Moreeh Khan

Calc 3 WH 11

1) $0 \le z \le 16 - x^2 - y^2$

$0 \le z \le 16 - r^2$

$0 \le r \le 4$

$0 \le \theta \le 2\pi$

$$V = \int_0^{2\pi} \int_0^4 \int_0^{16-r^2} 1 \cdot r \, dz \, dr \, d\theta$$

$$V = \int_0^{2\pi} \int_0^4 (16 - r^2) r \, dr \, d\theta$$

$$V = \int_0^{2\pi} \int_0^4 16r - r^3 \, dr \, d\theta$$

$$= \int_0^{2\pi} \left[8r^2 - \frac{r^4}{4} \right]_0^4 d\theta = \int_0^{2\pi} 128 - 64$$

$$= \left[64\theta \right]_0^{2\pi} = \boxed{128\pi = V}$$

2) $0 \le x^2 + y^2 \le 4$ $\quad x = r\cos\theta$

$0 \le r^2 \le 4$ $\quad y = r\sin\theta$

$0 \le z \le 10 - r\cos\theta - \sin\theta$

$0 \le r \le 2$

$0 \le \theta \le 2\pi$

$$V = \int_0^{2\pi} \int_0^2 \int^{10 - r\cos\theta - \sin\theta} r \, dz \, dr \, d\theta$$

$$= \int_0^{2\pi} \int_0^2 (10 - r\cos\theta - r\sin\theta) r \, dr \, d\theta$$

$$= \int_0^{2\pi} \int_0^2 (10r - r^2\cos\theta - r^2\sin\theta) \, dr \, d\theta$$

2. cont.

$$V = \int_0^{2\pi} \left[5r^2 - \frac{1}{3}r^3\cos\theta - \frac{1}{3}r^3\sin\theta\right]_0^2 d\theta$$

$$= \int_0^{2\pi} 20 - \frac{8}{3}\cos\theta - \frac{8}{3}\sin\theta \, d\theta$$

$$= \left[20\theta - \frac{8}{3}\sin\theta + \frac{8}{3}\cos\theta\right]_0^{2\pi}$$

$$= 20(2\pi) - \frac{8}{3}\sin(2\pi) + \frac{8}{3}\cos(2\pi)$$
$$- \left[20(0) - \frac{8}{3}\sin(0) + \frac{8}{3}\cos(0)\right]$$

$$= 40\pi + \cancel{\frac{8}{3}} - \cancel{\frac{8}{3}} = \boxed{40\pi}$$

3) $0 \le r \le 3$ $\quad x = \rho\sin\varphi\cos\theta$
$0 \le \varphi \le \pi/2$ $\quad y = \rho\sin\varphi\sin\theta$
$0 \le \theta \le \pi/2$ $\quad z = \rho\cos\varphi$

$$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^3 \rho\sin\varphi\cos\theta \cdot \rho\sin\varphi \, d\rho\, d\varphi\, d\theta$$

$$= \int_0^{\pi/2}\int_0^{\pi/2}\int_0^3 \rho^3\sin\varphi^2\cos\theta \, d\rho\, d\varphi\, d\theta$$

$$= \int_0^{\pi/2}\int_0^{\pi/2} \left[\frac{1}{4}\rho^4\sin\varphi^2\cos\theta\right]_0^3 d\varphi\, d\theta$$

$$= \int_0^{\pi/2}\int_0^{\pi/2} \frac{81}{4}\sin\varphi^2\cos\theta \, d\varphi\, d\theta$$

3 cont.

3) $\frac{81}{4} \int_0^{\pi/2} \int_0^{\pi/2} \frac{1}{2}(1+\cos 2\varphi) \cos\theta \, d\varphi \, d\theta$

$= \frac{81}{8} \int_0^{\pi/2} \left[\left(\varphi - \frac{1}{2}\sin 2\varphi\right) - \cos\theta \right]_0^{\pi/2}$

$= \frac{81}{8} \int_0^{\pi/2} \frac{\pi}{2} \cos\theta \, d\theta = \frac{27\pi}{4} \left[\sin\theta \right]_0^{\frac{\pi}{2}}$

$= \boxed{\frac{81\pi}{16}}$