3D simulations of induced polarization effects in time domain electromagnetic data using Stretched Exponential

Seogi Kang and Douglas W. Oldenburg

Department of Earth, Ocean and Atmospheric Sciences, University of British Columbia, B.C. V6T 1Z4, Canada

SUMMARY

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1 INTRODUCTION

Earth materials are electrically polarizable resulting in polarization charge buildup when electrical field is applied. This often called induced polarization (IP) effects, and effectively be expressed frequency-dependent resistivity or conductivity, which is complex-valued. A number of mathematical models have been suggested, but mainly two simple models can be representative.

2 FORMULATION

Complex conductivity model in Laplace domain can be expressed as

$$\sigma(s) = \sigma_\infty + \triangle\sigma(s) \tag{1}$$

where $s = \imath\omega$ is the Laplace transform variable. Inverse Laplace transform of $\sigma(s)$ yields:

$$\mathcal{L}^{-1}[\sigma(s)] = \sigma(t) = \sigma_\infty\delta(t) + \triangle\sigma(t) \tag{2}$$

where $\delta(t)$ is a Dirac-Delta function and $\triangle\sigma(t) = \mathcal{L}^{-1}[\triangle\sigma(s)]$. Maxwell's equations can be written as

$$\vec{\nabla} \times \vec{e} = -\frac{\partial\vec{b}}{\partial t} \tag{3}$$

$$\vec{\nabla} \times \mu^{-1}\vec{b} - \vec{j} = \vec{j}_s \tag{4}$$

Considering a time-dependent conductivity, Ohm's Law can be written as

$$\vec{j} = \int_0^t \sigma(t-u)\vec{e}(u)du \tag{5}$$

And substituting Eq. 2 yields

$$\vec{j} = \sigma_\infty \vec{e} + \int_0^t \triangle\sigma(t-u)\vec{e}(u)du \tag{6}$$

Using backward euler method, we respectively discretize Eqs. 3 and 4 in time:

$$\vec{\nabla} \times \vec{e}^{\,(n)} = -\frac{\vec{b}^{(n)} - \vec{b}^{(n-1)}}{\triangle t^{(n)}} \tag{7}$$

$$\vec{\nabla} \times \mu^{-1}\vec{b}^{(n)} - \vec{j}^{(n)} = \vec{j}_s^{(n)} \tag{8}$$

where $\triangle t^{(n)} = t^{(n)} - t^{(n-1)}$. To discretize integration part in Eq. 6, we use trapezoidal rule:

$$\int_{t^{(k-1)}}^{t^{(k)}} \triangle\sigma(t-u)\vec{e}(u)du = \frac{\triangle t^{(k)}}{2}\left(\triangle\sigma(t^{(n)} - t^{(k-1)})\vec{e}^{\,(k-1)} + \triangle\sigma(t^{(n)} - t^{(k)})\vec{e}^{\,(k)}\right) \tag{9}$$

Hence the Ohm's Law shown in Eq. 6 can be discretized as

$$\vec{j}^{(n)} = \sigma_\infty \vec{e}^{\,(n)} + \sum_{k=1}^n \frac{\triangle t^{(k)}}{2}\left(\triangle\sigma(t^{(n)} - t^{(k-1)})\vec{e}^{\,(k-1)} + \triangle\sigma(t^{(n)} - t^{(k)})\vec{e}^{\,(k)}\right) \tag{10}$$

This can be rewritten as

$$\vec{j}^{(n)} = \left(\sigma_\infty + \gamma(\triangle t^{(n)})\right)\vec{e}^{\,(n)} + \vec{j}_{pol}^{(n-1)} \tag{11}$$

where the polarization current, $\vec{j}_{pol}^{(n-1)}$ is

$$\vec{j}_{pol}^{(n-1)} = \sum_{k=1}^{n-1} \frac{\triangle t^{(k)}}{2}\left(\triangle\sigma(t^{(n)} - t^{(k-1)})\vec{e}^{\,(k-1)} + \triangle\sigma(t^{(n)} - t^{(k)})\vec{e}^{\,(k)}\right)$$

$$+\kappa(\triangle t^{(n)})\vec{e}^{\,(n-1)} \tag{12}$$

Using mimetic finite volume approach, we correspondingly discretize Eqs. 7, 8, and 11

$$\mathbf{C}\mathbf{e}^{(n)} = -\frac{\mathbf{b}^{(n)} - \mathbf{b}^{(n-1)}}{\triangle t^{(n)}} \tag{13}$$

$$\mathbf{C}\mathbf{M}_{\mu^{-1}}^f\mathbf{b}^{(n)} - \mathbf{M}^e\mathbf{j}^{(n)} = \mathbf{s}_e^{(n)} \tag{14}$$

$$\mathbf{M}^e\mathbf{j}^{(n)} = \mathbf{M}_A^{e\,(n)}\mathbf{e}^{(n)} + \mathbf{j}_{pol}^{(n-1)} \tag{15}$$

where

$$\mathbf{j}_{pol}^{(n-1)} = \sum_{k=1}^{n-1} \frac{\triangle t^{(k)}}{2}\left(\mathbf{M}_{\triangle\sigma(n,k-1)}^e\vec{e}^{\,(k-1)} + \mathbf{M}_{\triangle\sigma(n,k)}^e\mathbf{e}^{(k)}\right)$$

$$+\mathbf{M}_\kappa^e\mathbf{e}^{(n-1)} \tag{16}$$

We can rearrange above equations to solve either **e** or **b**. First consider solving **e** hence we remove **b**.

$$\left(\mathbf{C}^T\mathbf{M}^f_{\mu^{-1}}\mathbf{C} + \frac{1}{\triangle t^{(n)}}\mathbf{M}^e_A{}^{(n)}\right)\mathbf{e}^{(n)}$$

$$= -\frac{1}{\triangle t^{(n)}}(\mathbf{s}^{(n)}_e - \mathbf{s}^{(n-1)}_e) + \frac{1}{\triangle t^{(n)}}\mathbf{M}^e\mathbf{j}^{(n-1)} - \frac{1}{\triangle t^{(n)}}\mathbf{j}^{(n-1)}_{pol} \qquad (17)$$