

Ordered Multi-Signatures with Public-Key Aggregation from SXDH Assumption

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Version 2025/11/26

@ IWSEC 2025, Fukuoka, Japan

(Interactive) Multi-Signatures and Ordered Multi-Signatures

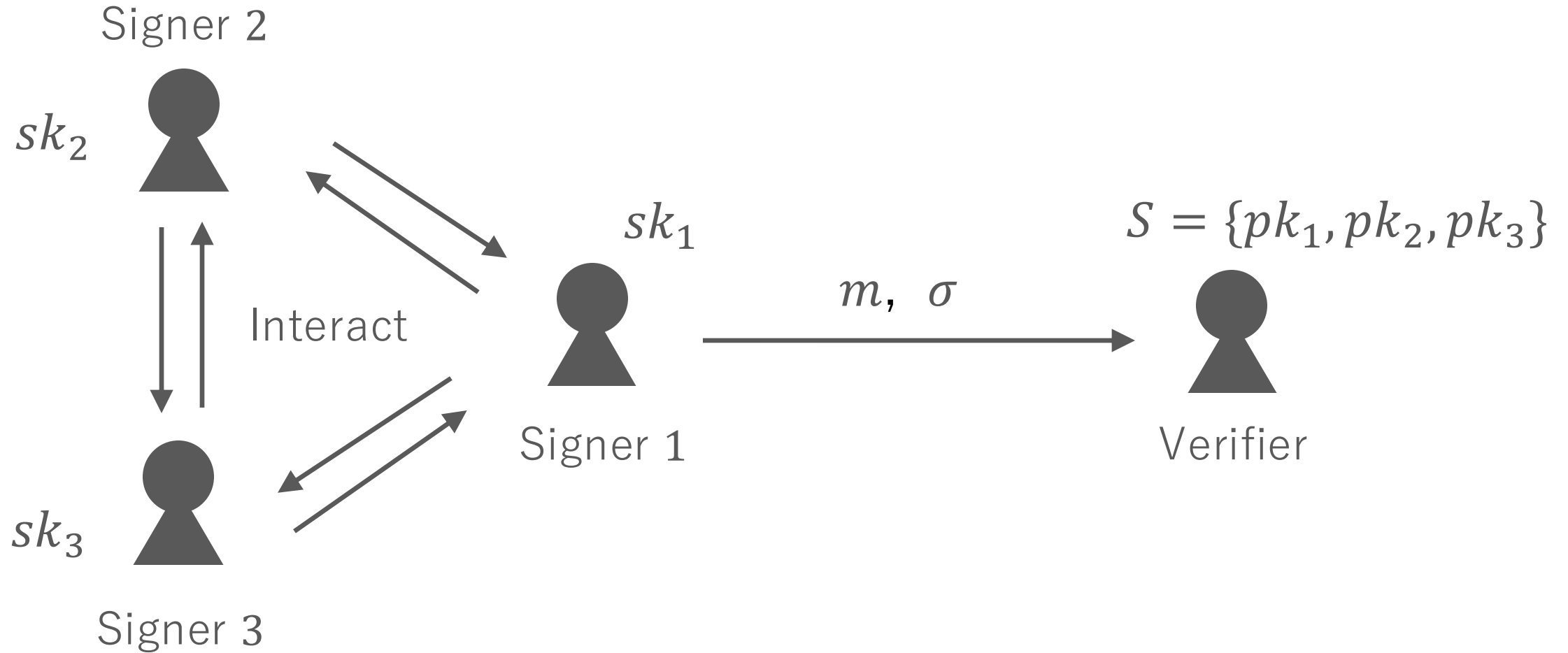
(Interactive) Multi-Signature (MS) [IN83]

3



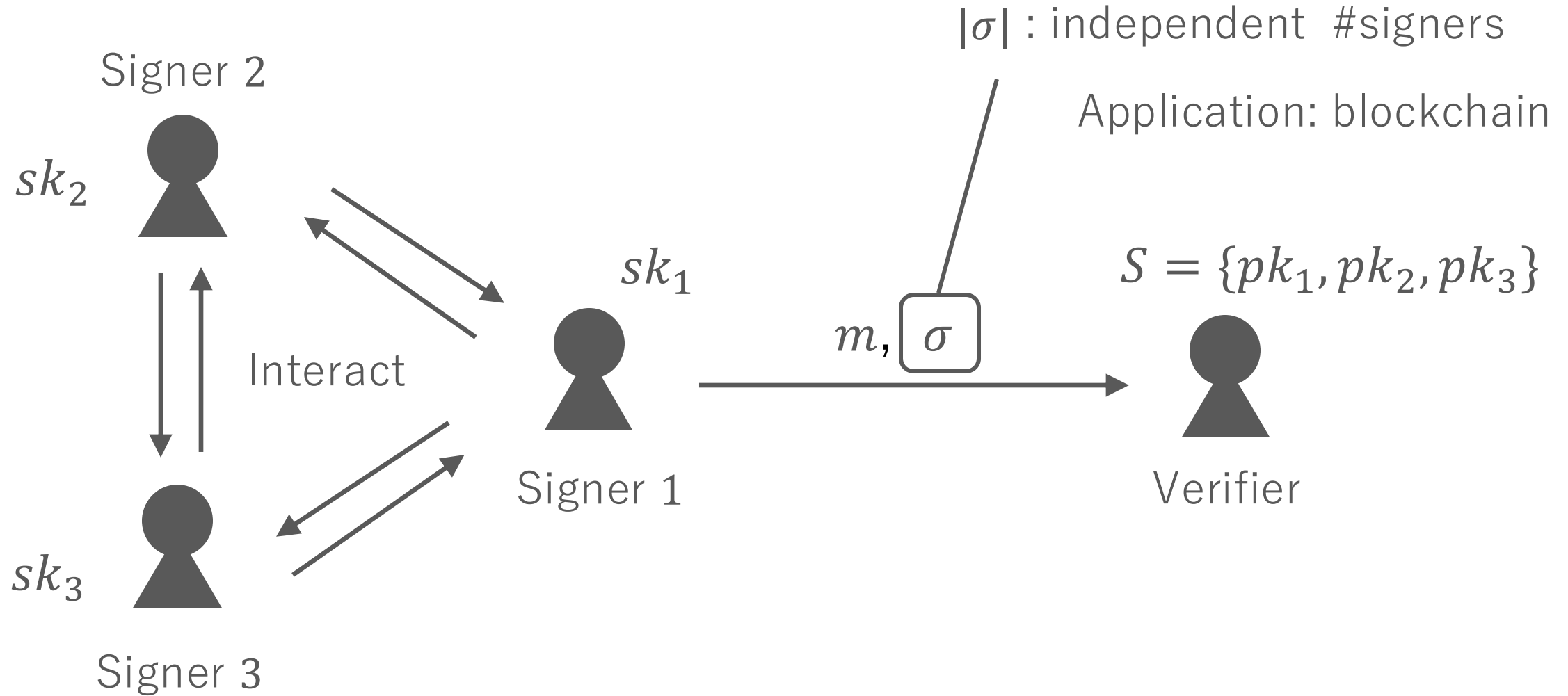
(Interactive) Multi-Signature (MS) [IN83]

4



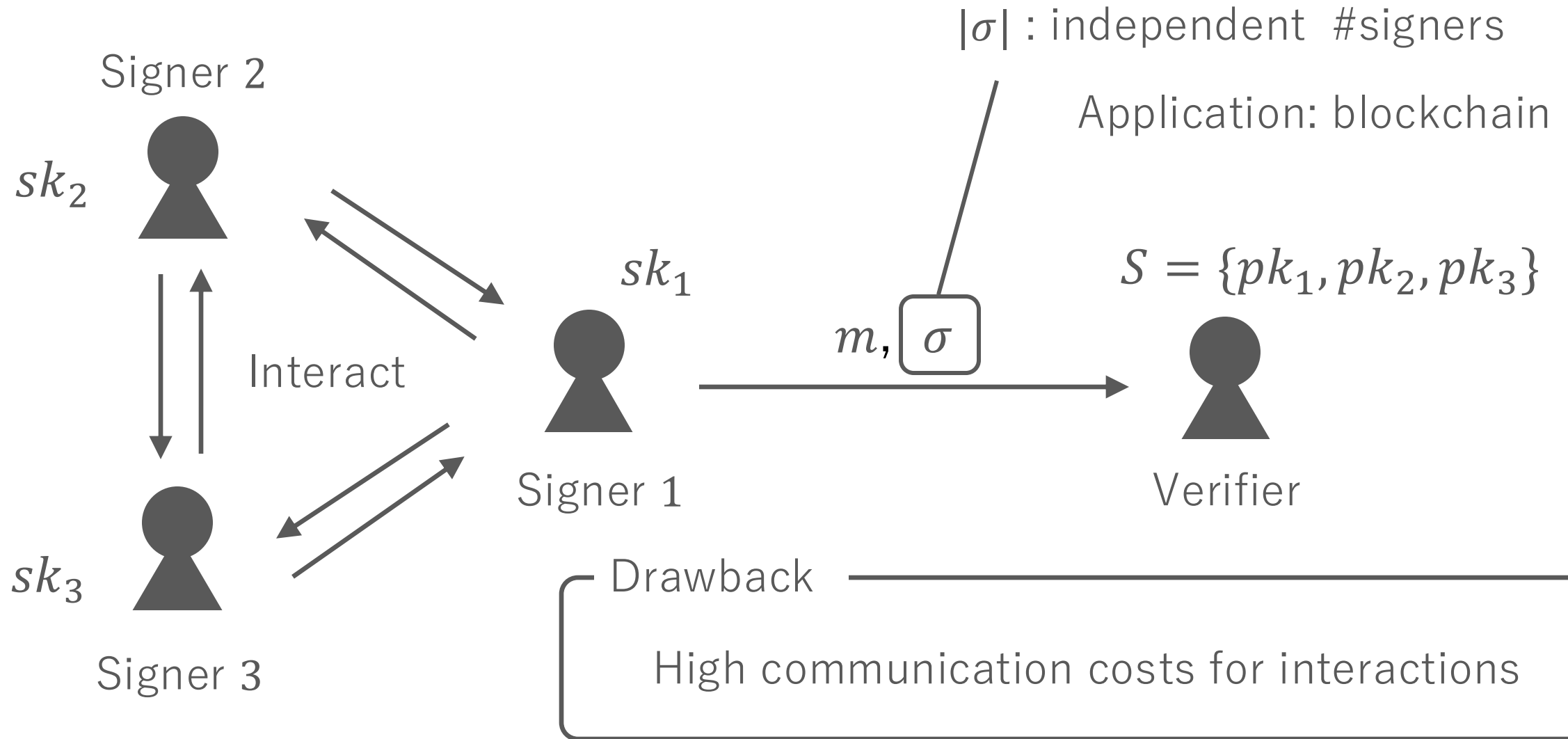
(Interactive) Multi-Signature (MS) [IN83]

5



(Interactive) Multi-Signature (MS) [IN83]

6



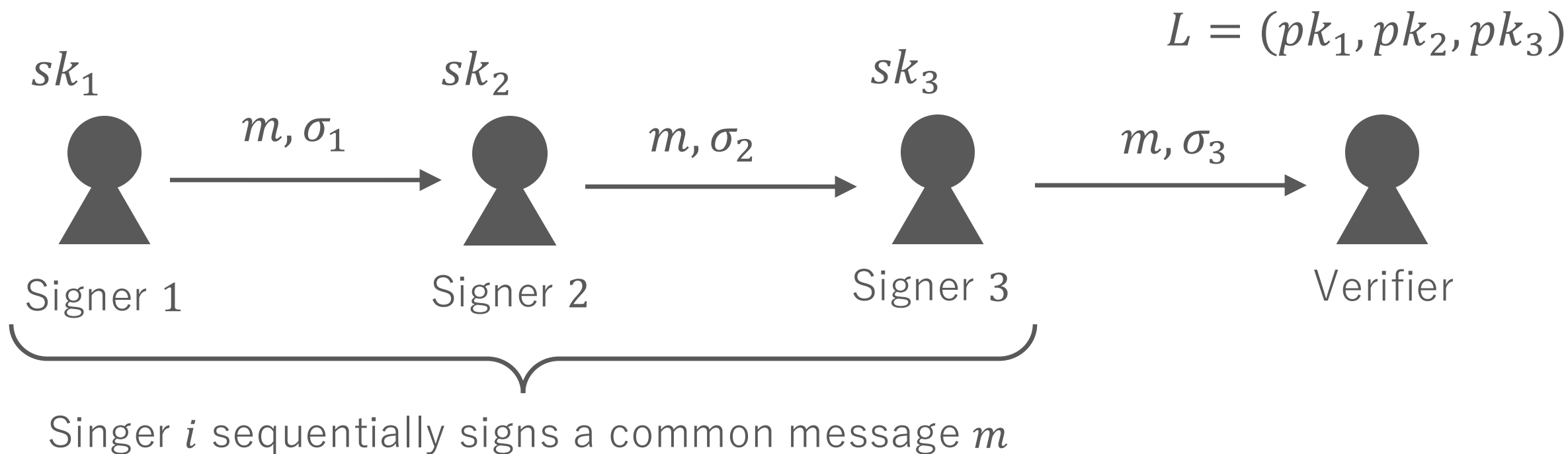
Ordered Multi-Signature (OMS) [BGOY07]

7



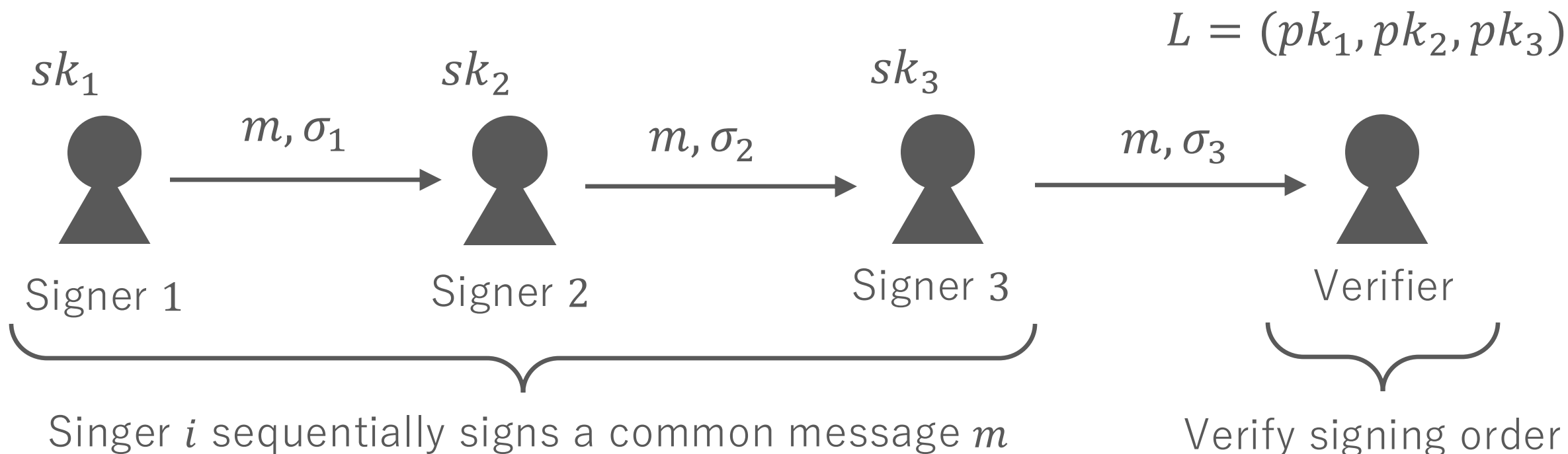
Ordered Multi-Signature (OMS) [BGOY07]

8



Ordered Multi-Signature (OMS) [BGOY07]

9

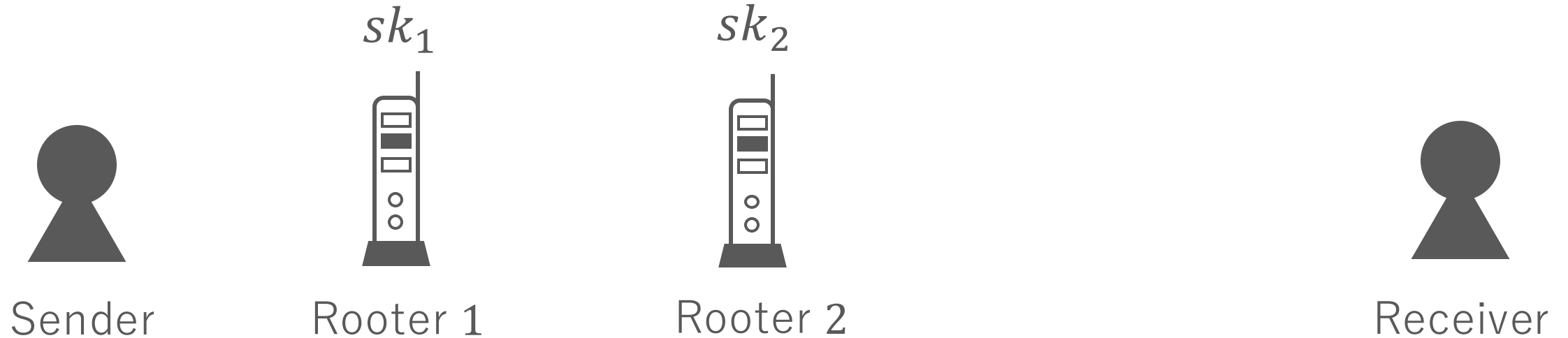


Application: Network rooting

Application of OMS for Network Rooting

10

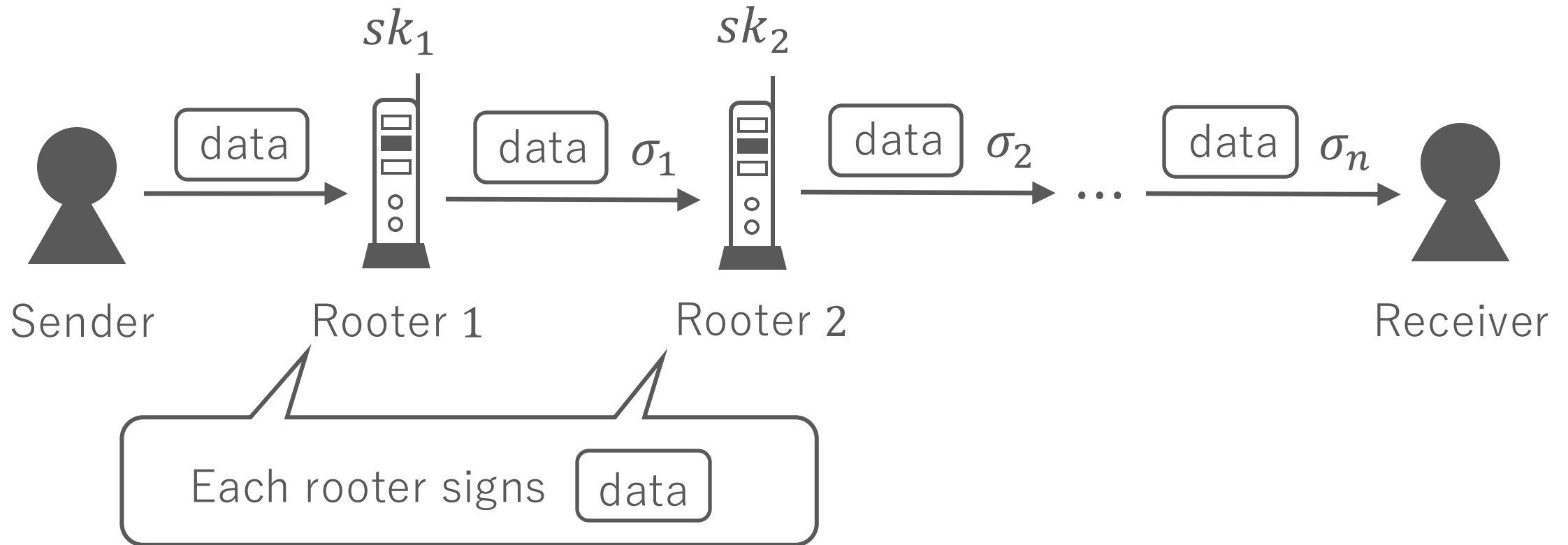
The sender sends the packet data data to the receiver.



Application of OMS for Network Rooting

11

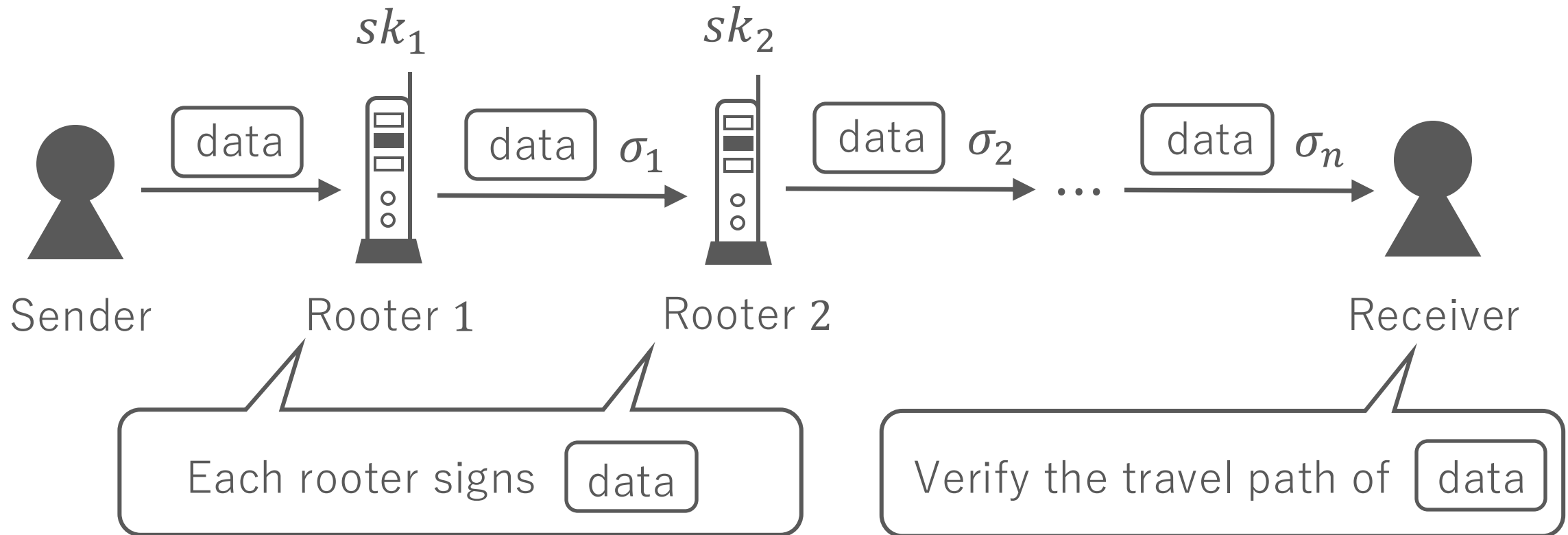
The sender sends the packet data data to the receiver.



Application of OMS for Network Rooting

12

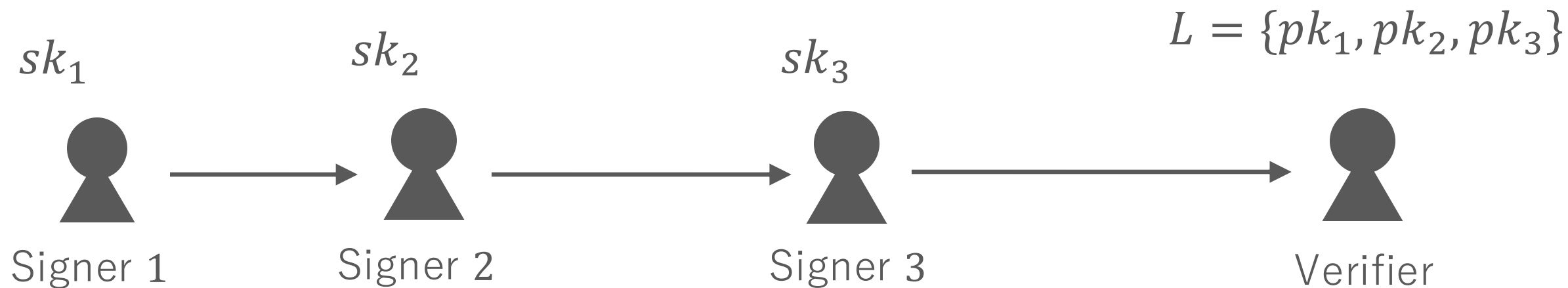
The sender sends the packet data data to the receiver.



Sequential Aggregate Signatures and Ordered Multi-Signatures

Sequential Aggregate Signature (SAS) [LMRS04]

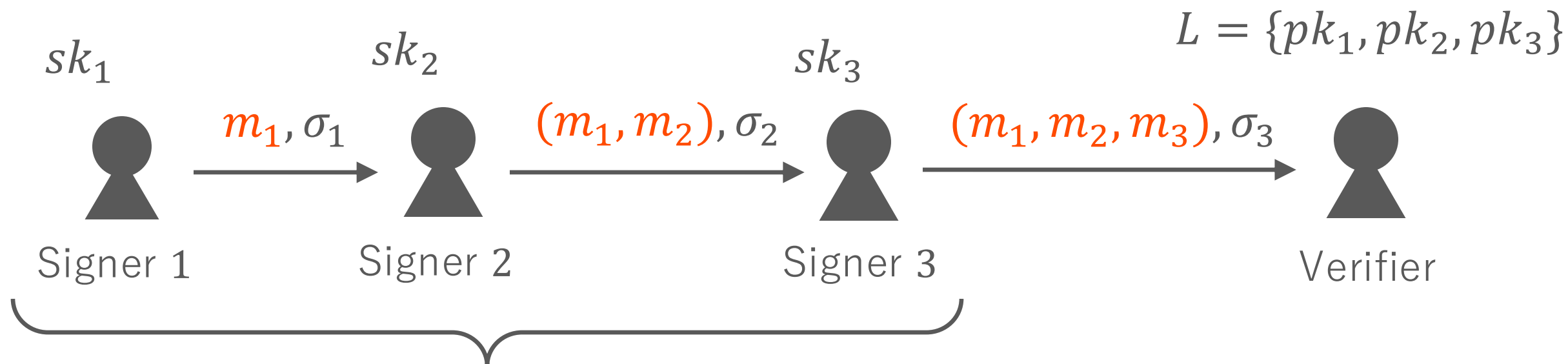
14



Two main difference between SAS and OMS !

Sequential Aggregate Signature (SAS) [LMRS04]

15

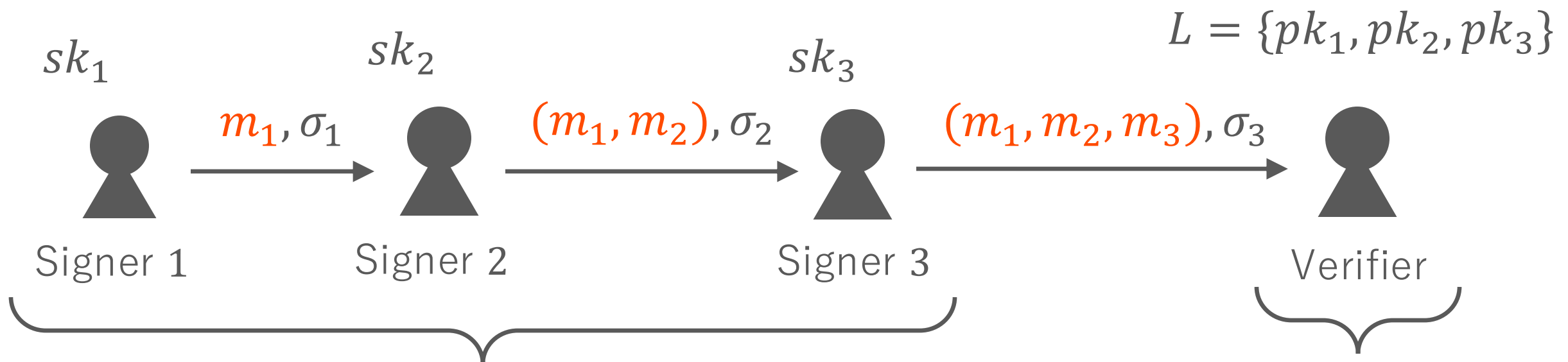


Singer i sequentially signs distinct message m_i

These are different from OMS.

Sequential Aggregate Signature (SAS) [LMRS04]

16



Singer i sequentially signs distinct message m_i

Not support
signing order verification

These are different from OMS.

Boldyreva et al.'s Transformation: From SAS to OMS

17

In SAS, simply signing a common message m does not yield OMS.

SAS does not support the signing order verification.

Boldyreva et al.'s Transformation: From SAS to OMS

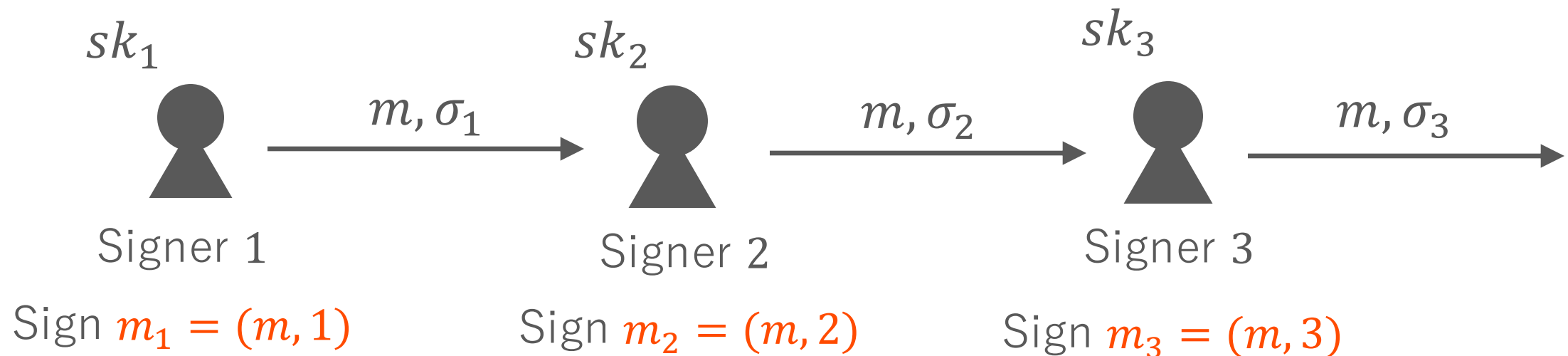
18

In SAS, simply signing a common message m does not yield OMS.

SAS does not support the signing order verification.

Boldyreva et al. [BGOY07] OMS is obtained from SAS !!

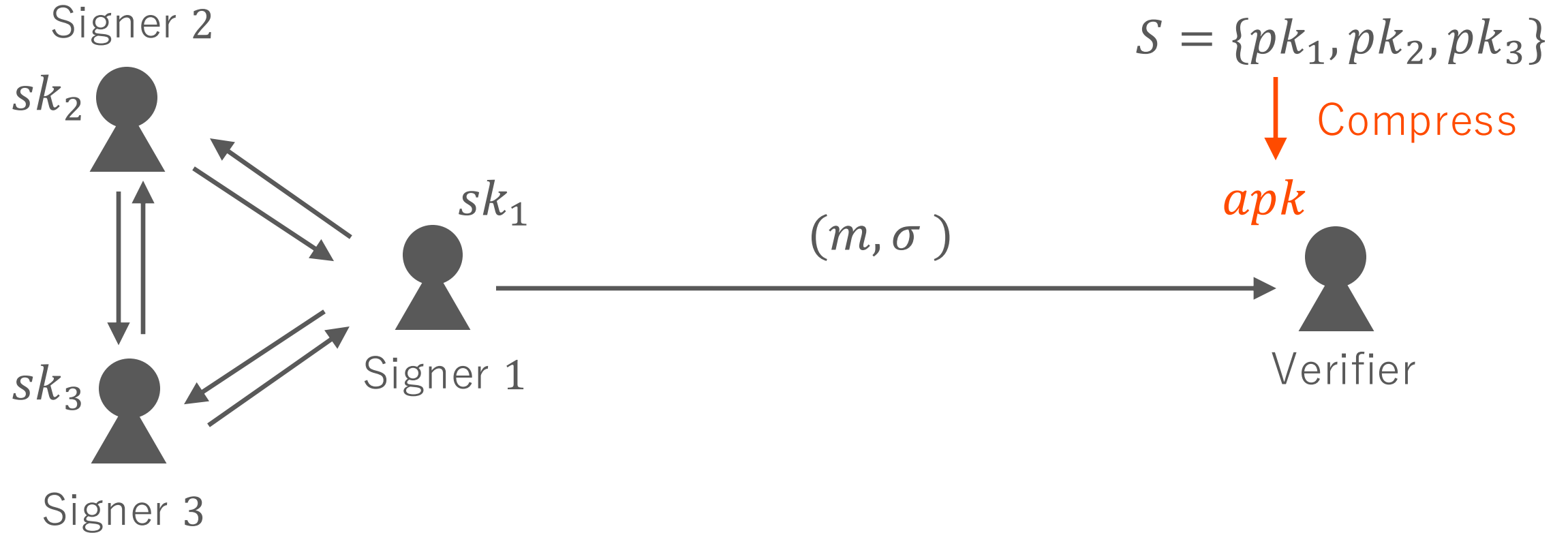
Idea : singer i signs $m_i = (m, i)$.



Public Key Aggregation for MS

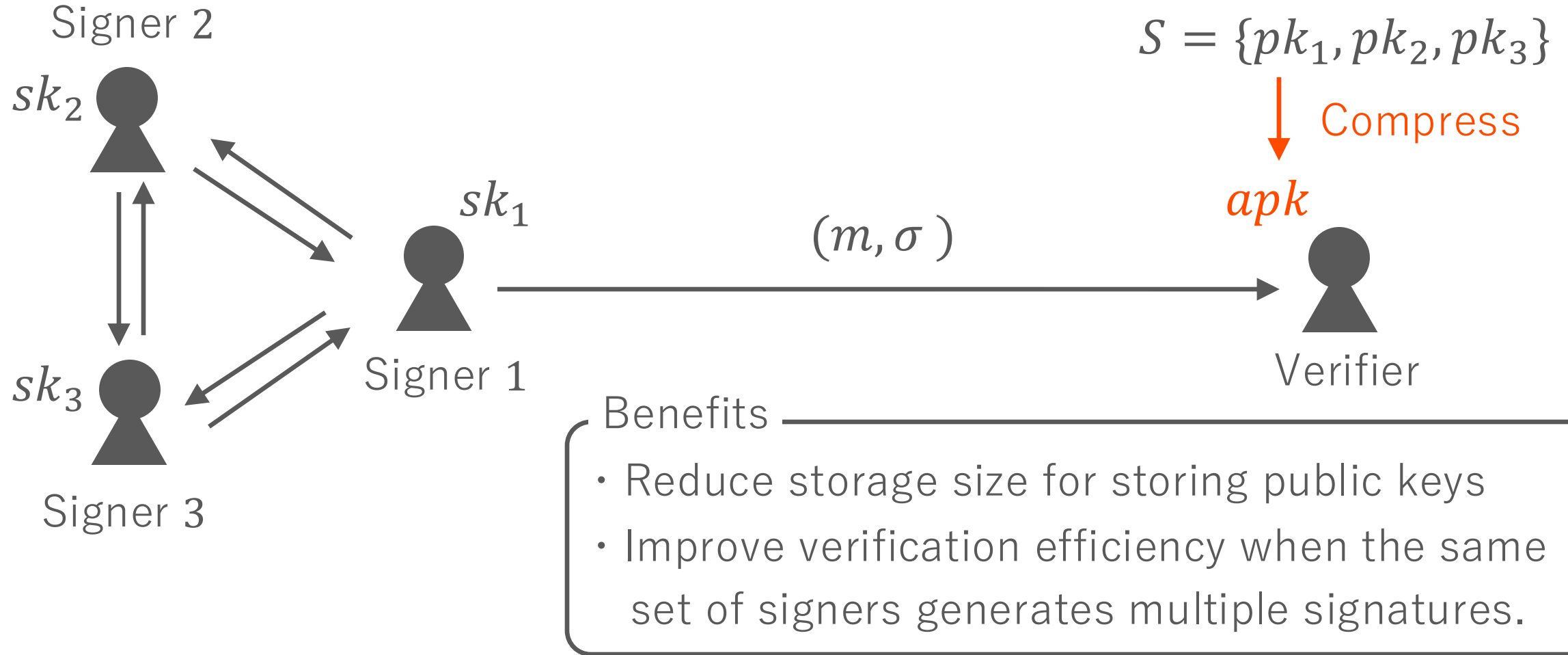
Motivation: Public Key Aggregation in MS [MPSW19]

20



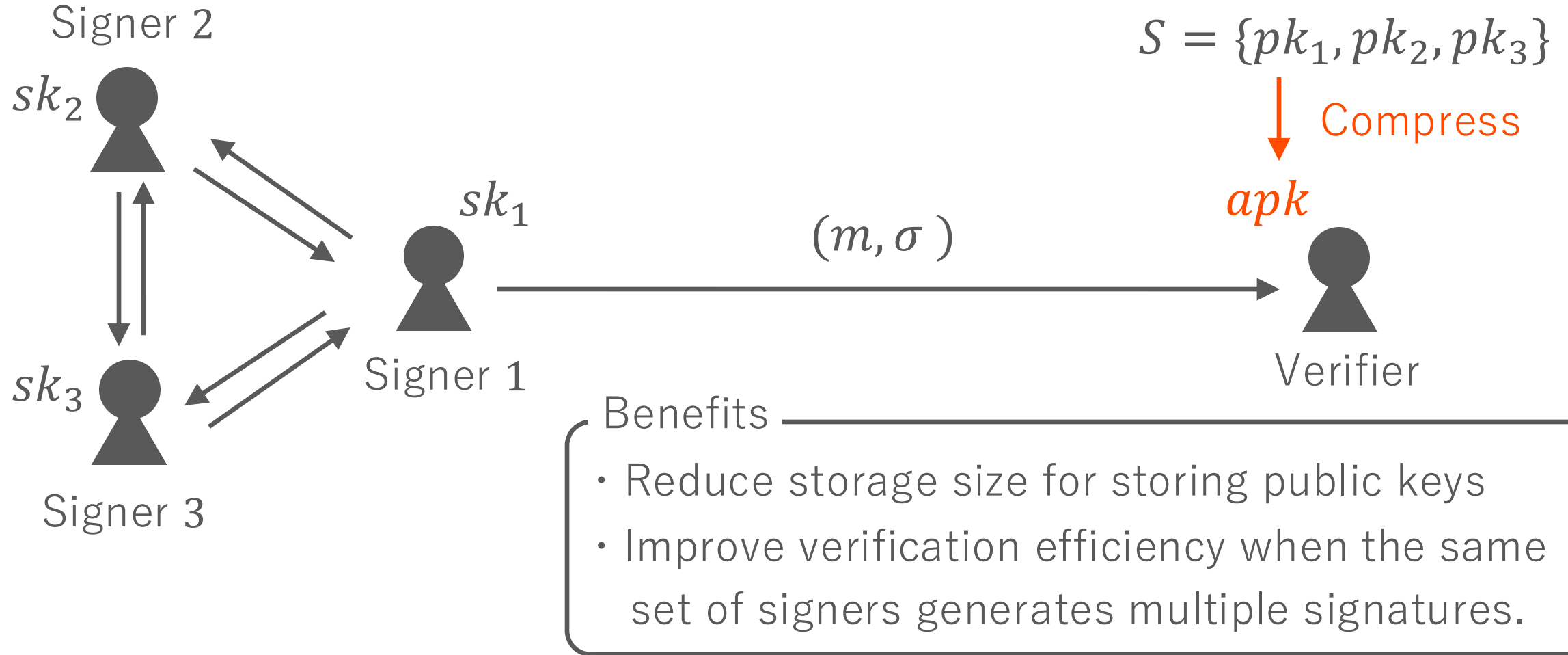
Motivation: Public Key Aggregation in MS [MPSW19]

21



Motivation: Public Key Aggregation in MS [MPSW19]

22



In OMS, some schemes support public key aggregation.

Pairing-Based OMS (OMS from SAS via Boldyreva trans) without the ROM ²³

Scheme	Assumption	pp size	pk size	sig size	PK Agg
OMS [YMO13]	CDH	$(m + 3) \mathbb{G} $	$3 \mathbb{G} $	$3 \mathbb{G} $	YES
SAS [LLY13]	LRSW (Interactive)	$2 \mathbb{G} $	$ \mathbb{G} $	$3 \mathbb{G} $	No
SAS [PS16]	PS (Interactive)	$2 \mathbb{G} + 2 \tilde{\mathbb{G}} $	$ \tilde{\mathbb{G}} $	$2 \mathbb{G} $	No
SAS [McD20]	2-PS (Interactive)	$2 \mathbb{G} + 2 \tilde{\mathbb{G}} $	$2 \tilde{\mathbb{G}} $	$2 \mathbb{G} $	YES
SAS [CK20]	SXDH	$4 \mathbb{G} + 3 \tilde{\mathbb{G}} $	$ \tilde{\mathbb{G}} $	$3 \mathbb{G} $	No

[YMO13] Yanai, Mambo, and Okamoto. An ordered multisignature scheme under the CDH assumption without random oracles. ISC 2013

[LLY13] Lee, Lee, and Yung. Aggregating cl-signatures revisited: Extended functionality and better efficiency. FC 2013

[PS16] Pointcheval and Sanders. Short randomizable signatures. CT-RSA 2016

[McD20] McDonald. The landscape of pointcheval-sanders signatures: Mapping to polynomial-based signatures and beyond. IACR Cryptol. ePrint Arch. 2020

[CK20] Chatterjee and Kabaleeshwaran. From rerandomizability to sequential aggregation: Efficient signature schemes based on SXDH assumption. ACISP2020

Pairing-Based OMS (OMS from SAS via Boldyreva trans) without the ROM ²⁴

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SAS [CK20]	SXDH	$4 \mathbb{G} + 3 \tilde{\mathbb{G}} $	$ \tilde{\mathbb{G}} $	$3 \mathbb{G} $	No

The number of group element in public parameter is **linear in message bit length**.

Scheme	Assumption	pp size	pk size	sig size	PK Agg
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The LRSW, PS, and 2-PS assumption are **interactive assumptions**.

Scheme	Assumption	pp size	pk size	sig size	PK Agg
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SAS [CK20]	SXDH	$4 \mathbb{G} + 3 \tilde{\mathbb{G}} $	$ \tilde{\mathbb{G}} $	$3 \mathbb{G} $	No

Does not support public key aggregation.

Pairing-Based OMS (OMS from SAS via Boldyreva trans) without the ROM ²⁷

Scheme	Assumption	pp size	pk size	sig size	PK Agg
OMS [YMO13]	CDH	$(m + 3) \mathbb{G} $	$3 \mathbb{G} $	$3 \mathbb{G} $	YES
SAS [LLY13]	LRSW (Interactive)	$2 \mathbb{G} $	$ \mathbb{G} $	$3 \mathbb{G} $	No
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Question

Compact pp size OMS with **PK aggregation** in the **standard assumptions**.

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SAS [CK20]	SXDH	$4 \mathbb{G} + 3 \tilde{\mathbb{G}} $	$ \tilde{\mathbb{G}} $	$3 \mathbb{G} $	No
Our Scheme	SXDH	$4 \mathbb{G} + 3 \tilde{\mathbb{G}}$	$2 \tilde{\mathbb{G}}$	$3 \mathbb{G}$	YES



How to Obtain Scheme

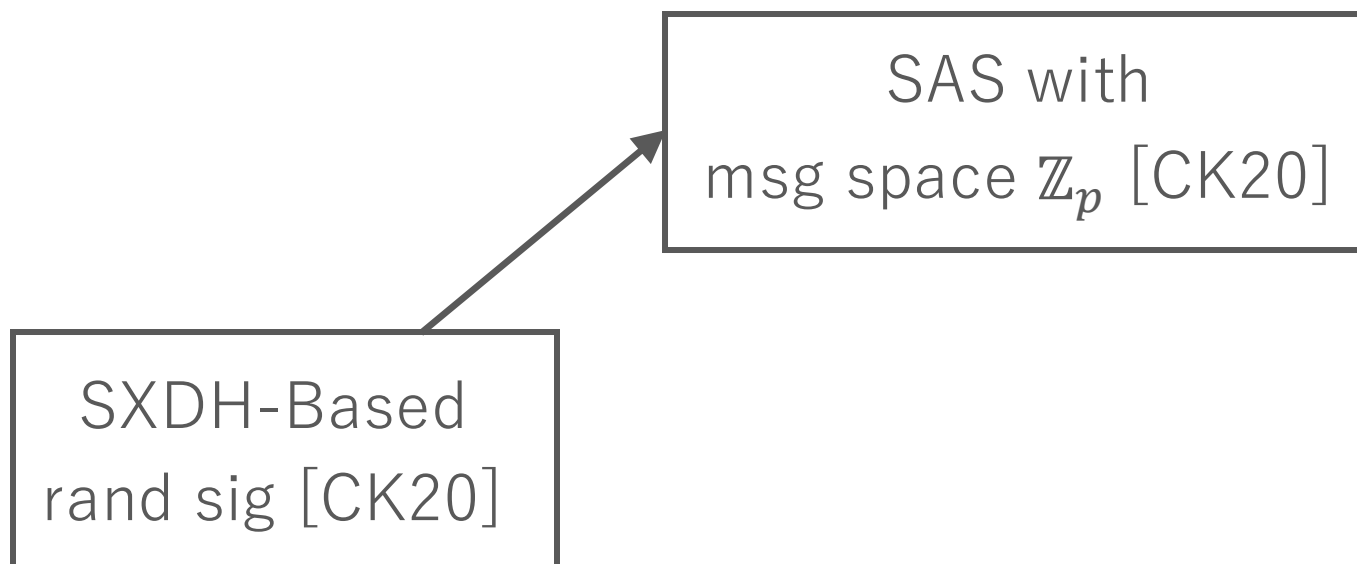
SXDH-Based SAS [CK20] and OMS

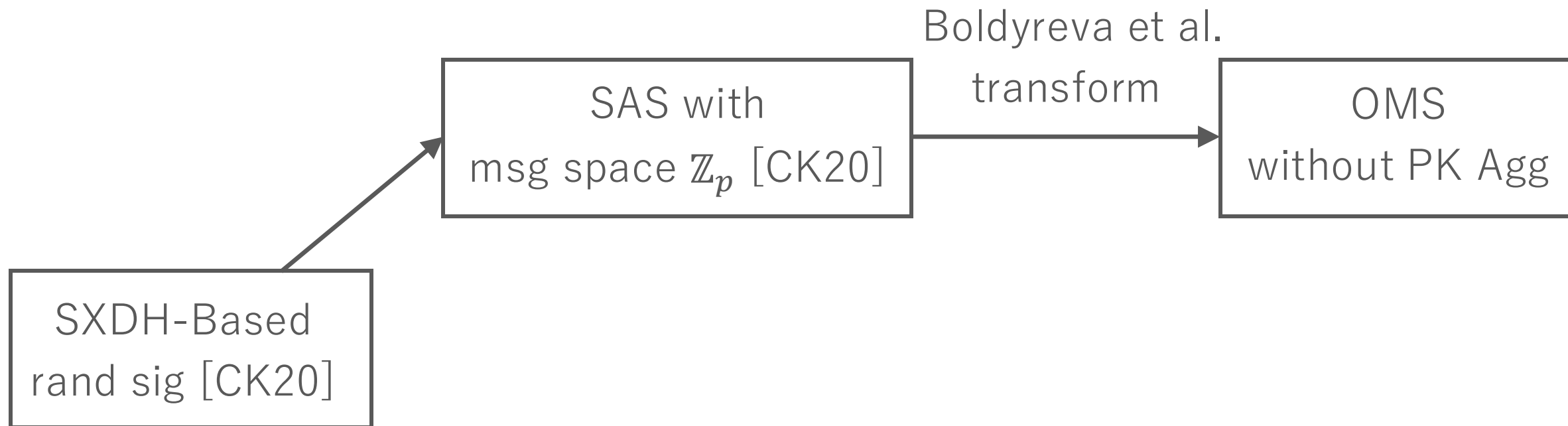
30

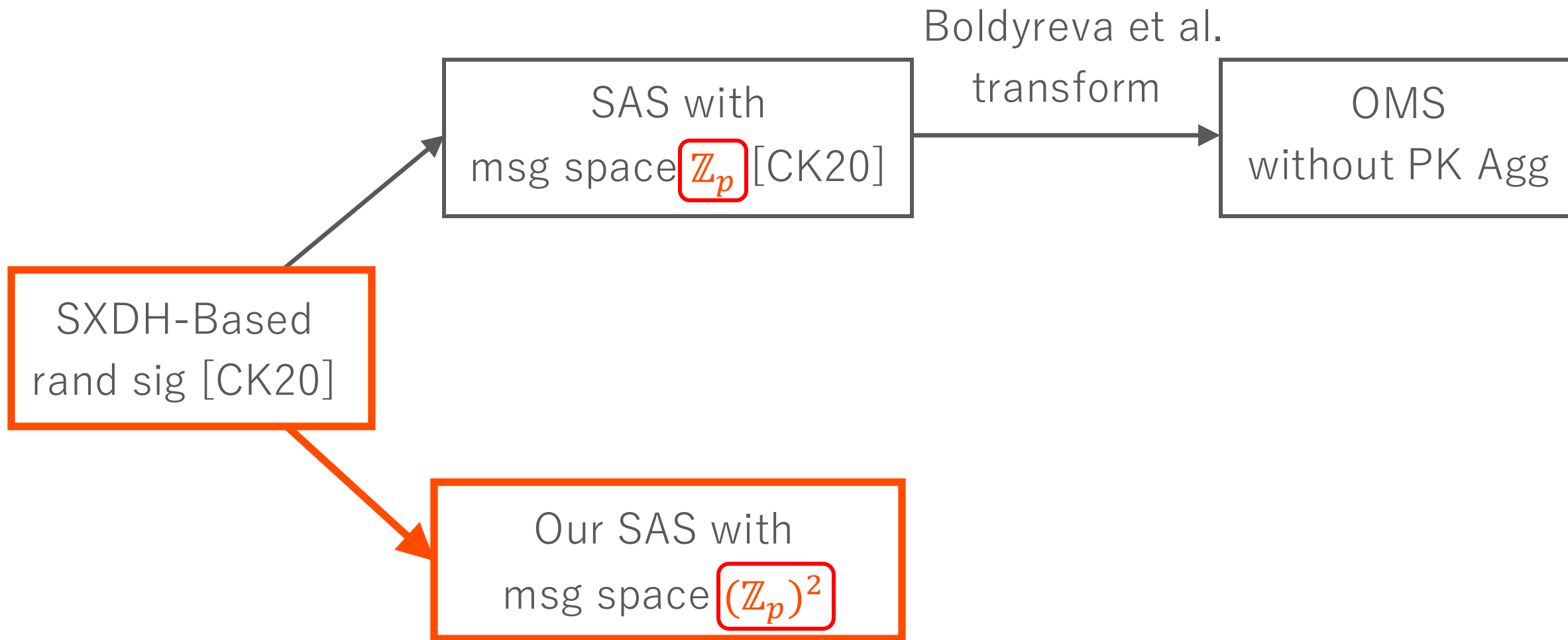
SXDH-Based
rand sig [CK20]

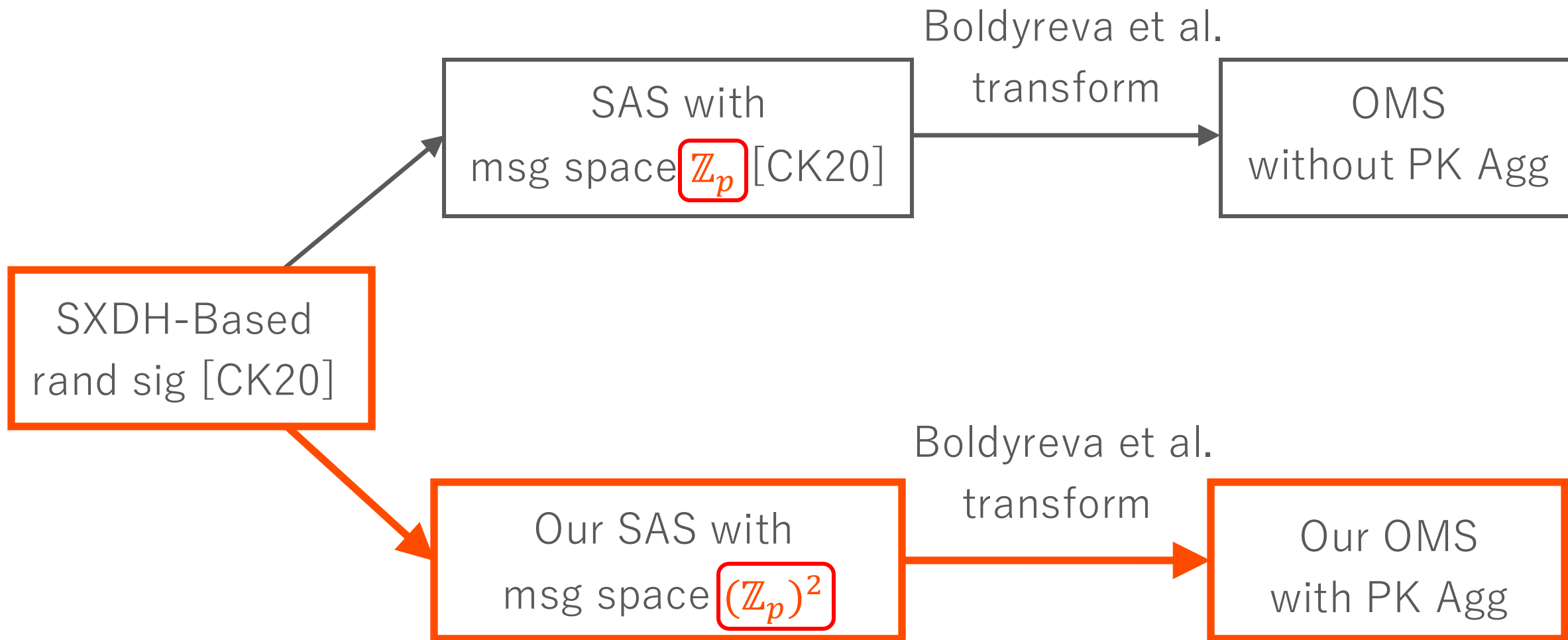
SXDH-Based SAS [CK20] and OMS

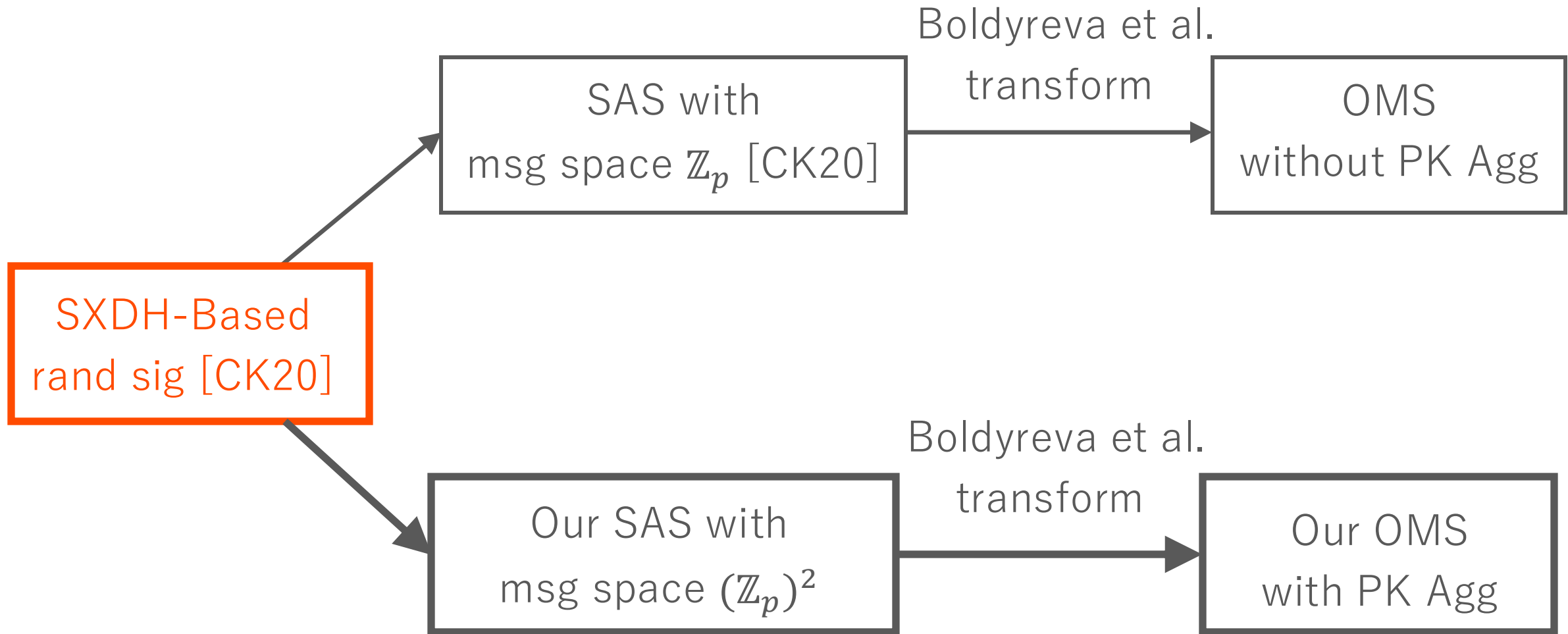
31











SXDH-Based Randomizable Signature for $\text{Msg } (\mathbb{Z}_p)^2$ [CK20]

36

$$pk = (\tilde{H}, \tilde{D}, \tilde{U}, \tilde{V}, \tilde{W}) \quad sk = (x_1, x_2, y_1, y_2, z_1, z_2)$$

$$d \leftarrow \mathbb{Z}_p^*, \tilde{D} = \tilde{H}^d, \tilde{U} = \tilde{H}^{x_2 - dx_1}, \tilde{V} = \tilde{H}^{y_2 - dy_1}, \tilde{W} = \tilde{H}^{z_2 - dz_1}$$

SXDH-Based Randomizable Signature for $\text{Msg } (\mathbb{Z}_p)^2$ [CK20]

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Signature $\sigma = (A, B, C)$ on message (m_1, m_2)

$$r \leftarrow \mathbb{Z}_p^*, A \leftarrow G^r, B \leftarrow A^{x_1 + m_1 y_1 + m_2 z_1}, C \leftarrow A^{x_2 + m_1 y_2 + m_2 z_2}$$

SXDH-Based Randomizable Signature for Msg $(\mathbb{Z}_p)^2$ [CK20]

38

$$pk = (\tilde{H}, \tilde{D}, \tilde{U}, \tilde{V}, \tilde{W}) \quad sk = (x_1, x_2, y_1, y_2, z_1, z_2)$$

$$d \leftarrow \mathbb{Z}_p^*, \tilde{D} = \tilde{H}^d, \tilde{U} = \tilde{H}^{x_2 - dx_1}, \tilde{V} = \tilde{H}^{y_2 - dy_1}, \tilde{W} = \tilde{H}^{z_2 - dz_1}$$

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Verification of $\sigma = (A, B, C)$ on message (m_1, m_2)

$$e(A, \tilde{U} \tilde{V}^{m_1} \tilde{W}^{m_2}) \cdot e(B, \tilde{D}) = e(C, \tilde{H}) ?$$

Randomizable Property

A signature $\sigma = (A, B, C)$ on message (m_1, m_2) is refreshed without a signing key sk .

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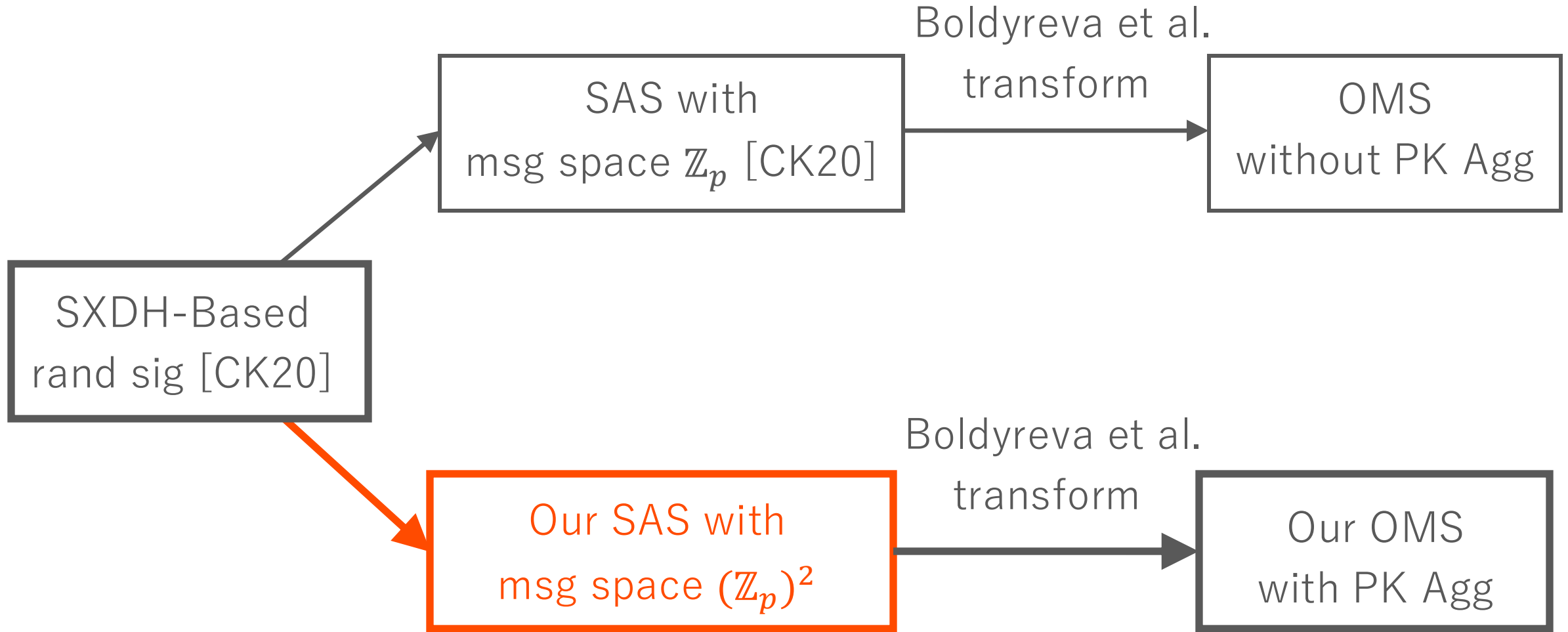
Signature $\sigma = (A, B, C)$ on message (m_1, m_2)



Randomizable property

$$r' \leftarrow \mathbb{Z}_p^*, A' \leftarrow A^{r'}, B' \leftarrow B^{r'}, C' \leftarrow C^{r'}$$

Refreshed signature $\sigma' = (A', B', C')$ on message (m_1, m_2)



How to Derive SAS with msg space $(\mathbb{Z}_p)^2$

Randomizable Signature [CK20] for Msg $(\mathbb{Z}_p)^2$



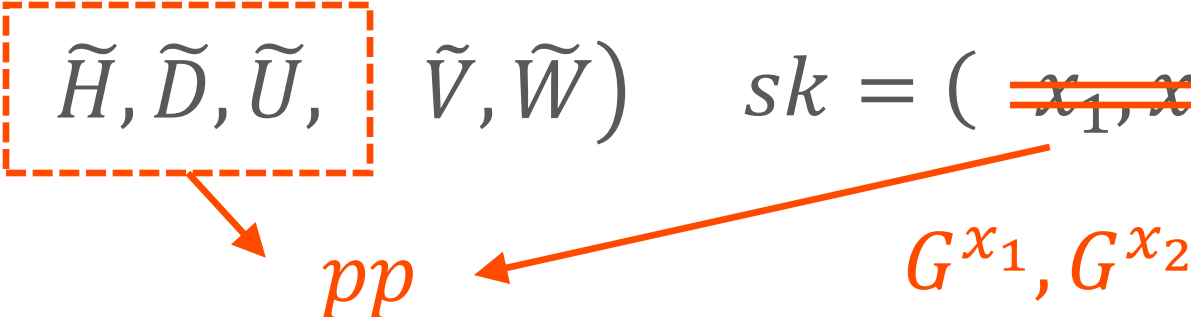
- PK sharing technique
- Modify signing algorithm

Our SAS for Msg $(\mathbb{Z}_p)^2$


$$pk = (\tilde{H}, \tilde{D}, \tilde{U}, \tilde{V}, \tilde{W}) \quad sk = (x_1, x_2, y_1, y_2, z_1, z_2)$$

PK Sharing Technique

$$pk = (\tilde{H}, \tilde{D}, \tilde{U}, \tilde{V}, \tilde{W}) \quad sk = (\cancel{x_1}, \cancel{x_2}, y_1, y_2, z_1, z_2)$$


 pp G^{x_1}, G^{x_2}

$$pk = (\tilde{H}, \tilde{D}, \tilde{U}, \tilde{V}, \tilde{W}) \quad sk = (\cancel{x_1}, \cancel{x_2}, y_1, y_2, z_1, z_2)$$



pp G^{x_1}, G^{x_2}

PK sharing technique

$$pp = (G, \tilde{H}, \tilde{D}, \tilde{U}, X_1 = G^{x_1}, X_2 = G^{x_2})$$

Singer i 's pk_i and sk_i

$$pk_i = (\tilde{V}_i, \tilde{W}_i) \quad sk_i = (y_{i,1}, y_{i,2}, z_{i,1}, z_{i,2})$$

Modifying Signing for Signer 1

Signer 1 signs $(m_{1,1}, m_{1,2})$ with $sk_1 = (y_{1,1}, y_{1,2}, z_{1,1}, z_{1,2})$

$$r_1 \leftarrow \mathbb{Z}_p^*, A_1 \leftarrow G^{r_1},$$

$$B_1 \leftarrow (X_1 G^{m_{1,1}y_{1,1} + m_{1,2}z_{1,1}})^{r_1}, C_1 \leftarrow (X_2 G^{m_{1,1}y_{1,2} + m_{1,2}z_{1,2}})^{r_1}$$

Generated signature $\sigma_1 = (A_1, B_1, C_1)$

$$A_1 = G^{r_1}$$

$$B_1 = A_1^{x + m_{1,1}y_{1,1} + m_{1,2}z_{1,1}}$$

$$C_1 = A_1^{x + m_{1,1}y_{1,2} + m_{1,2}z_{1,2}}$$

hold.

Modifying Signing for Signer $i \geq 2$

Signer i signs $(m_{i,1}, m_{i,2})$ with $sk_i = (y_{i,1}, y_{i,2}, z_{i,1}, z_{i,2})$

and updates $\sigma_{i-1} = (A_{i-1}, B_{i-1}, C_{i-1})$.

$$r_i \leftarrow \mathbb{Z}_p^*, A_i \leftarrow A_{i-1}^{r_i},$$

$$B_i \leftarrow (A_{i-1}^{m_{i,1}y_{i,1} + m_{i,2}z_{i,1}})^{r_i} \cdot B_{i-1}^{r_i}$$

$$C_i \leftarrow (A_{i-1}^{m_{i,1}y_{i,2} + m_{i,2}z_{i,2}})^{r_i} \cdot C_{i-1}^{r_i}$$

Updated signature $\sigma_i = (A_i, B_i, C_i)$

$$A_i = G^{\Pi r_k}, \quad B_i = A_i^{x + \sum m_{k,1}y_{k,1} + \sum m_{k,1}z_{k,1}}$$

$$C_i = A_i^{x + \sum m_{k,1}y_{k,2} + \sum m_{k,1}z_{k,2}} \quad \text{hold.}$$

Modifying Signing for Signer $i \geq 2$

Signer i signs $(m_{i,1}, m_{i,2})$ with $sk_i = (y_{i,1}, y_{i,2}, z_{i,1}, z_{i,2})$

and updates $\sigma_{i-1} = (A_{i-1}, B_{i-1}, C_{i-1})$.

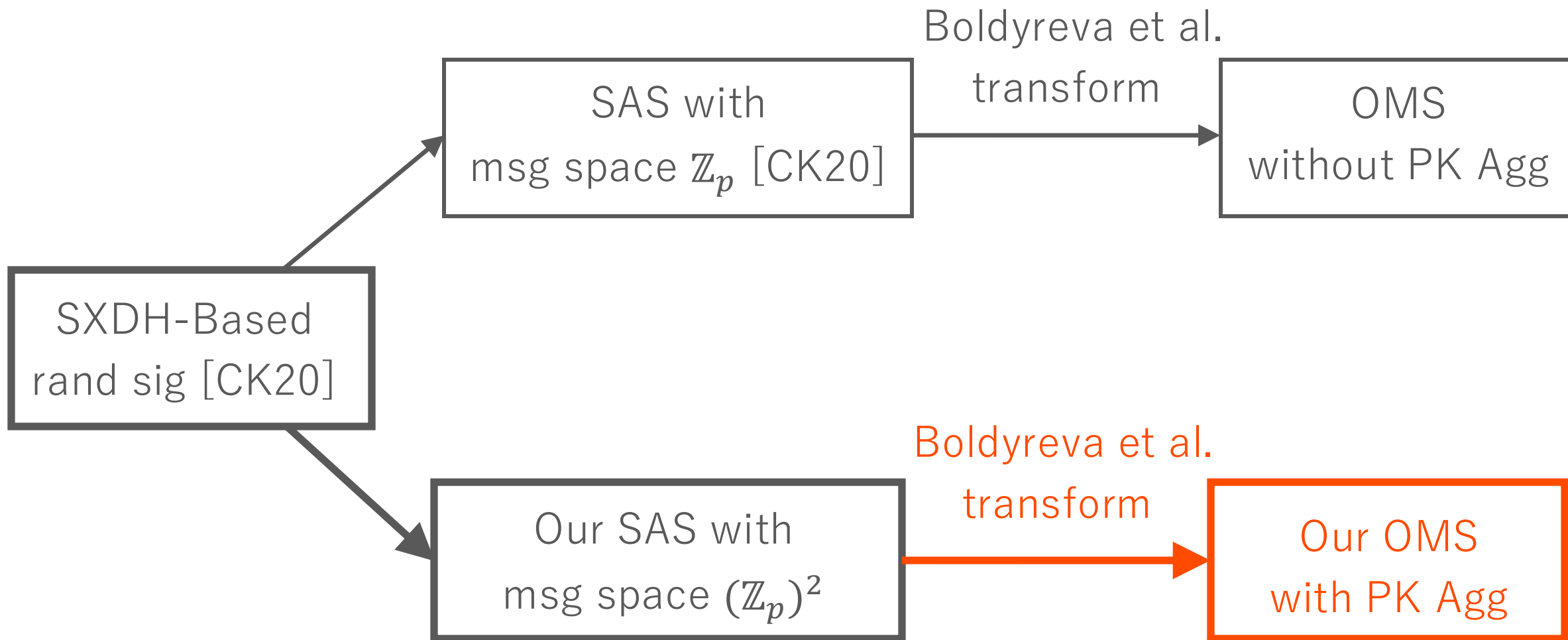
$$\begin{aligned}
 r_i &\leftarrow \mathbb{Z}_p^*, A_i \leftarrow \boxed{A_{i-1}^{r_i}}, \\
 B_i &\leftarrow (A_{i-1}^{m_{i,1}y_{i,1} + m_{i,2}z_{i,1}})^{r_i} \cdot \boxed{B_{i-1}^{r_i}}, \\
 C_i &\leftarrow (A_{i-1}^{m_{i,1}y_{i,2} + m_{i,2}z_{i,2}})^{r_i} \cdot \boxed{C_{i-1}^{r_i}}.
 \end{aligned}$$

Randomization of σ_{i-1}

Updated signature $\sigma_i = (A_i, B_i, C_i)$

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$$C_i = A_i^{x + \sum m_{k,1}y_{k,2} + \sum m_{k,1}z_{k,2}} \quad \text{hold.}$$



Singer i signs (m, i) with $sk_i = (y_{i,1}, y_{i,2}, z_{i,1}, z_{i,2})$
and updates $\sigma_{i-1} = (A_{i-1}, B_{i-1}, C_{i-1})$.

Updated signature $\sigma_i = (A_i, B_i, C_i)$

$$A_i = G^{\Pi r_k}$$

$$B_i = A_i^{x + \sum m y_{k,1} + \sum k z_{k,1}}$$

$$C_i = A_i^{x + \sum m y_{k,2} + \sum k z_{k,2}} \quad \text{hold.}$$

$$pk_i = (\tilde{V}_i, \tilde{W}_i) \quad \sigma_i = (A_i, B_i, C_i)$$

$$A_i = G^{\Pi r_k} \quad B_i = A_i^{x + \sum m y_{k,1} + \sum k z_{k,1}} \quad C_i = A_i^{x + \sum m y_{k,2} + \sum k z_{k,2}}$$

Verification of σ_i

$$e\left(A_i, \tilde{U} \cdot \left(\prod \tilde{V}_k\right)^m \cdot \prod \tilde{W}_k^k\right) \cdot e(B_i, \tilde{D}) = e(C_i, \tilde{H}) ?$$

$$pk_i = (\tilde{V}_i, \tilde{W}_i) \quad \sigma_i = (A_i, B_i, C_i)$$

$$A_i = G^{\Pi r_k} \quad B_i = A_i^{x + \sum m y_{k,1} + \sum k z_{k,1}} \quad C_i = A_i^{x + \sum m y_{k,2} + \sum k z_{k,2}}$$

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$$e\left(A_i, \tilde{U} \cdot \left(\prod \tilde{V}_k\right)^m \cdot \prod \tilde{W}_k^k\right) \cdot e(B_i, \tilde{D}) = e(C_i, \tilde{H}) ?$$

$$pk_i = (\tilde{V}_i, \tilde{W}_i) \quad \sigma_i = (A_i, B_i, C_i)$$

$$A_i = G^{\Pi r_k} \quad B_i = A_i^{x + \sum m y_{k,1} + \sum k z_{k,1}} \quad C_i = A_i^{x + \sum m y_{k,2} + \sum k z_{k,2}}$$

Verification of σ_i

$$e \left(A_i, \tilde{U} \cdot \boxed{(\Pi \tilde{V}_k)^m} \cdot \boxed{\Pi \tilde{W}_k^k} \right) \cdot e(B_i, \tilde{D}) = e(C_i, \tilde{H}) ?$$

$$apk = (\Pi \tilde{V}_k, \Pi \tilde{W}_k^k)$$

We construct the ordered multi-signature scheme with key-aggregation.

Our scheme offers compact pp , pk , and σ .

The security is proven under the SXDH assumption without the ROM.

Future work

Tightly secure OMS with PK agg in the standard model.

Thank you for listening !