Note on how to differentiate X

$$I = (i, \sigma), J = (j, \sigma')$$

$$F_{IJ} = f_{ij} \times \alpha(\sigma, \sigma')$$

$$X = F_{IJ} - F_{JI} = f_{ij} \times \alpha(\sigma, \sigma') - f_{ji} \times \alpha(\sigma', \sigma)$$

$$Y_{IJ} = \left(\frac{\partial X}{\partial f_{ab}}\right) = \delta_{ia}\delta_{jb} \times \alpha(\sigma, \sigma') - \delta_{ja}\delta_{ib} \times \alpha(\sigma', \sigma)$$

$$\sum_{M,L} (X^{-1})_{ML} Y_{ML} = \sum_{m,l,\sigma,\sigma'} (X^{-1})_{m\sigma,l\sigma'} Y_{l\sigma',m\sigma}$$

$$= \sum_{\sigma,\sigma'} (X^{-1})_{m\sigma,l\sigma'} [\delta_{la}\delta_{mb} \times \alpha(\sigma, \sigma') - \delta_{ma}\delta_{lb} \times \alpha(\sigma', \sigma)]$$

$$= -2 \sum_{\sigma,\sigma'} \alpha(\sigma, \sigma')(X^{-1})_{a\sigma,b\sigma'}$$

f_{ab}が虚部なら前にiがつくだけ

Note on how to differentiate X

$$\alpha(\uparrow,\uparrow) = -cs \qquad c = \cos(\beta/2),\, s = \sin(\beta/2)$$

$$\alpha(\uparrow,\downarrow) = cc$$

$$\alpha(\downarrow,\uparrow) = -ss$$

$$\alpha(\downarrow,\downarrow) = cs$$

$$(X^{-1})_{IJ} = -(X^{-1})_{JI} \text{ (skew symmetry)}$$

Note on how to differentiate X

$$\frac{\partial \mathrm{Pf}[A(x)]}{\partial x} = \frac{1}{2}\mathrm{Pf}[A(x)]\mathrm{Tr}\left[A^{-1}\frac{\partial A(x)}{\partial x}\right]$$

$$\mathrm{Pf}[A(x+\delta x)] = \mathrm{Pf}[A(x) + \frac{\partial A(x)}{\partial x}\delta x + O(\delta x^2)]$$

$$\sim \mathrm{Pf}[(I + \frac{1}{2}A^{-1}\frac{\partial A(x)}{\partial x}\delta x)^T A(x)(I + \frac{1}{2}A^{-1}\frac{\partial A(x)}{\partial x}\delta x) + O(\delta x^2)]$$

$$= \det(I + \frac{1}{2}A^{-1}\frac{\partial A(x)}{\partial x}\delta x + O(\delta x^2))\mathrm{Pf}[A(x)]$$

$$= \mathrm{Pf}[A(x)](1 + \frac{1}{2}\mathrm{Tr}\left[A^{-1}\frac{\partial A(x)}{\partial x}\right]\delta x + O(\delta x^2))$$