Deferred Public Pension and Longevity Risk for a Household in Retirement

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Abstract In recent years, managing longevity risk, the risk of outliving one's wealth, is a very important issue for a household in retirement. Hibiki and Oya(2015) show that this risk can be hedged by combining private life pension and public pension. However, private life pension is expensive due to high expense loading. Kenjoh *et al.*(2017) describe qualitatively the advantage of deferred public pension and thus we could hedge appropriately this risk with deferral receipt of public pension and private term pension to compensate income during the period before receiving public pension. To the best of our knowledge, there are no studies that evaluate quantitatively the advantage of deferred public pension for hedging longevity risk in Japan. In this paper, we evaluate it by an optimization model and suggest strategies retired households can choose.

keywords : public pension, deferral option, longevity risk, optimization model

1. Introduction

According to the Central Council for Financial Services Information(2018), 79.2% of household in Japan are afraid of life in retirement because of insufficient pension, insurance and financial assets, and no preparations after retirement. In recent years, managing longevity risk of outliving one's wealth is a very important issue for a household in retirement. There are a lot of researches of retirement planning for household, and it is shown that it is effective to purchase private life annuity in addition to public pension annuity in order to avoid longevity risk in many literatures. Many researches have been done for deferred life annuity (Horneff *et al.*, etc.) as well as immediate life annuity (Hibiki and Oya (2015), etc.). It is indicated that the deferred life annuity is superior to the immediate life annuity because of the cheaper price and the effectiveness of managing longevity risk. However, the private life annuity has a problem that it is more expensive than the term annuity. We show the ratio of loading premium to net premium for the actual annuities sold in Japan (5-year, 10-year and 15-year term annuity and life annuity with 10-year guarantee) in Figure 1. [1]

The loading premium of life annuity is larger than those of term annuity. This shows that the private life annuity is a relatively expensive as a tool of hedging longevity risk.

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[1] We employ the standard mortality table 2007 for the expected mortality rate after annuitization, and the standard mortality table 2018 for the expected mortality rate of life insurance. The guaranteed interest rate is set at 0.75%. We obtain the actual data from the website and brochure of Mitsui Sumitomo Aioi Life Insurance Company, Ltd. as of July 31, 2018. We use the pension products with level premium, which start paying benefit at age 65.

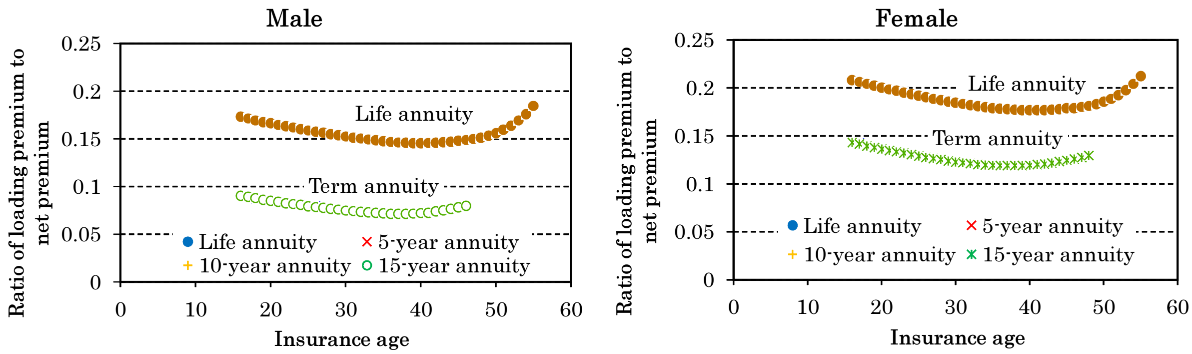


Figure 1: Ratio of loading premium to net premium

Recently, there are some previous studies with respect to the deferral options of public pension. Farrar *et al.*(2012) examine the value of incentives to defer the UK state pension by comparing the values of pension receipt at the end of the deferral period for three kinds of choice; no deferral choice (value of lump sum) and two post-deferral choice (values of the lump sum and the total enhanced pension stream). Rose(2015) examines the rate of return on delaying social security retirement benefits for individuals and for married couples in the U.S. when the start of the benefits is postponed. Genest-Gregoire *et al.*(2018) compare the differences among the cases for various starting age of receiving pension benefits through the simulations under the Canadian public pension system. They show that the efficient use of public pension benefit deferrals can reduce the cost of the precautionary behavior. Moizer *et al.*(2018) construct the state pension deferral system dynamics model, and examine the impact of pension deferral upon the sustainability of the UK's state pension system. In Japan, Kenjoh *et al.*(2017) describe that the private term annuity is purchased until the pensionable age of deferred public pension, and the longevity risk should be hedged by the public pension as shown in the right graph of Figure 2, instead of purchasing private life annuity in addition to public pension annuity for all retirement periods as shown in the left graph. The reasons are that the cost of public pension is cheaper than the private pension and the adverse selection of public pension is difficult to occur because it is mandatory. On the other hand, only one to two percentages of pension beneficiary currently use the deferral option according to Ministry of Health, Labour and Welfare(2017b)(MHLW hereafter). We examine the impact on the public pension system by the increase in the fraction of using the deferral option which is occurred by the household's behavioral change.

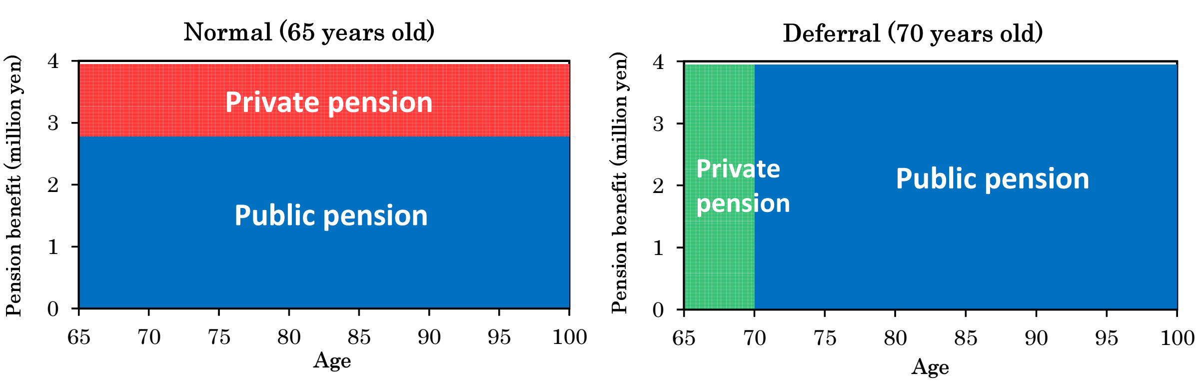


Figure 2: Comparison of cashflow in retirement

We attempt to evaluate the usefulness of deferred pension benefit in a simple setting.

Table 1 shows the private pension premiums which give the same cashflow for age 65 and 70 as in Figure 2.[2]

Table 1: Private pension premium for each pensionable age

pensionable age	65 years old	70 years old
private pension premium(million yen)	29.896	19.738

The private pension premium of life annuity is 29.896 million yen for 65 pensionable age because the life annuity is relatively expensive, whereas the premium of term annuity is 19.738 million yen for 70 pensionable age to complement the income until the receipt of pension benefit. This shows the use of deferral option is effective due to the reduction of the premium. Figure 3 shows the expected present value of old-age basic pension benefit for each pensionable age. The full amount of old-age basic pension benefit is 779,300 yen per year as of April 2018. The mortality rate is estimated by the model introduced in Section 2.5, and the guaranteed rate is 0.75%.

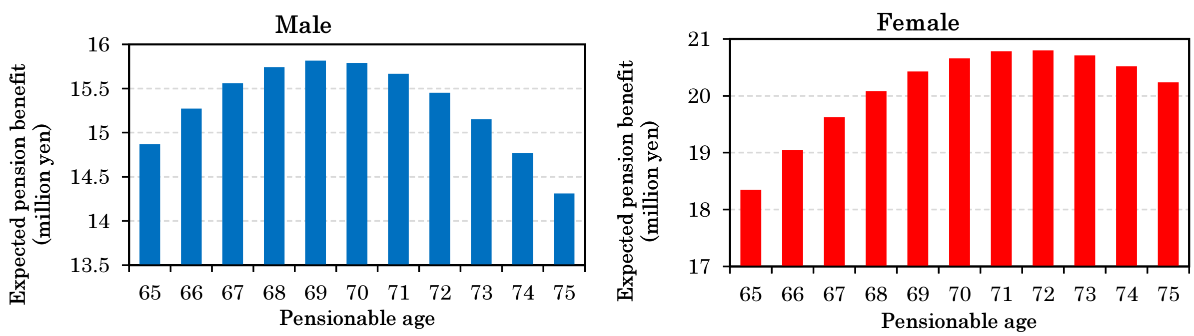


Figure 3: Expected present value of old-age basic pension benefit for each pensionable age

The expected pension benefits increase for male and female by deferring the receipts. The maximum deferred age is 69 for male, and 72 for female.[3] Although it is simple setting, we confirm the usefulness of deferral option of public pension under the assumption of current pension system and mortality rate.

However, some previous papers are studied qualitatively for the deferral option of public pension in Japan (Kenjoh *et al.*, and so on). To the best of our knowledge, we cannot find the previous researches which evaluate the longevity risk quantitatively involving the household to examine the effect of deferral option of public pension. In this paper, we examine it using the practical model in the public pension system in Japan. Specifically, we examine the usefulness of the combination of deferred public annuity and term private annuity by comparing it with the combination of normal public pension and life private annuity in order to reduce longevity risk in retirement quantitatively using the optimization model, and we propose the strategies that households should choose in the future. Moreover, we examine the impacts on the public pension finance due to changes of household behavior whereas we conduct the analysis under the current system. We investigate how to determine the increment rate for deferred pension in consideration of both the sustainability of public pension system and

[2]The private pension premiums are calculated by the parameters of term and life annuities in Section 5.1.

[3]The current pensionable age is 70 at a maximum, but we show the results under the assumption of 75 pensionable age.

longevity risk of household, and we attempt to build a win-win relationship between them. We also examine the impacts due to changes of asset mix and survivor pension system. It is difficult to manage actual household assets in retirement, because assets include saving money and investment asset except annuities as shown in Figure 2 when receiving public pension. However, we can give an appropriate and easy-to-understand message that the role of private pension is a support until receiving the public pension benefit whereas the public pension plays a role to manage longevity risk by using the deferral options. We also expect it leads to the better design of retirement planning.

This paper is organized as follows. We explain the modeling for retirement planning in Section 2. We overview the model of public pension finance in Section 3. We formulate the optimization model in retirement in Section 4. We conduct the numerical analysis in Section 5. Using the deferral option of public pension is effective to manage longevity risk in retirement quantitatively, and we propose the strategies that households should choose in the future. Moreover, we examine the impacts on the public pension finance due to changes of household behavior, asset mix, and survivor pension system. Section 6 provides our concluding remarks.

2. Modeling for retirement planning

We define a household, and describe income and consumption expenditure. We explain how to calculate investment rate of return, interest rate, and mortality rate. These parameters are used in the simulation based optimization model in Section 4. We attach a superscript (i) to random and path dependent parameters in order to formulate a model in the simulated path approach. The description $(i = 1, \ldots, I)$ is omitted, where I is the number of paths.

2.1. Model Structure

We define a household as a family composed of a householder and a spouse. Suppose the householder is just retired at 65 years old, and the spouse is also 65 years old. The planning period is 35 years from now (retirement of householder) for the 100-year life. One period is a year, and therefore we formulate the model for a 35-year problem. We calculate cash flow streams until the time when a family disappears because of death of householder and spouse or 100 years old. We assume the household income is public pension annuity, and the expenditures are minimum living cost and medical expenses. These cashflows occur at the end of each year. For example, we assume the household receives the money at age 66 for the cashflow associated with public pension benefit and expenditures from 65 to 66 years old. We invest on a risk-free asset and four risky assets: domestic stock and bond, and foreign stock and bond. We make decision of how much we purchase private pension and life insurance at time 0. We assume immediate private annuity with a lump-sum payment.

We purchase the life annuity of receiving from next year when we start to receive the public pension benefit from the standard age of 65 years old. Otherwise, we purchase the term annuity of receiving in the period until the start age of deferred public pension benefit. For example, we purchase five-year annuity of receiving from 66 to 70 years old when we defer to receive the public pension benefit, and decide to receive it at age 70. We purchase the 15-year life insurance with level payments.

Medical expenses and nursing care expenses is expected to increase as the average life expectancy grows in the future. However, the amount of self-pay is limited by the high-cost medical expense benefit and the payment of an allowance for high-cost long-term care service. Therefore, medical insurance and long-term care insurance are not included for a simple setting. On the other hand, it might be important to consider medical insurance and care insurance because residential expenses for nursing homes are not covered by the payment of an allowance for high-cost long-term care service. This is our future research.

For simplicity, non-financial assets such as houses and durable consumer goods are not included. The household income and expenditure are affected by the mortality risk. Parameters associated with the time of death are as follows.

$\tau^{(i)}_{AM,t}$: one if a householder is alive on path i at time t and zero otherwise

$\tau^{(i)}_{LM,t}$: one if a householder dies on path i at time t and zero otherwise

$\tau^{(i)}_{AF,t}$: one if a spouse is alive on path i at time t and zero otherwise

$\tau^{(i)}_{LF,t}$: one if a spouse dies on path i at time t and zero otherwise

$\tau^{(i)}_{A,t}$: one if any family member is alive on path i at time t, and zero otherwise, which is calculated as $\tau^{(i)}_{A,t} = \tau^{(i)}_{AM,t} + \tau^{(i)}_{AF,t} - \tau^{(i)}_{AM,t} \times \tau^{(i)}_{AF,t}$.

2.2. Household income

The income of the retired household consists of public and private pension benefits, and life insurance amount for a householder and a spouse.

2.2.1. Public pension and deferred receipt

Japanese public pension system has a two-tier structure. The lower tier is a national pension (basic pension) that all citizens are obligated to join, and the upper tier is a employee pension that is enrolled by company and government employees. In principle, the pensionable age of both systems is 65 years old.[4] However, we have advance and deferral options of pensionable age for public pension, and it can be determined in the range from age 60 to 70.[5] The decrement rate is 0.5% per month for advance option, and the increment rate is 0.7% per month for deferral option. The pension benefit increased by 42 % can be received by deferring the receipt for five years, and therefore it is expected that the longevity risk is managed effectively. On the other hand, the household has a possibility that the spouse may be exposed to a large income risk if the householder who receives the deferred pension benefit dies early. We show the summary of public pension systems for major countries in Table 2.[6]

The current increment rate has not been revised in Japan since it was calculated based on the average life expectancy and interest rate environment at the time due to the revision of pension system in 2000. Therefore, despite the fact that Japan's average life expectancy is one of the highest in the world, the increment rate is relatively larger than those of major countries. In the U.K., the increment rate of 10.4% was revised to 5.8 % in 2016, and the increment rate is expected to be revised in the future in Japan. In Section 5.3, we examine the impact on the household for the revision of increment rate for deferral option.

We calculate the disposal income except private annuity ($P^{(i)'}_t$) in consideration of the macroeconomic slide mechanism, survivor pension, and payment and exemption of income tax, resident tax, and social insurance premium for public pension. The public pension

[4] The second insured person who satisfies the old-age basic pension qualification can receive a specially-paid old-age employees pension from the age of 60 to 64. Gradually, the pensionable age has been raised to 65 years with the fixed portion being from 2001 to 2013 and the earnings-related component being from 2013 to 2025 (five years behind for woman).

[5] For selecting advance option, it is necessary to start receiving both the old-age basic pension and the old-age employees pension simultaneously.

[6] The table is generated according to social security and national government websites, World Health Organization(2018). We show the pensionable age as of September 24, 2018.

Table 2: Summary of public pension systems for major countries

	Country	Japan	U.S.A.	U.K.	Germany	France
pensionable age		age 65	age 67	age 68	age 67	age 62
advance option from		age 60	age 62	impossible	age 63	impossible
(reduction rate/year)		6.0%	6.7% for 3 years & 5% later		3.6%	
deferral option until		age 70	age 70	no limit	no limit	no limit
(increment rate/year)		8.4%	8.0%	5.8%	6.0%	5.0%
average life	male	81.1 years	76.0 years	79.7 years	78.7 years	80.1 years
expectancy	female	87.1 years	81.0 years	83.2 years	83.3 years	85.7 years

benefit cannot be received after the household is diminished. The disposal income of path i at time t is

$$P_t^{(i)} = \begin{cases} P_t^{(i)'}(1+T_D r_D)\tau_{A,t}^{(i)} & (t > T_D) \\ 0 & (t \leq T_D) \end{cases}, \tag{2.1}$$

where T_D is an advance year, and r_D is an increment rate per year for deferral option.

2.2.2. Private pension

There are a lot of types of private pension, but we assume that the members of household purchase the private annuity with a lump-sum payment at age 65. Specifically, the life private annuity is purchased for receiving the normal public pension, and term private annuity with the deferred period is purchased for receiving the deferred public pension. To the best of our knowledge, there are no private life annuity and less than ten-year annuity sold in Japan. Therefore, we calculate the ratio of loading premium to net premium from the ten-year annuity and life annuity with ten-year guaranteed period actually sold in Japan. We generate the virtual term annuities with one to five years, and calculate the premiums with a lump-sum payment. However, when we apply the ratio of loading premium to net premium of the products with level payments to the product with a lump-sum payment, the rate of receipt (cumulative pension receipt divided by total premium paid) is less than 100% for the term annuity with a one-year to five-year maturity even if a woman survives to maturity because of the low guaranteed rate under the low interest rate environment. This shows an unrealistic result. Therefore, the term annuity was set so that the rate of receipt can be 100% when both men and women survived to maturity. Actually, the ratio is set to be smaller than this value, and the strictest level (an unfavorable level if the public pension is deferred) is assumed for the analysis.

2.2.3. Life insurance

There are a lot of types of life insurance, but we assume that the members of household purchase the 15-year term insurance without cash value in this paper. We then employ the data of "Bridge", 15-year term insurance with level payment, actually sold by OLIX life insurance corporation in Japan.

2.3. Household expenditures

Household expenditures consists of minimum living cost, medical expenses, pension premium and life insurance premium purchased at time 0.

2.3.1. Minimum living cost

The minimum living cost is the cost required for daily life, and it is dependent on the current income. We estimate the relationship between public pension benefit P(million yen) and minimum living cost C_d(million yen) using the national survey of family income and expenditure 2014 by the Statistic Bureau, the Ministry of Internal Affairs and Communications(2015)(SB-MIAC, hereafter).[7] However, we remove the data of public pension benefit with less than 0.8 million yen because of few number of data. We also remove the data with more than 5.2 million yen because the minimum living cost becomes flat for the household with a large pension income. We assume that the minimum living cost is flat for public pension income of more than 3.96 million yen per year. The estimated regression is

$$C_d = \begin{cases} 0.98808 + 0.659P & (P \leq 3.96) \\ 3.59772 & (3.96 \leq P) \end{cases} \qquad R^2 = 0.984. \tag{2.2}$$

We assume the minimum living cost is constant at each time, and is calculated based on the normal pension benefit even if the public pension is deferred. However, it is dependent on the inflation, and it can be reduced to κ times after the death of one of the couple. The minimum living cost of path i at time t is expressed by

$$C_{d,t}^{(i)} = C_d \cdot \prod_{k=1}^{t}(1 + f_k^{(i)}) \cdot \tau_{A,t}^{(i)} \cdot \left\{ \kappa + \tau_{AM,t}^{(i)} \tau_{AF,t}^{(i)} (1 - \kappa) \right\} \tag{2.3}$$

where $f_t^{(i)}$ is inflation rate of path i at time t

2.3.2. Medical expense

The increase in medical expense is a risk factor in retirement, and it is defined separately from the minimum living cost. We assume the medical expense follows a log-normal distribution, and they are autocorrelated between time $t-1$ and t. We define the total medical expenses of a householder and a spouse $(h_{M,t}^{(i)},\ h_{F,t}^{(i)})$ as medical expense of age $a_0 + t$ at time t $(h_{M,(a_0+t)},\ h_{F,(a_0+t)})$ multiplied by the random number $(\epsilon_{M,t}^{(i)},\ \epsilon_{F,t}^{(i)})$.[8] We use the data of the national medical expense per person of MHLW(2017a), and the medical expenses are dependent on the inflation. The total medical expenses of a householder and a spouse are

$$h_{M,t}^{(i)} = h_{M,(a_0+t)} \cdot \prod_{k=1}^{t}(1 + f_k^{(i)}) \cdot \epsilon_{M,t}^{(i)} \cdot \tau_{AM,t}^{(i)}, \tag{2.4}$$

$$h_{F,t}^{(i)} = h_{F,(a_0+t)} \cdot \prod_{k=1}^{t}(1 + f_k^{(i)}) \cdot \epsilon_{F,t}^{(i)} \cdot \tau_{AF,t}^{(i)}. \tag{2.5}$$

The medical expense of a household of path i considering the self-pay ratio and the amount of self-pay limit is denoted by $H_t^{(i)}$. The amount of self-pay limit is dependent on the household income. The self-pay ratio is 0.3 for age of less than 70, 0.2 for age 70 or more and less than 75, and 0.1 for age of more than 75, The amount of self-pay limit per month

[7] We employ the data of one-month income and expenditure per household ranked by the pension benefits of public and governmental pension, and corporate and individual pension for the married couple with 65-year husband and 60-year wife. In this paper, the minimum living cost is defined as the consumption expenditure minus healthcare cost.

[8] $h_{M,a}$ and $h_{F,a}$ are medical expenses of male and female of age a at time 0, respectively. These are constant except inflation rates in the planning period.

is 57.6 thousand yen regardless of age. The medical expense of path i at time t is

$$H_t^{(i)} = \min((h_{M,t}^{(i)} + h_{F,t}^{(i)}) \cdot SP_t,\ 69.12) \tag{2.6}$$
$$\text{where } SP_t = \begin{cases} 0.3 \quad (t = 1, \ldots, 5) & : \text{age of less than 70} \\ 0.2 \quad (t = 6, \ldots, 10) & : \text{age 70 or more and less than 75} \\ 0.1 \quad (t = 11, \ldots, 35) & : \text{age of more than 75} \end{cases}$$

where SP_t is the self-pay ratio at time t.

2.4. Investment asset

We assume a retired household invests in J risky assets and a risk-free asset. A risk-free asset is denoted by r_0 at time 0, and $r_t^{(i)}(t = 1, \ldots, T-1)$ at time t. The rate of return of j-th risky asset is denoted by $\mu_{jt}^{(i)}(j = 1, \ldots, J,\ t = 1, \ldots, T)$. The rate of return of the portfolio of path i at time t is

$$\mu_{P1}^{(i)} = \sum_{j=1}^{J} \mu_{j1}^{(i)} x_{j0} + r_0 x_{00}, \tag{2.7}$$
$$\mu_{Pt}^{(i)} = \sum_{j=1}^{J} \mu_{jt}^{(i)} x_{j,t-1} + r_{t-1}^{(i)} x_{0,t-1} \quad (t = 2, \ldots, T), \tag{2.8}$$

where x_{0t} is a weight of risk-free asset at time t, and x_{jt} is a weight of j-th risky asset.

We generate random samples of risk-free rate $r_t^{(i)}$, and rate of return of j-th asset $\mu_{jt}^{(i)}$ using the Monte Carlo method, and we calculated the rates of return of the portfolio by substituting them into Equations (2.7) and (2.8). We assume the risk-free rate is one-year spot rate of JGB(Japanese government bond) expressed by the interest rate model, and the rate of return of risky asset follows a multivariate normal distribution, The discount rate is calculated by the spot rate at time 0.

2.4.1. Interest rate model

We express term structures of interest rate (yield curves) using the Nelson-Siegel model. We utilize the dynamic Nelson-Siegel model developed by Nelson-Siegel(1987) and modified by Diebold and Li(2006) to express dynamics of term structure in the future. The model includes three factors that express components of a yield curve called "level", "slope" and "curvature". The spot rate of maturity τ at time t is expressed as,

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau}\right) + \beta_{3t}\left(\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau}\right), \tag{2.9}$$

where β_1, β_2, and β_3 denote the factor values of "level", "slope" and "curvature", respectively. We estimate the term structure parameters β_{1t}, β_{2t}, and β_{3t}. We employ the forty spot rates from 0.5 to 20 years with 0.5-year increments in between which are derived using "Interest rate information" from 1993 to 2015 published by Ministry of Finance, through spline interpolation of annual par yield of one to ten, fifteen, and twenty years.[9] As with Diebold and Li(2006), we assume the value of λ is constant because the impact on the form of yield curve is limited. The value of λ is determined so that the coefficient of β_3 can be maximized at 2.5-year maturity in Diebold and Li(2006). On the other hand, each value of λ_τ^* is calculated so that the coefficient of β_3 can be maximized for each maturity τ in

[9] We use the data before 2015 in order to remove the impact on the negative interest rate which has been introduced by the Bank of Japan since January 2016.

Nakatani(2014). In this paper, we determine λ using the same method as Nakatani(2014). Specifically, we calculate the square root of mean squared error between the estimated value and actual value when the yield curve is estimated using λ^*_τ, and λ^*_n corresponding to maturity n with minimum square root among λ^*_τ is used as λ. We estimate the parameters as follows.

1. Estimating λ
   We estimate λ with reference to Nakatani(2014), and obtain $\hat{\lambda} = 0.299$.
2. Estimating β_{1t}, β_{2t}, and β_{3t}
   We estimate $\hat{\beta}_{1t}$, $\hat{\beta}_{2t}$, and $\hat{\beta}_{3t}$ by the minimum least square method using $\hat{\lambda}$.
$$y_t(\tau) = \hat{\beta}_{1t} + \hat{\beta}_{2t} H_2 + \hat{\beta}_{3t} H_3 + \epsilon_t, \tag{2.10}$$
   where H_2 and H_3 are loading factors of β_{2t} and β_{3t}, respectively.
3. Estimating AR(1) model
   We estimate $\hat{c}_j$ and $\hat{\gamma}_j$ by the minimum least square method using $\hat{\beta}_{1t}$, $\hat{\beta}_{2t}$, and $\hat{\beta}_{3t}$.
$$\beta_{jt} = \hat{c}_j + \hat{\gamma}_j \beta_{j,t-1} + \epsilon_{jt} \quad (j = 1, 2, 3) \tag{2.11}$$

We show the result in Table 3.

Table 3: Estimated paramets and coefficient of determination

	$\hat{c}_j$(intercept)	$\hat{\gamma}_j$	R^2
β_{1t}	0.00430	0.818	0.792
β_{2t}	-0.00826	0.689	0.464
β_{3t}	-0.00890	0.632	0.394

We forecast the future value of β by the parameters estimated by the above procedures, and estimate future yield curve using Equation (2.9). We generate sample paths of risk-free rate $r_t^{(i)} = \max(0.00001, y_t^{(i)}(1))$.[10]

2.4.2. Estimating rate of return of risky assets

Asset classes are domestic stock (DS), domestic bond (DB), foreign stock (FS), and foreign bond (FB). Each index is TOPIX for DS, FTSE Japan GBI LCL for DB, MSCI International index for FS, and FTSE Non-JPY World Government Bond Index in JPY term for FB. We assume the rates of return are normally distributed, and estimate the expected values and standard deviations of rate of return, and correlations between rates of return. We show the expected rates of return and standard deviations for risky assets, estimated using the historical annual rate of return from 1993 to 2015 in Table 4.

Table 4: Expected rates of return and standard deviations for risky assets

	DS	DB	FS	FB
Expected rate of return	3.13%	2.73%	7.52%	6.79%
Standard deviation	24.43%	3.08%	18.53%	12.41%

We show the correlation matrix among rate of return of risky assets and error terms of AR(1) model of interest rate in Table 5.

[10] We set the minimum value of deposit interest rate as of July 31, 2018 as the lower bound.

Table 5: Correlation matrix among risky assets and error terms of interest rate model

	DS	DB	FS	FB	β_1	β_2	β_3
DS	1.000	-0.159	0.551	0.001	-0.025	-0.071	0.200
DB		1.000	0.082	0.362	-0.268	-0.415	-0.394
FS			1.000	0.290	0.329	-0.340	-0.047
FB				1.000	-0.047	-0.163	-0.263
β_1					1.000	-0.645	0.099
β_2						1.000	0.161
β_3							1.000

2.5. Mortality rate

We forecast the future mortality rates of Japanese using a well-known Lee-Carter method in order to generate the dynamic life table involving the calendar year effect explicitly. Lee and Carter(1992) formulate the log-mortality rate of age x at time t, as

$$\ln m_{xt} = a_x + b_x k_t + \epsilon_{xt} \quad (x = 0, \ldots, \omega; t = 1, \ldots, T) \tag{2.12}$$

where ε_{xt} is a random error, and ω is a maximum age in the mortality rate data. The parameter a_x shows the log-mortality rate of age x which is independent of the calendar year, k_t shows the calendar year effect to the mortality rate which is dependent on time t, and b_x shows the sensitivity of the mortality rate of age x for the parameter k_t.

We estimate the three kinds of parameters a_x, b_x, and k_t, using the Japanese Mortality Database from 1970 to 2015 of National Institute of Population and Social Security Research. However, we remove 1995 and 2011 data due to avoiding the impacts of the Great Hanshin-Awaji Earthquake and the Great East Japan Earthquake. Lee and Carter(1992) calculate the future values of k_t for mortality rate in the U.S.A., using the ARIMA(0,1,0) model. On the other hand, it is more natural to assume that the mortality rate is expected to be improved not rapidly but gradually in the future because it is caught up with developed countries rapidly after the war, and reached the highest level in the world. The actual slope also becomes gradually gentle. Komatsu(2002) adopts the average of the estimates for the exponential function $k_t = \alpha_1 + \alpha_2 \exp\left(\frac{t+\alpha_4}{\alpha_3}\right)$, and logarithmic function $k_t = \beta_1 + \beta_2 \ln(t + \beta_3)$, where α_n and β_n are constant values. In this paper, the future values of k_t is calculated using the same method as Komatsu(2002), but the exponential function is set to $k_t = \alpha_1 + \alpha_2 \exp(t + \alpha_3)$, from the viewpoint of solution stability.

We calculate the survival rate of each 65-year-old individual who was alive as of 2015, using a dynamic life table, and it is shown in Figure 4.

The survival rate calculated using the dynamic life table has the increasing tendency for both male and female, compared to the 22nd life table by MHLW(2015), and the longevity risk is larger in the future. Therefore, it is important to examine a more effective method of hedging longevity risk.

3. Modeling public pension finance

The increase in the number of receiving the deferred pension may affect the public pension finance. We construct the model of public pension finance with reference to Yokoyama(2013) in order to examine the effect. We overview the model of public pension finance in Figure 5.

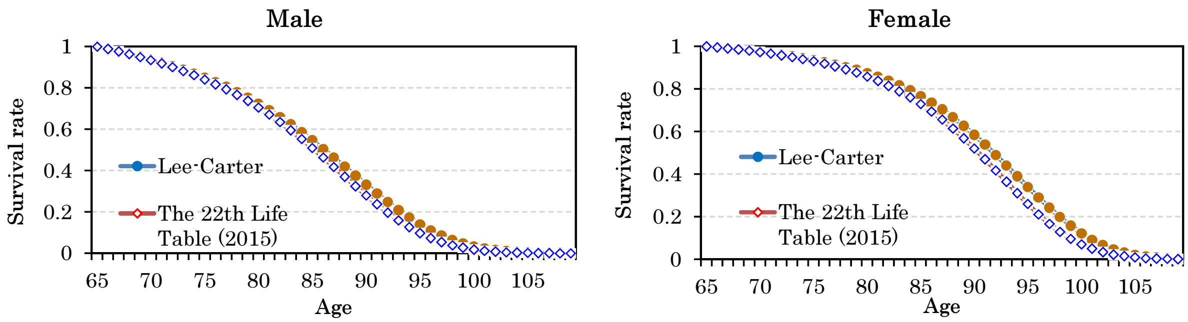


Figure 4: Survival functions of age 65 as of 2015

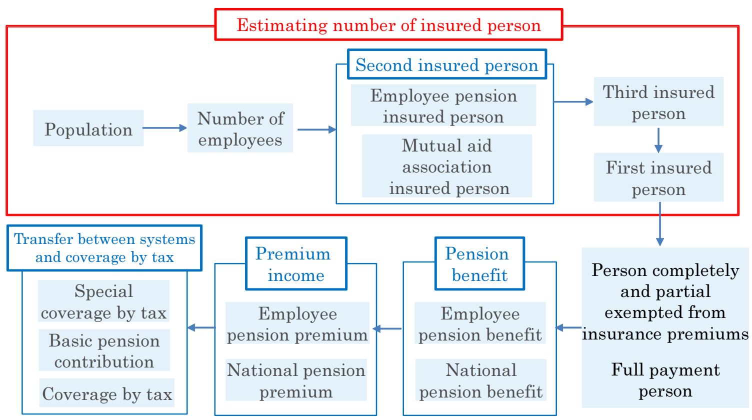


Figure 5: Overview of modeling public pension finance

At first, we estimate the number of employees from the population data, and the number of the second, third, and first insured person sequentially. Next we estimate the number of persons completely and partial exempted from insurance premiums, and the number of full payment persons. We calculate the pension benefits, premium income, and transfer between systems and coverage by tax. We use the public database published by MHLW(2014) as much as possible. We show the estimation results using the model for the medium-level birth and death estimation, and case E of economic assumption of MHLW(2014) in Figure 6. We find these have the similar results to those of MHLW(2014)[11].

4. Optimization model

In this paper, we formulate the optimization model in the simulation-based approach. The private pension plays an important role as well as the public pension, and the life insurance is important to evaluate the longevity risk because it play a role to hedge the mortality risk of householder in retirement. Therefore, we set the numbers of units of private pension and life insurance as decision variables. The asset mix decision is also important for retirement planning, but we give well-known strategies for asset mix as shown in Section 5.1(2) to focus on the effect of deferral options. This makes the optimization problem formulate simply, and we can analyze this issue easily. We show the notations and formulations as follows.

4.1. Notations

(1)Subscript/Superscript

[11] As with Yokoyama(2013), slight adjustments are made to the extent that the creditability is maintained so that there is no significant difference from those of the basic case of the MHLW(2014).

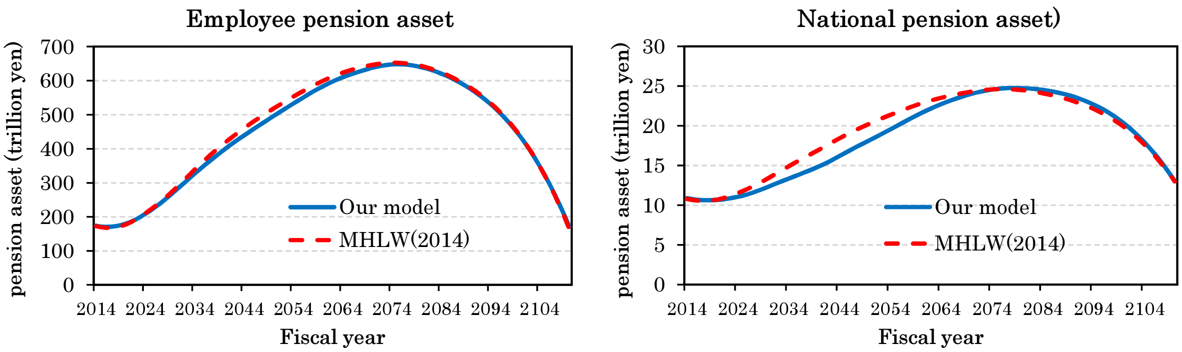


Figure 6: Comparison of estimation results with MHLW(2014): employee pension asset(left) and national pension asset (right)

i : path $(i = 1, \ldots, I)$. I is the number of paths.

t : time $(t = 0, \ldots, T)$. T is the number of planning periods.

The descriptions of $(t = 1, \ldots, T)$ for time t, and $(i = 1, \ldots, I)$ for path i are omitted hereafter.

(2) Parameters

$\mu_{Pt}^{(i)}$: rate of return of portfolio on path i at time t

df_t : discount factor used for cashflow at time t

W_0 : initial amount of wealth

$P_t^{(i)}$: disposable income on path i at time t (except private pension)

$C_{d,t}^{(i)}$: minimum living expense on path i at time t

$H_t^{(i)}$: medical expense on path i at time t

A_M, A_F : private pension premiums per unit of a householder and a spouse at time 0

a_M, a_F : private pension incomes per unit of a householder and a spouse at time 0

L_M, L_F : premiums of level payment life insurance per unit of a householder and a spouse bought at time 0

θ_M, θ_F : life insurance money per unit of a householder and a spouse bought at time 0

T_A, T_L : maturity of private pension and life insurance

κ : relative consumption level after a householder or a spouse died

$\omega_{R,t}$: weight associated with risk at time t where $\sum_{t=1}^{T} \omega_{R,t} = 1$

$W_{G,t}$: target wealth at time t

(3) Decision variables

x_M, x_F : numbers of units of private pension for a householder and a spouse at time 0

y_M, y_F : numbers of units of life insurance for a householder and a spouse at time 0

$W_t^{(i)}$: wealth on path i at time t

$q_t^{(i)}$: shortfall from target wealth on path i at time t

$\eta_{Wt}^{(i)}$: one for $W_t^{(i)} > 0$ on path i at time t and zero otherwise

4.2. Formulation

We formulate the optimization model of minimizing the first order lower partial moment (LPM(1)) in order to evaluate the longevity risk in retirement.

(1) Income, expenditure, and cashflow of insurance

We show the cashflows of private pension A_0^-, $A_t^{+(i)}$ in Equations (4.1) and (4.2), the cashflows of life insurance L_0^-, $L_t^{-(i)}$, $L_t^{+(i)}$ in Equations (4.3) to (4.5), and the overall cashflows D_0^-, $D_t^{-(i)}$, $D_t^{+(i)}$, $D_t^{(i)}$ in Equations (4.6) to (4.9). The cash inflow of household consists of public and private pension benefits and amount of life insurance money, and cash outflow consists of minimum living cost, medical expenditures and life insurance premiums.

$$\begin{aligned}
A_0^- &= A_M x_M + A_F x_F && (4.1)\\
A_t^{+(i)} &= a_M x_M \tau_{AM,t}^{(i)} + a_F x_F \tau_{AF,t}^{(i)} \quad (t = 1, \ldots, T_A) && (4.2)\\
L_0^- &= L_M y_M + L_F y_F && (4.3)\\
L_t^{-(i)} &= L_M y_M \tau_{AM,t}^{(i)} + L_F y_F \tau_{AF,t}^{(i)} \quad (t = 1, \ldots, T_L - 1) && (4.4)\\
L_t^{+(i)} &= \theta_M y_M \tau_{LM,t}^{(i)} + \theta_F y_F \tau_{LF,t}^{(i)} \quad (t = 1, \ldots, T_L) && (4.5)\\
D_0^- &= A_0^- + L_0^- && (4.6)\\
D_t^{-(i)} &= L_t^{-(i)} + C_{d,t}^{(i)} + H_t^{(i)} && (4.7)\\
D_t^{+(i)} &= A_t^{+(i)} + L_t^{+(i)} + P_t^{(i)} && (4.8)\\
D_t^{(i)} &= D_t^{+(i)} - D_t^{-(i)} && (4.9)
\end{aligned}$$

(2) Formulation

$$\textbf{Minimize} \quad \frac{1}{I} \sum_{i=1}^{I} \sum_{t=1}^{T} \tau_{A,t}^{(i)} \omega_{R,t} df_t q_t^{(i)} \tag{4.10}$$

$$\textbf{subject to} \tag{4.11}$$

$$W_t^{(i)} = \begin{cases} \left(1 + \mu_{P1}^{(i)}\right) (W_0 - D_0^-) + D_1^{(i)} & (t = 1) \\ \left(1 + \mu_{Pt}^{(i)} \eta_{W,t-1}^{(i)}\right) W_{t-1}^{(i)} + D_t^{(i)} & (t = 2, \ldots, T) \end{cases} \tag{4.12}$$

$$\eta_{Wt}^{(i)} = \begin{cases} 1 & \text{for } W_t^{(i)} > 0 \\ 0 & \text{otherwise} \end{cases} \qquad (t = 1, \ldots, T-1) \tag{4.13}$$

$$W_t^{(i)} + q_t^{(i)} \geq W_{G,t};\ q_t^{(i)} \geq 0 \tag{4.14}$$

$$x_M \geq 0;\ x_F \geq 0 \tag{4.15}$$

$$y_M \geq 0;\ y_F \geq 0 \tag{4.16}$$

$$\text{Equations (4.1) to (4.9)} \tag{4.17}$$

Equation(4.12) shows the wealth at each time under the assumption that we invest on assets when the wealth of path i at time $t-1$ is larger than zero ($W_{t-1}^{(i)} > 0$). The formulation is non-linear programming problem because $\eta_{Wt}^{(i)}$ of Equation(4.13) is dependent on $W_t^{(i)}$. Therefore, we propose the iterative algorithm to solve the problem approximately in the next section. Equation(4.14) is the constraint used to calculate the LPM(1), and Equations (4.15) and (4.16) are non-negativity constraints with respect to the numbers of units of private pension and life insurance for a householder and a spouse at time 0.

4.3. Iterative algorithm

As shown in the previous section, we propose the iterative algorithm to solve the problem approximately.

Step 1 : We solve the problem under the condition of $\eta_{Wt(0)}^{(i)} = 1$, and we calculate $W_{t(0)}^{(i)*}$. We set $k = 1$.

Step 2 : We solve the problem subject to $\eta^{(i)}_{Wt(k)} = 1$ for $W^{(i)*}_{t(k-1)} > 0$, or $\eta^{(i)}_{Wt(k)} = 0$ for $W^{(i)*}_{t(k-1)} \leq 0$, and we calculate $W^{(i)*}_{t(k)}$.

Step 3 : Stop if $\frac{1}{I \cdot T} \sum_{i=1}^{I} \sum_{t=1}^{T} (\mid \eta^{(i)}_{Wt(k)} - \eta^{(i)}_{Wt(k-1)} \mid)$ is lower than a tolerance. Otherwise, set $k \to k+1$, and return to Step 2.

Empirically, the value almost converges after two iterations in Step 2 in the numerical analysis in Section 5, and we obtain the approximate solutions.[12] Therefore we conduct the analysis fixing the number of iterations instead of the convergence condition in Step 3.

5. Numerical Analysis

5.1. Setting

We conduct the numerical analysis for a hypothetical household. All of the problems are solved using Numerical Optimizer Ver.20.1.0 — mathematical programming software package developed by NTT DATA Mathematical System, Inc. — on Windows 10 personal computer which is Hewlett-Packard Company HP Z420 Workstation, Xeon E5-1603 with 2.80 GHz and 64 GB memory. Basic parameters for the household and model parameters of public pension finance are shown below.

(1) Income and expenditure of household

The household income consists of the old-age basic pension (0.78 million yen) and old-age employee pension (1.22 million yen) for a householder, and the old-age basic pension (0.78 million yen) for a spouse.[13] The expenditure consists of the minimum living cost (2.82 million yen) and medical expenditure.

(2) Investment assets and their proportions

The investment weight for risky assets decreases relatively because the risk tolerance decreases with age. We adopt the investment rule of the "100 minus age" percentage for stock, which is a well-known simple method for risky assets in the U.S.[14] In addition, we also conduct the sensitivity analysis, using the actual investment weights in Japan, instead of this rule. As shown in Section 2.4.2, we employ the so-called traditional four assets; domestic stock, domestic bond, foreign stock and foreign bond. A stock is a relatively risky asset among them, and therefore we invest a half amount of the "100 minus age" percentage in domestic and foreign stocks, respectively. We invest a 70% of the "age" percentage in domestic bond, and a 30% in foreign bond.[15] We employ the parameter values for each asset estimated in Section 2.4.2. We invest in the abovementioned asset mix without a risk-free asset in the analysis after Section 5.2 except Section 5.4. However, we conduct the analysis using the asset mix with a risk-free asset in Section 5.4 in order to reflect the current situation in Japan that there is little investment in risky assets. We employ the parameter values of interest rate model (Nelson-Siegel model) estimated in Section 2.4.1.

(3) Initial wealth and target wealth

[12] The algorithm does not guarantee to derive the global optimal solutions. Moreover, the local optimality is not also theoretically guaranteed, but empirically the value almost converges. This algorithm is a heuristic one.

[13] We use the full amount for the old-age basic pension as of April 2018, and the average amount of employee pension beneficiary of male for old-age employee pension, according to MHLW(2017b). The amount of benefit is shown for 65 pensionable age, and it increases for the deferred pension.

[14] Whether the receipt of benefits is deferred or not might affect the asset allocation. However, we simply adopt this investment rule to focus on the effect of deferral option in this paper. The development and analysis of the model to solve the optimal asset allocation problem are our future tasks.

[15] We set the fraction between stock and bond in reference to asset mix of the Government Pension Investment Fund in Japan.

We assume the household has 15.52 million yen due to savings from income in the working age and retirement allowances. This value is based on "Family income and expenditure survey(2017)"by SB-MIAC(2017). We consider that the situation that the household has a negative wealth is exposed to a longevity risk, and the target amount is set to zero at all times.

(4) Private pension

We generate the virtual private pensions by employing the data of "& LIFE individual annuity insurance", life annuity with 10-year guaranteed period, actually sold by Mitsui Sumitomo Aioi Life Insurance Company, Limited in Japan, standard mortality table 2007 after annuitization, standard mortality table 2018 for life insurance, and 0.75% guaranteed rate. The amounts of benefit per unit of a householder and a spouse are $a_M, a_F = 0.9$ million yen, and the corresponding premiums are shown in Table 6.

Table 6: Parameter for private pension(unit: million yen)

Maturity	life	1 year	2 years	3 years	4 years	5 years
Male	21.195	0.9	1.8	2.7	3.6	4.5
Female	27.785	0.9	1.8	2.7	3.6	4.5

(5) Life insurance

We employ the data of "Bridge", 15-year term insurance with level payment, actually sold by OLIX life insurance corporation in Japan. The amounts of life insurance money per unit of a householder and a spouse are $\theta_M, \theta_F = 10$ million yen, and the corresponding premiums are $L_M = 236.7$ thousand yen per year for a householder, and $L_F = 116.5$ thousand yen per year for a spouse.[16]

(6) Other parameters

We show the other parameters used in the numerical analysis in Table 7.

Table 7: Setting parameters

Parameter	Value
Planning period (year)	$T = 35$
Relative consumption level after a householder or a spouse died	$\kappa = 0.60$
Weight of risk averse coefficient	$\omega_{R,t} = \frac{1}{T}$
Inflation rate	$f = 0.0065$
Number of paths	$I = 10,000$

The value of relative consumption level κ is based on "Family income and expenditure survey(2017)"by SB-MIAC(2017).

The inflation rate is dependent on time t and path i in the model, but we assume it is constant for simplicity in the numerical analysis, and it is set to 0.65%. This value is based on the average consumption price index from 1983 to 2017 by SB-MIAC. We show the average values of objective function values, private pension premium, and life insurance premium calculated using ten random seeds to decrease the sampling error in the numerical analysis.

[16] Ratios of loading premium to net premium are 15.6% for male and 15.9% for female under the assumption that the expected mortality rate is based on the standard mortality table 2018, and the guaranteed rate is 0.75%.

(7) Parameters of public pension finance model

We adopt the birth/death medium estimates and case E of long-term economic assumptions of MHLW(2014) as a basic case in the public pension finance model, The long-term economic assumptions are shown in Table 8.[17] Cases E1 to E3 are set originally in this paper, and used to analyze how real wage growth rate, real investment yield, and labor force participation rate affect public pension finance when deferred payments are promoted.

Table 8: Long-term economic assumptions

	Inflation rate	Wage growth rate(real)	Investment yield(real)	Labor force participation rate(LFPR)
Case C	1.6%	1.8%	3.2%	Increase in LFPR
Case E	1.2%	1.3%	3.0%	
Case G	0.9%	1.0%	2.2%	Decrease in LFPR
Case E1	1.2%	**2.3%**	3.0%	Increase in LFPR
Case E2	1.2%	1.3%	**2.2%**	
Case E3	1.2%	1.3%	3.0%	**Decrease in LFPR**

5.2. Basic Analysis

We solve the optimization problem for the case of receiving the conventional public pension at the age of 65, and the Case 1 to 5 of starting the receipt of deferred benefits at the age of 66 to 70, in order to demonstrate the usefulness of deferral option. We purchase the life annuity when we start to receive the public pension benefit at the age of 65. Otherwise, we purchase the term annuity of receiving in the period until the start age of deferred public pension benefit. The objective function values are shown in Table 9.

Table 9: Objective function values for each pensionable age

	Normal	Case 1	Case 2	Case 3	Case 4	Case 5
Pensionable age	age 65	age 66	age 67	age 68	age 69	age 70
Maturity of private pension	life	1 year	2 years	3 years	4 years	5 years
Objective function value	0.2807	0.0587	0.0169	0.0131	0.0505	2.1702

We find that Case 3 where the public pension is deferred until age 68 has the smallest objective function value, and we can hedge the longevity risk more effectively by the deferral receipt. We show private annuity premiums and life insurance premiums paid at time 0 in Figure 7. Examining the private pension premium paid at time 0 in the left side of Figure 7, we find more private pensions are purchased as the deferred years become longer from Cases 1 to 4. The household avoids investing in risky assets, and purchases the private annuity because a stable annuity income can be obtained by the deferred receipt. On the other hand, the amount of private pension decreases in Case 5 because the initial wealth is not enough to defer the receipt, and we need to avoid running out of wealth during the deferred period. Examining the life insurance premium paid at time 0 in the right side of Figure 7, we find the amounts of life insurance are purchased in Cases 3 to 5 where the public pensions are deferred. The spouse is exposed to a large income reduction risk and longevity

[17] MHLW(2014) provides the ranges of real wage growth rate and investment yield, but we set the values in Table 8 which are included in the range. These are values relative to the wage growth rate.

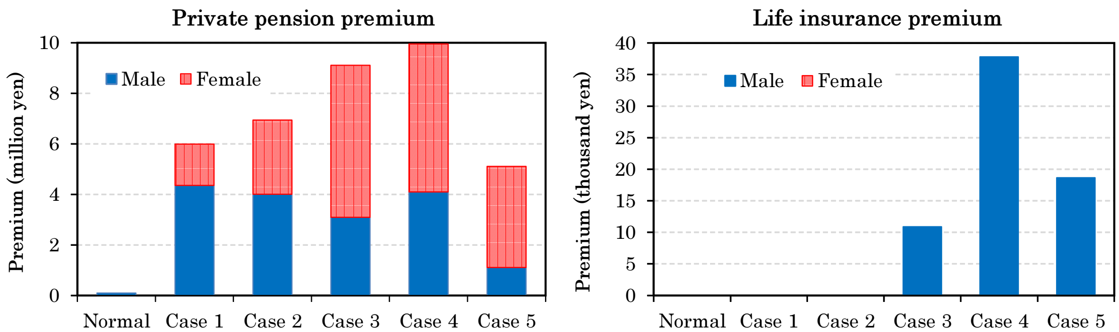


Figure 7: Private pension premium and life insurance premium at time 0

risk if the householder who defers the receipt of public pension benefit died early, because the current survivor's pension system calculates the survivor's annual amount based on the original amounts without the deferral premium. Instead of private life insurance, we might need to replace the role of life insurance by a public social security system, in considering the aging society in the future. It is expected to be improved by reforming the survivor's pension system so that the benefit can be calculated based on the amounts actually received instead of the original amounts. However, pension expenditures are likely to become larger due to the system change. Therefore, we analyze the impact of the survivor's pension system change quantitatively from both the household and public pension finances in Section 5.6.

We show the expected income and expected balance at each pensionable age in Figure 8, and the LPM (1) and expected wealth at each time in Figure 9. The household is exposed to longevity risk at the end period because the public pension benefits are not enough to live for the whole period in the case of receiving it from the age of 65. On the other hand, the amounts of wealth significantly reduce until the start of deferred receipt in Cases 1 to 4, but the longevity risk can be hedged more appropriately due to sufficient income after the start of receipt. However, the savings are exhausted before the start of receipt in Case 5 where the receipt of benefit is deferred until age 70. The household cannot cover the expenditures by savings only at the age of 65, and it is exposed to longevity risk. We find the deferred receipt of public pensions is effective for hedging longevity risk, however it is also important for individuals to promote the asset formation. The expected wealth at each time is increasing in the normal case, but the household is exposed to a large longevity risk in the end period. We examine the impact of deferred public pension on households after retirement. We show the terminal wealth in Table 10 and the cumulative probability of wealth in Figure 10.

Table 10: Statistics of terminal wealth

	Normal	Case 1	Case 2	Case 3	Case 4	Case 5
Average	2,611.70	2,448.12	2,326.03	2,050.83	1,532.02	1,394.82
Median	2,378.24	2,246.91	2,140.57	1,891.55	1,419.36	1,280.57
Standard deviation	1,469.62	1,238.47	1,135.12	997.98	763.37	871.34
Skewness	1.21	1.24	1.26	1.21	1.17	1.13
Minimum	−1,023.16	−646.27	−556.96	−581.67	−651.86	−781.65
Maximum	15,909.64	13,752.60	12,094.74	9,865.55	6,559.28	7,589.48

According to Table 10 and Figure 10, the average terminal wealth in the normal case is

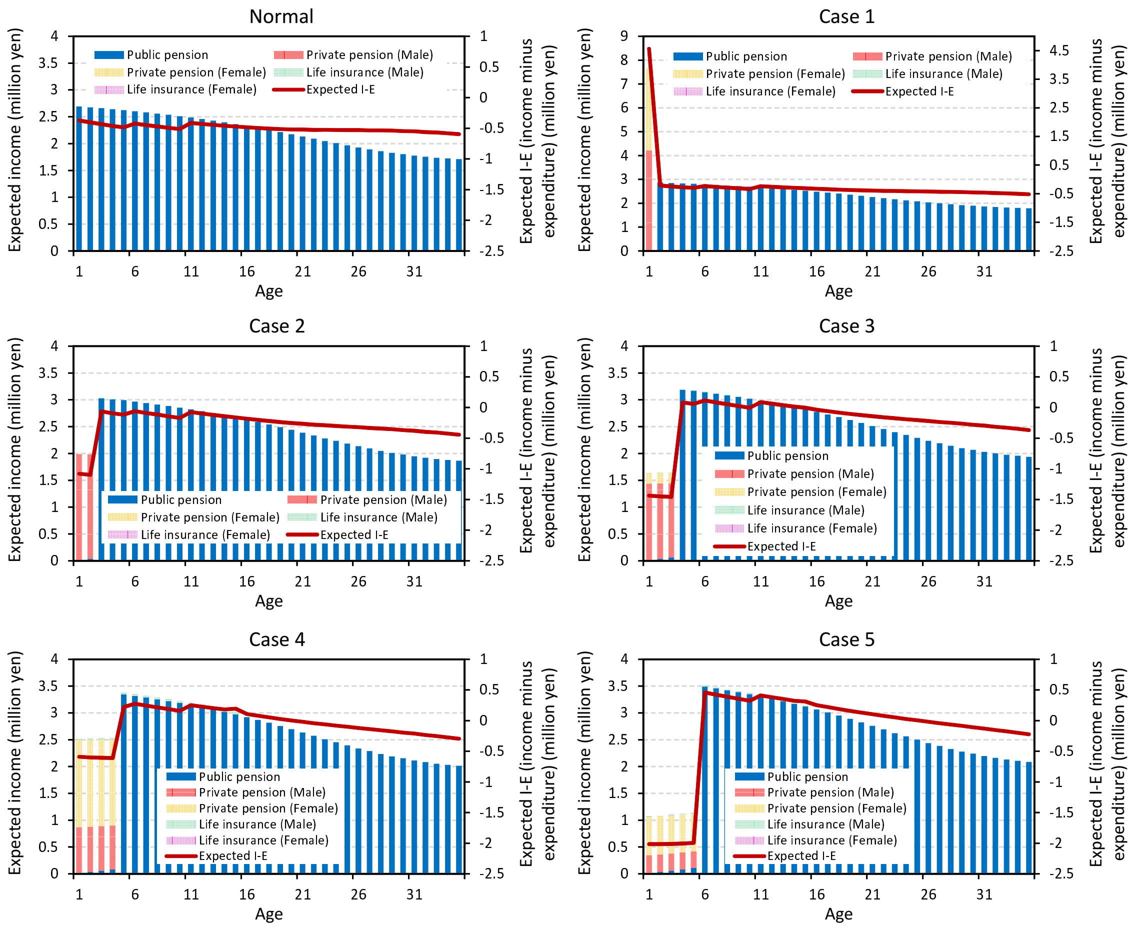


Figure 8: Expected income and expected income minus expenditure for each pensionable age

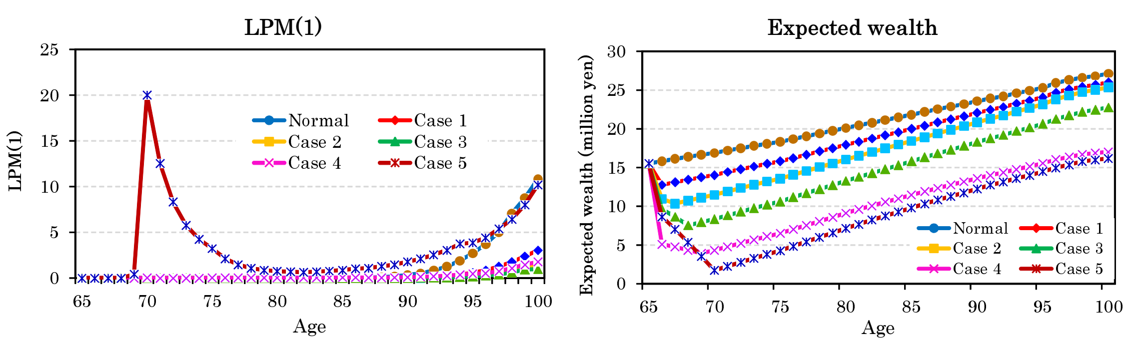


Figure 9: LPM(1) and expected wealth for each age

the largest by investing more amounts of risky assets because the income is not enough and private life annuity is expensive due to the high loading premiums. However, the terminal wealth has a large downside risk. On the other hand, it is possible to obtain a stable income after the start of receipt by receiving the deferral benefit, and the terminal wealth of deferral cases has a smaller downside risk than the normal case because we do not need to take investment risk during the deferred period. Therefore, the deferral option is useful for retired households to hedge longevity risk because it guarantees enough income to live. In addition, it can give flexible opportunities of selecting the pensionable age for the retired

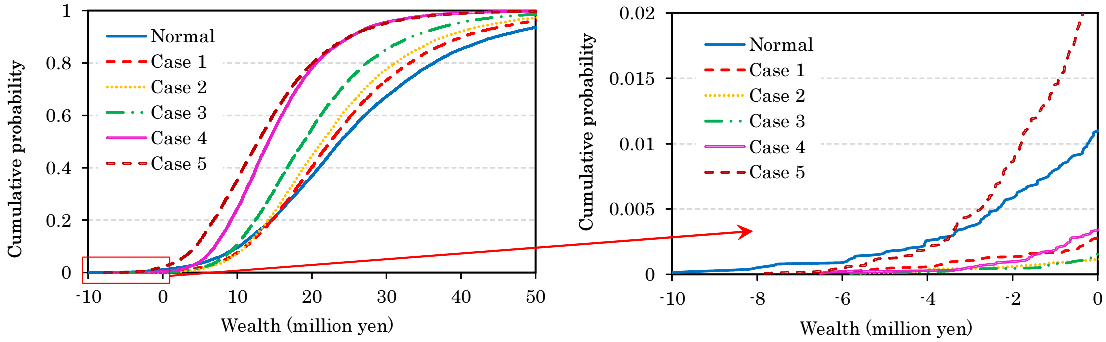


Figure 10: Cumulative probability of terminal wealth

households.

5.3. Sensitivity analysis of the increment rate for deferred pension

We find the deferred receipt of public pension benefit is useful for hedging longevity risk according to the basic analysis in Section 5.2. However, there is a concern that the increment rate for deferred pension may change for households. The current increment rate calculated based on the average life expectancy and interest rate has not been revised since the 2000 revision of the pension system, and it is not mathematically equivalent due to the increase in the average life in recent years. Therefore, we examine the impact on Cases 1 to 4 for the different increment rates which are less than the current 0.7 % because the objective function values of these cases are smaller than that of normal case of receiving a public pension from age 65 in the basic analysis in Section 5.2. We show the objective function values for the five increment rates in Figure 11.

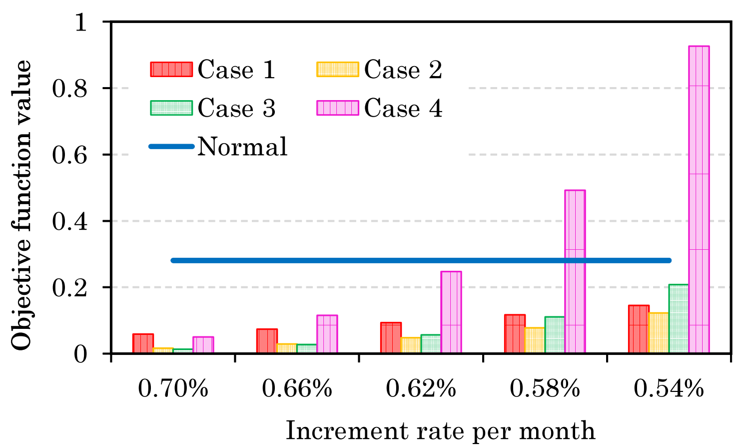


Figure 11: Objective function values for each increment rate

Even if it is reduced from the current level of 0.7% to 0.54%, we find that the deferred receipt is still useful in Cases 1 to 3. On the other hand, the reduction of the increment rate for deferred pension is highly sensitive to the objective function values of Case 4. It is necessary to set an appropriate rate to consider the sustainability of the pension system whereas the low deferred receipts do not become effective for hedging longevity risk. We need to design the system in consideration of the trade-off relationship. However, it should be noted that it is important to employ public pension deferral option, taking into account the household attributes.

5.4. Effect of asset mix

We find the effectiveness of deferral receipts under the well-known simple investment rule of the "100 minus age" percentage for stock in the abovementioned analysis. However, according to SB-MIAC(2017), stocks and stock investment trusts only account for 5.54% of savings, and bonds and public bond investment trusts account for 1.52 %, with deposits and savings account for the majority. Therefore, suppose that the investment ratio is 5.54 % for stocks (domestic : foreign = 1 : 1) and 1.52% for bonds (domestic : foreign = 7 : 3) over the planning period, with risk-free asset accounting for 92.94%, based on the current situation of elderly people in Japan. We solve the problem as well as the basic analysis in Section 5.2. We show the objective function values in Table 11.

Table 11: Objective function values for each pensionable age

	Normal	Case 1	Case 2	Case 3	Case 4	Case 5
Pensionable age	age 65	age 66	age 67	age 68	age 69	age 70
Maturity of private pension	life	1 year	2 years	3 years	4 years	5 years
Objective function value	7.3098	2.2920	0.7517	0.3938	0.3993	4.2679

The objective function values are larger than those of Table 9 in all cases, and we find investing in risky assets helps the reduction of longevity risks for elderly households even a simple investment rule, such as "100 minus age" percentage for stock, which decreases risk tolerance gradually with age. Case 3 in which the public pension is deferred until age 68 has the smallest objective function value as well as the basic analysis in Section 5.2, whereas the normal case receiving benefit from age 65 has the largest objective function value. We show the LPM(1) and expected wealth at each time in Figure 12.

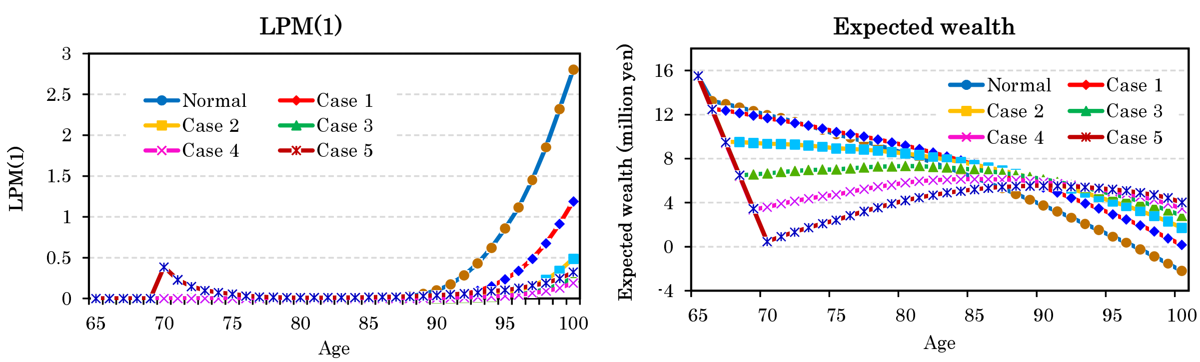


Figure 12: LPM(1) and expected wealth for each age

The household is exposed to a large longevity risk at the end of planning period in the normal case receiving public pension benefit from age 65. This is because the household invests the most amount of wealth in the risk-free asset even starting the early receipt of pension benefit. The current low interest rate environment makes investment income and public pension income difficult, and the amounts of wealth are exhausted in the end of planning period. On the other hand, Case 5 has a large amount of LPM(1) at the beginning of the planning period because of the insufficient initial wealth, but we can manage longevity risk appropriately at the end of the planning period by receiving the deferred pension benefit. We show private annuity premiums and life insurance premiums paid at time 0 in Figure 13.

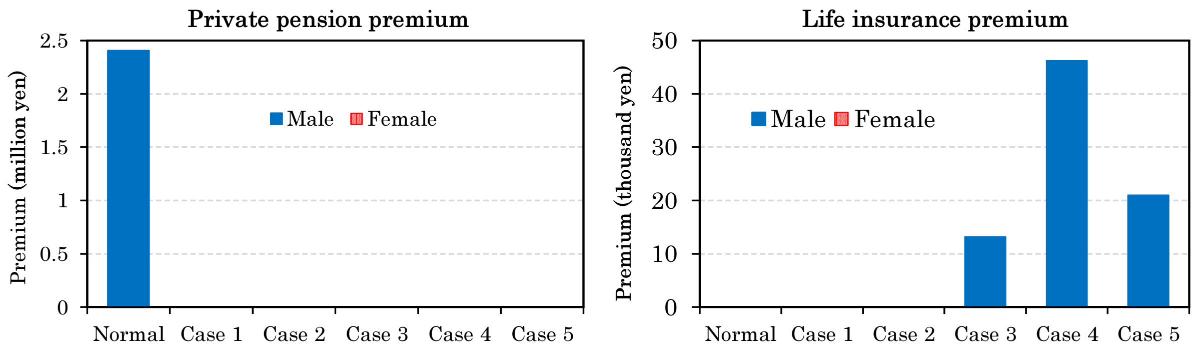


Figure 13: Private pension premium and life insurance premium at time 0

The household does not purchase the private life annuity in the normal case of receiving the public pension from age 65 in the basic analysis of Section 5.2, but purchases the private life annuity for male. Life insurance premiums slightly increase, compared with the basic analysis. This might be due to the fact that the household is exposed to larger longevity risk and income reduction risk by low investment returns, and it needs to hedge risks with private life annuities and life insurance.

Considering the investment environment in Japan, the increment rate of 8.4 % due to deferred payments is very attractive for many elderly people even with insufficient financial literacy. The deferral option also gives attractive and flexible decision opportunity for the retired households because of the reduction of investment risk. On the other hand, individuals need to make efforts by themselves, and it is also important to promote information dissemination and investment education in order to properly utilize public pension deferral option and investments.

5.5. Impact on public pension finances

The public pension finance may be affected by changing the beneficiary behavior of households to hedge longevity risk. If it gives a large impact, it is necessary to revise the increment rate for deferred pension to ensure the sustainability of the public pension. We examine the impact on public pension finance when the ratio of deferred recipients of public pension increases.

Suppose current pensioners from the age of 65 shift to the ratio of new recipients at each age over 30 years, as shown in Cases A1 to A5 in the left side of Figure 14. We show the employees' pension asset in the right side of Figure 14.[18] We show the balance sheet of employees' pension finance in each case in Table 12.

We find that pension benefits and basic pension contributions increase as the proportion of deferred beneficiaries increases, and it results in worsening the employees' pension finance. As the number of deferred beneficiaries increases, the amounts of benefit decrease and the employees' pension asset increases in the short term, but the increase in deferred benefit affects the finance significantly in the long term. Therefore, it is necessary to properly discuss how to reduce the increment rate for deferred pension in order to build a win-win relationship that ensures the sustainability of public pension finance while hedging the longevity risk of the household. When the proportion of deferred recipients increases as Case A3, we show the employees' pension asset for the four cases where the deferred increment rates are reduced from the current level of 0.7 % in Figure 15. We show the balance sheet

[18] We conduct the same analysis as the national pension fund, but we omit it because of the similar results.

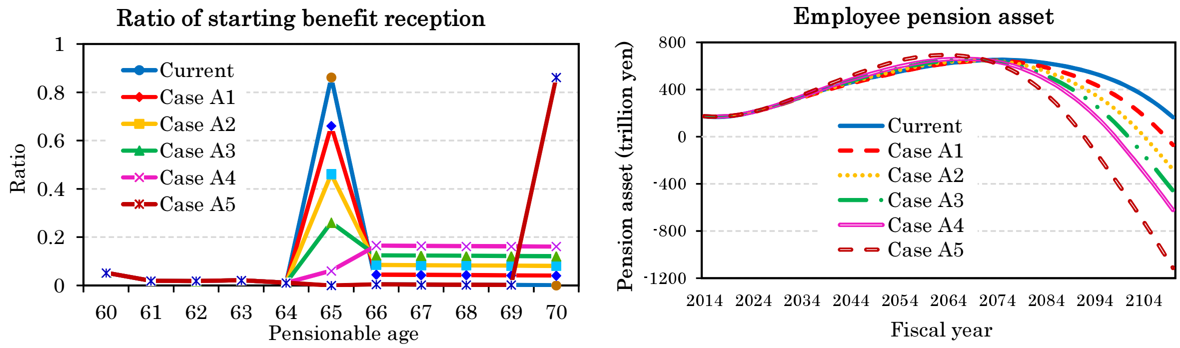
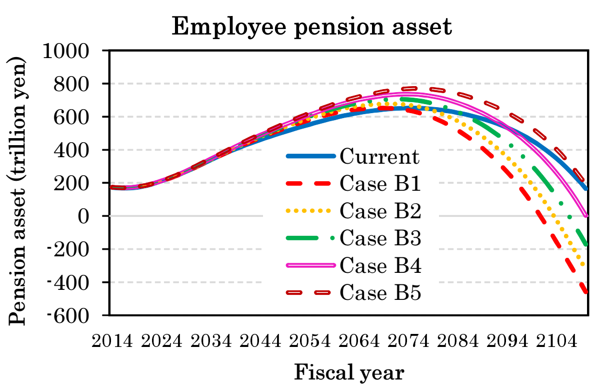


Figure 14: Ratio of starting benefit reception for each pensionable age and employees' pension asset

Table 12: Balance sheet of employees' pension finance

Unit: trillion yen	Current	Case A1	Case A2	Case A3	Case A4	Case A5
Premium	1,332.8	1,332.8	1,332.8	1,332.8	1,332.8	1,332.8
Tax revenue	368.1	369.0	369.9	370.9	371.8	374.8
Pension asset	173.8	173.8	173.8	173.8	173.8	173.8
Total asset	1,874.6	1,875.6	1,876.5	1,877.4	1,878.4	1,881.4
Pension benefit	1,141.5	1,145.0	1,148.5	1,151.9	1,155.4	1,167.7
Contribution	726.3	728.2	730.1	732.0	733.8	739.9
Total liability	1,867.9	1,873.2	1,878.6	1,883.9	1,889.2	1,907.6
Net debt	−6.8	−2.3	2.1	6.5	10.9	26.2

of employees' pension finance in each case in Table 13.



	Deferral ratio	Increment rate/month
Case B1		0.70%(*1)
Case B2		0.68%
Case B3	Case A3	0.66%
Case B4		0.64%
Case B5		0.62%

(*1) Current rate

Figure 15: Impact on employees' pension asset by reducing increment rate

We find that pension finance can be maintained under the assumption of our research, even if the number of deferred beneficiaries increases by decreasing the increment rate for deferred pension. As described above, even if the increment rate is reduced, we can manage the longevity risk by receiving deferred benefits for retired households. Therefore, it is important to design the system appropriately in consideration of both household and pension finances. We show increment rate that can maintain the current pension financial level in Table 14 when the proportion of deferred beneficiaries increase from Case A1 to A5 in each case of economic assumptions C, E, and G.

Table 13: Balance sheet of employees' pension financing

Unit: trillion yen	Current	Case B1	Case B2	Case B3	Case B4	Case B5
Premium	1,332.8	1,332.8	1,332.8	1,332.8	1,332.8	1,332.8
Tax revenue	368.1	370.9	370.3	369.8	369.2	368.7
Pension asset	173.8	173.8	173.8	173.8	173.8	173.8
Total asset	1,874.6	1,877.4	1,876.9	1,876.3	1,875.8	1,875.2
Pension benefit	1,141.5	1,151.9	1,149.0	1,146.0	1,143.1	1,140.1
Contribution	726.3	732.0	730.9	729.8	728.7	727.6
Total liability	1,867.6	1,883.9	1,879.9	1,875.8	1,871.8	1,867.7
Net debt	−6.8	6.5	3.0	−0.5	−4.0	−7.5

Table 14: Increment rate per month to maintain current financing level

	Deferral ratio	Case A1	Case A2	Case A3	Case A4	Case A5
Economic assumption	Case C	0.66%	0.64%	0.63%	0.62%	0.59%
	Case E	0.66%	0.64%	0.62%	0.61%	0.59%
	Case G	0.64%	0.61%	0.60%	0.58%	0.56%
	Case E1	0.65%	0.63%	0.62%	0.61%	0.57%
	Case E2	0.63%	0.60%	0.59%	0.57%	0.54%
	Case E3	0.66%	0.64%	0.62%	0.61%	0.59%

The increment rate of the economic assumption of Case E can be reduced to 0.66% per month for the percentage of deferred recipients of Case A1, but 0.59% for Case A5 where all people start receiving from the age 70 instead of the age 65. Therefore, the more the deferred receipt is promoted, the lower the increment rate needs to be. It is necessary to reduce the increment rate most in Case G (low-growth case), especially 0.56% per month in combination with Case A5.

Next, we analyze the impact of long-term economic assumptions on public pension finance when deferred benefits are promoted. Case E1 is the case where only the wage increase rate is set to a large value by 1.0% in Case E. Case E2 is the case where only the investment yield is set to a large value by 0.8% in Case E, and Case E3 is not the case of the increase in the labor force participation rate in Case E, but the case of the decrease. We find that it is necessary to set the lower increment rates in case E1 than those of Case E. Therefore, the public pension finance becomes worse even the increase in the wage growth rate by 1.0%, if deferred receipts are promoted. However, the effect is not so large because the increment rate is only reduced by 0.1% to 0.2% per month. We also find that it is necessary to reduce the increment rates in Case E2 more than those of Case E. The public pension finance gets worse by the effect of lower investment yield and promoted deferred receipt. Case E3 has the same results as Case E. This shows the labor force participation rate does not impact on public pension finance even if deferred payments are promoted. The investment yield gives a major impact on the public pension finance as you can see from the fact that Case G has lower thresholds than Cases C and E.

5.6. Effect of changing survivor's pension system

In the basic analysis of Section 5.2, the surviving spouse is exposed to significant longevity risk in the current survivor's pension system when the householder receives the deferred benefits and dies early because survivor's pension benefit is calculated based on the normal benefit without increment rate. This rule gives a huge risk generally for women with a long life expectancy, especially full-time housewives. Therefore, instead of the current rule, we examine the effects under the assumption that the survivor's pension system is revised to the rule that the system pays the benefit with increment rate.

We show the objective function values in Table 15, and the first-order LPMs at each time and cumulative probabilities of terminal wealth under the current and assumed survivor's pension system in Case 4 in Figure 16.

Table 15: Objective function values for each pensionable age

	Normal	Case 1	Case 2	Case 3	Case 4	Case 5
Pensionable age	age 65	age 66	age 67	age 68	age 69	age 70
Maturity of private pension	life	1 year	2 years	3 years	4 years	5 years
Objective function value	0.2807	0.0495	0.0098	0.0026	0.0022	1.3433

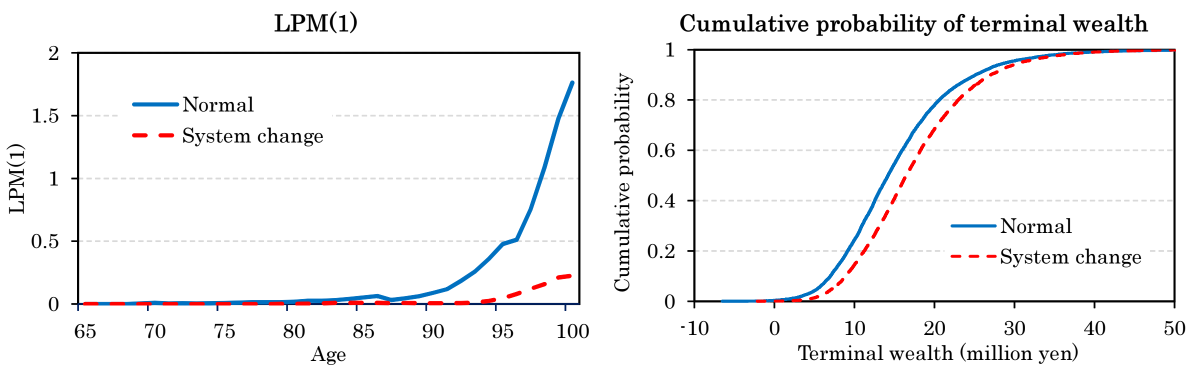


Figure 16: First-order LPMs at each time and cumulative probability of terminal wealth

We have the smallest objective function values in Case 4 under the assumed survivor's pension system, and the distribution of terminal wealth is shifted to the right. We find the deferred benefit in the assumed survivor's pension system is effective for hedging the longevity risk because the income risk of spouse is reduced and the first-order LPM of wealth at the end of the period becomes small, even if the householder who received the deferred benefit died early. However, the change in survivor's pension system gives the larger impact on the pension finances. Therefore, we examine the impacts on both household and public pension finance by the reduction of increment rate for deferred pension from the current level of 0.7%.

We show the objective function values for seven increment rates in Cases 1 to 4 where the objective function values are smaller than the normal case of receiving a public pension benefit from age 65 in Figure 17. We also show the employees' pension asset under the assumed survivor's pension system in Figure 18 when the increment rate is reduced from the current level of 0.7%, and the deferred beneficiary increases in Case A3.

The change in survivor's pension system has a large impact on households, and the deferred benefits are still useful in Cases 1 to 3 even if the increment rate is reduced from the current level of 0.7% to 0.5%. We find the public pension finance can be maintained even if the number of deferred beneficiaries increases under the assumption of the survivor's pension system with the reduction of the increment rate. Therefore, we also need to consider the change of the survivor's pension system, together with the promotion of the deferred receipt

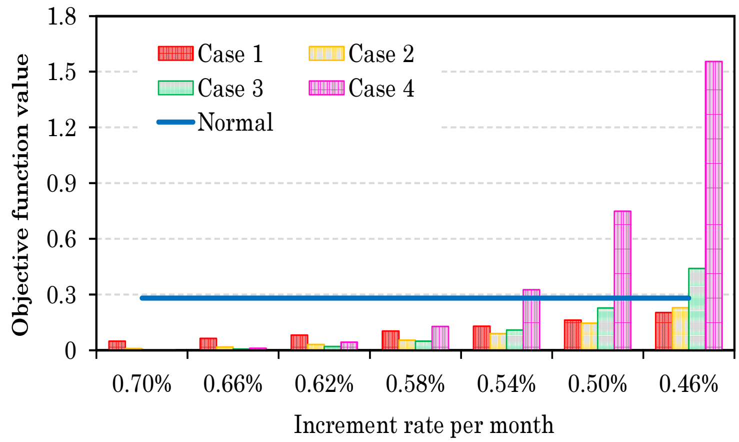
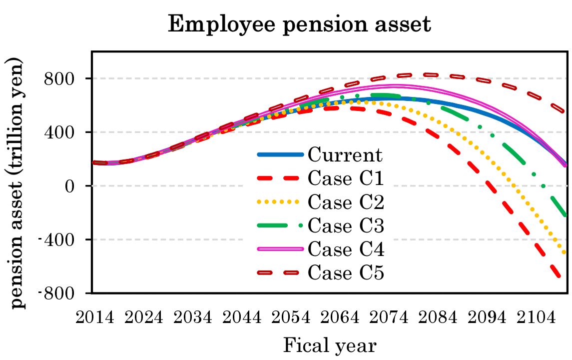


Figure 17: Objective function values for each increment rate



<table>
<tr><th></th><th>Deferral ratio</th><th>Increment rate/month</th><th>Survivor's pension system</th></tr>
<tr><td>Case C1</td><td rowspan="5">Case A3</td><td>0.70%(*1)</td><td rowspan="5">Change</td></tr>
<tr><td>Case C2</td><td>0.68%</td></tr>
<tr><td>Case C3</td><td>0.66%</td></tr>
<tr><td>Case C4</td><td>0.64%</td></tr>
<tr><td>Case C5</td><td>0.62%</td></tr>
</table>

(*1) Current rate

Figure 18: Impact on employees' pension asset by reducing increment rate according to the change of survivor's pension system

to hedge the longevity risk and the reduction of increment rates to maintain the public pension finance, in designing the system to guarantee the future life in retired households.

6. Concluding remarks

In recent years, managing longevity risk is a very important issue for retired households due to the recent aging of the population. In our paper, we formulate the optimization model, and we show quantitatively that it is effective to use a public pension deferral option after receiving a private periodic pension, instead of receiving a private life annuity together with a public pension. We find the deferral option is effective for managing longevity risk. We examine whether we build the win-win relationship between the retired household and public pension finance through the different methods of setting the increment rate for deferred pension in consideration of both the sustainability of public pensions and the longevity risk of households. We find that the public pension finance becomes worse if the number of deferred recipients increases at the current increment rate.

On the other hand, we find the deferred receipts are very useful for retired households in order to hedge longevity risk even if the increment rate is reduced, and the public pension finances also can be maintained by the reduction of increment rate. It is important to design the system appropriately in consideration of the trade-off relationship between household and public pension finances.

We examine how the asset mix impacts on the deferred benefit and longevity risk. When we use the current asset mix held by the elderly people in Japan, we find it is more important to receive the deferral public pension benefit in order to avoid longevity risk because of low investment returns. On the other hand, individuals need to make efforts by themselves, and it is also important to promote information dissemination and investment education in order to properly utilize public pension deferral option and investments.

In addition, we conduct the analysis to examine the effects under the assumption that the survivor's pension system is revised to the rule that the system pays the benefit with increment rate, instead of the current rule. We find the deferred benefit in the assumed survivor's pension system is effective for hedging the longevity risk because the income risk of spouse is reduced, even if the householder died early. Therefore, it is important to consider the change of the survivor's pension system, together with the promotion of the deferred receipt and the reduction of increment rates, in designing the system to guarantee the future life in retired household.

The issue of the declining birthrate and aging population has a great impact on public pensions, and it is necessary to give an appropriate and easy-to-understand message in order to remove the future anxiety. In our study, we examined the effect of deferred receipts quantitatively in consideration of both household and pension finance, and show that it is effective for hedging longevity risk. However, note that it is not always necessary to use the deferred option, and it is important for the households to select a strategy appropriately, taking into account their attributes.

We have an important point to pay attention to the deferred public pension system. High-income households can obtain the additional benefits due to the use of the deferral option, whereas it is difficult for low-income households to use the deferral option because they might not live without public pension after retirement. This may cause the reverse redistribution. Therefore, it is necessary to conduct the analysis in a more realistic framework in order to design public pension systems, but it is our future research. In addition, we need to conduct the analysis that takes into account social policies to make it possible to defer public pensions because the Concept meeting towards 100-year life era(2018) promotes the employment for the elderly. In addition, we need to model the cost of nursing care more clearly, and consider the medical insurance and long-term care insurance. We also need to formulate the optimization model of obtaining the optimal asset allocation for each case of deferred and regular payments. These are our future research.

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