submitted to *Geophys. J. Int.*

Simulations of induced polarization effects in time domain electromagnetic data

Seogi Kang and Douglas W. Oldenburg

Department of Earth, Ocean and Atmospheric Sciences, University of British Columbia, B.C. V6T 1Z4*, Canada*

SUMMARY

XXX

1 FORMULATION

Complex conductivity model in Laplace domain can be expressed as

$$\sigma(s) = \sigma_\infty + \triangle\sigma(s) \tag{1}$$

where $s = \imath\omega$ is the Laplace transform variable. Inverse Laplace transform of $\sigma(s)$ yields:

$$\mathcal{L}^{-1}[\sigma(s)] = \sigma(t) = \sigma_\infty\delta(t) + \triangle\sigma(t) \tag{2}$$

where $\delta(t)$ is a Dirac-Delta function and $\triangle\sigma(t) = \mathcal{L}^{-1}[\triangle\sigma(s)]$. Maxwell's equations can be written as

$$\vec{\nabla} \times \vec{e} = -\frac{\partial \vec{b}}{\partial t} \tag{3}$$

$$\vec{\nabla} \times \mu^{-1}\vec{b} - \vec{j} = \vec{j}_s \tag{4}$$

Considering a time-dependent conductivity, Ohm's Law can be written as

$$\vec{j} = \int_0^t \sigma(t-u)\vec{e}(u)du \tag{5}$$

And substituting Eq. 2 yields

$$\vec{j} = \sigma_\infty\vec{e} + \int_0^t \triangle\sigma(t-u)\vec{e}(u)du \tag{6}$$

Using backward euler method, we respectively discretize Eqs. 3 and 4 in time:

$$\vec{\nabla} \times \vec{e}^{\,(n)} = -\frac{\vec{b}^{(n)} - \vec{b}^{(n-1)}}{\triangle t^{(n)}} \tag{7}$$

$$\vec{\nabla} \times \mu^{-1}\vec{b}^{(n)} - \vec{j}^{(n)} = \vec{j}_s^{(n)} \tag{8}$$

where $\triangle t^{(n)} = t^{(n)} - t^{(n-1)}$. To discretize integration part in Eq. 6, we use trapezoidal rule:

$$\int_{t^{(k-1)}}^{t^{(k)}} \triangle\sigma(t-u)\vec{e}(u)du = \frac{\triangle t^{(k)}}{2}\Big(\triangle\sigma(t^{(n)} - t^{(k-1)})\vec{e}^{\,(k-1)} + \triangle\sigma(t^{(n)} - t^{(k)})\vec{e}^{\,(k)}\Big) \tag{9}$$

Hence the Ohm's Law shown in Eq. 6 can be discretized as

$$\vec{j}^{(n)} = \sigma_\infty \vec{e}^{\,(n)} + \sum_{k=1}^{n} \frac{\triangle t^{(k)}}{2}\Big(\triangle\sigma(t^{(n)} - t^{(k-1)})\vec{e}^{\,(k-1)} + \triangle\sigma(t^{(n)} - t^{(k)})\vec{e}^{\,(k)}\Big) \tag{10}$$

This can be rewritten as

$$\vec{j}^{(n)} = \Big(\sigma_\infty + \gamma(\triangle t^{(n)})\Big)\vec{e}^{\,(n)} + \vec{j}_{pol}^{(n-1)} \tag{11}$$

where the polarization current, $\vec{j}_{pol}^{(n-1)}$ is

$$\vec{j}_{pol}^{(n-1)} = \sum_{k=1}^{n-1} \frac{\triangle t^{(k)}}{2}\Big(\triangle\sigma(t^{(n)} - t^{(k-1)})\vec{e}^{\,(k-1)} + \triangle\sigma(t^{(n)} - t^{(k)})\vec{e}^{\,(k)}\Big) + \kappa(\triangle t^{(n)})\vec{e}^{\,(n-1)} \tag{12}$$

Using mimetic finite volume approach, we correspondingly discretize Eqs. 7, 8, and 11

$$\mathbf{C}\mathbf{e}^{\,(n)} = -\frac{\mathbf{b}^{(n)} - \mathbf{b}^{(n-1)}}{\triangle t^{(n)}} \tag{13}$$

$$\mathbf{C}\mathbf{M}^f_{\mu^{-1}}\mathbf{b}^{(n)} - \mathbf{M}^e\mathbf{j}^{(n)} = \mathbf{s}_e^{(n)} \tag{14}$$

$$\mathbf{M}^e\mathbf{j}^{(n)} = \mathbf{M}^{e\,(n)}_A\mathbf{e}^{\,(n)} + \mathbf{j}_{pol}^{(n-1)} \tag{15}$$

where

$$\mathbf{j}_{pol}^{(n-1)} = \sum_{k=1}^{n-1} \frac{\triangle t^{(k)}}{2}\Big(\mathbf{M}^e_{\triangle\sigma(n,k-1)}\vec{e}^{\,(k-1)} + \mathbf{M}^e_{\triangle\sigma(n,k)}\mathbf{e}^{\,(k)}\Big) + \mathbf{M}^e_\kappa\mathbf{e}^{\,(n-1)} \tag{16}$$

We can rearrange above equations to solve either $\mathbf{e}$ or $\mathbf{b}$. First consider solving $\mathbf{e}$ hence we remove $\mathbf{b}$.

$$\Big(\mathbf{C}^T\mathbf{M}^f_{\mu^{-1}}\mathbf{C} + \frac{1}{\triangle t^{(n)}}\mathbf{M}^{e\,(n)}_A\Big)\mathbf{e}^{(n)} = -\frac{1}{\triangle t^{(n)}}(\mathbf{s}_e^{(n)} - \mathbf{s}_e^{(n-1)}) + \frac{1}{\triangle t^{(n)}}\mathbf{M}^e\mathbf{j}^{(n-1)} - \frac{1}{\triangle t^{(n)}}\mathbf{j}_{pol}^{(n-1)} \tag{17}$$