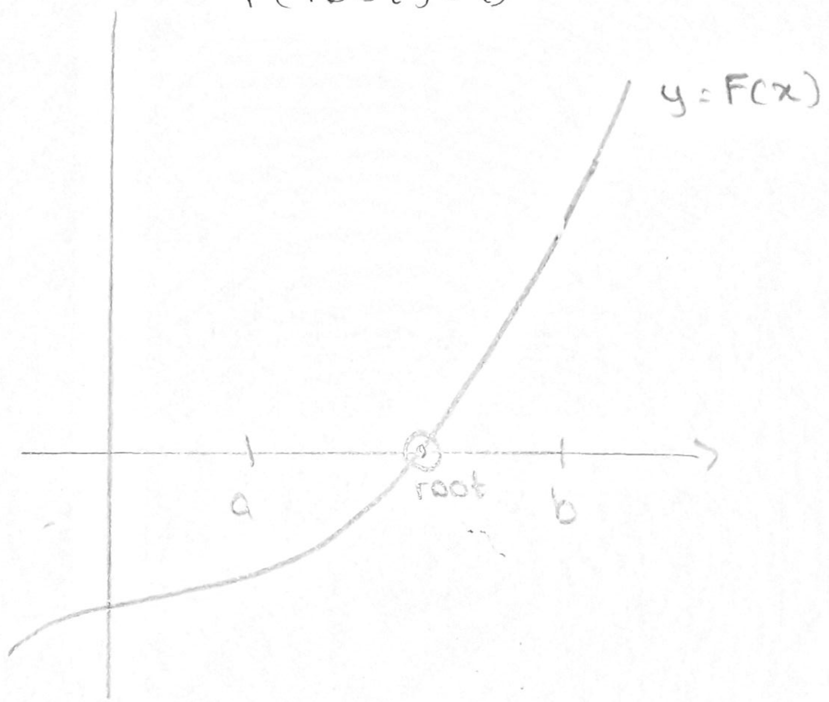
Bisection Method.

To find roots of,

$F(x) \longrightarrow [a, b]$

If you think a is a root, $F(a) = 0$

$F(\text{root}) = 0$



If root is in between a and b, if a has a positive value, b must be a negative value, vice versa.

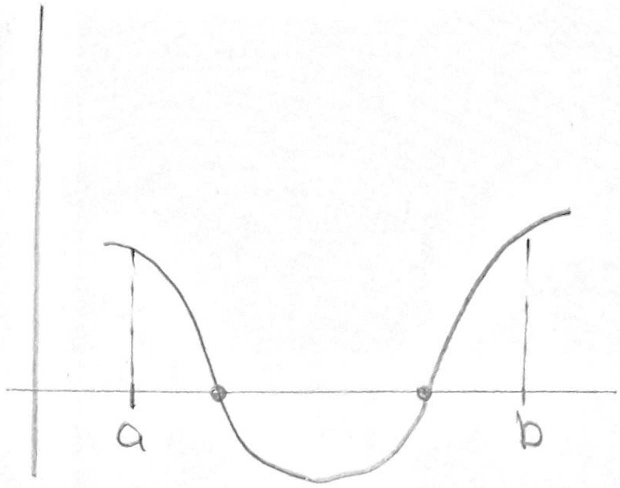
$F(a) . F(b) < 0.$

If $F(a) < 0$ then $F(b) > 0$

If $F(b) > 0$ then $F(a) < 0$

can you find the roots ~~of below diagram~~ in below ~~d~~ graph ? using Bisection Method ?

<u>NO</u> ← This is a weakness.

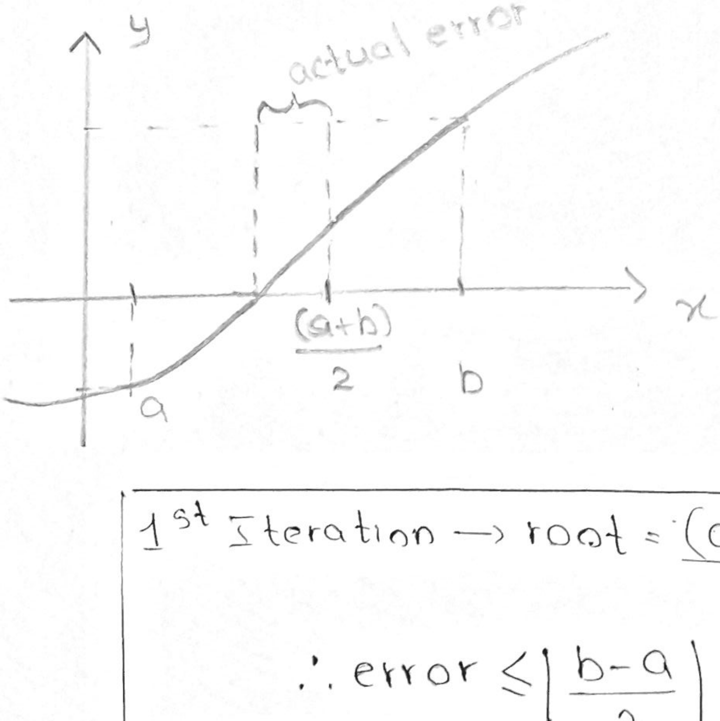


$F(a)\,F(b) > 0$

If $F(a) > 0$ then $F(b) > 0$

or

If $F(a) < 0$ then $F(b) < 0$



I → $F(x)$ is a continuous function

II → $F(a) \cdot F(b) < 0$

then you can use bisection method.

$$\boxed{\begin{array}{c} 1^{st} \text{ Iteration} \rightarrow \text{root} = \dfrac{(a+b)}{2} \\ \therefore \text{error} \leq \left| \dfrac{b-a}{2} \right| \end{array}}$$

$$\text{actual error} = \left| \frac{(a+b)}{2} - \text{root} \right|$$

$$\left| \frac{(a+b)}{2} - \text{root} \right| \leq \frac{(b-a)}{2}$$

$F\left(\frac{a+b}{2}\right) > 0$ depends on the value / graph

$a \qquad \frac{(a+b)}{2} \qquad b$

$F(a) < 0 \qquad F\left(\frac{a+b}{2}\right) > 0 \qquad F(b) > 0$

$$\underbrace{\qquad\qquad}_{\left[a,\ \frac{b+a}{2}\right]}$$

$$F\left(\frac{a+b}{2}\right) \cdot F(b) > 0$$

2nd Iteration $\rightarrow$ root $= \dfrac{\left(a + \frac{b+a}{2}\right)}{2}$

$$= \frac{(3a+b)}{4}$$

$$\text{error} \leq \frac{\left(\frac{a+b}{2} - a\right)}{2}$$

$$\text{error} \leq \frac{b-a}{4}$$

Tutorial 2.

(3)

$$F(x) = x - \sin(x) - \frac{1}{2}$$

(1) (i) $F(x)$ is a continuous function

Because $F(x) = x$ and $F(x) = \sin x$ is continuous.

(ii) a. $[-\pi, 0]$

$$F(-\pi) = (-\pi) - \sin(-\pi) - \frac{1}{2}$$
$$= -\pi - 0 - \frac{1}{2}$$
$$= -\pi - \frac{1}{2} < 0$$
$$F(0) = 0 - \sin(0) - \frac{1}{2}$$
$$= -\frac{1}{2} < 0$$

$F(-\pi) . F(0) > 0$

Can't use

b. $[0, 1]$

$$F(0) = 0 - \sin 0 - \frac{1}{2} = -\frac{1}{2} < 0$$

$$F(1) = 1 - \sin(1) - \frac{1}{2} = -0.34 < 0$$

Can't use. $F(0) . F(1) > 0$

c. $[0, \pi]$

$$F(0) = -\frac{1}{2} < 0$$

$$F(\pi) = \pi - \sin(\pi) - \frac{1}{2}$$
$$= 2.64 > 0$$

$F(0) . F(\pi) < 0$

Can use.

d. $\left[0, \frac{\pi}{4}\right]$

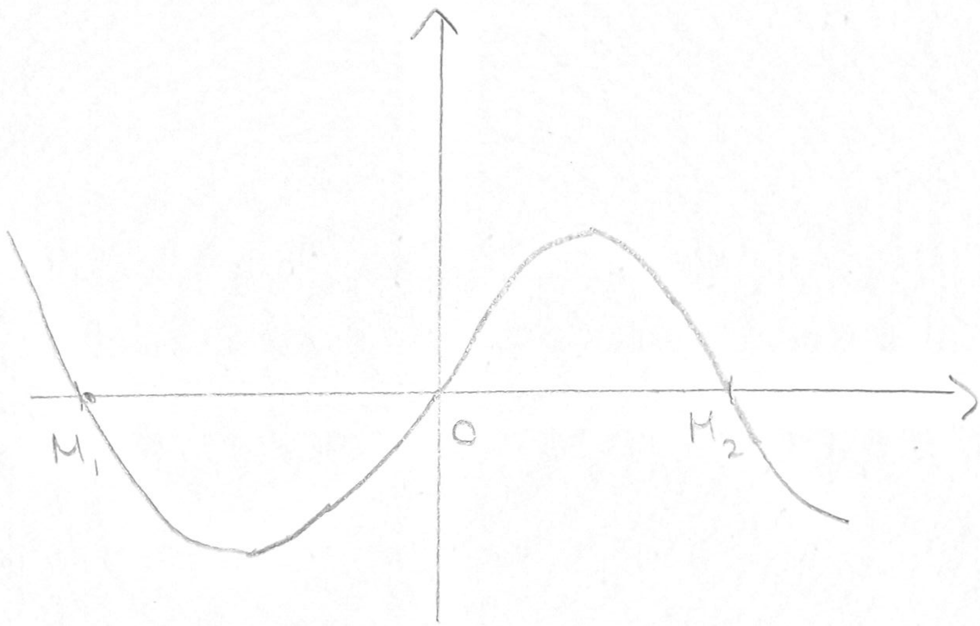
$$F(0) = -\frac{1}{2} < 0$$

$$F\left(\frac{\pi}{4}\right) = \frac{\pi}{4} - \sin(\pi/4) - \frac{1}{2}$$

$$= -0.42 < 0$$

$$F(0) \cdot F\left(\frac{\pi}{4}\right) > 0$$

$\therefore$ can't use.



(2)

(a) $f(-\pi/2) = -\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right) - \frac{1}{2} < 0$

$f(0) < 0$

can't use this interval

(b) $f(0) < 0$

✓ $f(\pi/2) = \frac{\pi}{2} - \sin\left(\frac{\pi}{2}\right) - \frac{1}{2} > 0$

can use this interval

$$\text{root} = \frac{0 + \pi/2}{2} = \frac{\pi}{4}$$

$$\text{error} \leqslant \frac{\pi/2 - 0}{2} = \frac{\pi}{4}$$

(c) $f(0) < 0$

$f(1/2) = \cancel{\frac{1}{2}} - \sin\left(\frac{1}{2}\right) - \cancel{\frac{1}{2}} < 0$

can't use this interval

(d) $f(0) < 0$

$f(\pi/8) = \frac{\pi}{8} - \sin\left(\frac{\pi}{8}\right) - \frac{1}{2} < 0$

can't use this interval

(3) $[0, \pi] \longrightarrow$ have roots in this

$$\text{error} \leq 10^{-2}$$

0^{th} iteration $\longrightarrow$ root $(\pi/2)$ $\left(\frac{0+\pi}{2}\right)$

$$\text{error} \leq (\pi/2) \left(\frac{\pi - 0}{2}\right)$$

1^{st} iteration $\longrightarrow [0, \pi/2]$

$$\text{root} \rightarrow \pi/4$$

$$\text{error} \leq \pi/4$$

$\vdots$

n^{th} iteration:

$$\text{error} \leq \frac{\pi}{2^{n+1}}$$

$$\frac{\pi}{2^{n+1}} = \frac{1}{100}$$

$$100\pi = 2^{n+1}$$

$$n+1 = \log_2(100\pi)$$

$$n = \log_2(100\pi) - 1$$

$$n = 7.295$$

$$\frac{\pi}{2^{n+1}} \leq 10^{-2}$$

$$\frac{\pi}{2^{n+1}} \leq \frac{1}{100}$$

$$100\pi \leq 2^{n+1}$$

$$\log_2(100\pi) \leq n+1$$

$$\log_2(100\pi) - 1 \leq n$$

$$7.295 \leq n$$

$$n = 8 //$$