Advanced Autonomous Systems

Localization Problem Using EKF

In this document we describe the EKF update step, for the particular problem of processing updates which are based on range and bearing observations, e.g. for map-based localization.

The states to be estimated are related to the pose of a mobile platform,

$$\mathbf{X} = \begin{bmatrix} x \\ y \\ \phi \end{bmatrix} \qquad \text{E1}$$

If observations, related to each observed landmark, are expressed as range and bearing:

$$obs = \begin{bmatrix} r \\ \alpha \end{bmatrix} \qquad \text{E2}$$

These are usually provided by sensors such as LIDAR (range and bearing), or monocular cameras (just bearing), or sonars (just range), or other similar sensors.

Range and bearing observation functions, associated to a particular landmark whose position (global) is (x_a, y_a), are expressed as follows:

$$h(\mathbf{X}) = \begin{bmatrix} h_1(x, y, \phi) \\ h_2(x, y, \phi) \end{bmatrix} = \begin{bmatrix} \sqrt{(x_a - x)^2 + (y_a - y)^2} \\ \text{atan2}(y_a - y, x_a - x) - \phi + \pi/2 \end{bmatrix}$$

$$\Downarrow \qquad \text{E3}$$

$$= \begin{bmatrix} \sqrt{(x_a - x)^2 + (y_a - y)^2} \\ \text{atan2}(y_a - y, x_a - x) - \phi + \pi/2 \end{bmatrix} = \begin{bmatrix} r \\ \alpha \end{bmatrix}$$

We can clearly see that both functions are functions of the state X.

Note: The angular (i.e. bearing) observation function may be different, depending on the coordinates' convention being used.

Note: The only point at which the angular observation function is undefined, is for the case $(x, y) = (x_{a,} y_a)$ (what would physically mean the UGV's body is invading the landmark's position!); a situation that would usually never happen. However, if it does happen, we will ignore those observations.

By linearization of the observation function, evaluated at the PRIOR expected value of the states, we obtain the following **H** matrix (expressed analytically),

$$\mathbf{H} = \left.\frac{\partial h(\mathbf{X})}{\partial \mathbf{X}}\right|_{\mathbf{X}=\hat{\mathbf{X}}^-} = \left.\begin{bmatrix} \frac{\partial h_1}{\partial x} & \frac{\partial h_1}{\partial y} & \frac{\partial h_1}{\partial \phi} \\ \frac{\partial h_2}{\partial x} & \frac{\partial h_2}{\partial y} & \frac{\partial h_2}{\partial \phi} \end{bmatrix}\right|_{\mathbf{X}=\hat{\mathbf{X}}^-} =$$

$$= \left.\begin{bmatrix} -\frac{(x_a - x)}{\sqrt{(x_a-x)^2+(y_a-y)^2}} & -\frac{(y_a - y)}{\sqrt{(x_a-x)^2+(y_a-y)^2}} & 0 \\ \frac{(y_a - y)}{(x_a-x)^2+(y_a-y)^2} & \frac{-(x_a - x)}{(x_a-x)^2+(y_a-y)^2} & -1 \end{bmatrix}\right|_{\mathbf{X}=\hat{\mathbf{X}}^-} \qquad \text{E4}$$

(Student should verify that the partial derivatives in Equation E4 are correct)

As the number of estimated states is three, and the number of observations, simultaneously processed, are two, then we have a rectangular **H** matrix, of size *2* by *3.*
This linearization is performed at the PRIOR expected value of the states (replacing these values in E4). You will also need the values of the positions of the associated landmark ((x_a, y_a): you get those from the known navigation Map!)
It is relevant to appreciate that each time that the scanning sensor allows to detect an OOI, we need to identify if that object does correspond to a known landmark, to be able to specify (x_a, y_a), in the related observation equations.

Suppose we know that the range measurement is polluted by Gaussian white noise, whose standard deviation is 10 cm, and that the angular error has standard deviation = 0.5 degrees; then the we propose the **R** matrix to be:

$$\mathbf{R} = \begin{bmatrix} (0.1)^2 & 0 \\ 0 & (0.5 \cdot \pi / 180)^2 \end{bmatrix}$$

Note that, in this case, we are using *meters* for expressing the range, and *radians* for the bearing angle. The non-diagonal elements of the Covariance of the measurement noise are assumed to be zero; this is because we know (or assume) that the noise in both measurements are independent. Are they really independent? → To be discussed in class.
If this is the case, we could process both measurements individually and sequentially.
The rest of the details in the implementation of this localizer will be discussed in the lecture.
If we had more observations (e.g. associated to other landmarks) we could process those in a sequence in which the POSTERIOR of one update would be the PRIOR of a subsequent one. In this way, the belief about the states is refined by combining all the sources of information. This procedure is adequate because we consider that the uncertainties that corrupt the observations associated to different landmarks are independent.

Additional matters to discuss in class

We discuss, in class, about additional relevant matters.

Other versions of localizers, we may use:

A range only localizer (“sonar like” case)
A bearing only localizer (“camera like” case)
Localizer that uses observations from a compass
Other cases

How do we initialize the EKF localizer?

Initial expected value and covariance matrix

Tuning the EKF localizer:

How do we propose the noises' covariance matrixes? (in our cases: for range and bearing observations)

Are those noises really GWN (Gaussian White Noise)?

(We discuss different cases: range, bearing, compass, etc.)

Verification:

How do we know if the estimation process does converge? (i.e. if the system is observable)

Matters related to the map:

If we are able/allowed to deploy the landmarks: where? How many?

What happens if our map surveying is not perfect? (i.e. the map we provide is corrupted as well)

Other type of landmarks (in addition to the "friendly" poles)

Can we run the localizer not having a process model? (or a very bad one)