# Mean-Variance Hedging with Neural Networks using Heston Model as The Underlying Sample Path Generator

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**Abstract**

We investigate the effectiveness of hedging basket swap option using the Heston Model within a contractual framework involving two parties. The contractual agreement stipulates that at a time $T$, if the value of the stock owned by party 1 exceeds that of party 2, a trade occurs. We make use of the Heston Model to optimise the hedging strategy, considering factors such volatility of the respective stocks, covariation of both stocks with their respective variances and covariation between the two stocks.
Specifically, we will explore how covariance between the two underlying stocks, as well as their respective covariances with volatilities influence option pricing and hedging outcomes. To estimate the parameters of the Heston Model, our first step is to discretise the Heston Model utilising Euler's discretisation method. Thereafter employ Maximum Likelihood Estimation on the joint multivariate normal distribution of the two stocks to estimate the parameters of the model.

**Keywords:** Heston Model, Neural Network, Basket Option, Maximum Likelihood Estimation, Delta Hedging.

## 1 Literature Review

The Heston Model, introduced in 1993, has become integral to quantitative finance by incorporating stochastic volatility, offering a more accurate portrayal of option pricing dynamics, particularly in volatile markets. Numerous studies have applied the Heston Model to price and hedge various options, including basket options. Li and Lin [1] emphasized the importance of precise parameter estimation for effective hedging strategies, while Huang et al. [2] advocated for dynamic delta hedging combined with variance swaps. Jiang et al. Wang and Chen [3] proposed a method combining maximum likelihood estimation with particle swarm optimization for better parameter estimation. Despite advancements, challenges persist in accurately estimating parameters and aligning strategies with contractual terms, warranting further research to enhance practical applicability.

## 2 Aims and Objectives

1 Generate prices of the stocks using Euler discretised multivariate Heston Model.
   - Simulate sample paths to represent the two stocks.

2 Perform Maximum Likelihood Estimation for the parameters of the multivariate Heston Model.
   - Define the likelihood function that represents the probability of observing the simulated stock prices given the model parameters.

3 Fit a Neural Network to Hedge

4 Do this on real world data

## 3 Project Milestone Deliverables

| Date | Description | Deliverable |
|---|---|---|
| 1 Sept17 | Start .... | |
| 10 Sep 17 | Still starting ... | |
| 31 Dec 18 | This is the end | The End |

Tab. 1: Key dates and deliverables for research project

## 4 References

[1] Lin, L., Li, Y. and Wu, J., 2018. The pricing of European options on two underlying assets with delays. Physica A: Statistical Mechanics and its Applications, 495, pp.143-151.

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[3] Wang, X., He, X., Bao, Y.,& Zhao, Y. (2018). Parameter estimates of Heston stochastic volatility model with MLE and consistent EKF algorithm. Science China Information Sciences, 61, 1-17.