Investigating the Distributions of SMOTE-Augmented Datasets

Statistics 98

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Outline

What is an unbalanced dataset?

Harvard College Admissions, Class of 2025

- 57,786 applicants
- 2,320 admitted
- Only 4.01% of our data represent admitted students

A General Framework

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  - "positive" $Y = 1$
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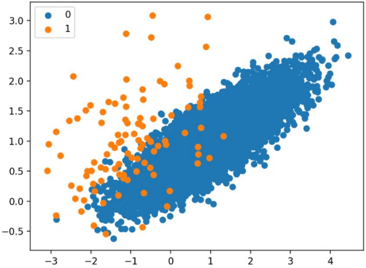
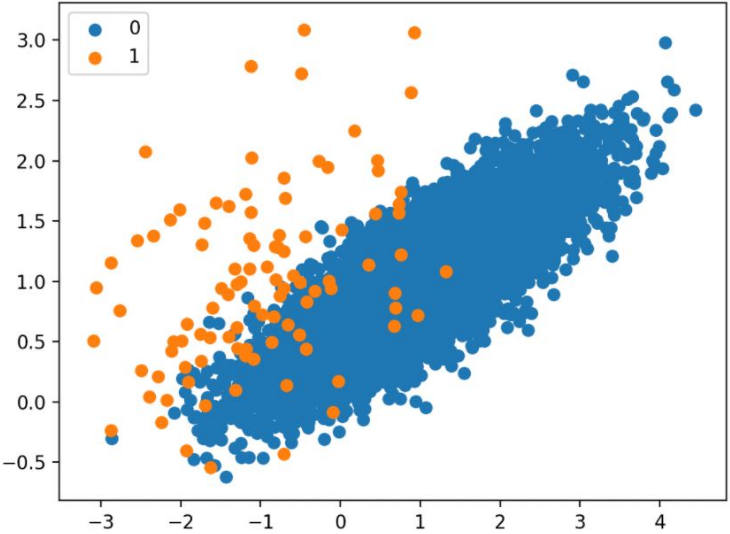
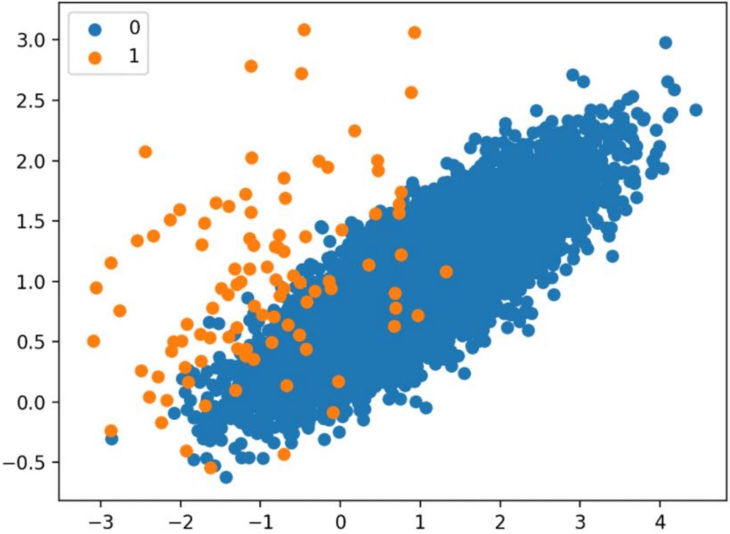
- One outcome is a "majority class" while the other is a "minority class"



Majority Under-sampling

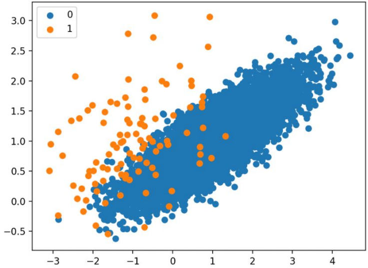


Minority Over-sampling



What is SMOTE?

- Synthetic Minority Over-sampling Technique



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- Algorithm:
  1. Select a point
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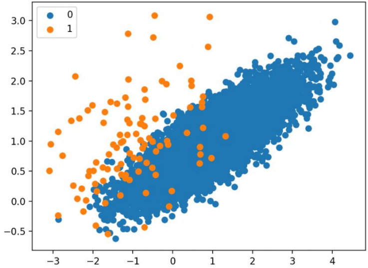

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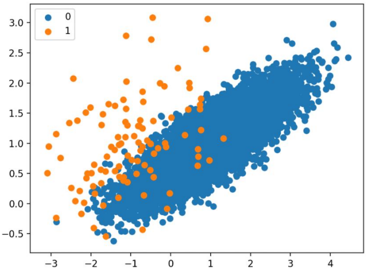
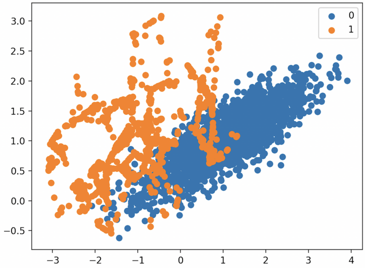
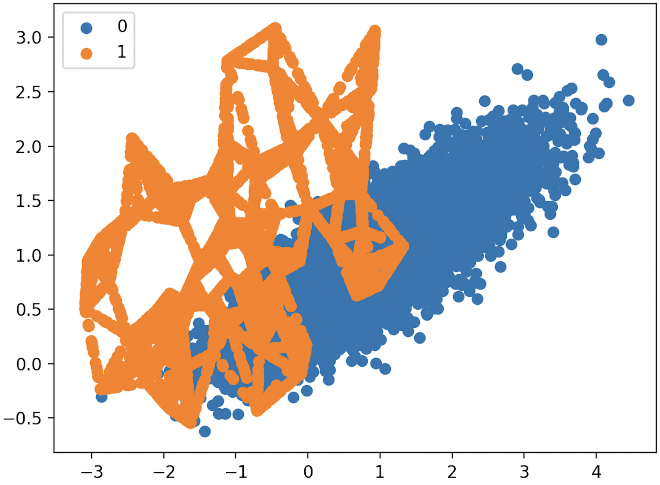




Too much SMOTE... IMPOSSIBLE!



Research Question

How does data augmentation with SMOTE affect the distribution of the predictors, and in turn affect the classification Model?

- Model:
$$\operatorname{logit}(P(Y=1)) = \beta_0 + \beta_1 X_1 + \beta_2 X_2$$
Set:
$$P(Y=1) = 0.05$$
- Over-sample using SMOTE, then check how does this affects the distribution of X_1 and X_2 and the logistical model predicting Y.

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- $\beta_0 = \ln(0.05/0.95) - 4.15$. This results in $\mathbb{E}(P(Y = 1)) = 0.05$
- $Y_i \sim \text{Bernolli}(P(Y_i = 1))$ where

$$\text{logit}(P(Y_i = 1)) = \beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i}$$

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- We oversample the training data using SMOTE to achieve a rate of $1 : 1$ of positive to negative data points. This will be our "level 0.5".

Recording Results

For each of the three levels, we record the following:

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- **Parameters of the Model:** We will record the estimated β_0, β_1, and β_2.
- **The Model's perfromance:** We will evaluate the accompanying logistic regression model on the test data by calculating the F1 score.

Simulation Results: Distribution of the Predictors

- $X_1 = \text{Expo}(1)$

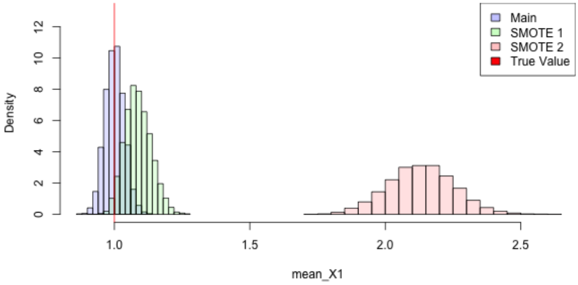

Figure: Distribution of the Mean of X_1

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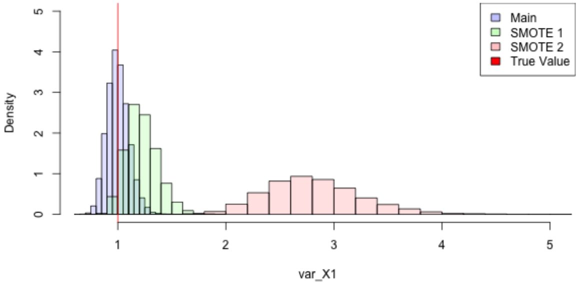

Figure: Distribution of the Variance of X_1

Simulation Results: Distribution of the Predictors

- $X_2 = \mathcal{N}(X_1, 1)$

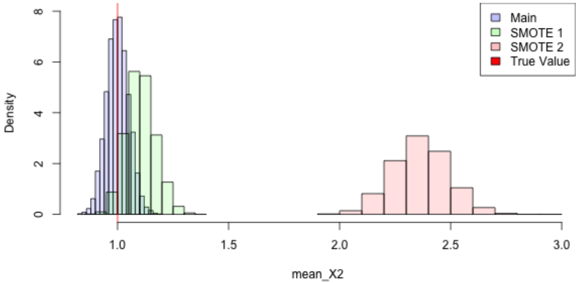

Figure: Distribution of the Mean of X_2

Simulation Results: Distribution of the Predictors

- $X_2 = \mathcal{N}(X_1, 1)$

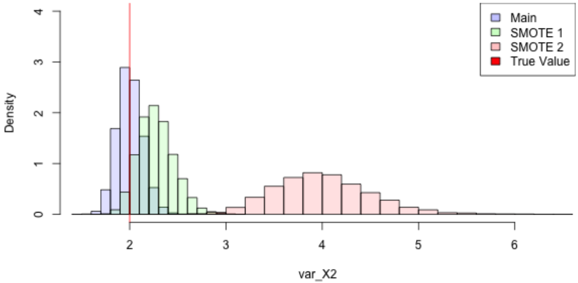

Figure: Distribution of the Variance of X_2

Simulation Results: Correlation between the Predictors

- Theoretical Correlation:

$$\mathbb{E}(X_1X_2) = \mathbb{E}(\mathbb{E}(X_1X_2|X_1)) = \mathbb{E}(X_1^2)$$

$$\text{Corr}(X_1, X_2) = \frac{\mathbb{E}(X_1X_2) - \mathbb{E}(X_1)E(X_2)}{\sigma_{X_1}\sigma_{X_2}} = \frac{1}{\sqrt{2}} \approx 0.707$$

- Correlation in the Simulation:

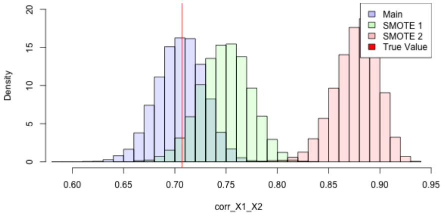

Figure: Distribution of the Correlation of X_1 and X_2

Simulation Results: Parameters of the Model

- $\beta_0 \approx -7$
- PRB (Percentage Relative Bias) for the three levels: -3.05%, 1.85%, 29.11%.

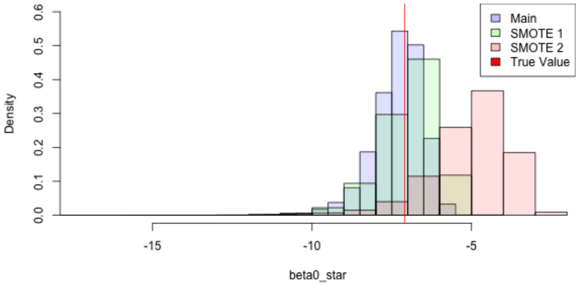

Figure: Distribution of the estimate of β_0

Simulation Results: Parameters of the Model

- Theoretical Coefficient: $\beta_1 = 1$
- Simulation:
  $\hat{\beta}_{1,0} = 1.03$, $\hat{\beta}_{1,0.1} = 1.06$, $\hat{\beta}_{1,0.5} = 1.15$.
  PRB: 3.16%, 6.49%, 15.43%.
  ANOVA test gives $F_{2,29997} = 291.2$ with $p < 2e - 16$.

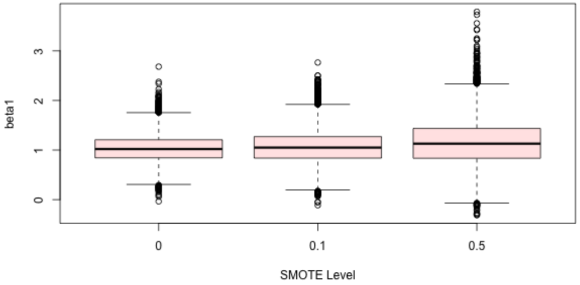

Figure: Distribution of the estimate of β

Simulation Results: Parameters of the Model

- Theoretical Coefficient: $\beta_2 = 1$
- Simulation:
  $\hat{\beta}_{2,0} = 1.03$, $\hat{\beta}_{2,0.1} = 1.08$, $\hat{\beta}_{2,0.5} = 1.18$.
  PRB: 3.41%, 8.13%, 18.35%.
  ANOVA test gives $F_{2,29997} = 463.2$ with $p < 2e - 16$.

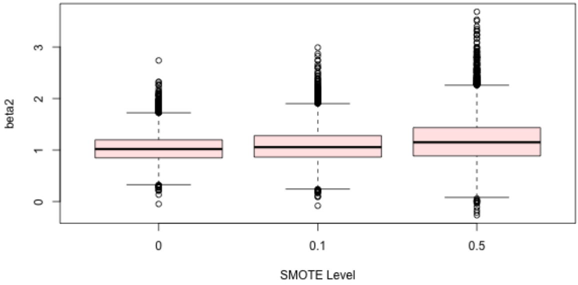

Figure: Distribution of the estimate of β

Simulation Results: The Model's Predictions

- $F1_0 = 0.603$, $F1_{0.1} = 0.629$, $F1_{0.5} = 0.460$.
  A 2-sample t-test comparing the original dataset to the 0.1 level procedures using Fisher's strict null gives $t_{19757} = 12.958$ with $p < 2e - 16$.

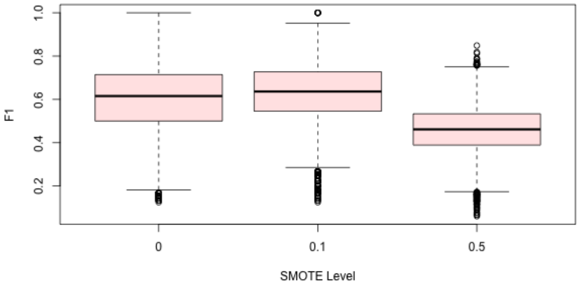

Figure: Distribution of the F1 score

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- SMOTE is useful for classification up to a certain extent
- Possible Next Steps: How much SMOTE is too much SMOTE? Can we tune our balancing levels to optimally improve predictions?