1 Introduction

Till now, I really could say nothing about that thesis... Almost no practical stuff is talked and implemented. In this manner, I am here.

1.1 Normal Distribution

For this part, the original source goes to `http://www.mathworks.com/help/stats/normal-distribution.html`.

1.1.1 Definition

The normal pdf is

$$y = f(x|\mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp^{\frac{-(x-\mu)^2}{2\sigma^2}} \tag{1}$$

1.1.2 Background

The normal distribution is a two-parameter family of curves. The first parameter, μ, is the mean. The second, σ, is the standard deviation. The standard normal distribution (written $\Phi(x)$) sets μ to 0 and σ to 1.

$\Phi(x)$ is functionally related to the error function erf.

$$erf(x) = 2\Phi(x \cdot \sqrt{2}) - 1 \tag{2}$$

The first use of the normal distribution was as a continuous approximation to the binomial.

The usual justification for using the normal distribution for modeling is the Central Limit Theorem, which states (roughly) that the sum of independent samples from any distribution with finite mean and variance converges to the normal distribution as the sample size goes to infinity.

1.1.3 Parameters

To use statistical parameters such as mean and standard deviation reliably, you need to have a good estimator for them. The maximum likelihood estimates (MLEs) provide one such estimator. However, an MLE might be biased, which means that its expected value of the parameter might not equal the parameter being estimated. For example, an MLE is biased for estimating the variance of a normal distribution. An unbiased estimator that is commonly used to estimate the parameters of the normal distribution is the *minimum variance unbiased estimator (MVUE)*. The MVUE has the minimum variance of all unbiased estimators of a parameter.

The MVUEs of parameters μ and σ^2 for the normal distribution are the sample mean and variance. The sample mean is also the MLE for μ. The following are two common formulas for the variance.

$$s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 \tag{3}$$

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \tag{4}$$

where $x_i = \sum_{i=1}^{n} \frac{x_i}{n}$.

Equation (3) is the maximum likelihood estimator for σ^2, and (4) is the MVUE.

As an example, suppose you want to estimate the mean, μ, and the variance, σ^2 of the heights of all fourth grade children in the United States. The function normfit returns the MVUE for μ, the square root of the MVUE for σ^2 and confidence intervals for μ and σ^2. Here is a playful example modeling the heights in inches of a randomly chosen fourth grade class.

```
height = normrnd(50,2,30,1);    % Simulate heights.
[mu,s,muci,sci] = normfit(height)
```

2 Multi-Variate Model

2.1 Definition

The probability density function of the d-dimensional multivariate normal distribution is given by

$$y = f(x, \mu, \sum) = \frac{1}{\sqrt{|\sum|2\pi^d}} \exp^{-\frac{1}{2}(x-\mu)\sum^{-1}(x-\mu)'} \tag{5}$$

where x and μ are 1-by-d vectors and $\sum$ is a d-by-d symmetric positive definite matrix. While it is possible to define the multivariate normal for singular $\sum$, the density cannot be written as above. Only random vector generation is supported for the singular case. Note that while most textbooks define the multivariate normal with x and μ oriented as column vectors, for the purposes of data analysis software, it is more convenient to orient them as row vectors, and Statistics Toolbox software uses that orientation.

2.2 Background

The multivariate normal distribution is a generalization of the univariate normal to two or more variables. It is a distribution for random vectors of correlated variables, each element of which has a univariate normal distribution. In the simplest case, there is no correlation among variables, and elements of the vectors are independent univariate normal random variables.

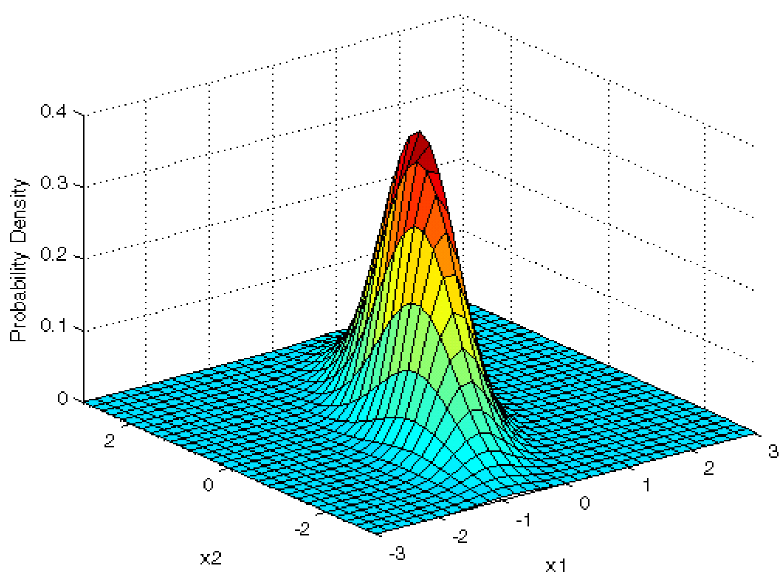
The multivariate normal distribution is parameterized with a mean vector, μ, and a covariance matrix, $\sum$. These are analogous to the mean μ and variance σ^2 parameters of a univariate normal distribution. The diagonal elements of $\sum$ contain the variances for each variable, while the off-diagonal elements of $\sum$ contain the covariances between variables.

The multivariate normal distribution is often used as a model for multivariate data, primarily because it is one of the few multivariate distributions that is tractable to work with.

2.3 Examples

This example shows the probability density function (pdf) and cumulative distribution function (cdf) for a bivariate normal distribution with unequal standard deviations. You can use the multivariate normal distribution in a higher number of dimensions as well, although visualization is not easy.

```
mu = [0  0];
Sigma = [.25  .3;  .3  1];
x1 = -3:.2:3; x2 = -3:.2:3;
[X1,X2] = meshgrid(x1,x2);
F = mvnpdf([X1(:) X2(:)],mu,Sigma);
F = reshape(F,length(x2),length(x1));
surf(x1,x2,F);
caxis([min(F(:))-.5*range(F(:)),max(F(:))]);
axis([-3 3 -3 3 0 .4])
xlabel('x1'); ylabel('x2');
zlabel('Probability Density');
```




```
F = mvncdf([X1(:) X2(:)],mu,Sigma);
F = reshape(F,length(x2),length(x1));
surf(x1,x2,F);
caxis([min(F(:))-.5*range(F(:)),max(F(:))]);
```

```
axis([-3 3 -3 3 0 1])
xlabel('x1'); ylabel('x2');
zlabel('Cumulative Probability');
```