS [PS16]	PS (Interactive)	$2 \mathbb{G} + 2 \tilde{\mathbb{G}} $	$ \tilde{\mathbb{G}} $	$2 \mathbb{G} $	No
SAS [McD20]	2-PS (Interactive)	$2 \mathbb{G} + 2 \tilde{\mathbb{G}} $	$2 \tilde{\mathbb{G}} $	$2 \mathbb{G} $	YES
SAS [CK20]	SXDH	$4 \mathbb{G} + 3 \tilde{\mathbb{G}} $	$ \tilde{\mathbb{G}} $	$3 \mathbb{G} $	No

[YMO13] Yanai, Mambo, and Okamoto. An ordered multisignature scheme under the CDH assumption without random oracles. ISC 2013

[LLY13] Lee, Lee, and Yung. Aggregating cl-signatures revisited: Extended functionality and better efficiency. FC 2013

[PS16] Pointcheval and Sanders. Short randomizable signatures. CT-RSA 2016

[McD20] McDonald. The landscape of pointcheval-sanders signatures: Mapping to polynomial-based signatures and beyond. IACR Cryptol. ePrint Arch. 2020

[CK20] Chatterjee and Kabaleeshwaran. From rerandomizability to sequential aggregation: Efficient signature schemes based on SXDH assumption. ACISP2020

Pairing-Based OMS (OMS from SAS via Boldyreva trans) without the ROM 24

Scheme	Assumption	pp size	pk size	sig size	PK Agg
OMS [YMO13]	CDH	$(m + 3) \mathbb{G} $	$3 \mathbb{G} $	$3 \mathbb{G} $	YES
SAS [LLY13]	LRSW (Interactive)	$2 \mathbb{G} $	$ \mathbb{G} $	$3 \mathbb{G} $	No
SAS [PS16]	PS (Interactive)	$2 \mathbb{G} + 2 \tilde{\mathbb{G}} $	$ \tilde{\mathbb{G}} $	$2 \mathbb{G} $	No
SAS [McD20]	2-PS (Interactive)	$2 \mathbb{G} + 2 \tilde{\mathbb{G}} $	$2 \tilde{\mathbb{G}} $	$2 \mathbb{G} $	YES
SAS [CK20]	SXDH	$4 \mathbb{G} + 3 \tilde{\mathbb{G}} $	$ \tilde{\mathbb{G}} $	$3 \mathbb{G} $	No

The number of group element in public parameter is **linear in message bit length.**

Scheme	Assumption	pp size	pk size	sig size	PK Agg
OMS [YMO13]	CDH	$(m + 3) \mathbb{G} $	$3 \mathbb{G} $	$3 \mathbb{G} $	YES
SAS [LLY13]	LRSW (Interactive)	$2 \mathbb{G} $	$ \mathbb{G} $	$3 \mathbb{G} $	No
SAS [PS16]	PS (Interactive)	$2 \mathbb{G} + 2 \tilde{\mathbb{G}} $	$ \tilde{\mathbb{G}} $	$2 \mathbb{G} $	No
SAS [McD20]	2-PS (Interactive)	$2 \mathbb{G} + 2 \tilde{\mathbb{G}} $	$2 \tilde{\mathbb{G}} $	$2 \mathbb{G} $	YES
SAS [CK20]	SXDH	$4 \mathbb{G} + 3 \tilde{\mathbb{G}} $	$ \tilde{\mathbb{G}} $	$3 \mathbb{G} $	No

The LRSW, PS, and 2-PS assumption are **interactive assumptions**.

Scheme	Assumption	pp size	pk size	sig size	PK Agg
OMS [YMO13]	CDH	$(m + 3) \mathbb{G} $	$3 \mathbb{G} $	$3 \mathbb{G} $	YES
SAS [LLY13]	LRSW (Interactive)	$2 \mathbb{G} $	$ \mathbb{G} $	$3 \mathbb{G} $	No
SAS [PS16]	PS (Interactive)	$2 \mathbb{G} + 2 \tilde{\mathbb{G}} $	$ \tilde{\mathbb{G}} $	$2 \mathbb{G} $	No
SAS [McD20]	2-PS (Interactive)	$2 \mathbb{G} + 2 \tilde{\mathbb{G}} $	$2 \tilde{\mathbb{G}} $	$2 \mathbb{G} $	YES
SAS [CK20]	SXDH	$4 \mathbb{G} + 3 \tilde{\mathbb{G}} $	$ \tilde{\mathbb{G}} $	$3 \mathbb{G} $	No

Does not support public key aggregation.

Scheme	Assumption	pp size	pk size	sig size	PK Agg
OMS [YMO13]	CDH	$(m + 3) \mathbb{G} $	$3 \mathbb{G} $	$3 \mathbb{G} $	YES
SAS [LLY13]	LRSW (Interactive)	$2 \mathbb{G} $	$ \mathbb{G} $	$3 \mathbb{G} $	No
SAS [PS16]	PS (Interactive)	$2 \mathbb{G} + 2 \tilde{\mathbb{G}} $	$ \tilde{\mathbb{G}} $	$2 \mathbb{G} $	No
SAS [McD20]	2-PS (Interactive)	$2 \mathbb{G} + 2 \tilde{\mathbb{G}} $	$2 \tilde{\mathbb{G}} $	$2 \mathbb{G} $	YES
SAS [CK20]	SXDH	$4 \mathbb{G} + 3 \tilde{\mathbb{G}} $	$ \tilde{\mathbb{G}} $	$3 \mathbb{G} $	No

Question

Compact pp size OMS with PK aggregation in the standard assumptions.

Our Result

Scheme	Assumption	pp size	pk size	sig size	PK Agg
OMS [YMO13]	CDH	$(m + 3) \mathbb{G} $	$3 \mathbb{G} $	$3 \mathbb{G} $	YES
SAS [LLY13]	LRSW (Interactive)	$2 \mathbb{G} $	$ \mathbb{G} $	$3 \mathbb{G} $	No
SAS [PS16]	PS (Interactive)	$2 \mathbb{G} + 2 \tilde{\mathbb{G}} $	$ \tilde{\mathbb{G}} $	$2 \mathbb{G} $	No
SAS [McD20]	2-PS (Interactive)	$2 \mathbb{G} + 2 \tilde{\mathbb{G}} $	$2 \tilde{\mathbb{G}} $	$2 \mathbb{G} $	YES
SAS [CK20]	SXDH	$4 \mathbb{G} + 3 \tilde{\mathbb{G}} $	$ \tilde{\mathbb{G}} $	$3 \mathbb{G} $	No
Our Scheme	SXDH	$4 \mathbb{G} + 3 \tilde{\mathbb{G}}$	$2 \tilde{\mathbb{G}}$	$3 \mathbb{G}$	YES

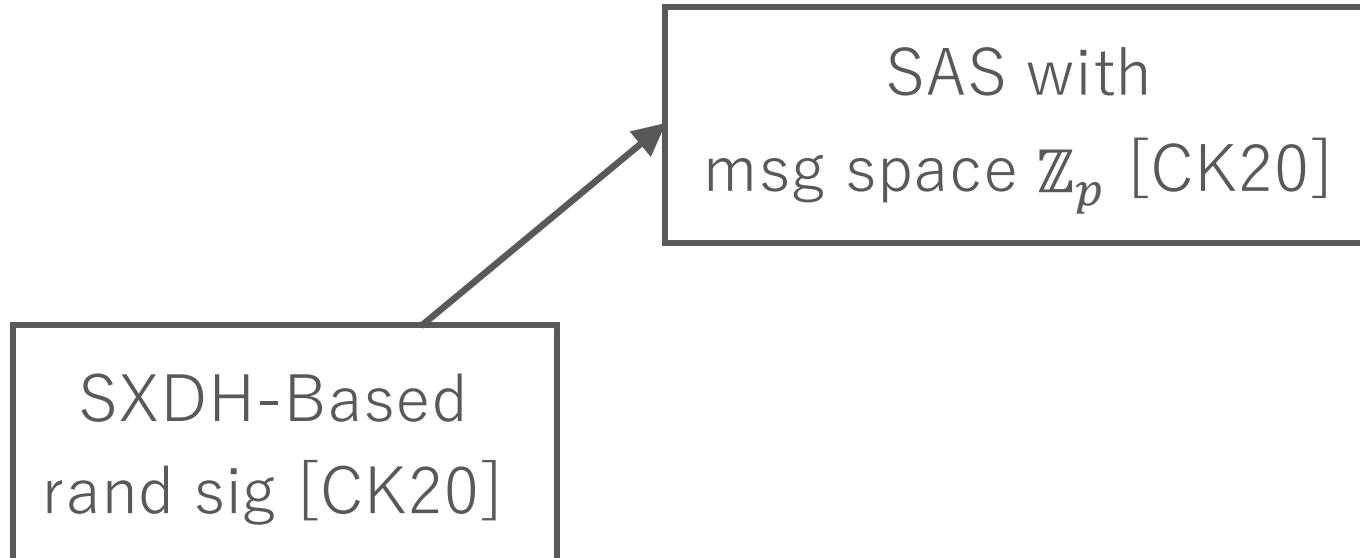
How to Obtain Scheme

SXDH-Based SAS [CK20] and OMS

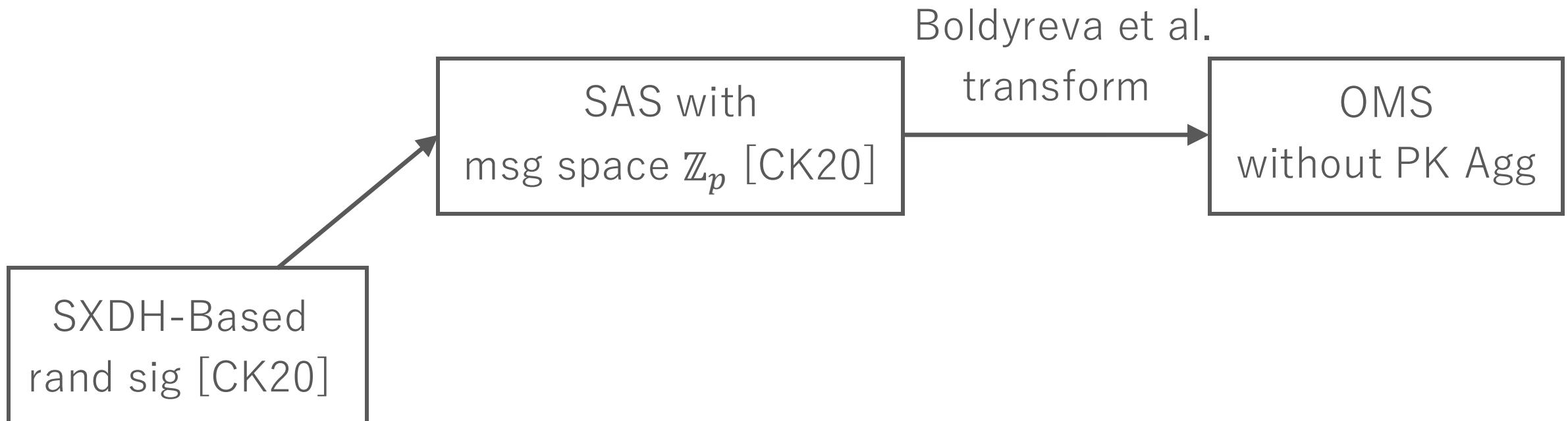
30

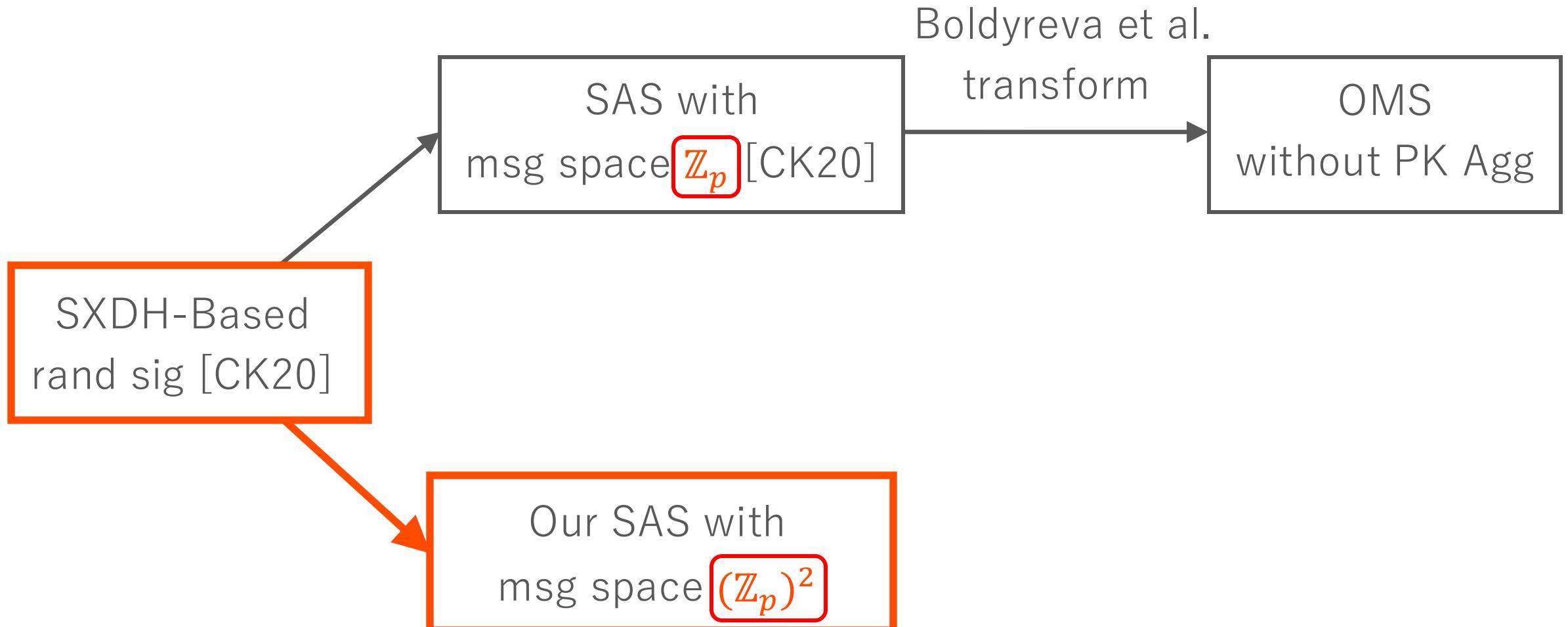
SXDH-Based
rand sig [CK20]

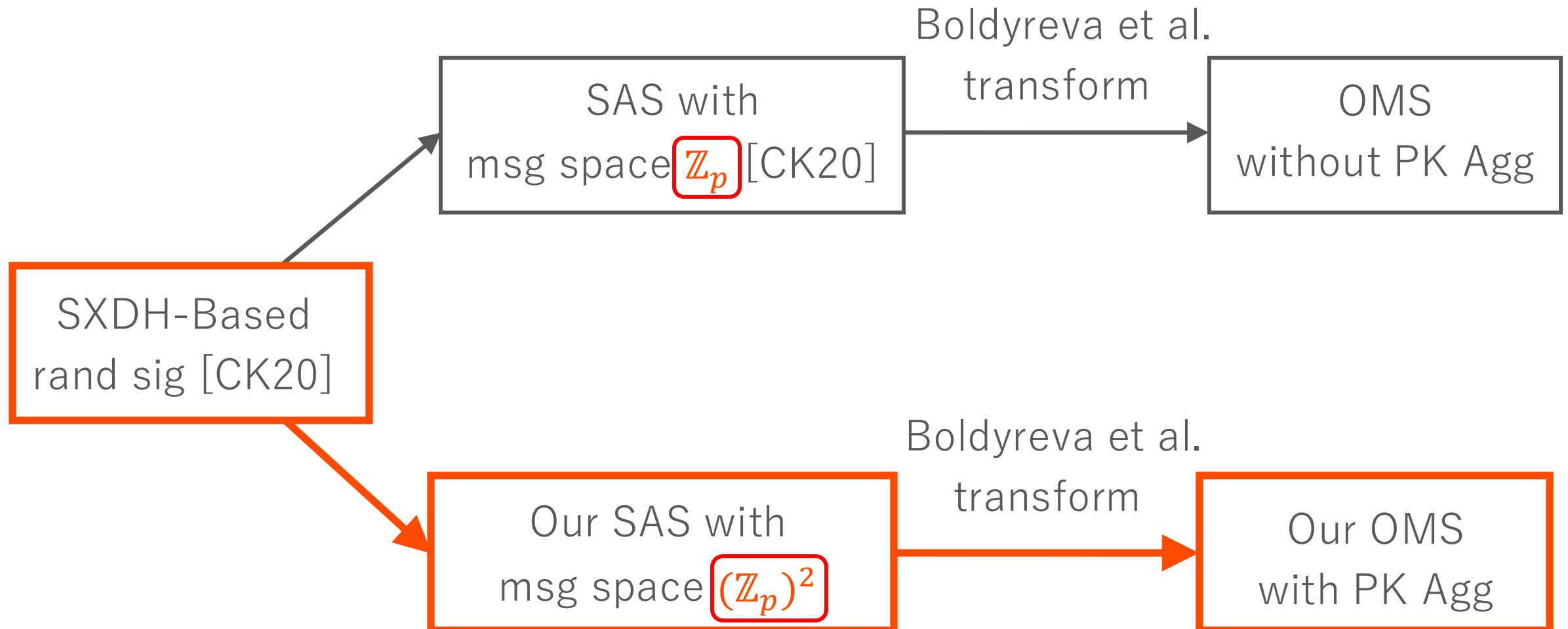
SXDH-Based SAS [CK20] and OMS



SXDH-Based SAS [CK20] and OMS

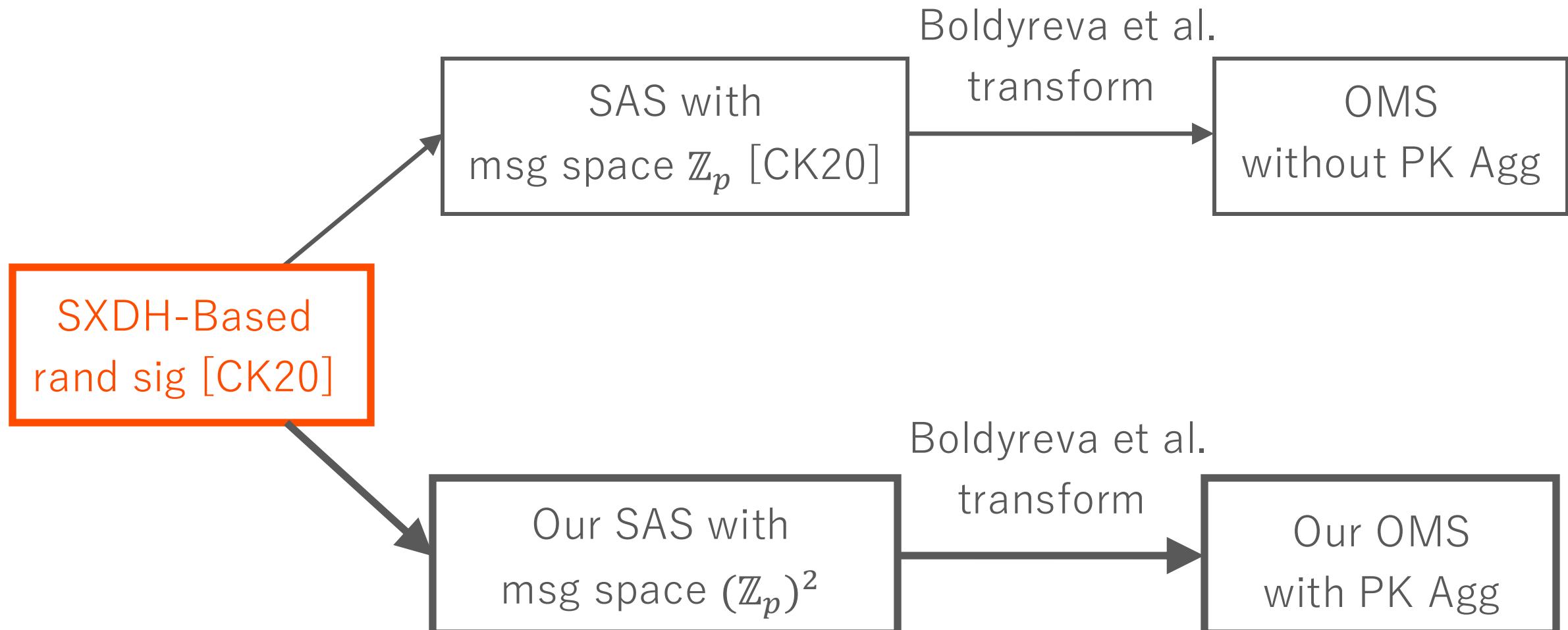






Starting Point: Based Randomizable Signature Scheme

35



SXDH-Based Randomizable Signature for $\text{Msg} \in (\mathbb{Z}_p)^2$ [CK20]

$$pk = (\tilde{H}, \tilde{D}, \tilde{U}, \tilde{V}, \tilde{W}) \quad sk = (x_1, x_2, y_1, y_2, z_1, z_2)$$

$$d \leftarrow \mathbb{Z}_p^*, \quad \tilde{D} = \tilde{H}^d, \quad \tilde{U} = \tilde{H}^{x_2 - dx_1}, \quad \tilde{V} = \tilde{H}^{y_2 - dy_1}, \quad \tilde{W} = \tilde{H}^{z_2 - dz_1}$$

SXDH-Based Randomizable Signature for $\text{Msg} \in (\mathbb{Z}_p)^2$ [CK20]

$$pk = (\tilde{H}, \tilde{D}, \tilde{U}, \tilde{V}, \tilde{W}) \quad sk = (x_1, x_2, y_1, y_2, z_1, z_2)$$

$$d \leftarrow \mathbb{Z}_p^*, \quad \tilde{D} = \tilde{H}^d, \quad \tilde{U} = \tilde{H}^{x_2 - dx_1}, \quad \tilde{V} = \tilde{H}^{y_2 - dy_1}, \quad \tilde{W} = \tilde{H}^{z_2 - dz_1}$$

Signature $\sigma = (A, B, C)$ on message (m_1, m_2)

$$r \leftarrow \mathbb{Z}_p^*, \quad A \leftarrow G^r, \quad B \leftarrow A^{x_1 + m_1 y_1 + m_2 z_1}, \quad C \leftarrow A^{x_2 + m_1 y_2 + m_2 z_2}$$

SXDH-Based Randomizable Signature for $\text{Msg} \in (\mathbb{Z}_p)^2$ [CK20]

$$pk = (\tilde{H}, \tilde{D}, \tilde{U}, \tilde{V}, \tilde{W}) \quad sk = (x_1, x_2, y_1, y_2, z_1, z_2)$$

$$d \leftarrow \mathbb{Z}_p^*, \quad \tilde{D} = \tilde{H}^d, \quad \tilde{U} = \tilde{H}^{x_2 - dx_1}, \quad \tilde{V} = \tilde{H}^{y_2 - dy_1}, \quad \tilde{W} = \tilde{H}^{z_2 - dz_1}$$

Signature $\sigma = (A, B, C)$ on message (m_1, m_2)

$$r \leftarrow \mathbb{Z}_p^*, \quad A \leftarrow G^r, \quad B \leftarrow A^{x_1 + m_1 y_1 + m_2 z_1}, \quad C \leftarrow A^{x_2 + m_1 y_2 + m_2 z_2}$$

Verification of $\sigma = (A, B, C)$ on message (m_1, m_2)

$$e(A, \tilde{U}\tilde{V}^{m_1}\tilde{W}^{m_2}) \cdot e(B, \tilde{D}) = e(C, \tilde{H}) ?$$

Randomizable Property

A signature $\sigma = (A, B, C)$ on message (m_1, m_2) is refreshed without a signing key sk .

Signature $\sigma = (A, B, C)$ on message (m_1, m_2)

Randomizable Property

A signature $\sigma = (A, B, C)$ on message (m_1, m_2) is refreshed without a signing key sk .

Signature $\sigma = (A, B, C)$ on message (m_1, m_2)



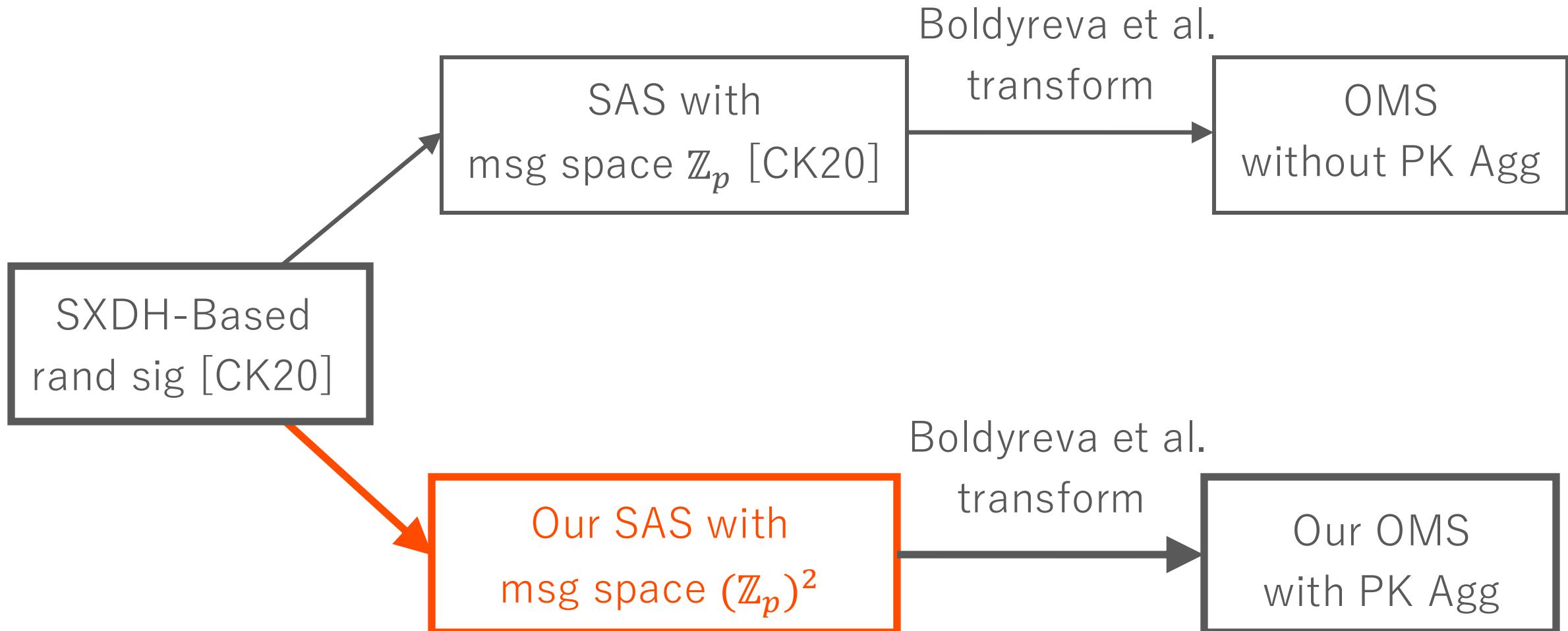
Randomizable property

$$r' \leftarrow \mathbb{Z}_p^*, A' \leftarrow A^{r'}, B' \leftarrow B^{r'}, C' \leftarrow C^{r'}$$

Refreshed signature $\sigma' = (A', B', C')$ on message (m_1, m_2)

From Based Signature Scheme to Our SAS

41



How to Derive SAS with msg space $(\mathbb{Z}_p)^2$

Randomizable Signature [CK20] for Msg $(\mathbb{Z}_p)^2$



- PK sharing technique
- Modify signing algorithm

Our SAS for Msg $(\mathbb{Z}_p)^2$

PK Sharing Technique

$$pk = (\tilde{H}, \tilde{D}, \tilde{U}, \tilde{V}, \tilde{W}) \quad sk = (x_1, x_2, y_1, y_2, z_1, z_2)$$

PK Sharing Technique

$$pk = (\boxed{\tilde{H}, \tilde{D}, \tilde{U}}, \tilde{V}, \tilde{W}) \quad sk = (\cancel{x_1, x_2}, y_1, y_2, z_1, z_2)$$

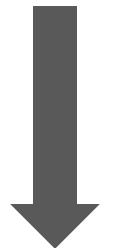
$\cancel{x_1, x_2}$



PK Sharing Technique

$$pk = (\boxed{\tilde{H}, \tilde{D}, \tilde{U}}, \tilde{V}, \tilde{W}) \quad sk = (\cancel{x_1, x_2}, y_1, y_2, z_1, z_2)$$

↓ ← ↑
 pp G^{x_1}, G^{x_2}



PK sharing technique

$$pp = (G, \tilde{H}, \tilde{D}, \tilde{U}, X_1 = G^{x_1}, X_2 = G^{x_2})$$

Singer i 's pk_i and sk_i

$$pk_i = (\tilde{V}_i, \tilde{W}_i) \quad sk_i = (y_{i,1}, y_{i,2}, z_{i,1}, z_{i,2})$$

Modifying Signing for Signer 1

Signer 1 signs $(m_{1,1}, m_{1,2})$ with $sk_1 = (y_{1,1}, y_{1,2}, z_{1,1}, z_{1,2})$

$$r_1 \leftarrow \mathbb{Z}_p^*, A_1 \leftarrow G^{r_1},$$

$$B_1 \leftarrow (X_1 G^{m_{1,1}y_{1,1} + m_{1,2}z_{1,1}})^{r_1}, C_1 \leftarrow (X_2 G^{m_{1,1}y_{1,2} + m_{1,2}z_{1,2}})^{r_1}$$

Generated signature $\sigma_1 = (A_1, B_1, C_1)$

$$A_1 = G^{r_1}$$

$$B_1 = A_1^{x + m_{1,1}y_{1,1} + m_{1,2}z_{1,1}}$$

$$C_1 = A_1^{x + m_{1,1}y_{1,2} + m_{1,2}z_{1,2}} \quad \text{hold.}$$

Modifying Signing for Signer $i \geq 2$

Signer i signs $(m_{i,1}, m_{i,2})$ with $sk_i = (y_{i,1}, y_{i,2}, z_{i,1}, z_{i,2})$
 and updates $\sigma_{i-1} = (A_{i-1}, B_{i-1}, C_{i-1})$.

$$r_i \leftarrow \mathbb{Z}_p^*, A_i \leftarrow {A_{i-1}}^{r_i},$$

$$B_i \leftarrow ({A_{i-1}}^{m_{i,1}y_{i,1} + m_{i,2}z_{i,1}})^{r_i} \cdot {B_{i-1}}^{r_i}$$

$$C_i \leftarrow ({A_{i-1}}^{m_{i,1}y_{i,2} + m_{i,2}z_{i,2}})^{r_i} \cdot {C_{i-1}}^{r_i}$$

Updated signature $\sigma_i = (A_i, B_i, C_i)$

$$A_i = G^{\prod r_k}, \quad B_i = {A_i}^{x + \sum m_{k,1}y_{k,1} + \sum m_{k,1}z_{k,1}}$$

$$C_i = {A_i}^{x + \sum m_{k,1}y_{k,2} + \sum m_{k,1}z_{k,2}} \quad \text{hold.}$$

Modifying Signing for Signer $i \geq 2$

Signer i signs $(m_{i,1}, m_{i,2})$ with $sk_i = (y_{i,1}, y_{i,2}, z_{i,1}, z_{i,2})$

and updates $\sigma_{i-1} = (A_{i-1}, B_{i-1}, C_{i-1})$.

$$r_i \leftarrow \mathbb{Z}_p^*, A_i \leftarrow [A_{i-1}]^{r_i},$$

$$B_i \leftarrow (A_{i-1}^{m_{i,1}y_{i,1} + m_{i,2}z_{i,1}})^{r_i} \cdot [B_{i-1}]^{r_i}$$

$$C_i \leftarrow (A_{i-1}^{m_{i,1}y_{i,2} + m_{i,2}z_{i,2}})^{r_i} \cdot [C_{i-1}]^{r_i}$$

Randomization of σ_{i-1}

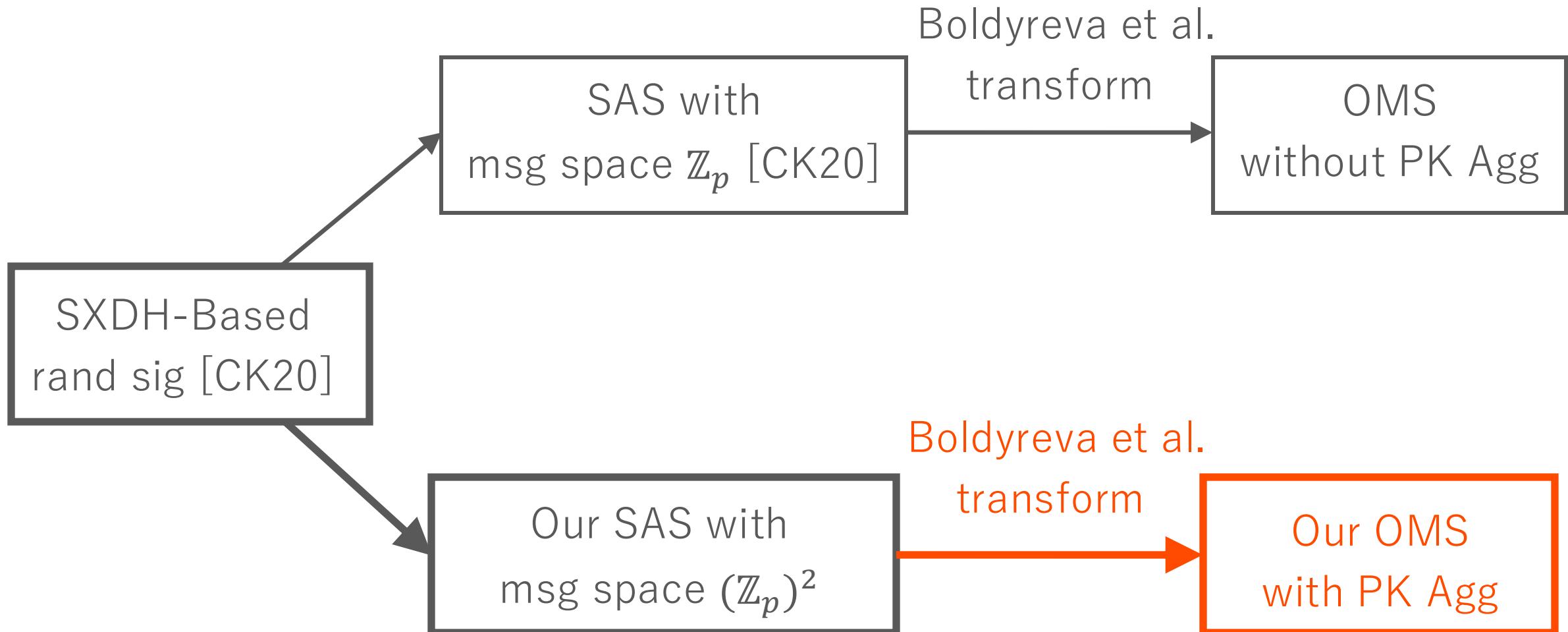
Updated signature $\sigma_i = (A_i, B_i, C_i)$

$$A_i = G^{\prod r_k}, \quad B_i = A_i^{x + \sum m_{k,1}y_{k,1} + \sum m_{k,1}z_{k,1}}$$

$$C_i = A_i^{x + \sum m_{k,1}y_{k,2} + \sum m_{k,1}z_{k,2}} \quad \text{hold.}$$

Our SAS to Our OMS with PK Agg

49



Boldyreva et al. Transformation

Singer i signs (m, i) with $sk_i = (y_{i,1}, y_{i,2}, z_{i,1}, z_{i,2})$

and updates $\sigma_{i-1} = (A_{i-1}, B_{i-1}, C_{i-1})$.

Updated signature $\sigma_i = (A_i, B_i, C_i)$

$$A_i = G^{\prod r_k}$$

$$B_i = A_i^{x + \sum \textcolor{red}{m} y_{k,1} + \sum \textcolor{red}{k} z_{k,1}}$$

$$C_i = A_i^{x + \sum \textcolor{red}{m} y_{k,2} + \sum \textcolor{red}{k} z_{k,2}} \quad \text{hold.}$$

PK Aggregation

$$pk_i = (\tilde{V}_i, \tilde{W}_i) \quad \sigma_i = (A_i, B_i, C_i)$$

$$A_i = G^{\prod r_k} \quad B_i = A_i^{x + \sum my_{k,1} + \sum kz_{k,1}} \quad C_i = A_i^{x + \sum my_{k,2} + \sum kz_{k,2}}$$

Verification of σ_i

$$e\left(A_i, \tilde{U} \cdot (\prod \tilde{V}_k)^m \cdot \prod \tilde{W}_k^k\right) \cdot e(B_i, \tilde{D}) = e(C_i, \tilde{H}) ?$$

PK Aggregation

$$pk_i = (\tilde{V}_i, \tilde{W}_i) \quad \sigma_i = (A_i, B_i, C_i)$$

$$A_i = G^{\prod r_k} \quad B_i = A_i^{x + \sum my_{k,1} + \sum kz_{k,1}} \quad C_i = A_i^{x + \sum my_{k,2} + \sum kz_{k,2}}$$

Verification of σ_i

$$e(A_i, \tilde{U} \cdot \boxed{(\prod \tilde{V}_k)^m} \cdot \boxed{\prod \tilde{W}_k^k}) \cdot e(B_i, \tilde{D}) = e(C_i, \tilde{H}) ?$$

PK Aggregation

$$pk_i = (\tilde{V}_i, \tilde{W}_i) \quad \sigma_i = (A_i, B_i, C_i)$$

$$A_i = G^{\prod r_k} \quad B_i = A_i^{x + \sum m y_{k,1} + \sum k z_{k,1}} \quad C_i = A_i^{x + \sum m y_{k,2} + \sum k z_{k,2}}$$

Verification of σ_i

$$e(A_i, \tilde{U} \cdot \left[(\prod \tilde{V}_k) \right]^m \cdot \left[\prod \tilde{W}_k^k \right]) \cdot e(B_i, \tilde{D}) = e(C_i, \tilde{H}) ?$$

$apk = (\prod \tilde{V}_k, \prod \tilde{W}_k^k)$

Conclusion

We construct the ordered multi-signature scheme with key-aggregation.

Our scheme offers compact pp , pk , and σ .

The security is proven under the SXDH assumption without the ROM.

Future work

Tightly secure OMS with PK agg in the standard model.

Thank you for listening !