清华大学 Tsinghua University **数学作业纸**

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hw4

t1. 解: $f(t) = e^{-at} \quad (a>0,\ t>0)$

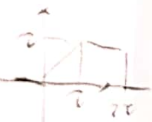
$$\mathcal{F}[f(t)] = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t}\, dt$$

$$= \int_{0}^{+\infty} e^{-(j\omega+a)t}\, dt$$

$$= \frac{1}{-(j\omega+a)} e^{-(j\omega+a)t}\Big|_{0}^{+\infty}$$

$$= \frac{1}{j\omega+a}$$

t2. 解: $f(t) = \begin{cases} t. & 0\le t<\tau \\ \tau. & \tau\le t<2\tau \\ 0. & \text{else} \end{cases}$



$$\mathcal{F}[f(t)] = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t}\, dt$$

$$= \int_{0}^{\tau} t(\cos\omega t - j\sin\omega t)\, dt + \int_{\tau}^{2\tau} \tau(\cos\omega t - j\sin\omega t)\, dt$$

其中 $\int t\cos\omega t = \frac{1}{\omega} t\sin\omega t + \frac{1}{\omega^2}\cos\omega t$ $\quad \int t\sin\omega t = -\frac{1}{\omega} t\cos\omega t + \frac{1}{\omega^2}\sin\omega t$

$$\therefore \text{上式} = \cancel{\frac{\tau}{\omega}\sin\omega\tau} + \frac{1}{\omega^2}\cos\omega\tau - \frac{1}{\omega^2}$$

$$- j\left(\cancel{-\frac{\tau}{\omega}\cos\omega\tau} + \frac{\sin\omega\tau}{\omega^2}\right)$$

$$+ \frac{\tau}{\omega}\left[\sin 2\omega\tau - \cancel{\sin\omega\tau} + j(\cos 2\omega\tau - \cancel{\cos\omega\tau})\right]$$

$$= \frac{e^{-j\omega\tau}-1}{\omega^2} + \frac{j\tau}{\omega} e^{-2j\omega\tau}$$

t3. 解 a. $\mathcal{F}\left[e^{-\frac{t^2}{20}}\right] = \int_{-\infty}^{+\infty} e^{-\frac{t^2}{20} - j\omega t}\, dt$

$$= 2\sqrt{5}\int_{-\infty}^{+\infty} e^{-t^2 - j2\sqrt{5}\omega t}\, dt$$

$$= 2\sqrt{5}\int_{-\infty}^{+\infty} e^{-5\omega^2} e^{-t^2 - j2\sqrt{5}\omega t + 5\omega^2}\, dt$$

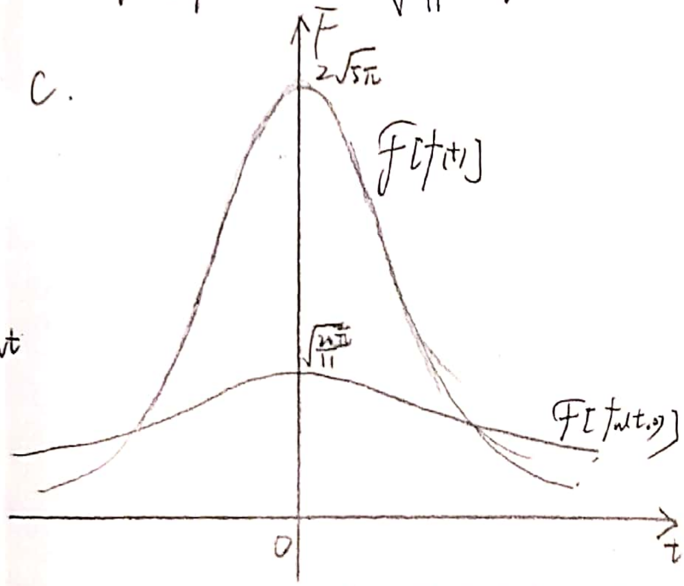
$$= \frac{2\sqrt{5\pi}}{e^{5\omega^2}}$$

b. $f_w(t,0) = f(t)\cdot w(t,0)$

$$= e^{-\frac{t^2}{20}} \cdot e^{-\frac{t^2}{2}} = e^{-\frac{11}{20}t^2}$$

$$\therefore \mathcal{F}[f_w(t,0)] = \sqrt{\frac{20\pi}{11}}\, e^{-\frac{5}{11}\omega^2}$$

c.



对比可见取窗函数后局部频谱更平缓