

数学作业纸

科目 离散

班级: 计72

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习题5

$$1. (9) \text{ 原式} = ((\forall x)(P(x) \wedge Q(x)) \wedge (\exists y)R(y)) \\ \vee ((\forall x)(P(x) \wedge Q(x)) \wedge (\exists y)S(y)) \\ = ((\forall x)(P(x) \wedge Q(x)) \wedge (\exists y)R(y)) \\ \vee ((\forall x)(P(x) \wedge Q(x)) \wedge (\exists y)S(y))$$

$$= (\forall x)(P(x) \wedge Q(x)) \wedge ((\exists y)R(y) \vee (\exists y)S(y)) \\ \wedge ((\forall x)(P(x) \wedge Q(x)) \vee (\exists y)S(y)) \\ \wedge ((\exists y)R(y) \vee (\exists y)S(y)) = \text{右}$$

$$= (\forall x)(P(x) \wedge Q(x)) \wedge (\exists y)(R(y) \vee S(y))$$

(8) $\frac{7}{\text{左}} = (\exists y)(\forall x)(P(x) \wedge Q(x))$
(题号与题标)
 $\Rightarrow (\exists x)(P(x) \wedge Q(x))$

(10) $\frac{7}{\text{右}} = ((\exists z)(\exists x) \neg P(x, z) \vee (\exists z)(\exists x)Q(x, z)) \\ \vee ((\exists z)(\exists y) \neg R(y, z) \vee (\exists z)(\exists y)S(y, z))$

$$= (\exists z)(\exists x)(\neg P(x, z) \vee Q(x, z)) \\ \vee (\exists z)(\exists y)(\neg R(y, z) \vee S(y, z))$$

$$= (\exists z)((\exists x)(\neg P(x, z) \vee Q(x, z)) \\ \vee (\exists y)(\neg R(y, z) \vee S(y, z)))$$

$$= (\exists z)(\exists x \exists y)((P(x, z) \rightarrow Q(x, z)) \\ \vee (R(y, z) \rightarrow S(y, z))) = \text{右}$$

2. (b) 不是

考虑域 $\{1, 2\}$ 上. $P(1) = F, P(2) = T$
 $Q(1) = T, Q(2) = F$

$$\text{则 } (\forall x)(P(x) \vee Q(x)) = T \\ (\forall x)P(x) \vee (\forall x)Q(x) = F$$

(7) 不是
有 (b) 之反例

(8) 不是

考虑域 $\{1, 2\}$ 上 $P(1, 1) = T, P(2, 2) = T$
 $P(1, 2) = F, P(2, 1) = F$

$$\text{则 } (\forall x)(\exists y)P(x, y) = T \\ (\exists y)(\forall x)P(x, y) = F$$

4. (4) 原式 $= ((\forall x) \neg P(x) \vee (\forall y)(Q(y) \\ \vee (\forall z)(R(z)))$

$$= \neg (\forall x)(\forall y)(\neg P(x) \vee Q(y)) \vee (\forall z)R(z)$$

$$= (\exists x)(\exists y)(P(x) \wedge \neg Q(y)) \vee (\forall z)R(z)$$

$$= (\exists x)(\exists y)(\forall z)((P(x) \wedge \neg Q(y)) \vee R(z))$$

8) 原式 $= (\forall x)(\neg P(x) \vee (\exists y)(Q(x, y) \\ \vee (\forall z)R(z)))$

$$= (\forall x)((\exists y)(\neg P(x) \vee Q(x, y)) \\ \vee (\forall z)R(z))$$

$$= ((\forall x)(\exists y)(\forall z) \\ ((\neg P(x) \vee Q(x, y)) \vee R(z)))$$

其 skolem 范式为 $(\forall x)(\forall z) \\ (\neg P(x) \vee Q(x, f(z)) \vee R(z))$



10. 其 skolem 范式为

$$(\forall x)(\forall z)(\exists v)(\exists u) p(x, a, z, f(x, z), u)$$

